Data Structures & Algorithms

Hashing

- Dictionary and Hashing
- Idealistic Hash Function
- Universal Hash Family
- Collision Resolution
  - Closed Addressing
  - Open Addressing
  - Prefect Hashing

Dictionary and Hashing

Dictionary Data Type

Dictionary. Given a universe U of possible elements, maintain a subset $S \subseteq U$ so that inserting, deleting, and searching in S is efficient.

Dictionary interface.

- `Create()`: Initialize a dictionary with $S = \phi$.
- `Insert(u)`: Add element $u \in U$ to S.
- `Delete(u)`: Delete u from S, if u is currently in S.
- `Lookup(u)`: Determine whether u is in S.

Challenge. Universe U can be extremely large so defining an array of size |U| is infeasible.

Applications. File systems, databases, Google, compilers, checksums P2P networks, associative arrays, cryptography, web caching, etc.

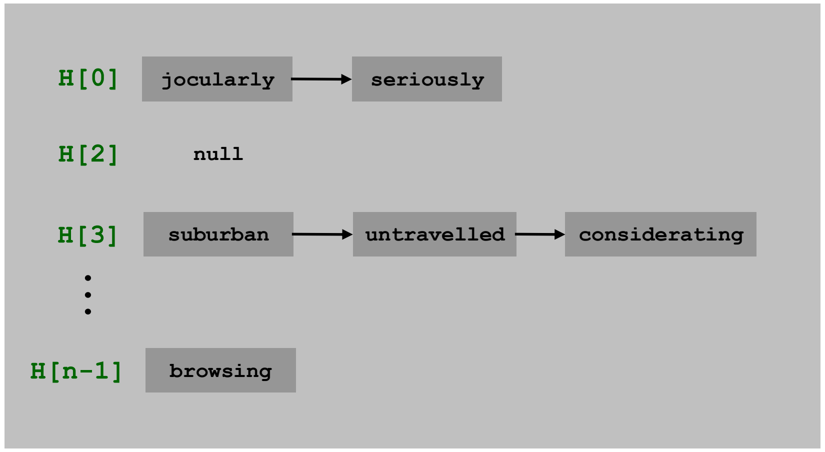
Hashing

Hash function. $h : U \rightarrow [n]$ where $[n] = \{ 0, 1, \ldots, n-1 \}$.

Hashing. Create an array H of size n. When processing element u, access array element H[h(u)].

Collision. When $h(u) = h(v)$ but $u \neq v$.

- A collision is expected after $\Theta(\sqrt{n})$ random insertions. This phenomenon is known as the "birthday paradox."
- Separate chaining: H[i] stores linked list of elements u with $h(u) = i$.



Idealistic Hash Function

A Sample Hash Function

A Sample function.

```
int h(String s, int n) {
   int hash = 0;
   for (int i = 0; i < s.length(); i++)
      hash = (31 * hash) + s[i];
   return hash % n;
}
```

hash function ala Java string library

Deterministic hashing. If $|U| \geq n^2$, then for any fixed hash function h, there is a subset $S \subseteq U$ of n elements that all hash to same slot. Thus, $\Theta(n)$ time per search in worst-case.

Q. But isn't the sample hash function good enough in practice?

Idealistic hash function

Idealistic hash function. Maps m elements uniformly at random to n hash slots.

- Running time depends on length of chains.
- Average length of chain = α = m / n.
- Choose n $\approx$ m $\Rightarrow$ on average O(1) per insert, lookup, or delete.

Theorem. If we map each element u.a.r to n hash slots, then for any k and any distinct numbers $x_1,\ldots,x_k$ we have

$$\Pr(h(x_1)=h(x_2)=\ldots=h(x_k))=1/n^{k-1}$$

And moreover, for any u and v we have Pr(h(u)=h(v))=1/n

There is a challenge (will be explained next) to reach this goal.

Challange

Challenge. Since we may see several occurrence of j, we should remember where j is mapped upon the arrival of the first occurrence of j.

Two methods to resolve this:

- We have to maintain all h(j) in a data structure which of course needs a linear data structure, or
- we use an explicit formula for the hash function. It seems we have a fix function and each input is uniquely mapped to an element of [n]

Challange

Offline fashion. Instead of mapping j u.a.r. to an element in [n] in the online fashion, we can do all randomness in the offline fashion,i.e., at the beginning, we specify h(j) u.a.t. instead of waiting to receive the first j and specify h(j).

```
    for (int j = 1; |U|; j++)
       h[j] = random(0,n-1);
```

Example. for planning a single-elimination knockout tournament over n teams, we can construct a tree with n leaves and put a random permutation of teams in the leaves or in each round the opponent of each team is specified u.a.r. The result is the same meaning that the probability that two teams meet each other is the same in both methods

Challange

- This method is equivalent to select a function h: from U to [n] u.a.r. from all functions from U to [n] (#such functions = $n^{|U|}$) .

- If we do that, we don't have an explicit formula for h and we need a data structure to maintain h.

Universal Hash Family

K-universal hash Family and Universal Hash Family

Approach. A practical solution is to put some functions (not all functions) with explicit formula into a set H (called a family of hash functions) and select one of them u.a.r. at the beginning (offline fashion).

```
i = random(1,|H|);
h = the i-th function of H
```

Remark. Even if a adversary knows your hash family (before running your program), he can not detect which hash function is used by your program when it starts running.

K-universal hash family: H is a k-universal hash family if for any k distinct number $x_1,\dots,x_k$ we have

$$\Pr(h(x_1)= h(x_2)=\dots=h(x_k)) \leq 1/n^{k-1}$$

2-Universal hash family: H is a 2-universal hash family if for any distinct number u and v we have

$$\Pr(h(u)=h(v)) \leq 1/n$$

K-universal hash Family and Universal Hash Family

- For most applications, having a 2-universal hashing family suffices.
- How to show the family H is 2-universal: for any distinct u and v, count the number of hash functions h that h(u) = h(v). Divide this number by |H|. If it always (for any distinct u and v) is equal or less than 1/n, H is universal.

Universal Hashing

.Ex. $U = \{ a, b, c, d, e, f \}$, $n = 2$.

	a	b	c	d	e	f
$h_1(x)$	0	1	0	1	0	1
$h_2(x)$	0	0	0	1	1	1

$H = \{h_1, h_2\}$
$\Pr_{h \in H} [h(a) = h(b)] = 1/2$
$\Pr_{h \in H} [h(a) = h(c)] = 1$
$\Pr_{h \in H} [h(a) = h(d)] = 0$
. . .

not 2-universal

	a	b	c	d	e	f
$h_1(x)$	0	1	0	1	0	1
$h_2(x)$	0	0	0	1	1	1
$h_3(x)$	0	0	1	0	1	1
$h_4(x)$	1	0	0	1	1	0

$H = \{h_1, h_2, h_3, h_4\}$
$\Pr_{h \in H} [h(a) = h(b)] = 1/2$
$\Pr_{h \in H} [h(a) = h(c)] = 1/2$
$\Pr_{h \in H} [h(a) = h(d)] = 1/2$
$\Pr_{h \in H} [h(a) = h(e)] = 1/2$
$\Pr_{h \in H} [h(a) = h(f)] = 0$
. . .

2-universal

Universal Hashing

Universal hashing property. Let H be a 2-universal class of hash functions; let $h \in H$ be chosen uniformly at random from H; and let $u \in U$. For any subset $S \subseteq U$ of size at most n, the expected number of items in S that collide with u is at most 1.

Pf. For any element $s \in S$, define indicator random variable $X_s = 1$ if $h(s) = h(u)$ and 0 otherwise. Let X be a random variable counting the total number of collisions with u.

$$E_{h \text{ in } H}[X] = E[\sum X_s] \underset{\substack{\uparrow \\ \text{linearity of expectation}}}{=} \sum E[X_s] \underset{\substack{\uparrow \\ X_s \text{ is a 0-1 random variable}}}{=} \sum \Pr[X_s = 1] \underset{\substack{\uparrow \\ \text{universal} \\ (\text{assumes } u \notin S)}}{\leq} \sum \frac{1}{n} = |S| \frac{1}{n} \leq 1$$

Designing a Universal Family of Hash Functions

Theorem. [Chebyshev 1850] There exists a prime between n and 2n.

Modulus. Choose a prime number $p \approx n$. $\longleftarrow$ no need for randomness here

Integer encoding. Identify each element $u \in U$ with a base-p integer of r digits: $x = (x_1, x_2, \ldots, x_r)$.

Hash function. Let A = set of all r-digit, base-p integers. For each $a = (a_1, a_2, \ldots, a_r)$ where $0 \leq a_i < p$, define

$$h_a(x) = \left(\sum_{i=1}^{r} a_i x_i \right) \mod p$$

Hash function family. $H = \{ h_a : a \in A \}$.

Selecting a u.a.r hash function: we must determine a_i

```
     for (int i = 1; r; i++)
        a[i] = random(0,p-1)
```

Designing a Universal Class of Hash Functions

Theorem. H = { h_a : a $\in$ A } is a universal class of hash functions.

Pf. Let x = (x_1, x_2, …, x_r) and y = (y_1, y_2, …, y_r) be two distinct elements of U. We need to show that Pr[$h_a(x) = h_a(y)$] $\leq$ 1/n.

- Since x $\neq$ y, there exists an integer j such that $x_j \neq y_j$.
- We have $h_a(x) = h_a(y)$ iff

$$a_j \underbrace{(y_j - x_j)}_{z} = \underbrace{\sum_{i \neq j} a_i (x_i - y_i)}_{m} \mod p$$

- Can assume a was chosen uniformly at random by first selecting all coordinates a_i where i $\neq$ j, then selecting a_j at random. Thus, we can assume a_i is fixed for all coordinates i $\neq$ j.
- Since p is prime, a_j z = m mod p has at most one solution among p possibilities. $\longleftarrow$ see lemma on next slide
- Thus Pr[$h_a(x) = h_a(y)$] = 1/p $\leq$ 1/n. ▪

Number Theory Facts

Fact. Let p be prime, and let $z \neq 0 \bmod p$. Then $\alpha z = m \bmod p$ has at most one solution $0 \leq \alpha < p$.

Pf.

- Suppose α and β are two different solutions.
- Then $(\alpha - \beta)z = 0 \bmod p$; hence $(\alpha - \beta)z$ is divisible by p.
- Since $z \neq 0 \bmod p$, we know that z is not divisible by p; it follows that $(\alpha - \beta)$ is divisible by p.
- This implies $\alpha = \beta$. ▪

Bonus fact. Can replace "at most one" with "exactly one" in above fact.
Pf idea. Euclid's algorithm.

Collision Resolution

Three General Ways

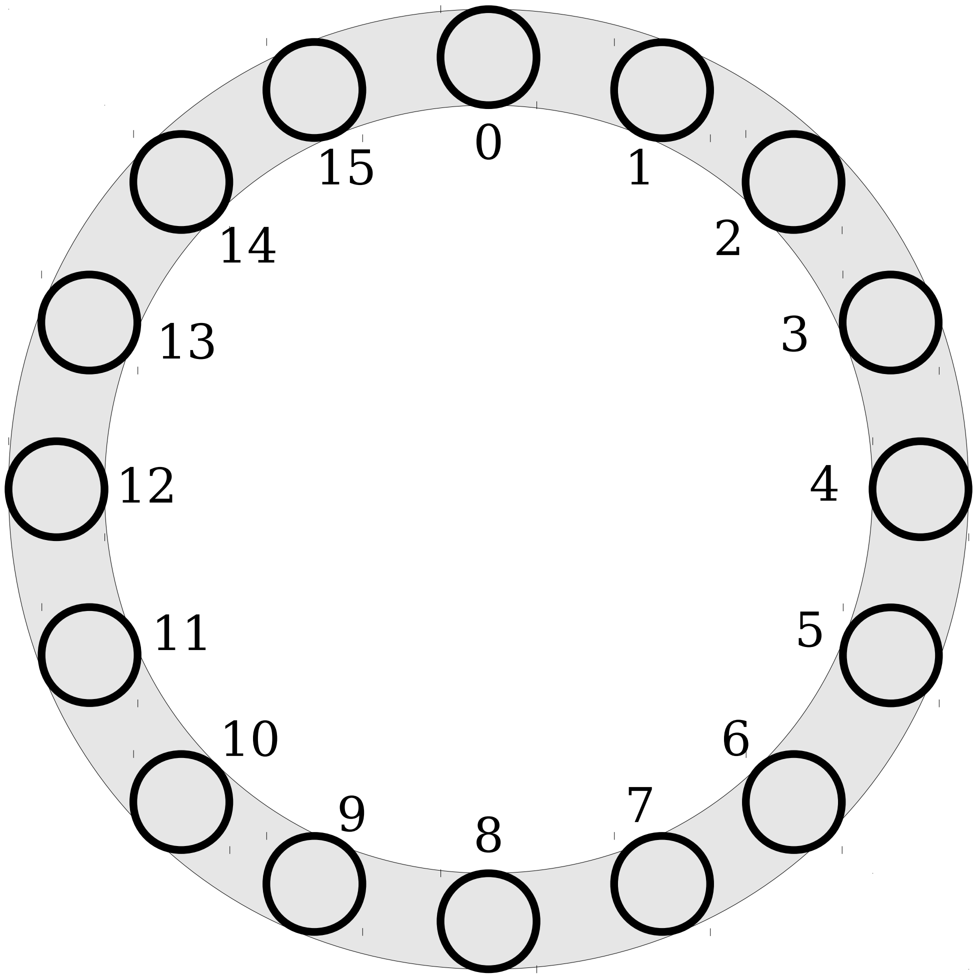
Closed addressing: Store all colliding elements in an auxiliary data structure like a linked list or BST (For example, standard chained hashing introduced at the beginning).

Open addressing: Allow elements to overflow out of their target slot and into other slot (For example, linear probing hashing).

Perfect hashing: Do something clever with multiple hash functions to eliminate collisions (For example, Cuckoo Hashing).

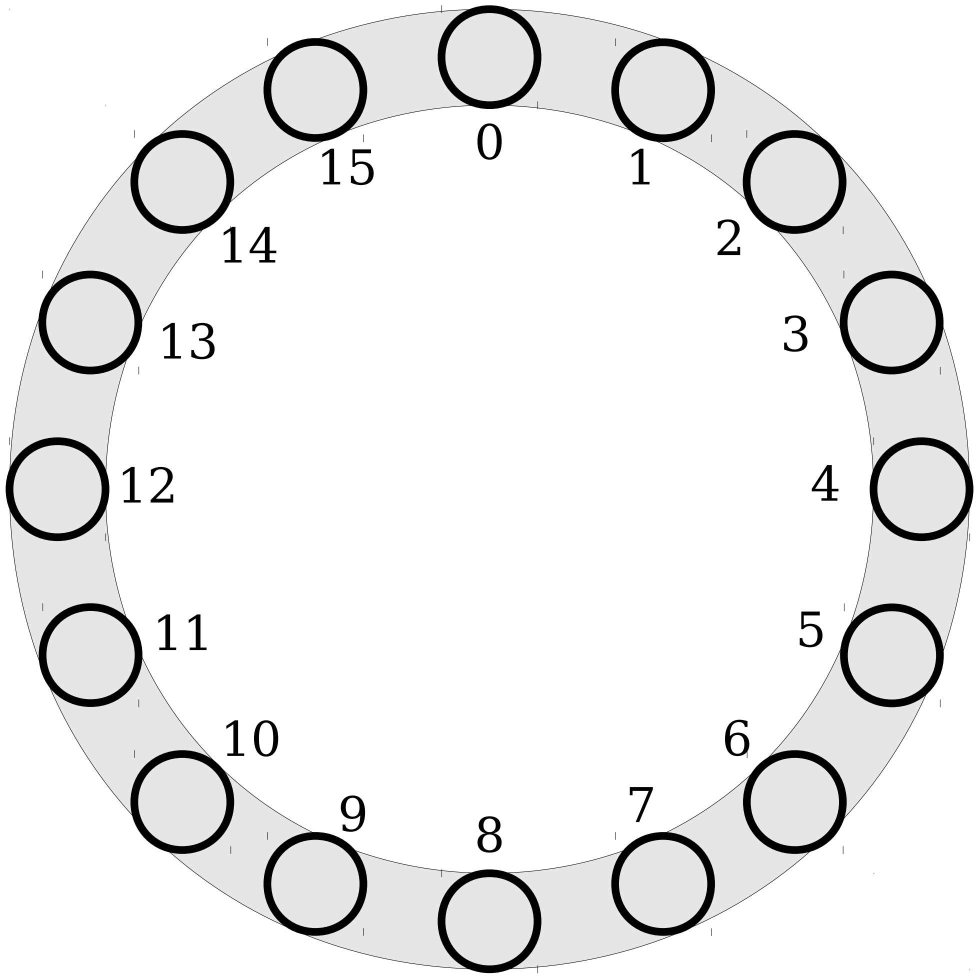
Linear Probing

- ***Linear probing*** is a simple open-addressing hashing strategy.
- We maintain an array of ***slots***, which we think of as forming a ring.



Linear Probing

- To insert an element, compute its hash code and try to place it at the slot with that number.



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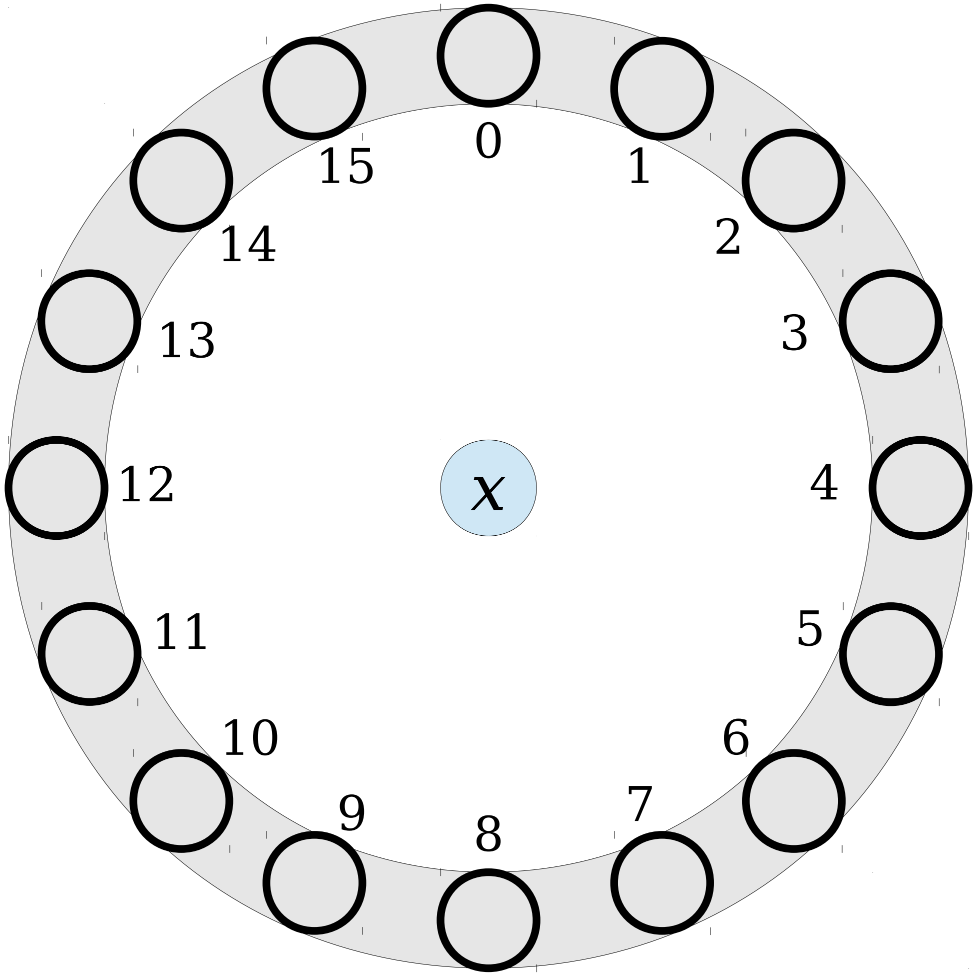

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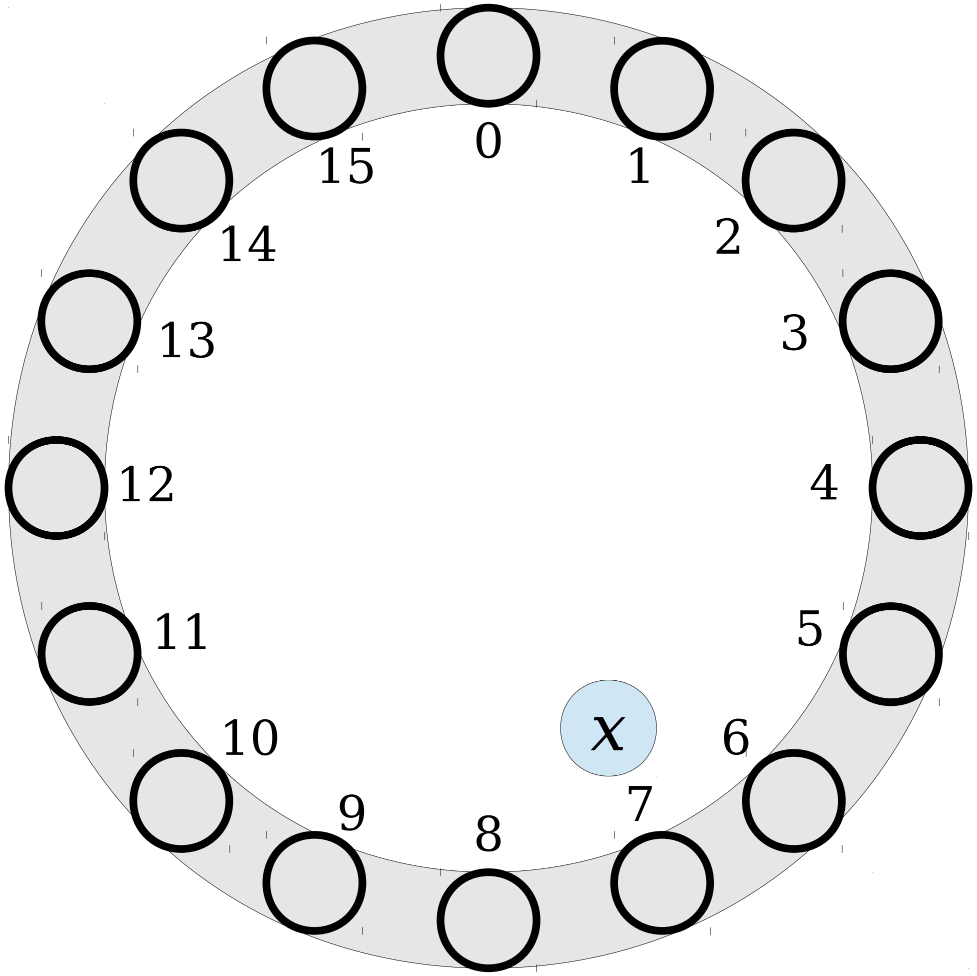

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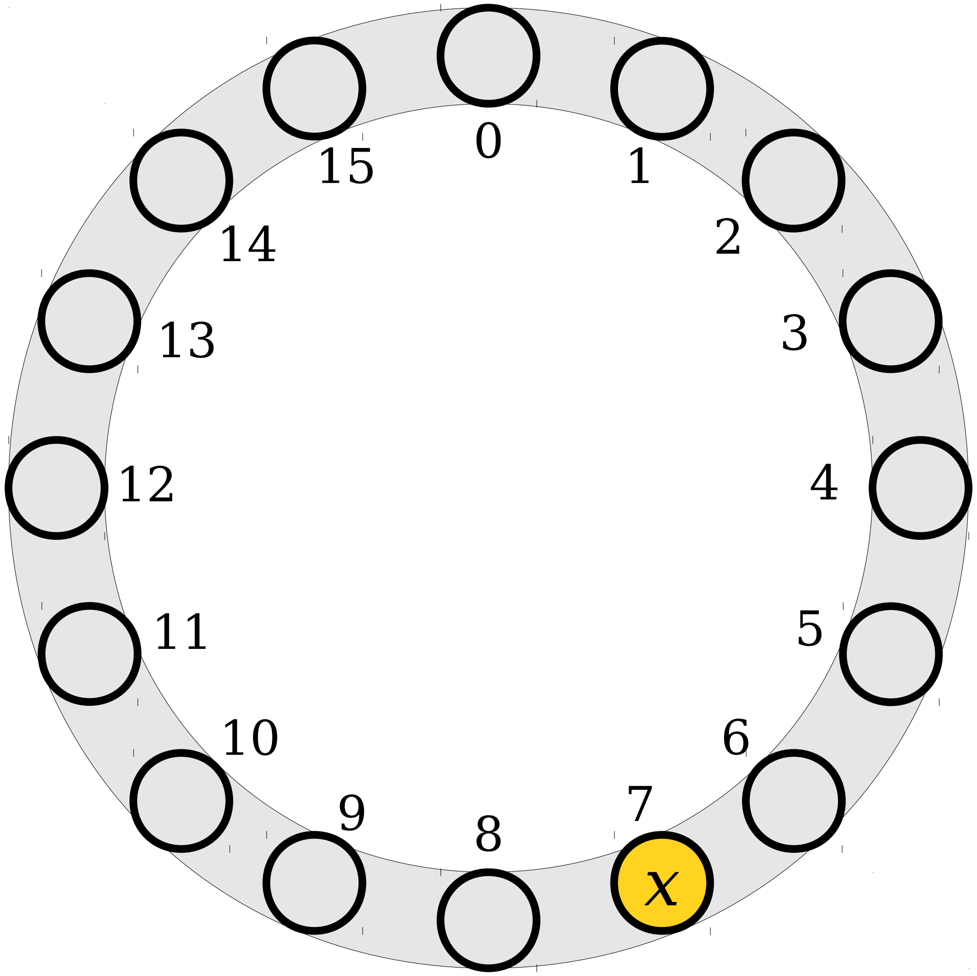

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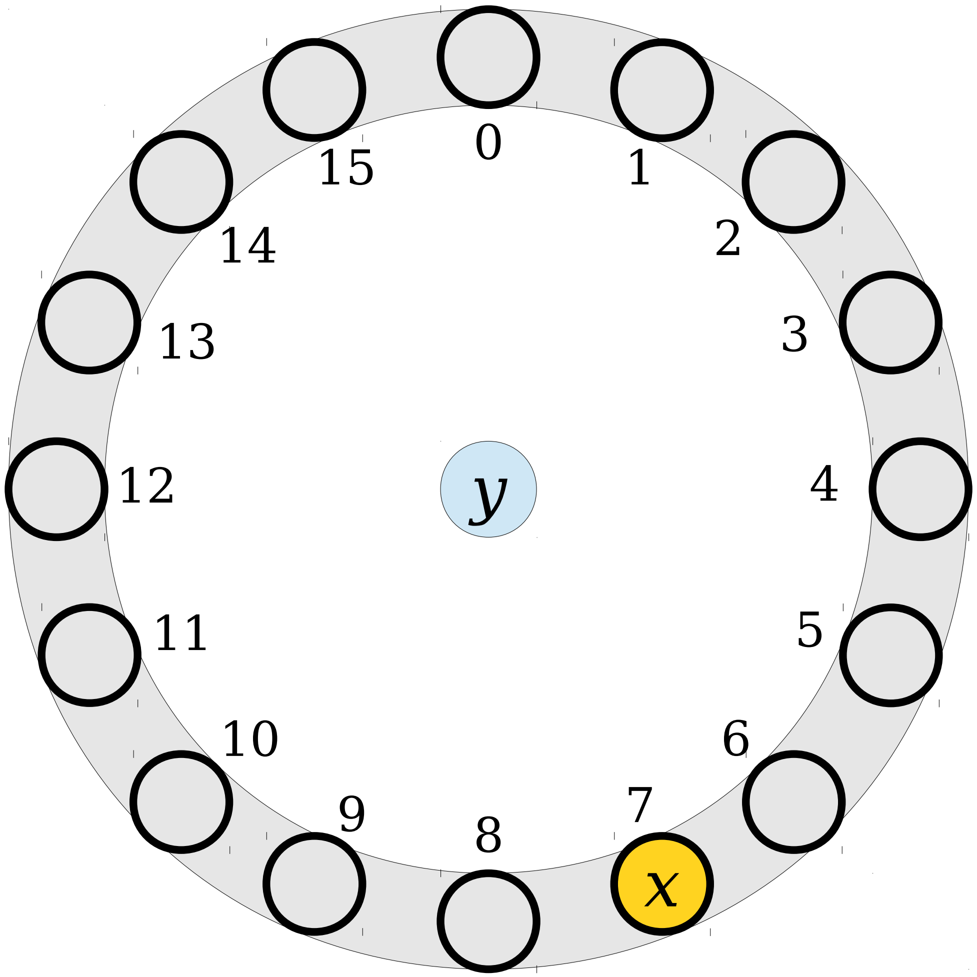

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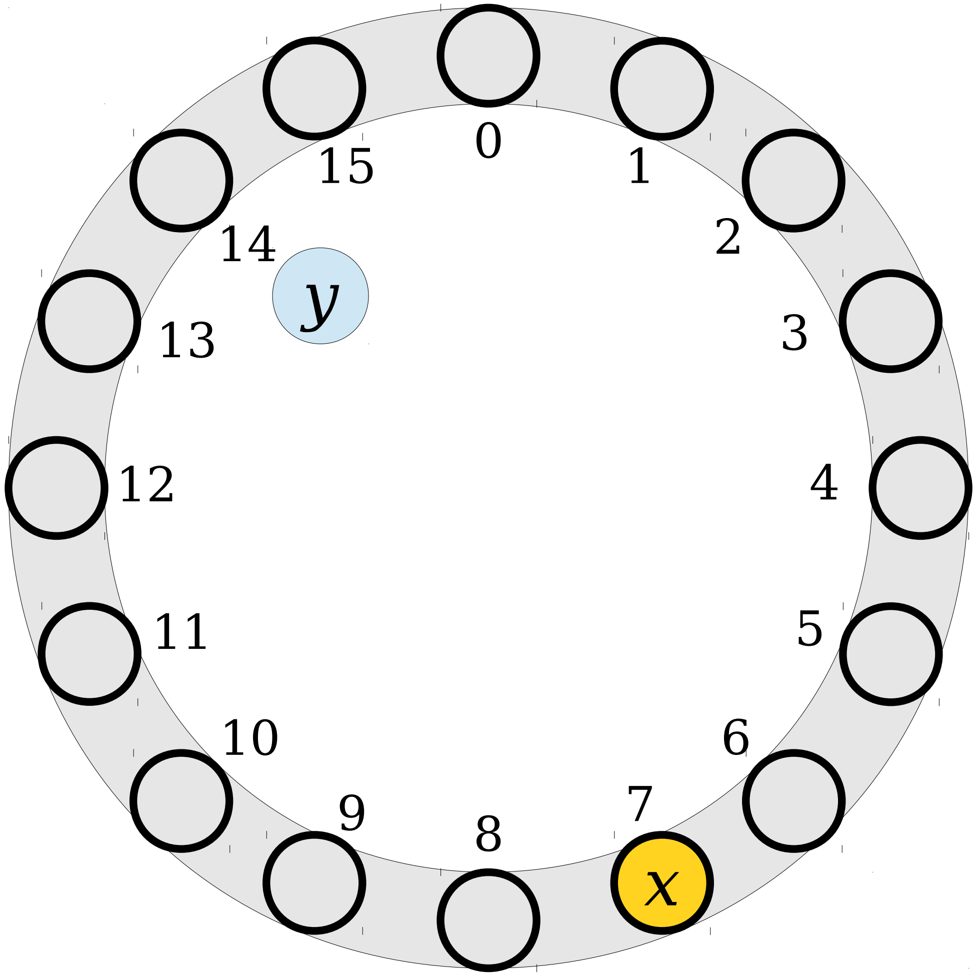

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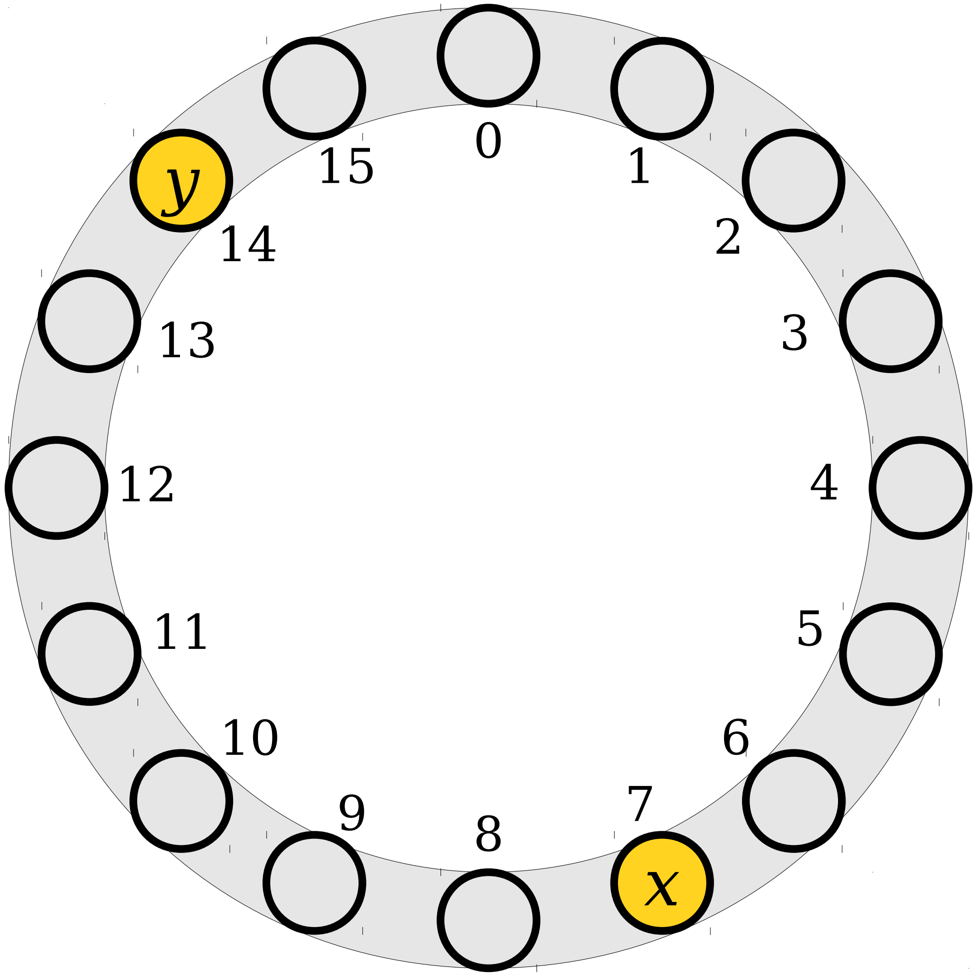

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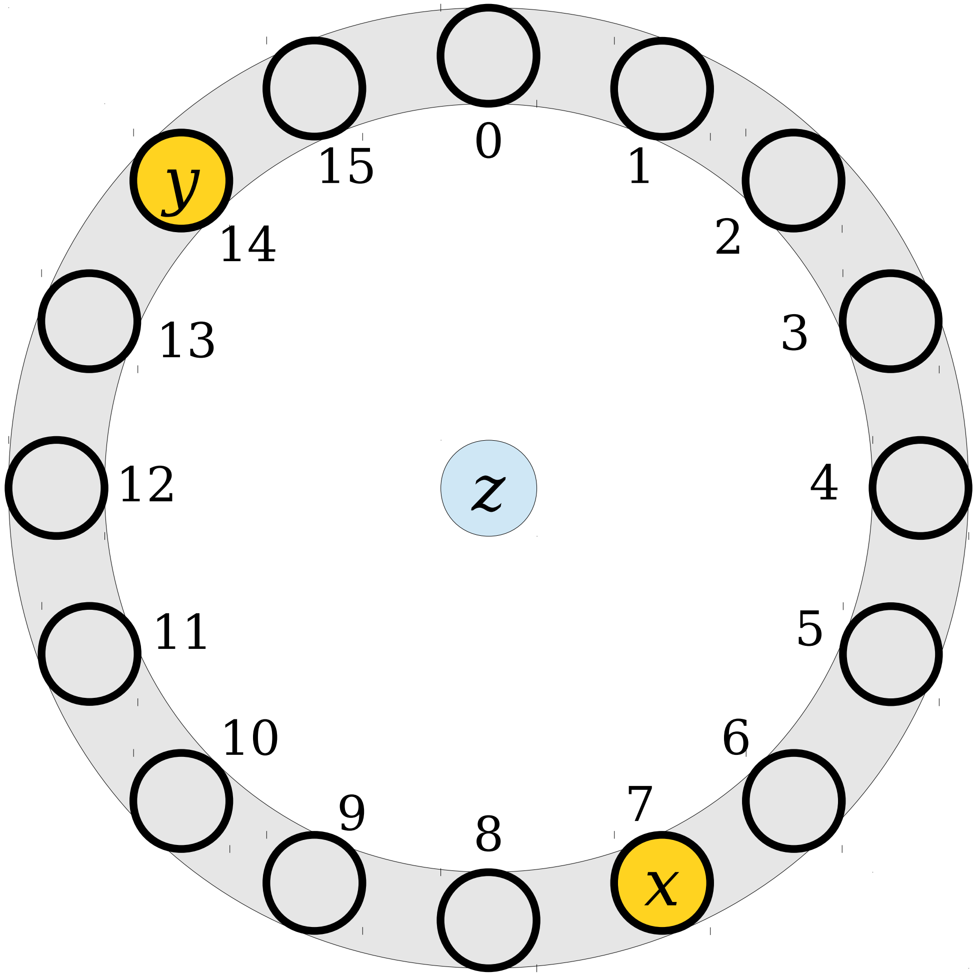

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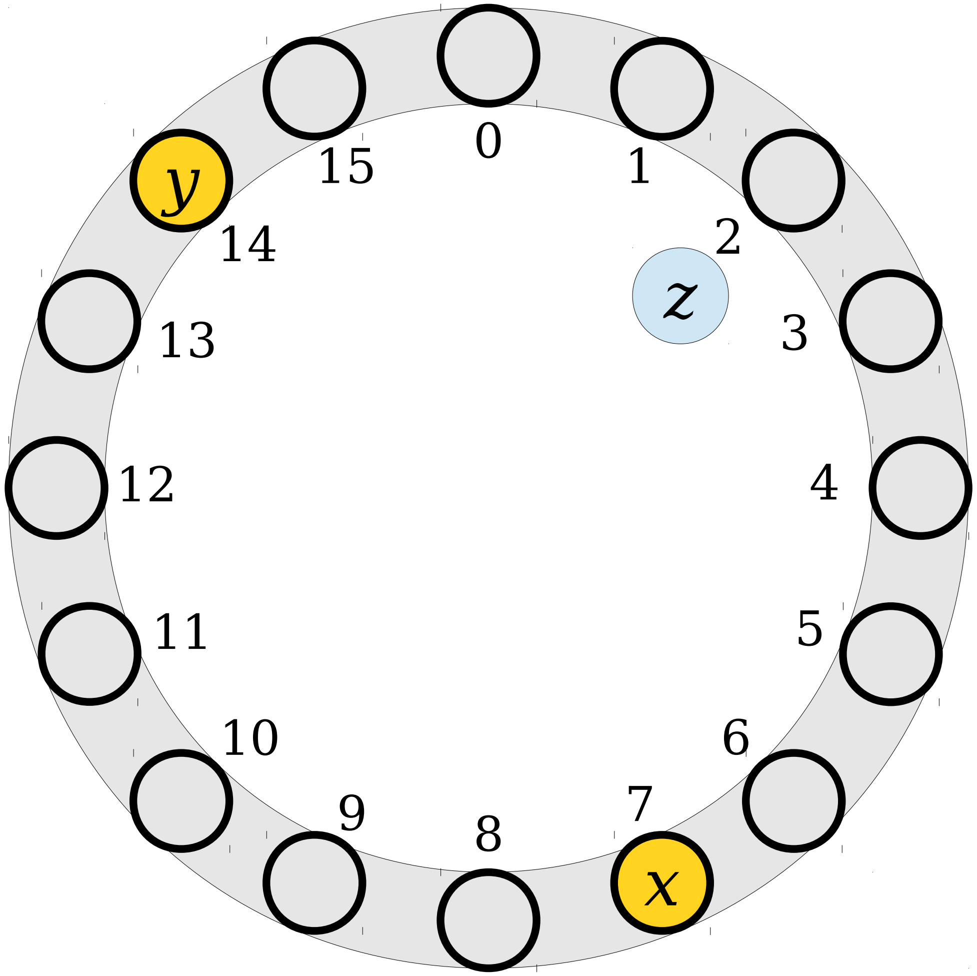

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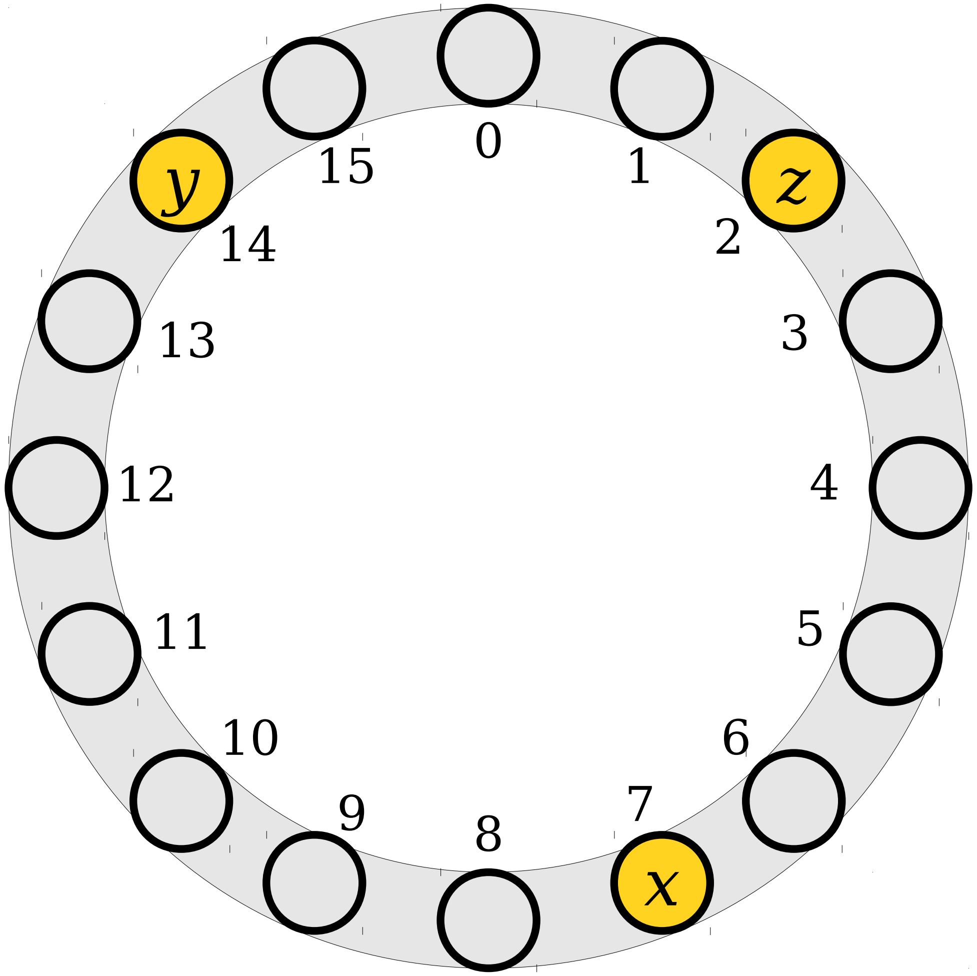

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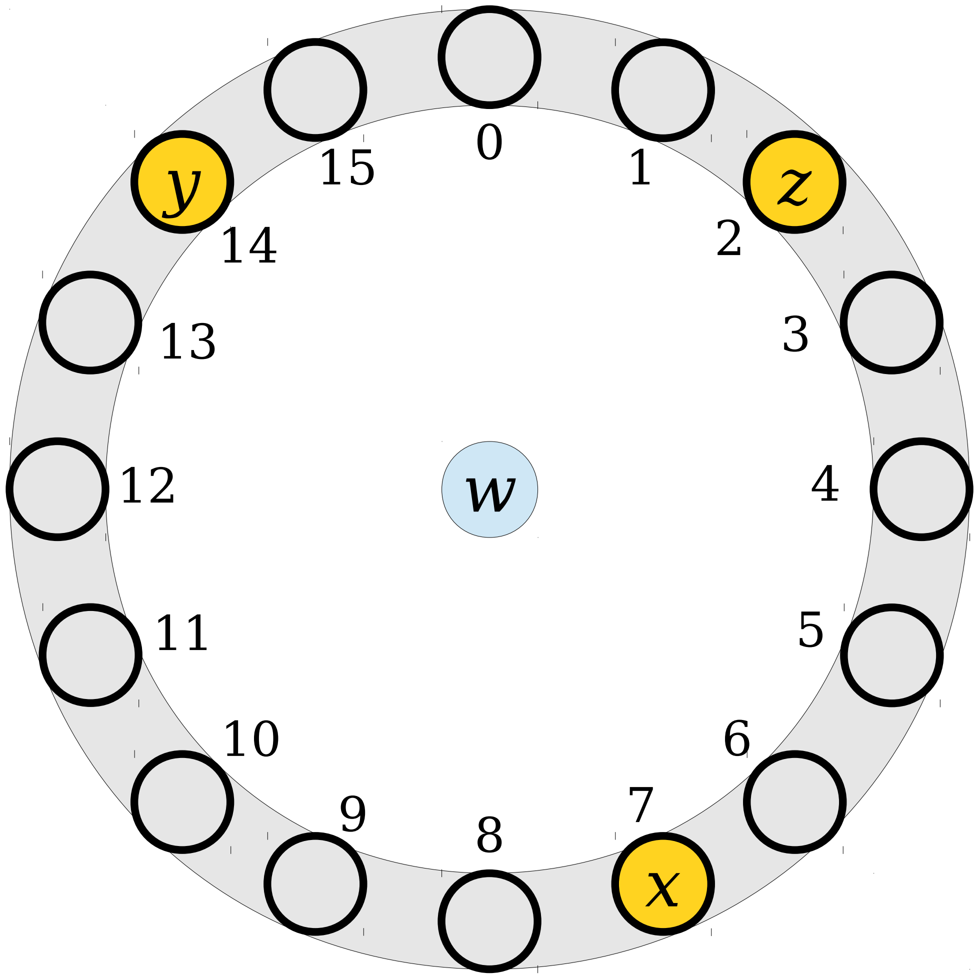

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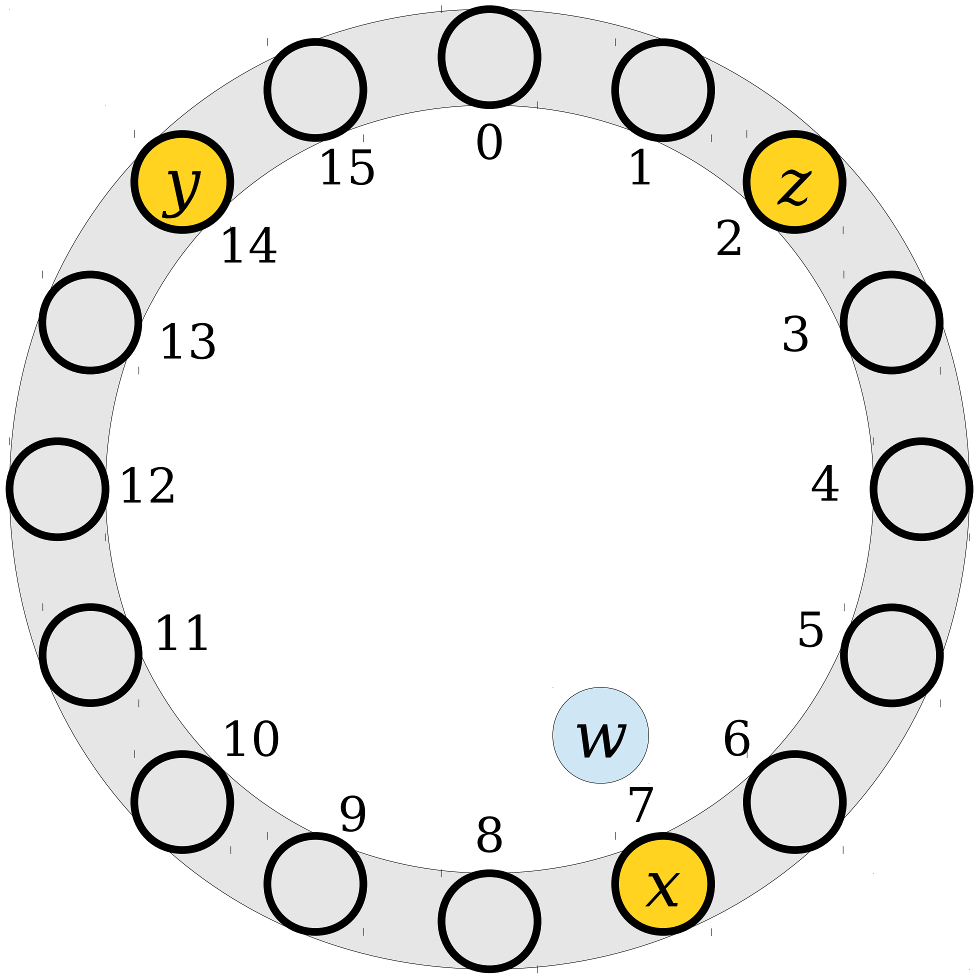

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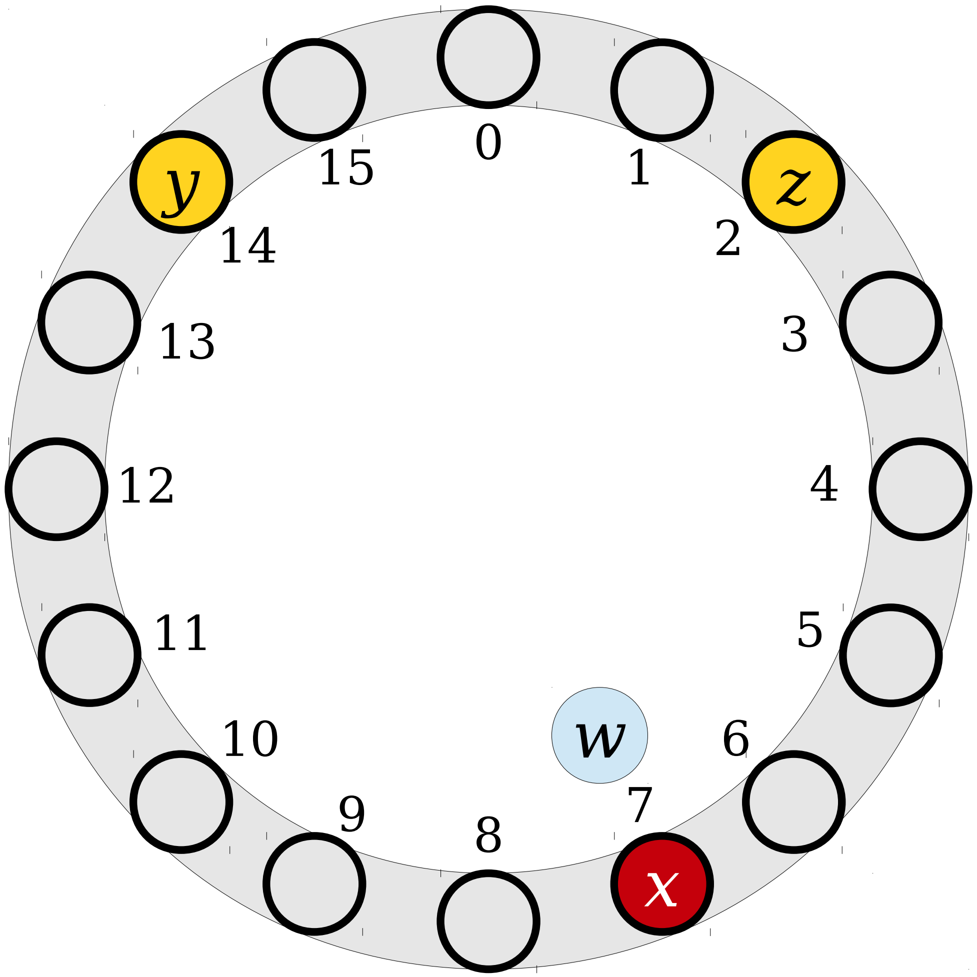

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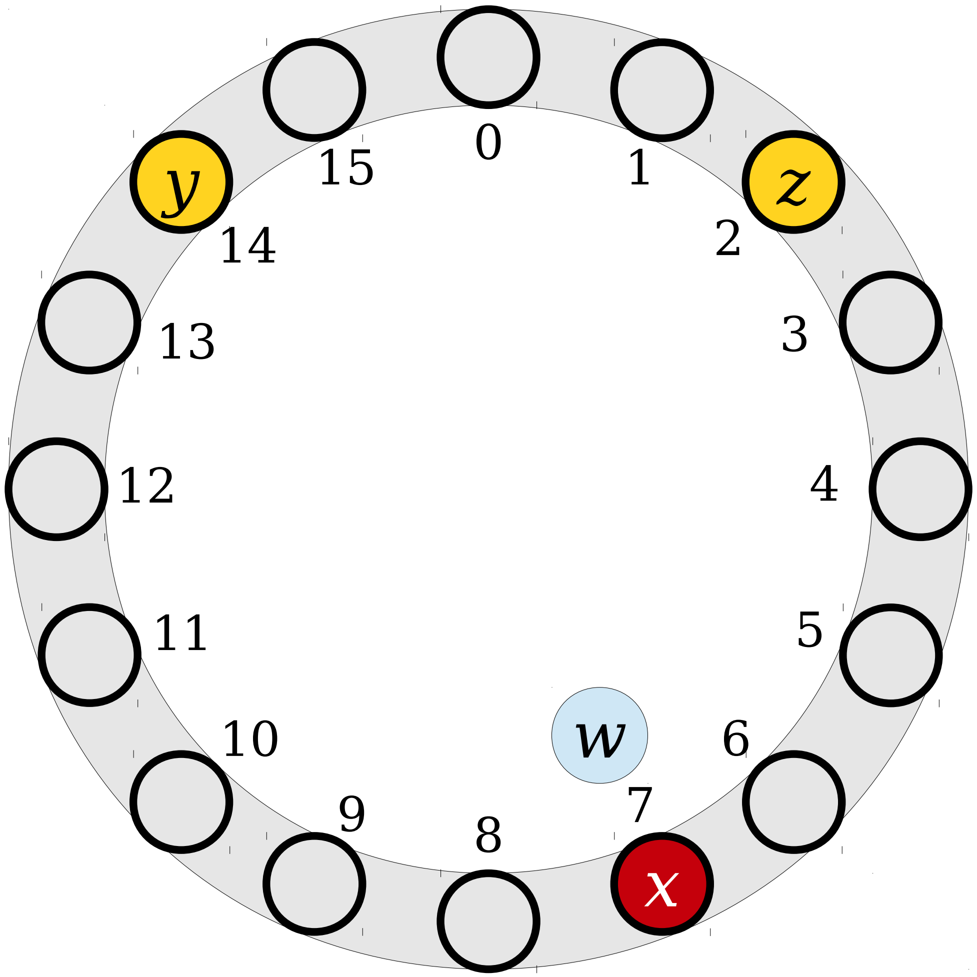

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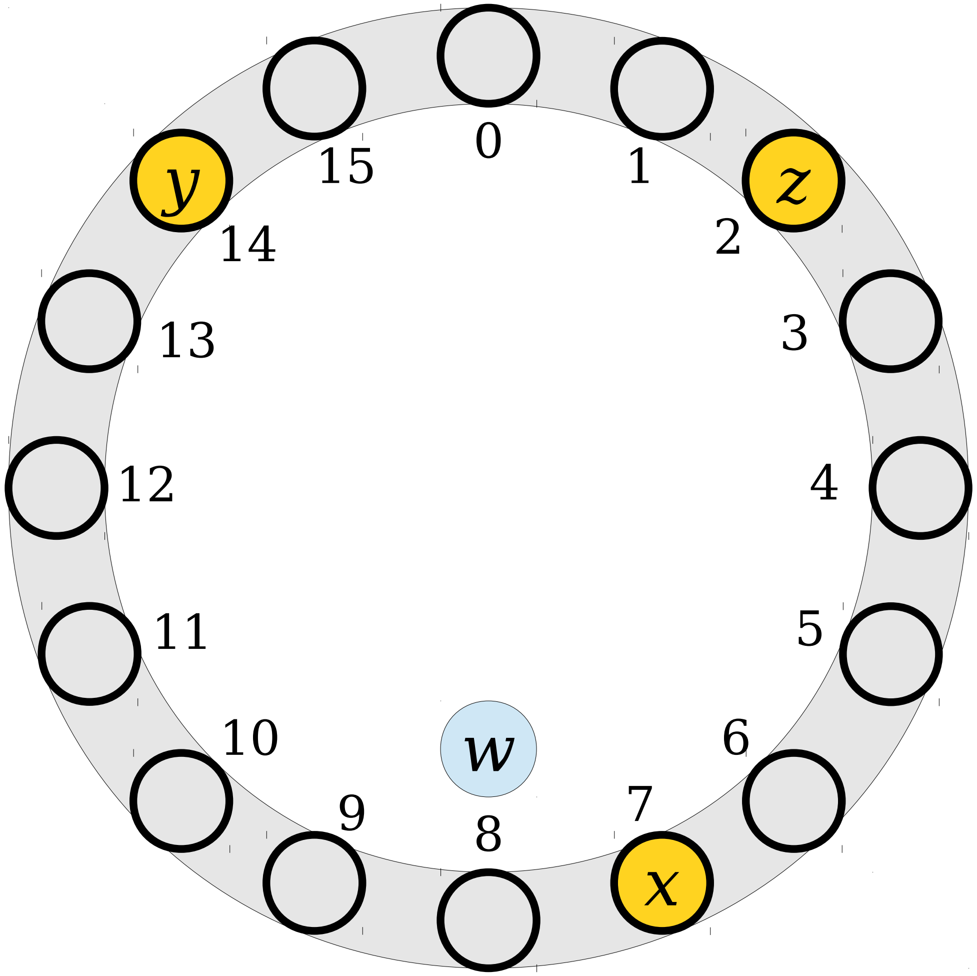

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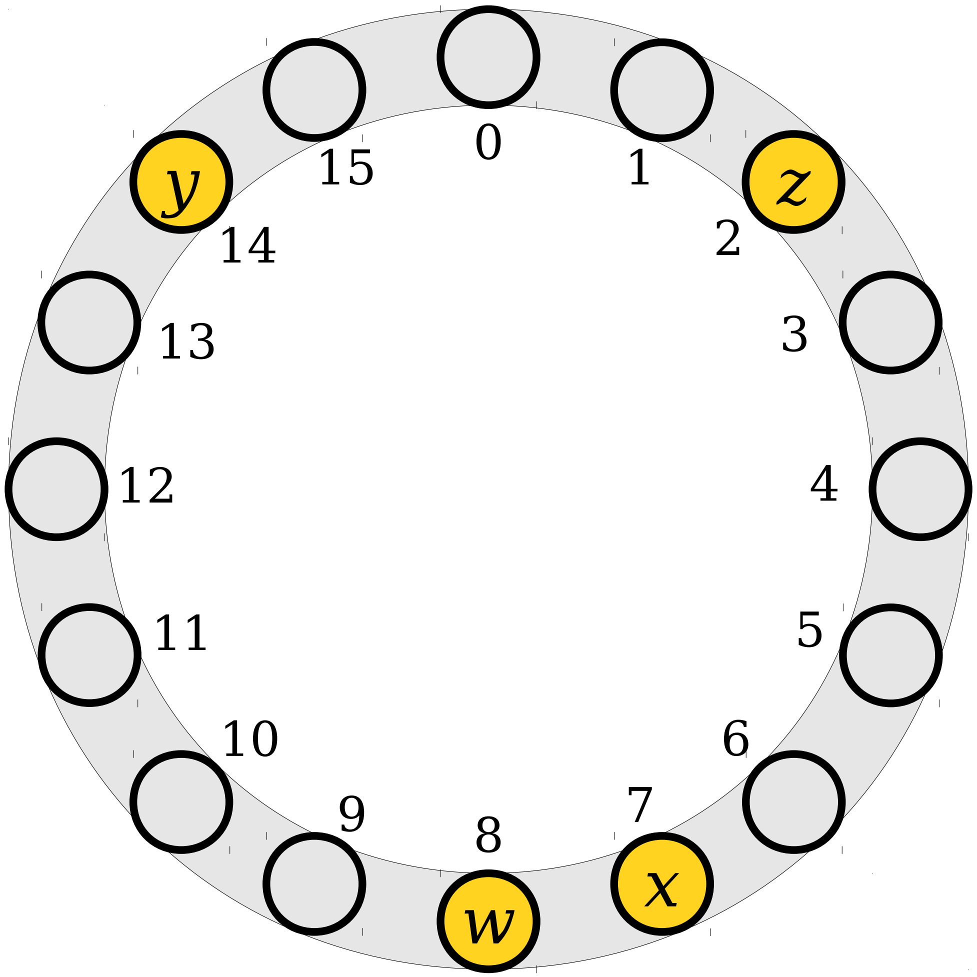

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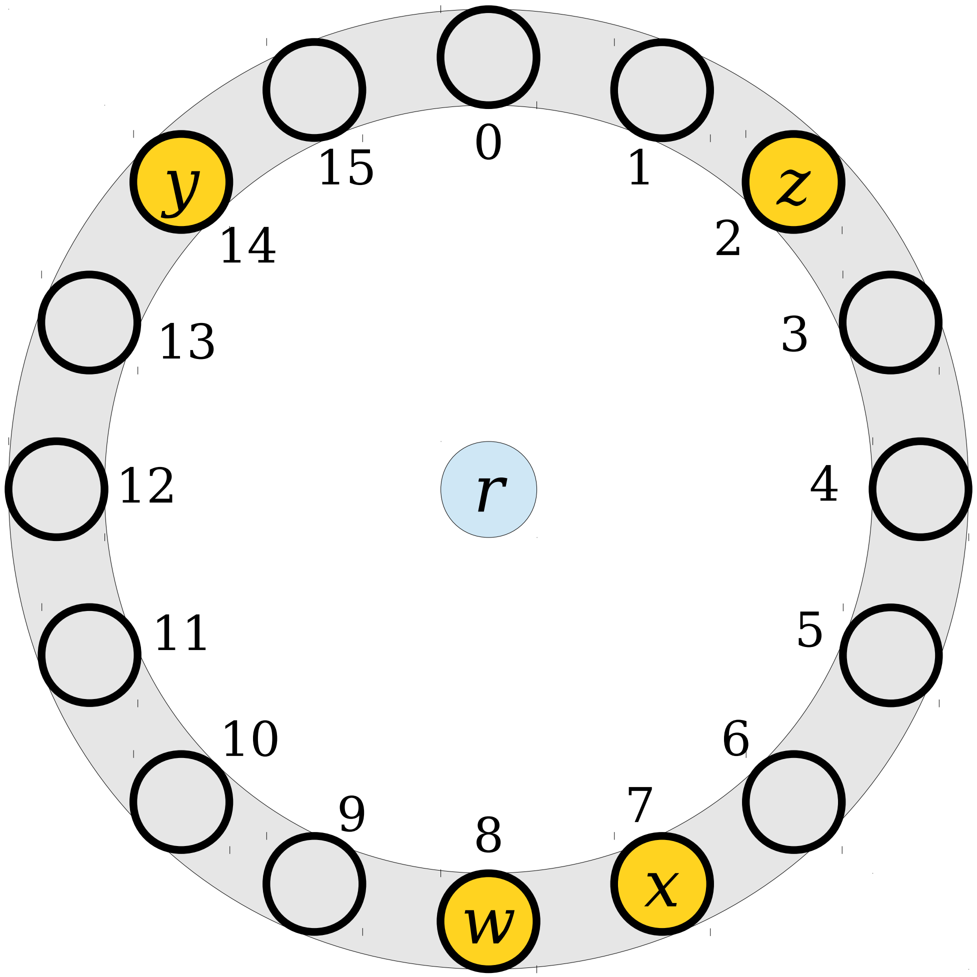

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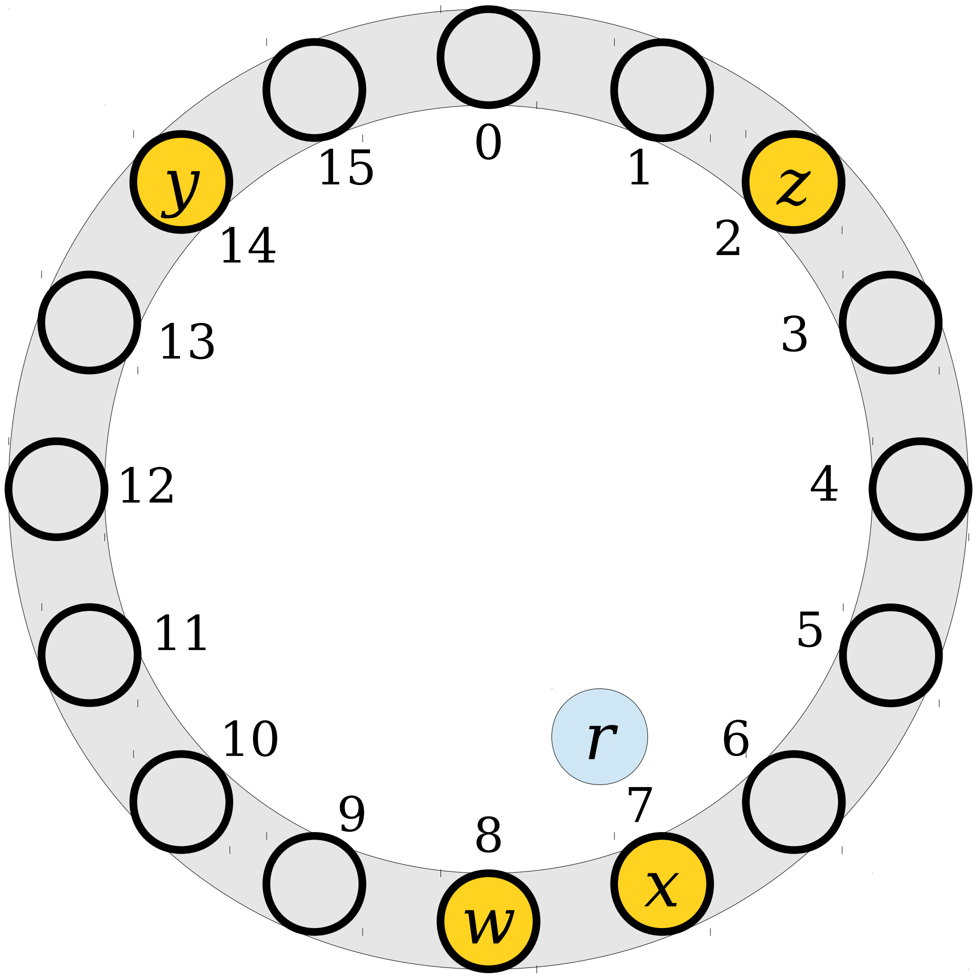

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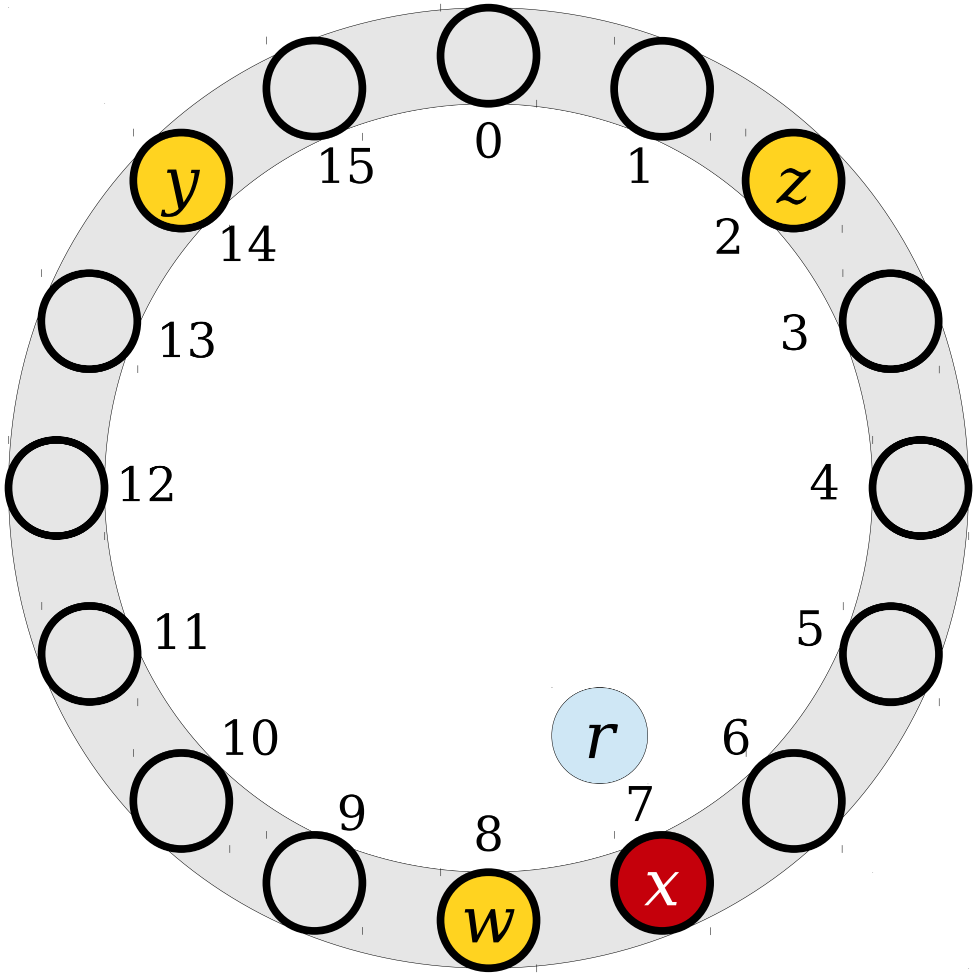

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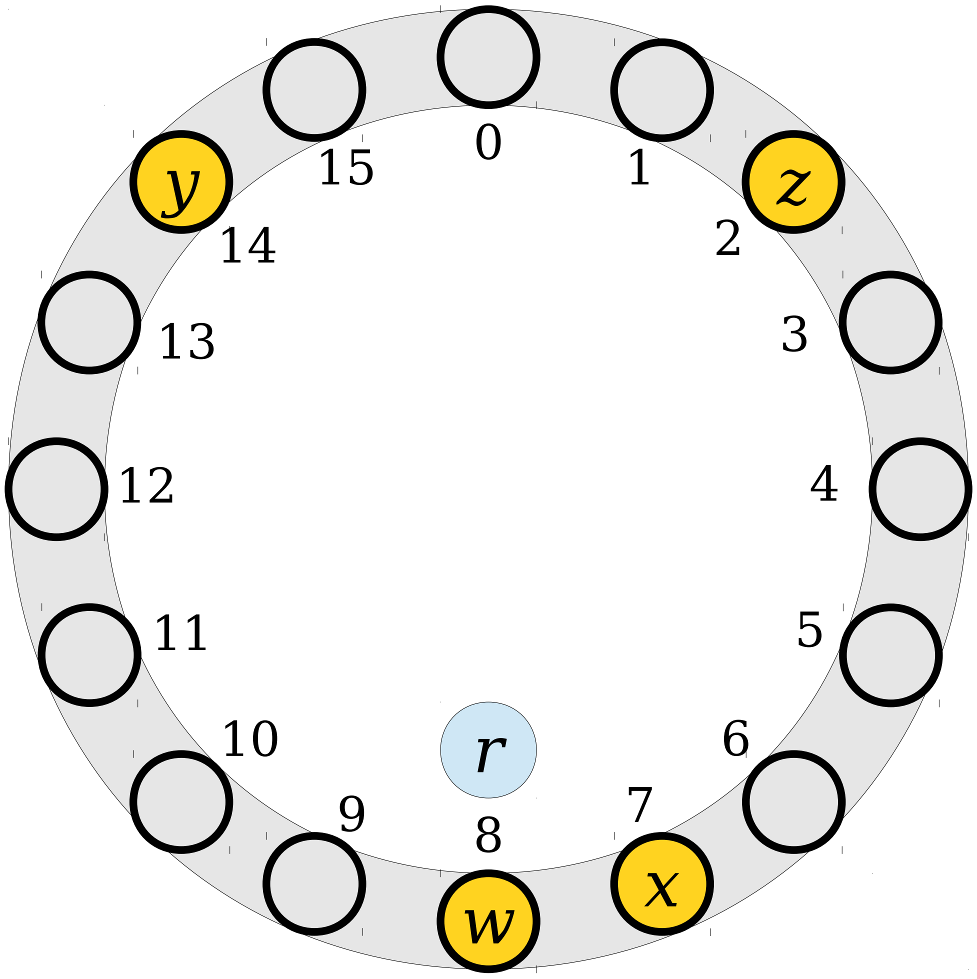

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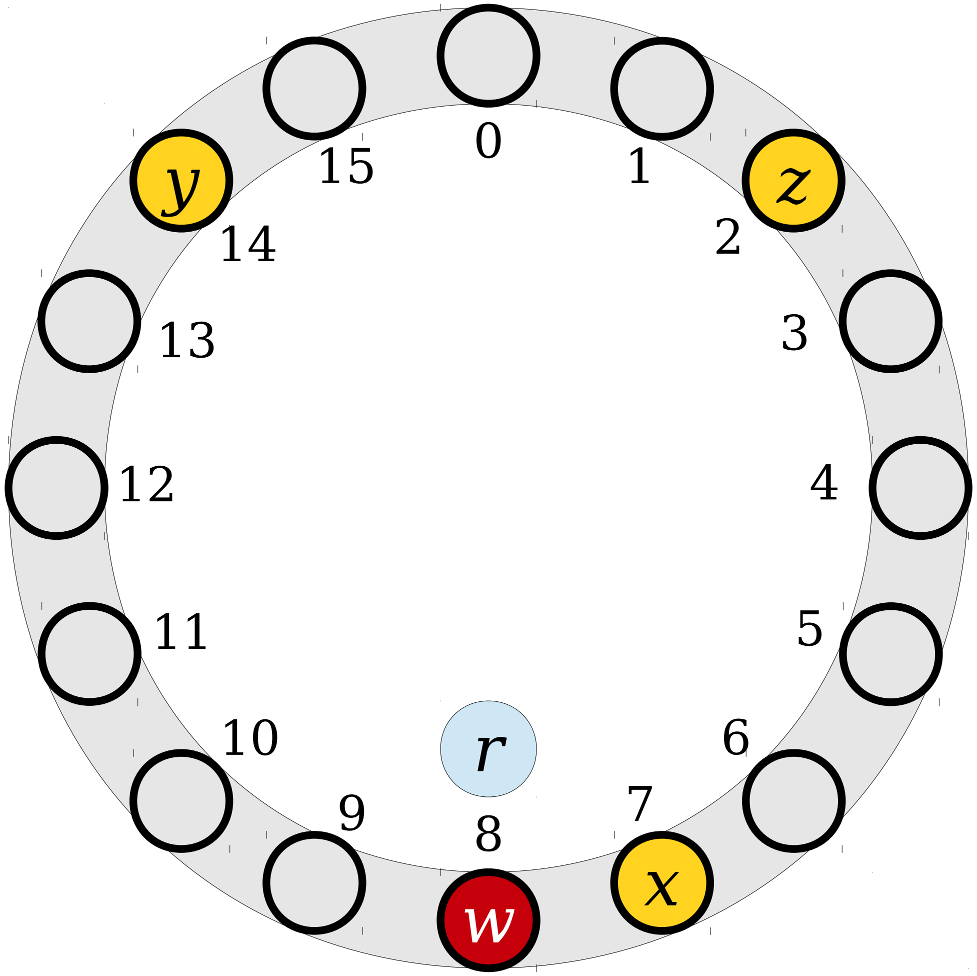

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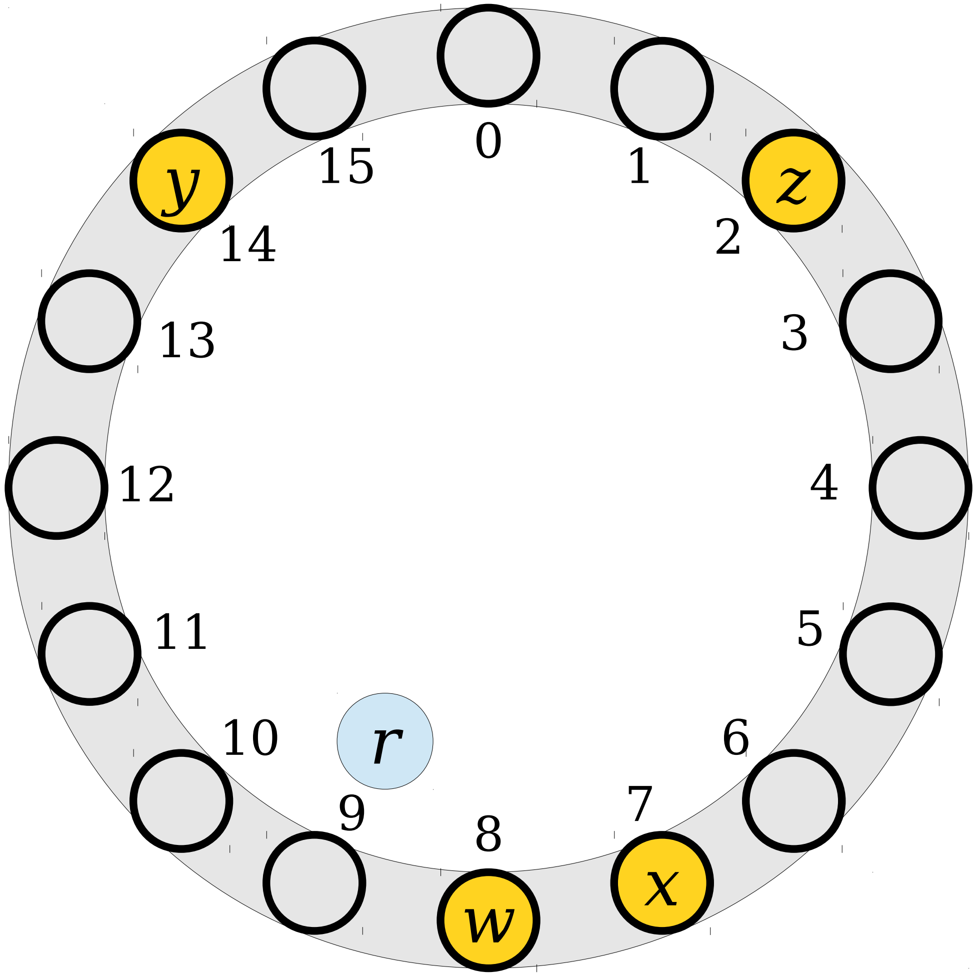

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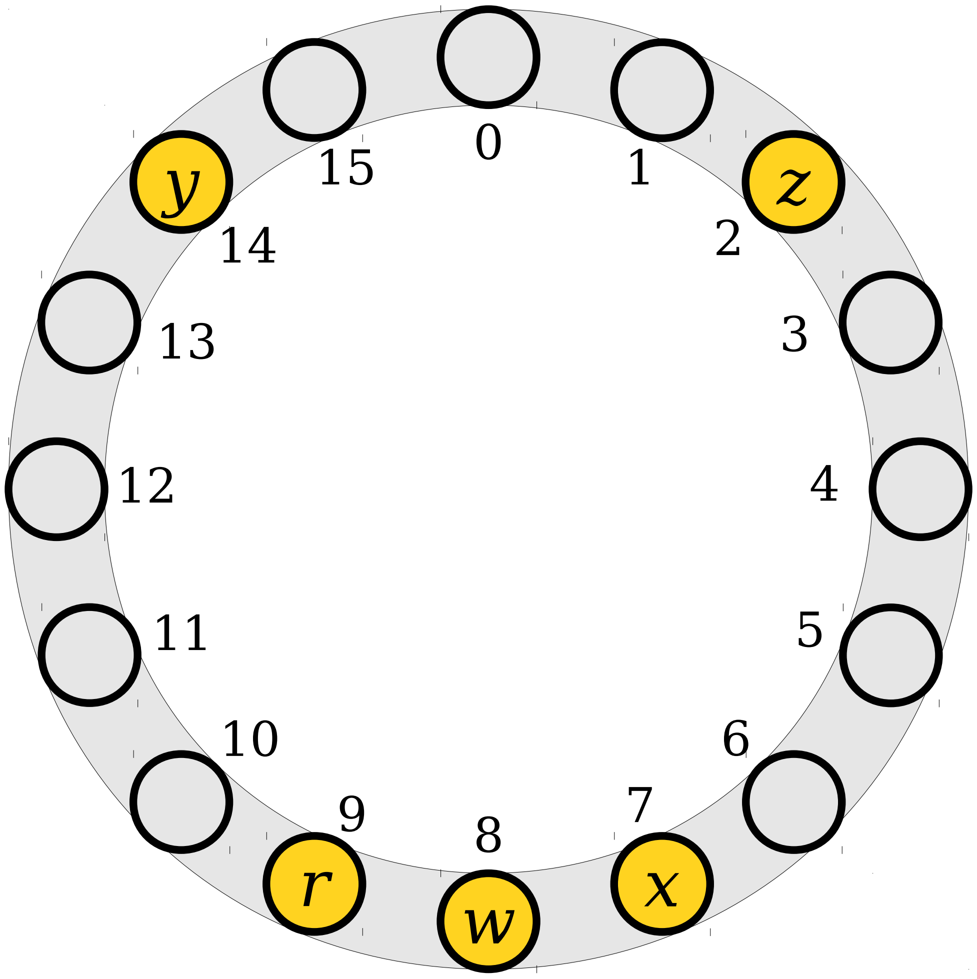

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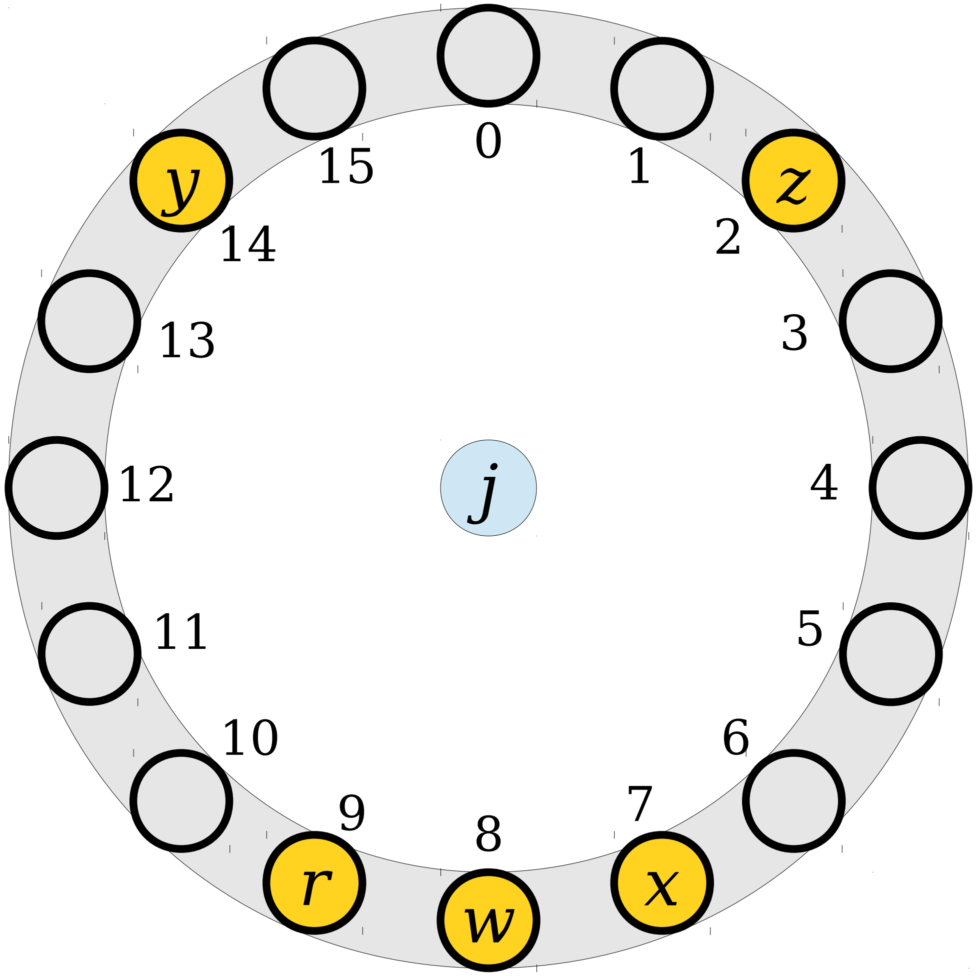

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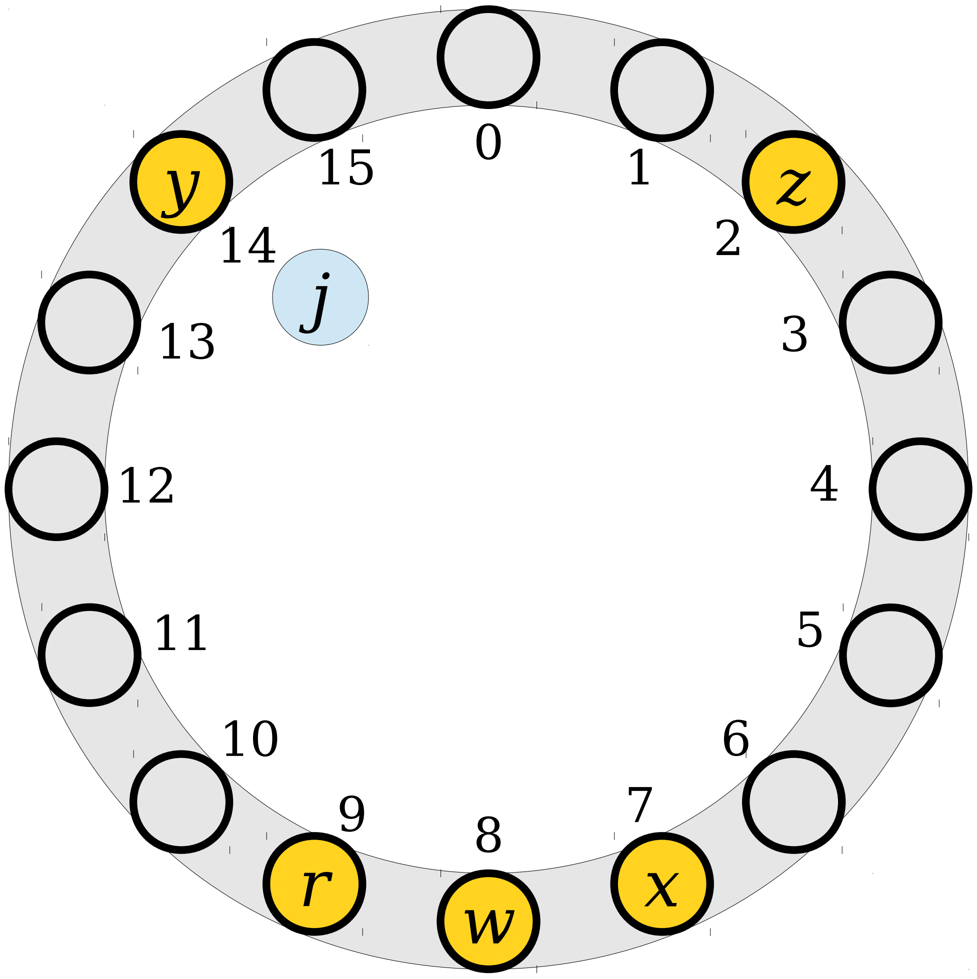

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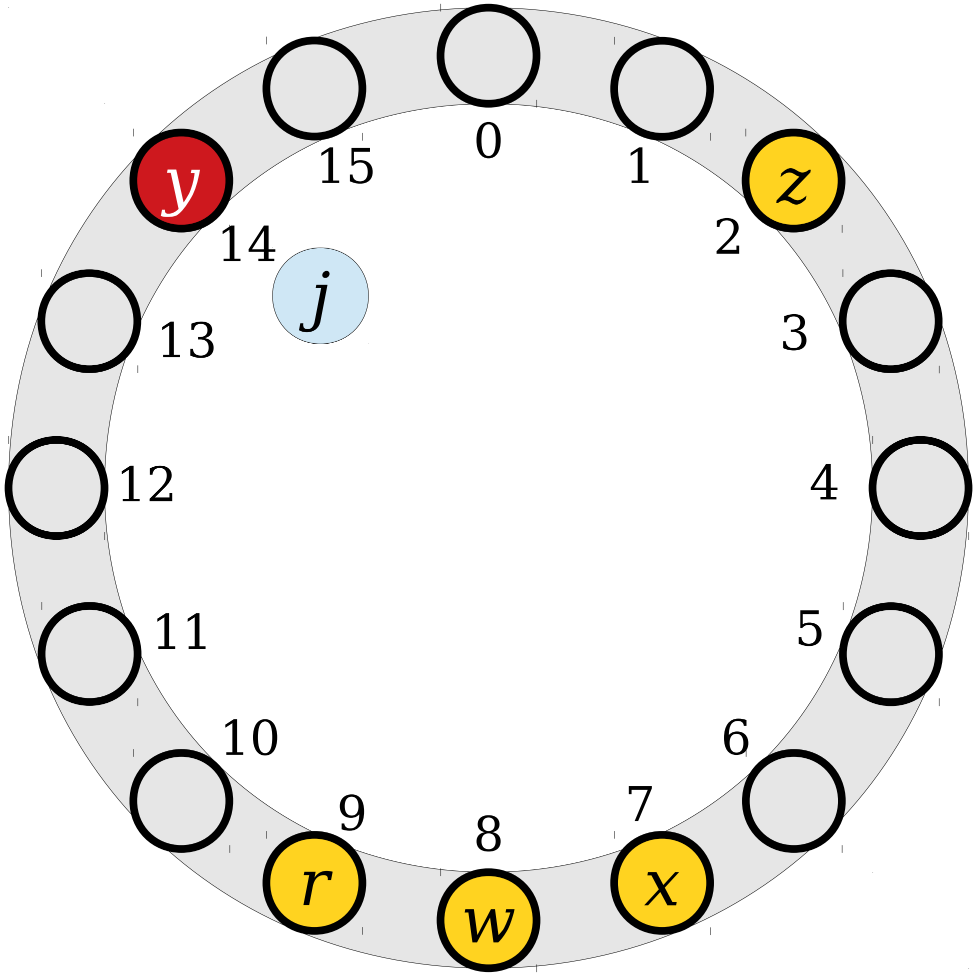

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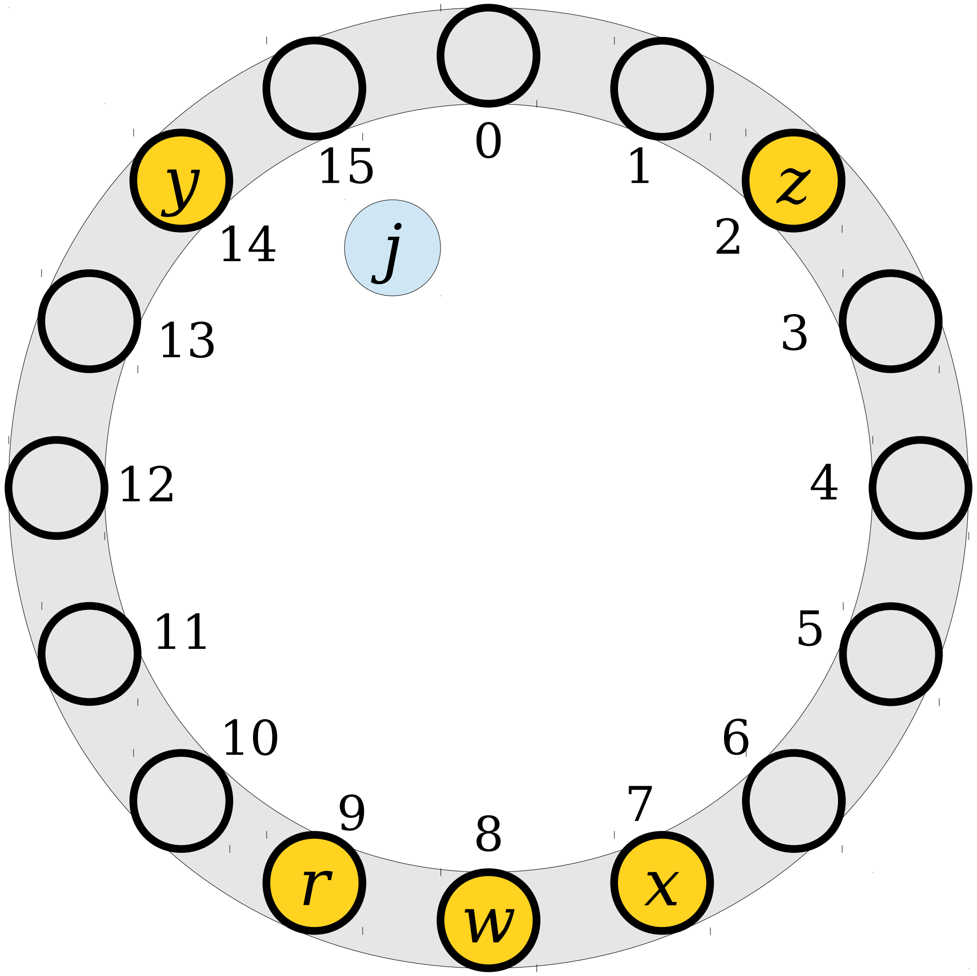

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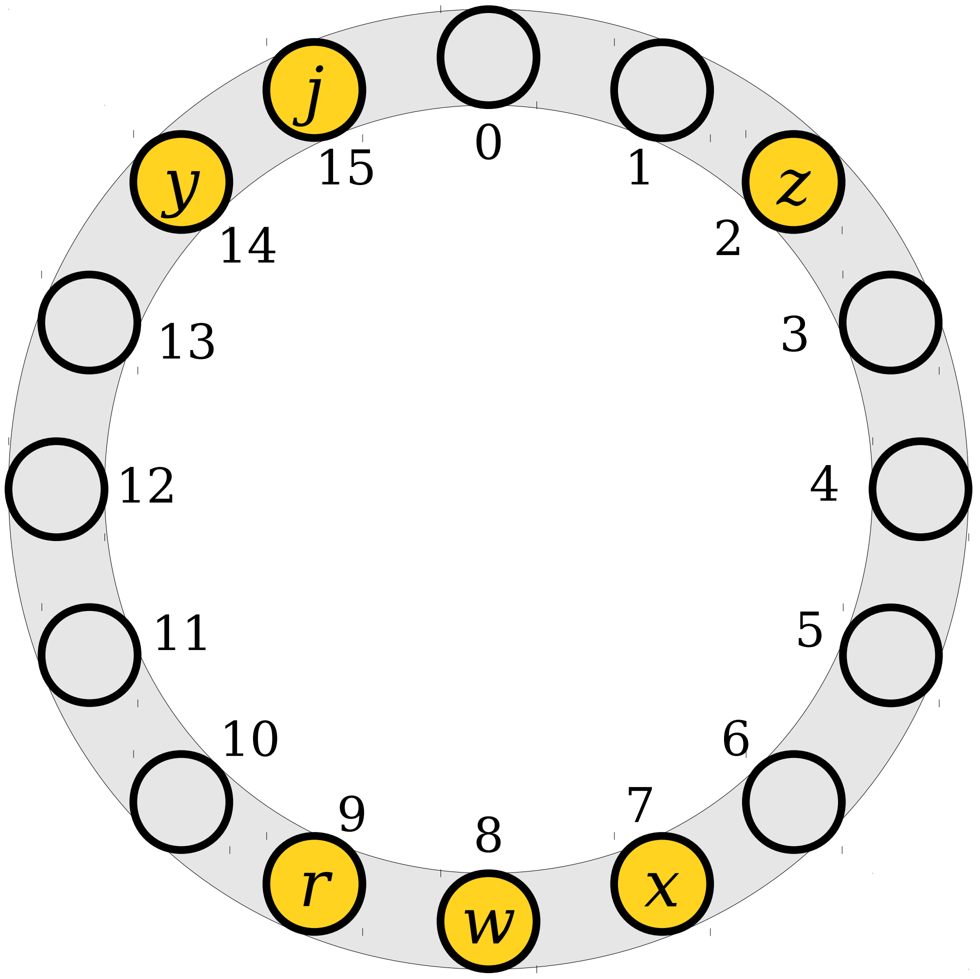

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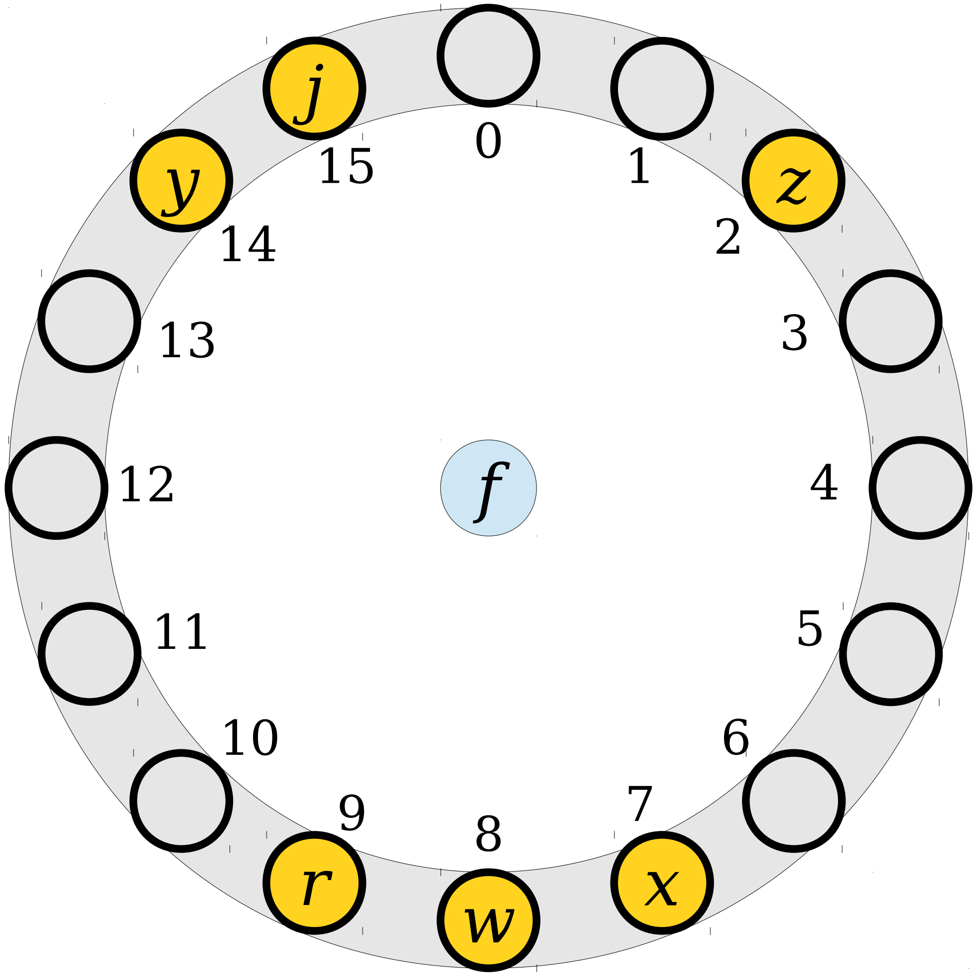

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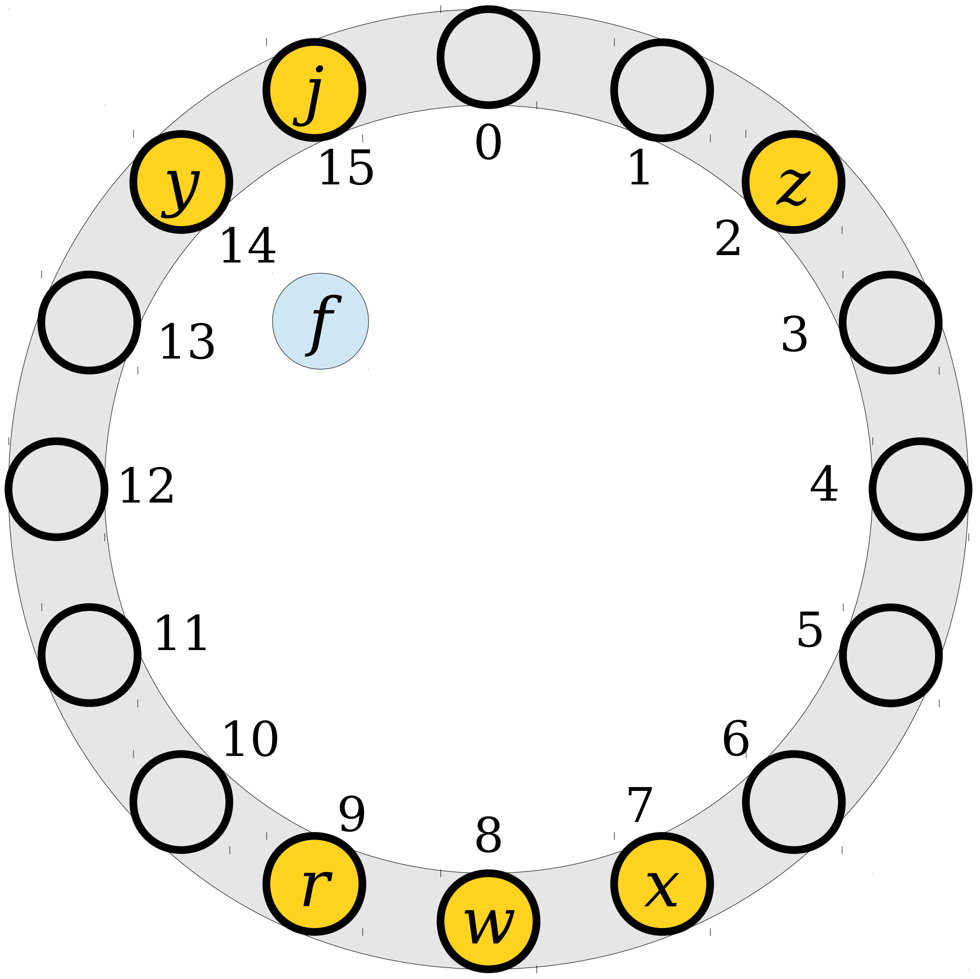

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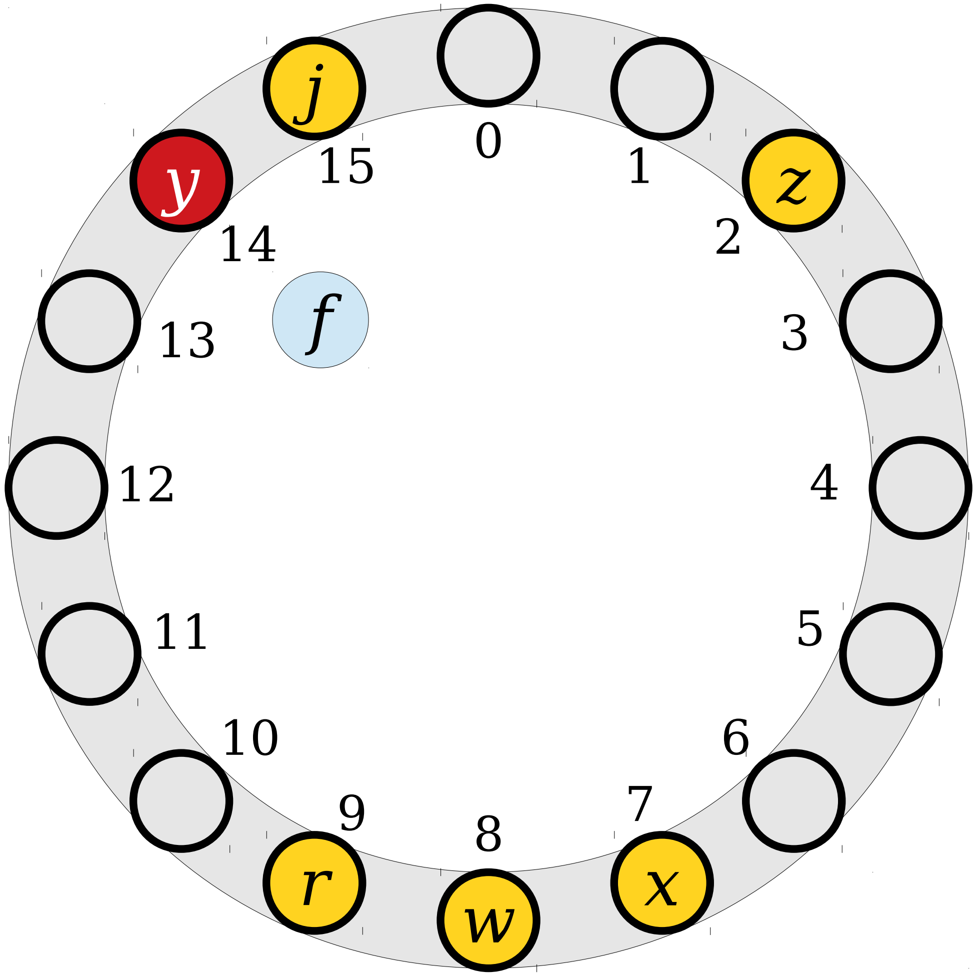

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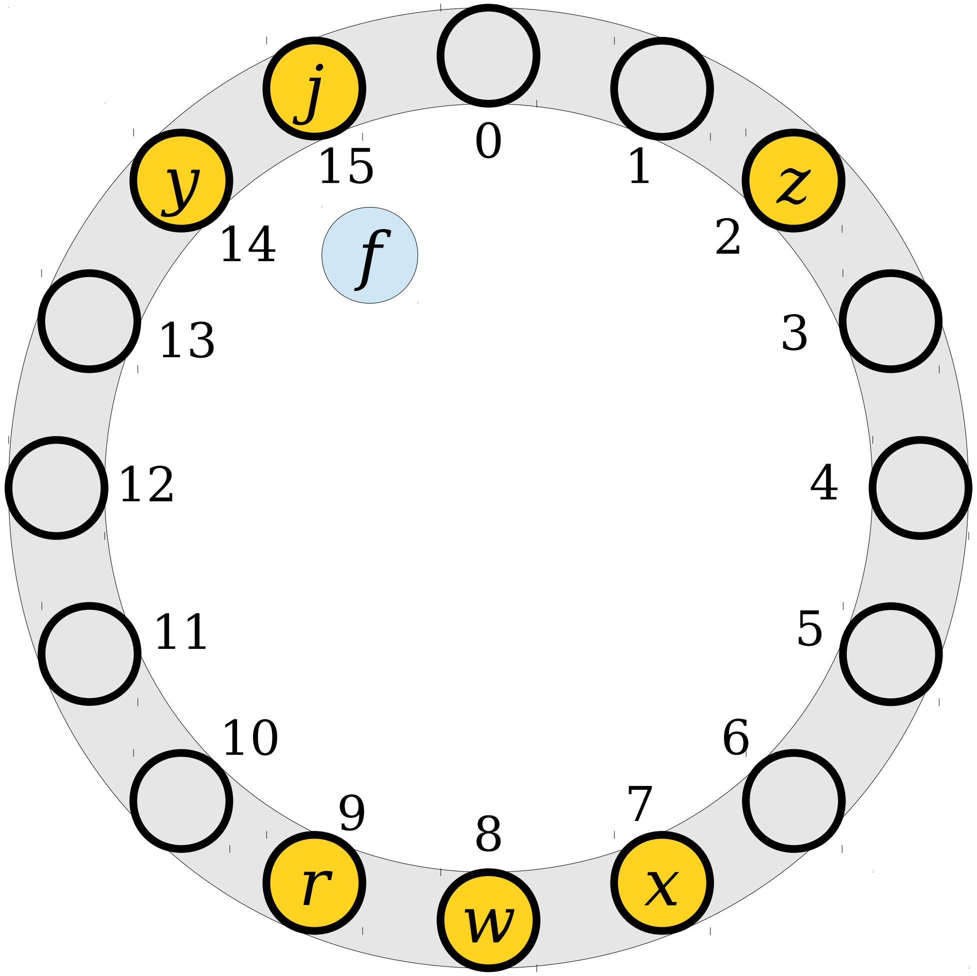

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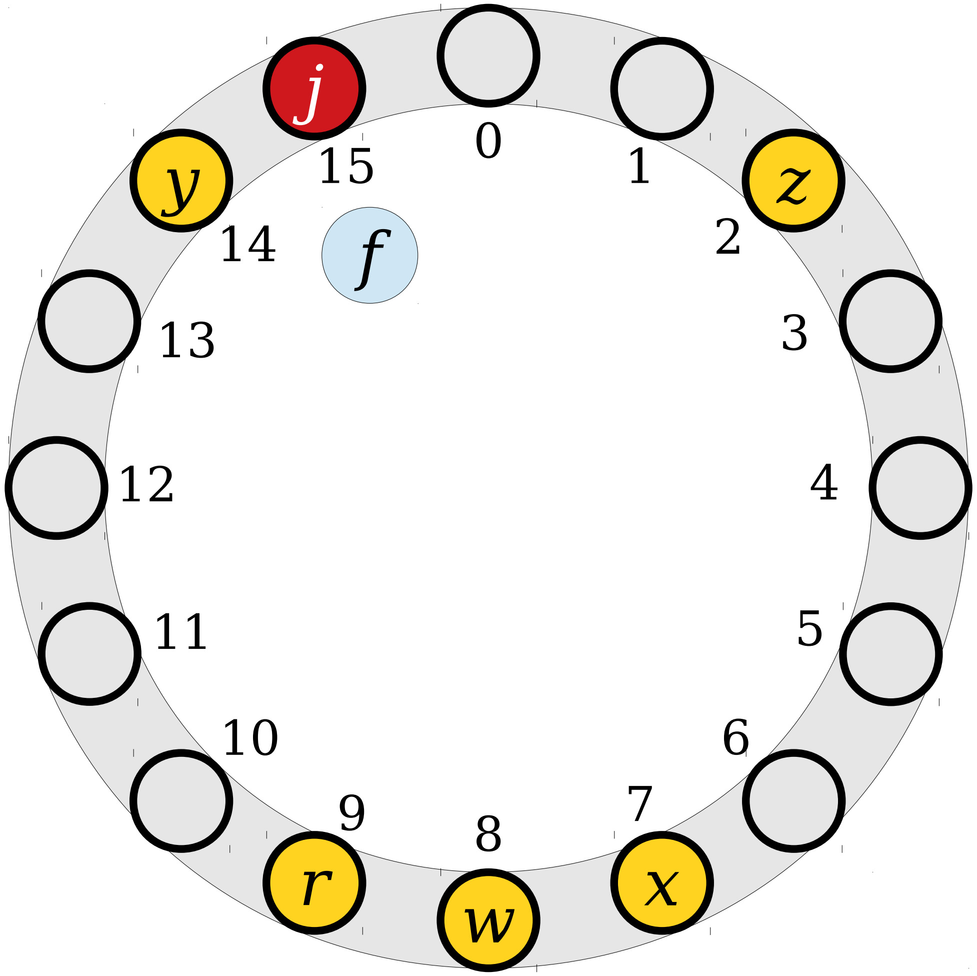

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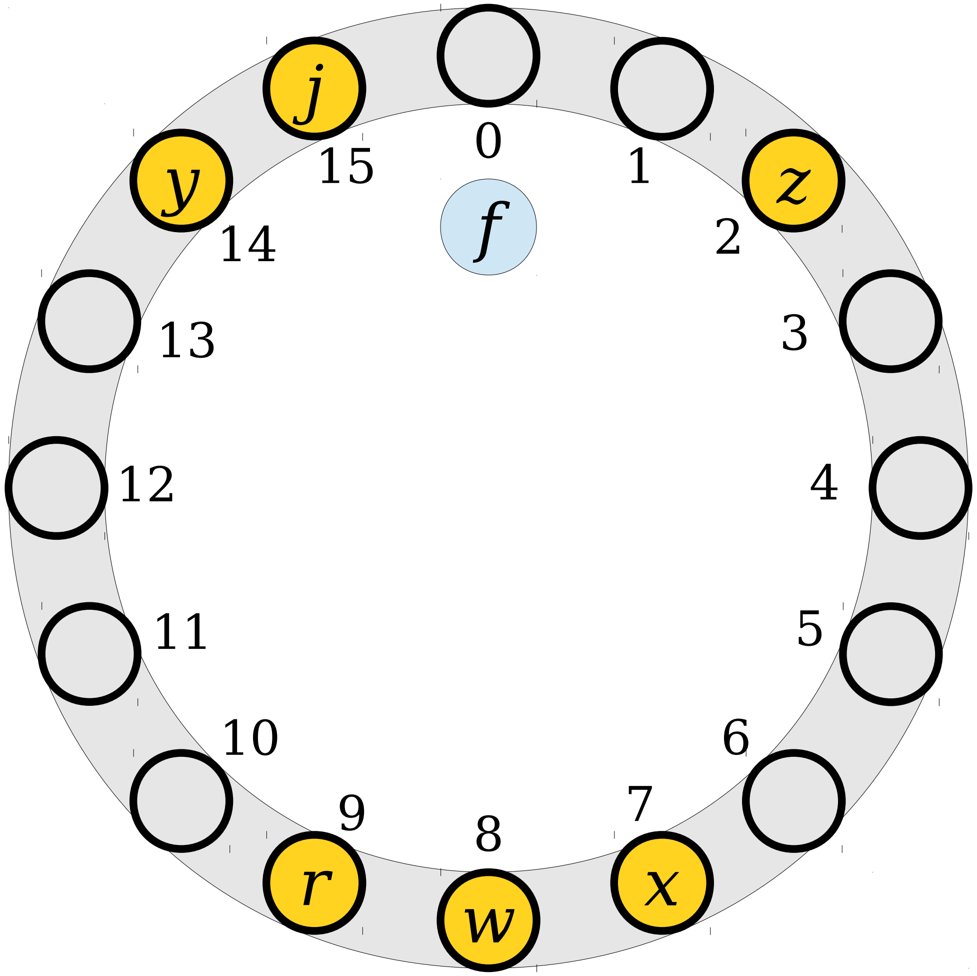

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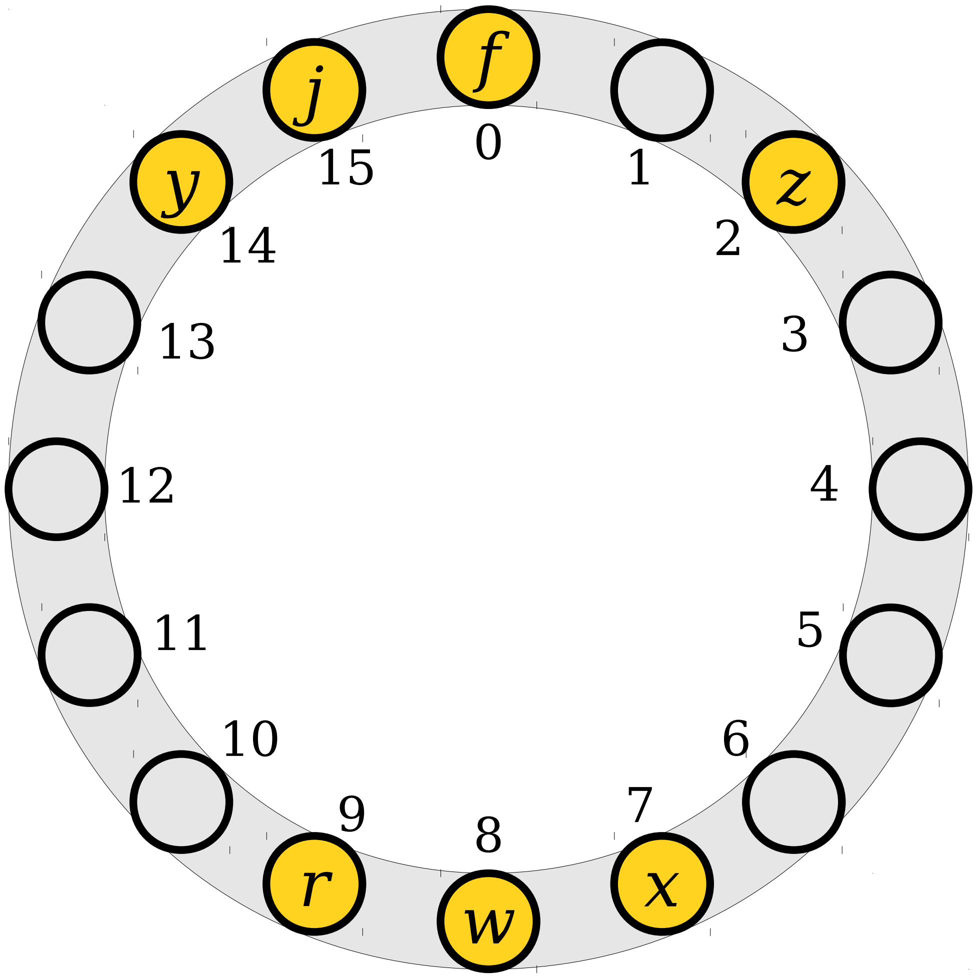

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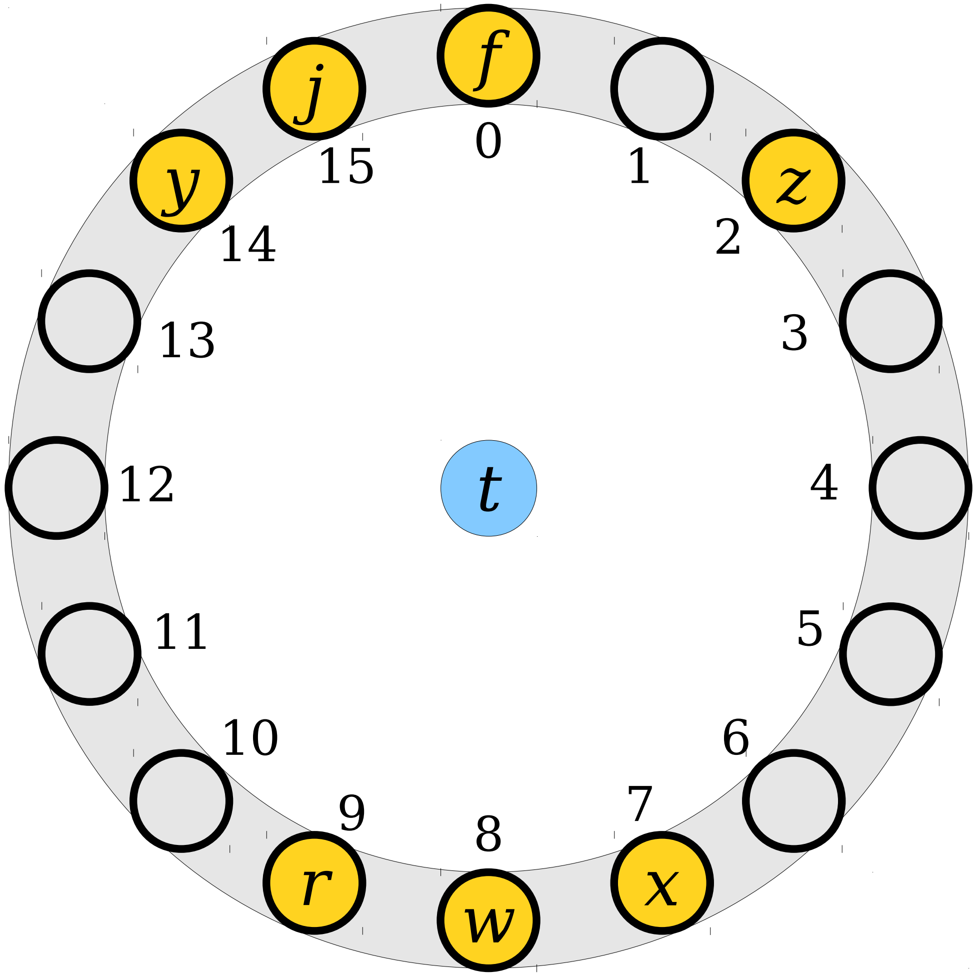

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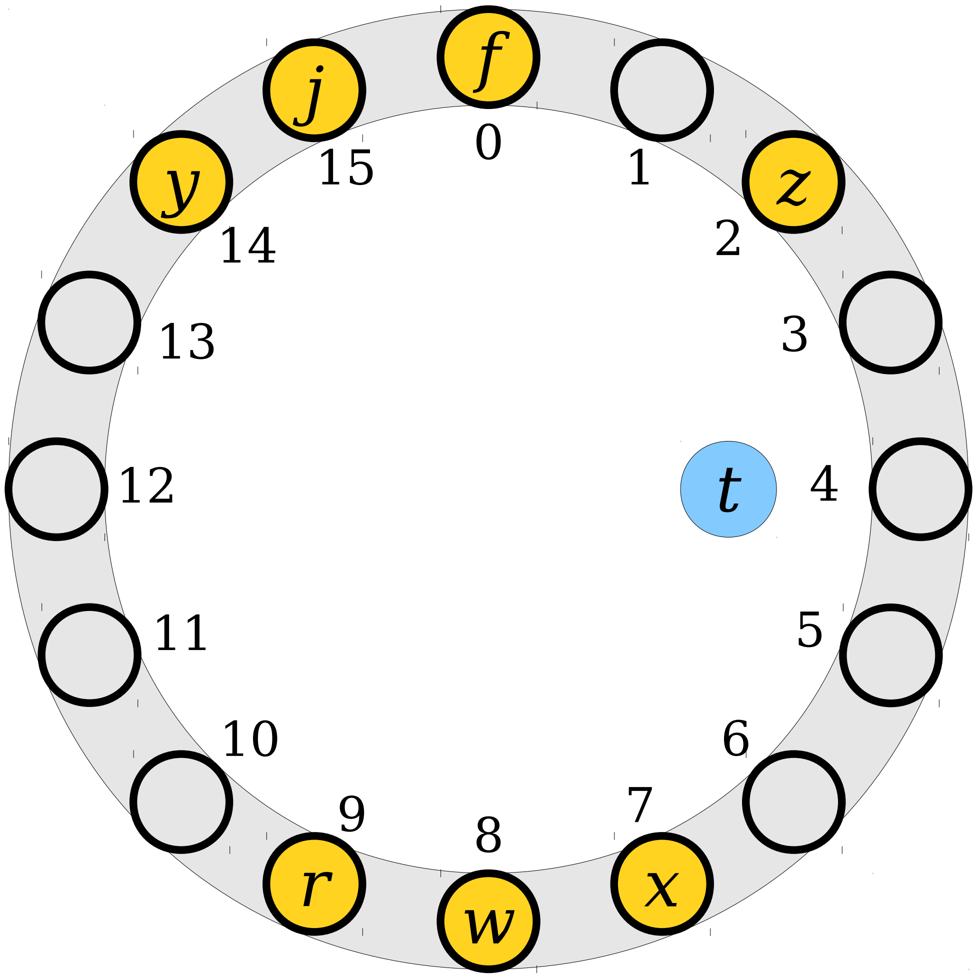

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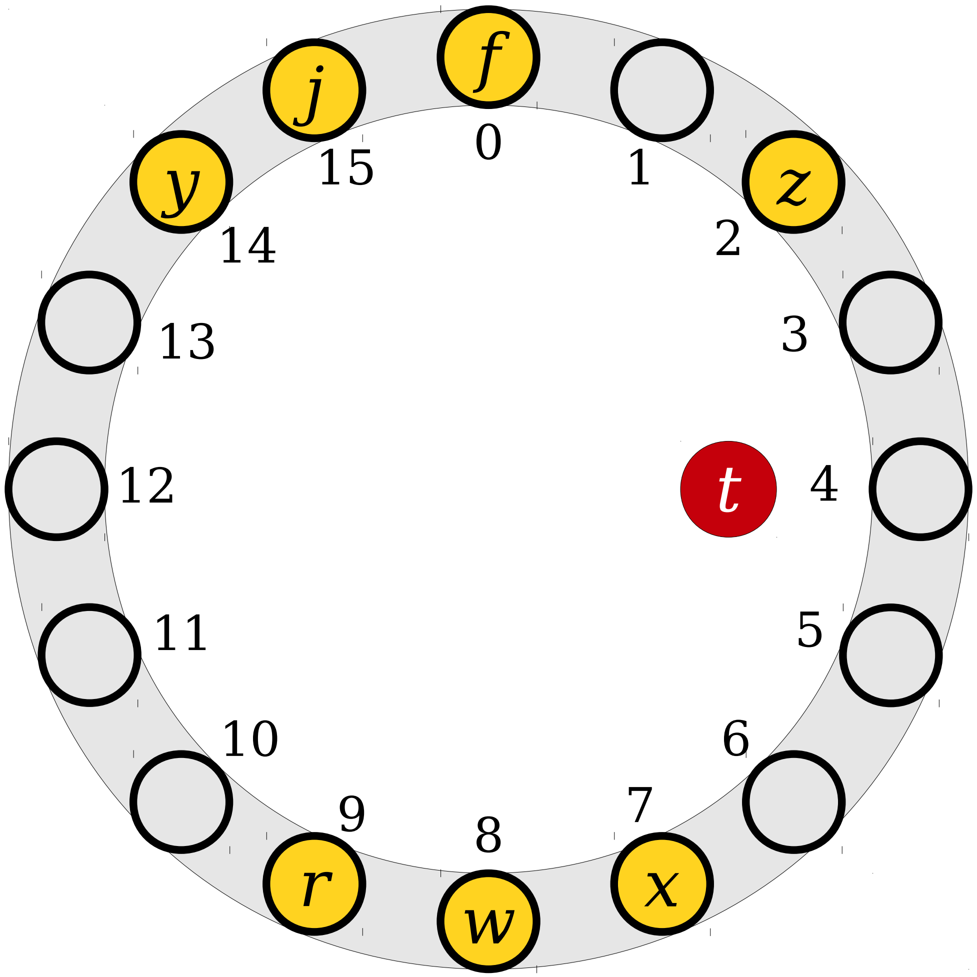

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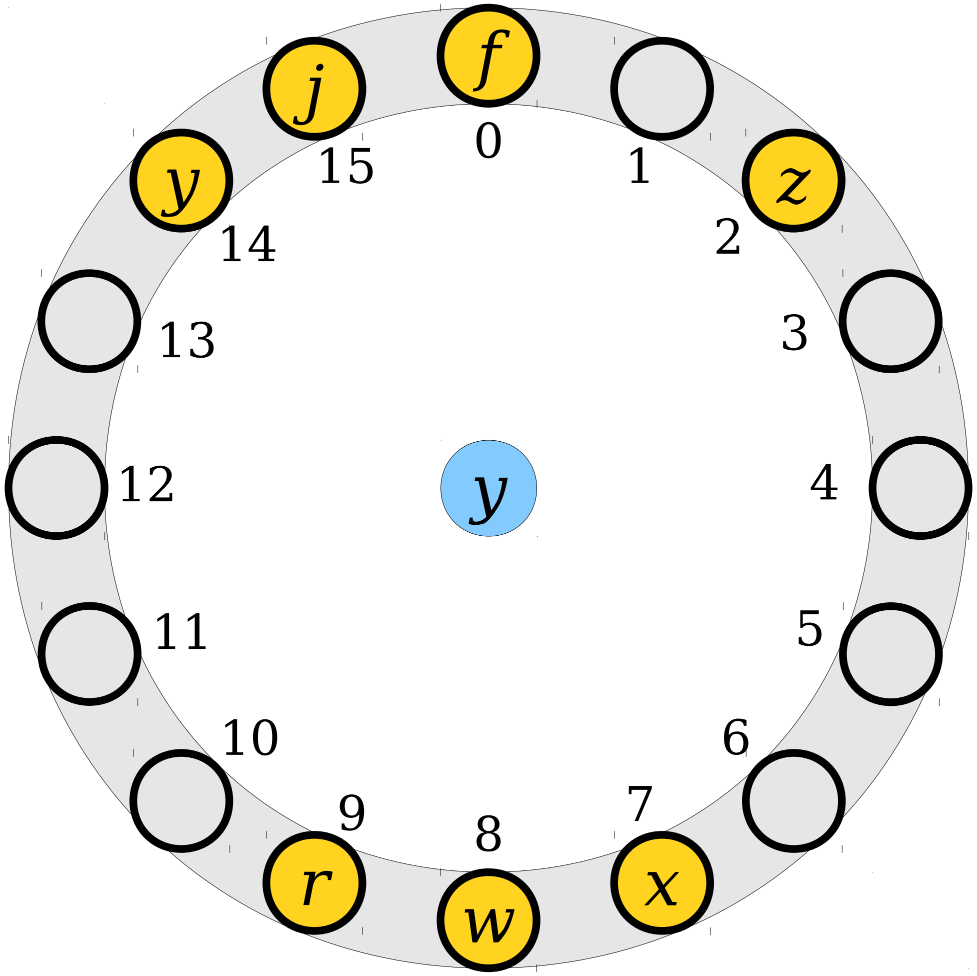

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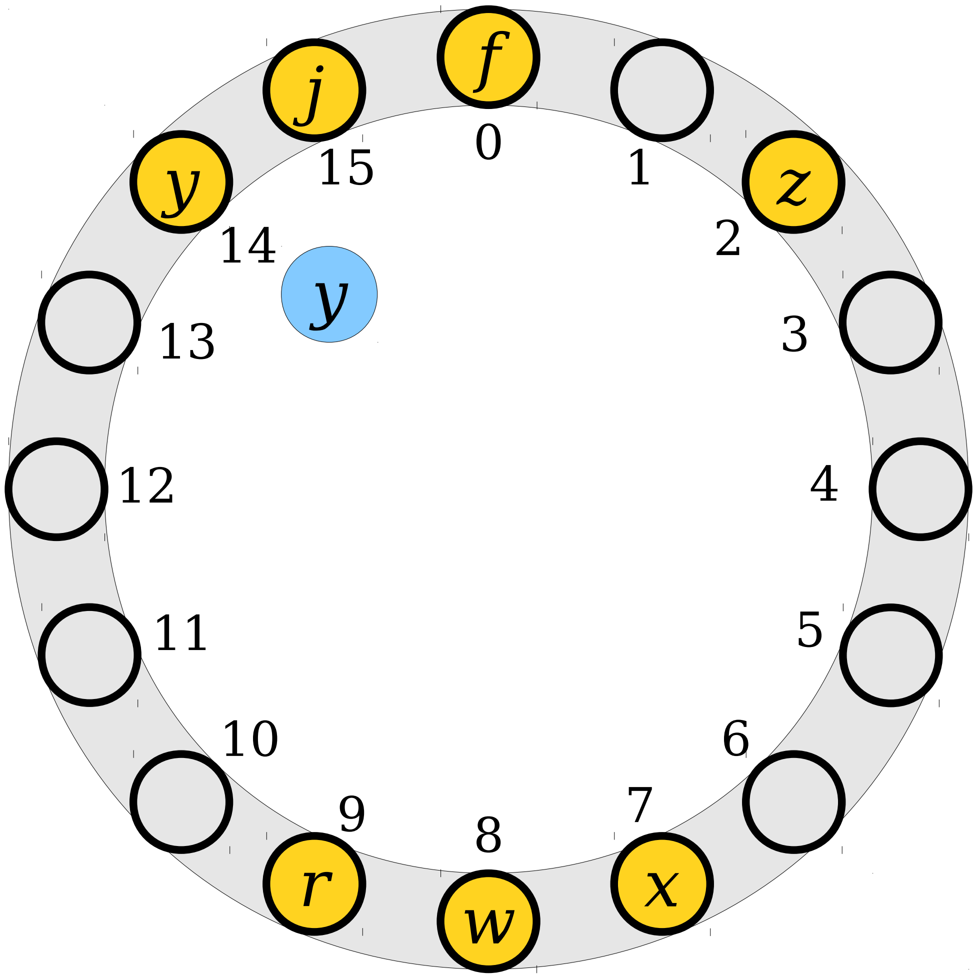

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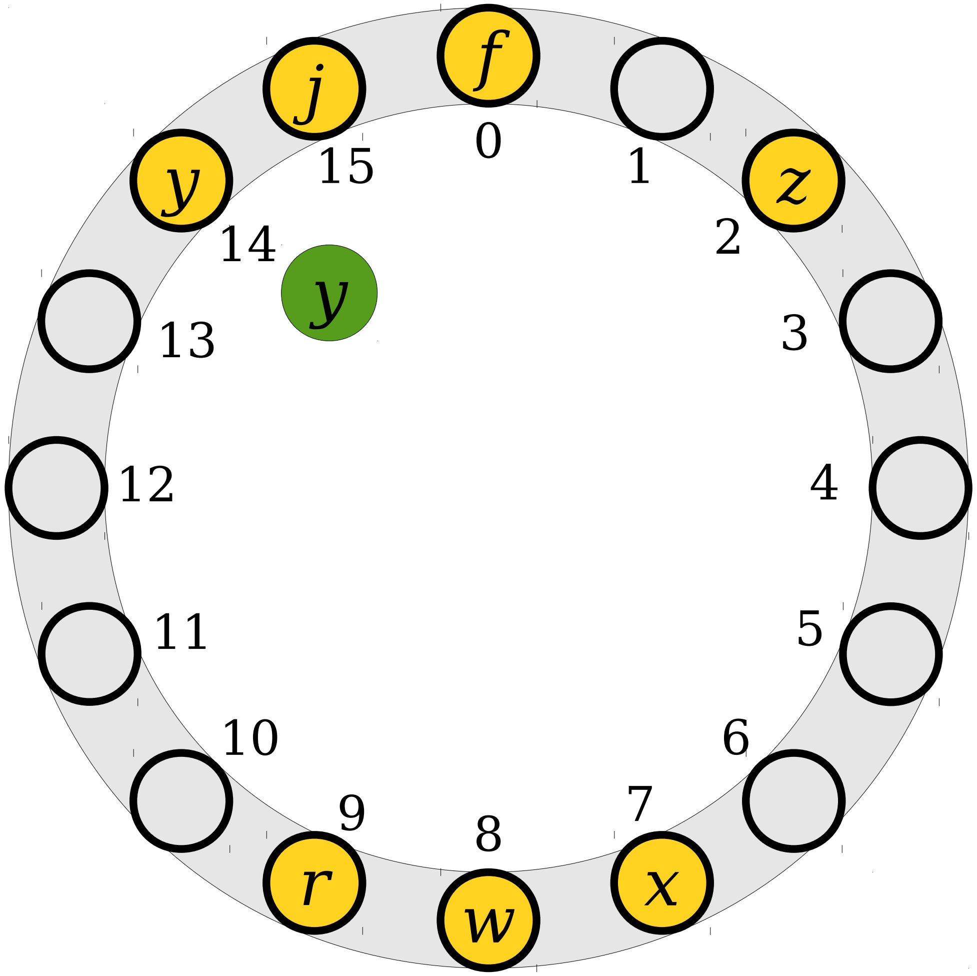

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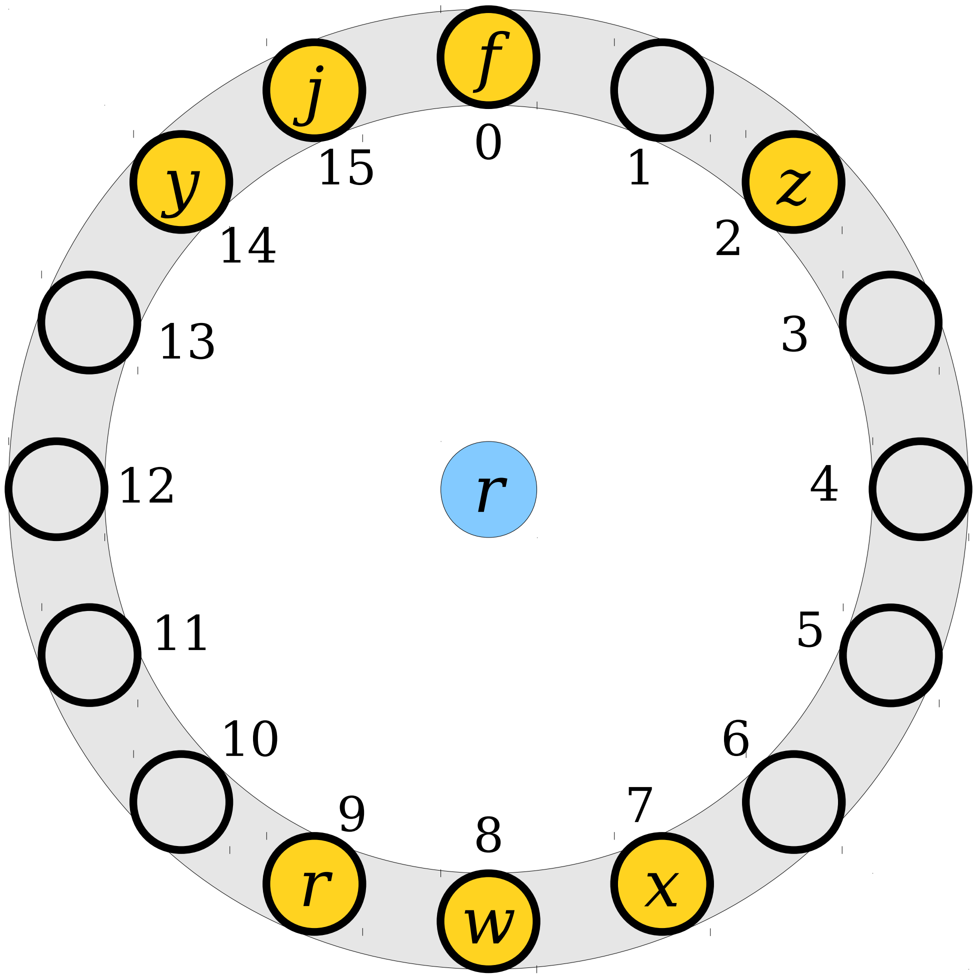

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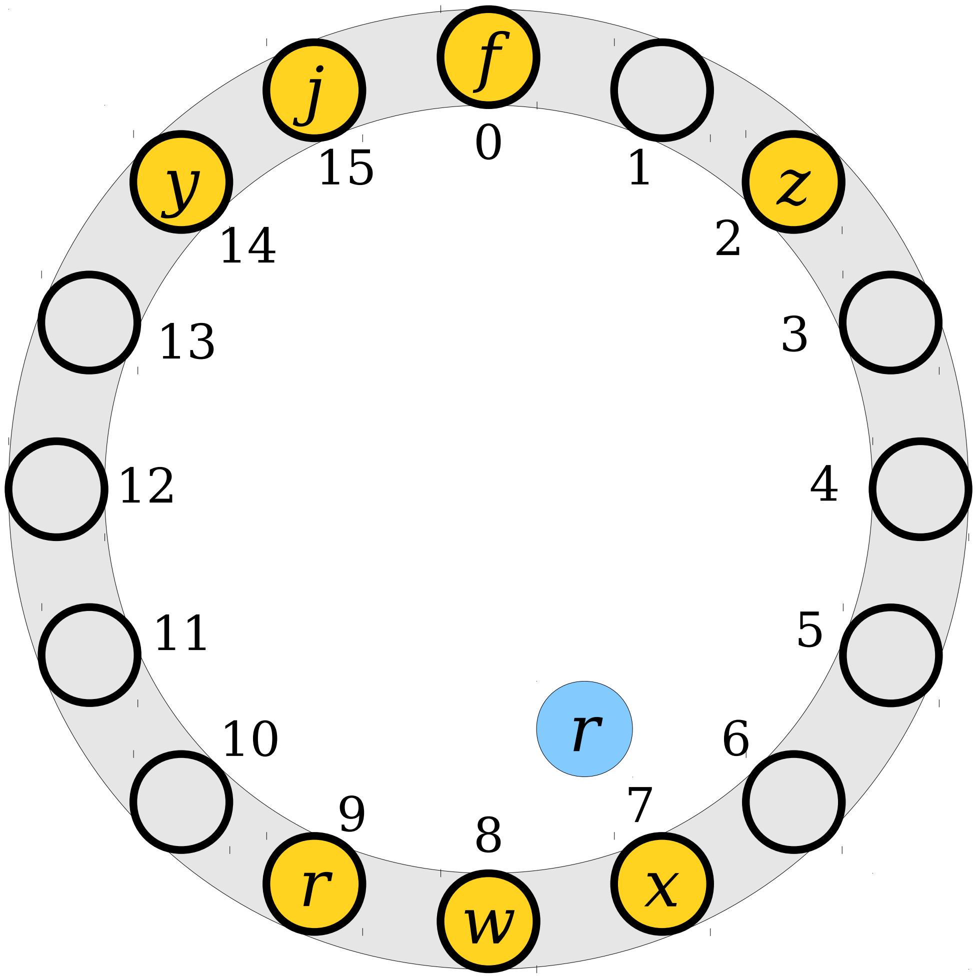

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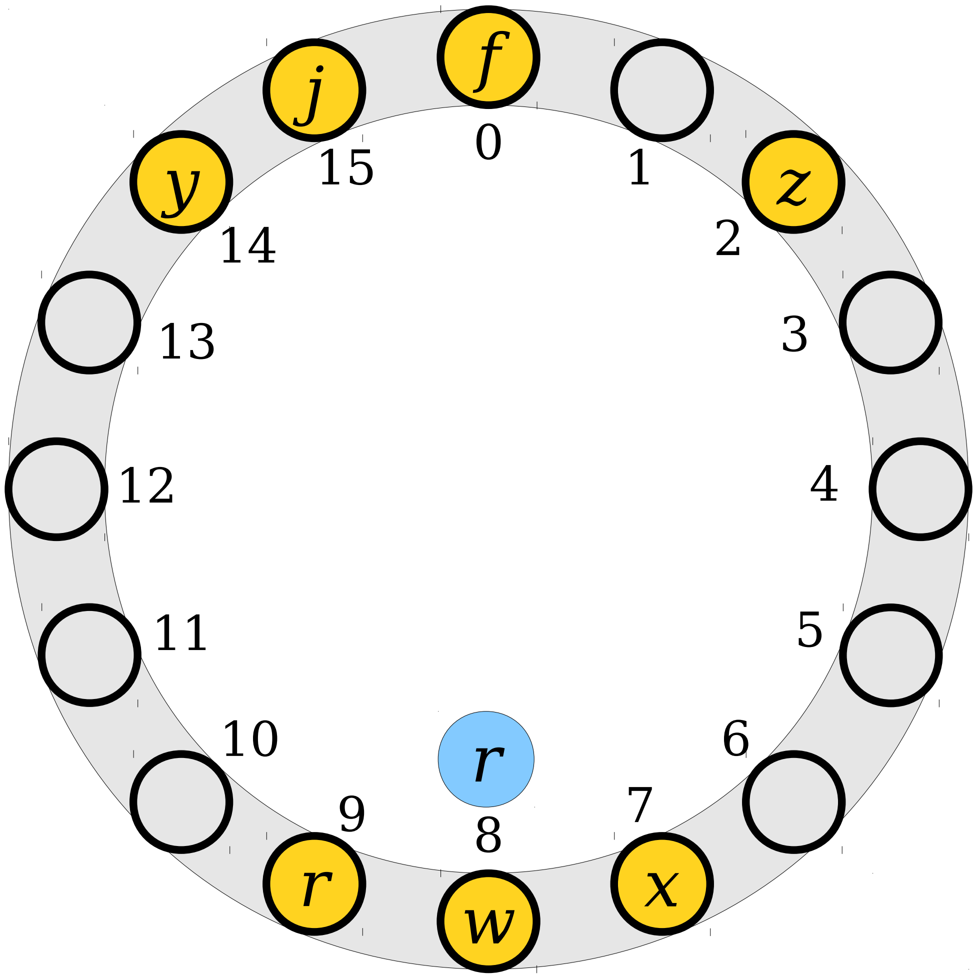

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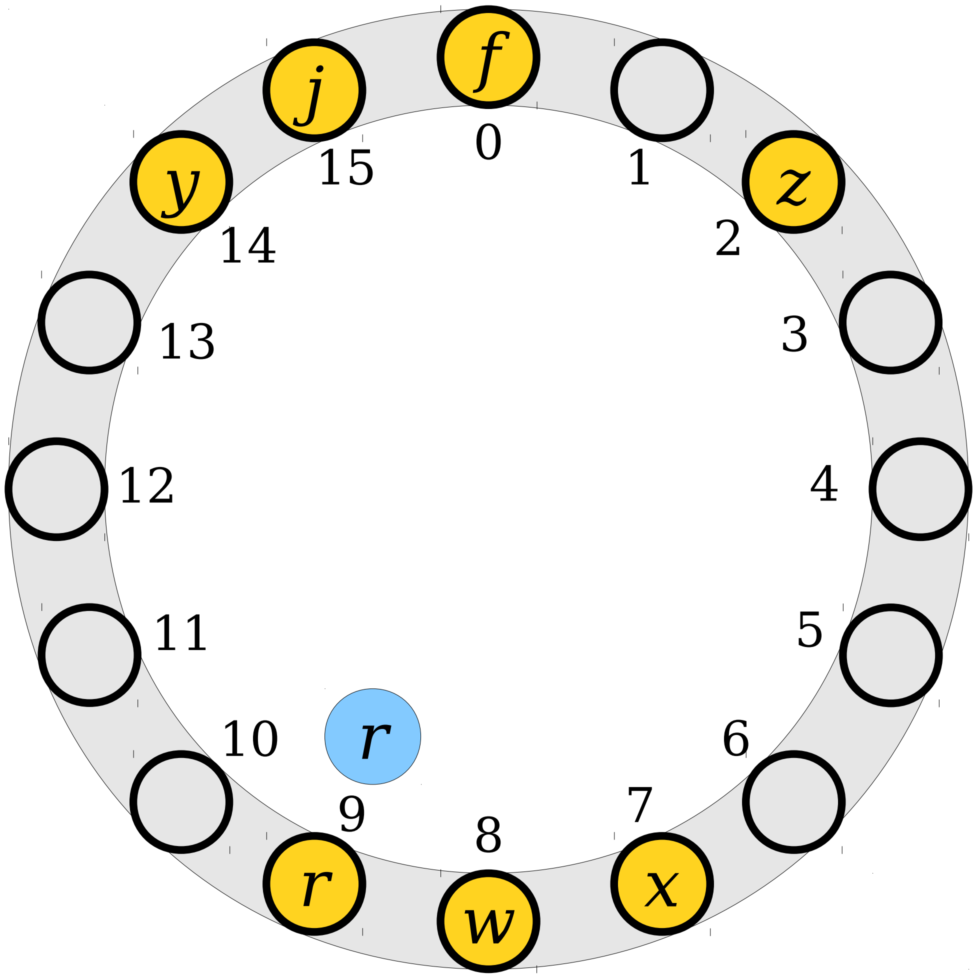

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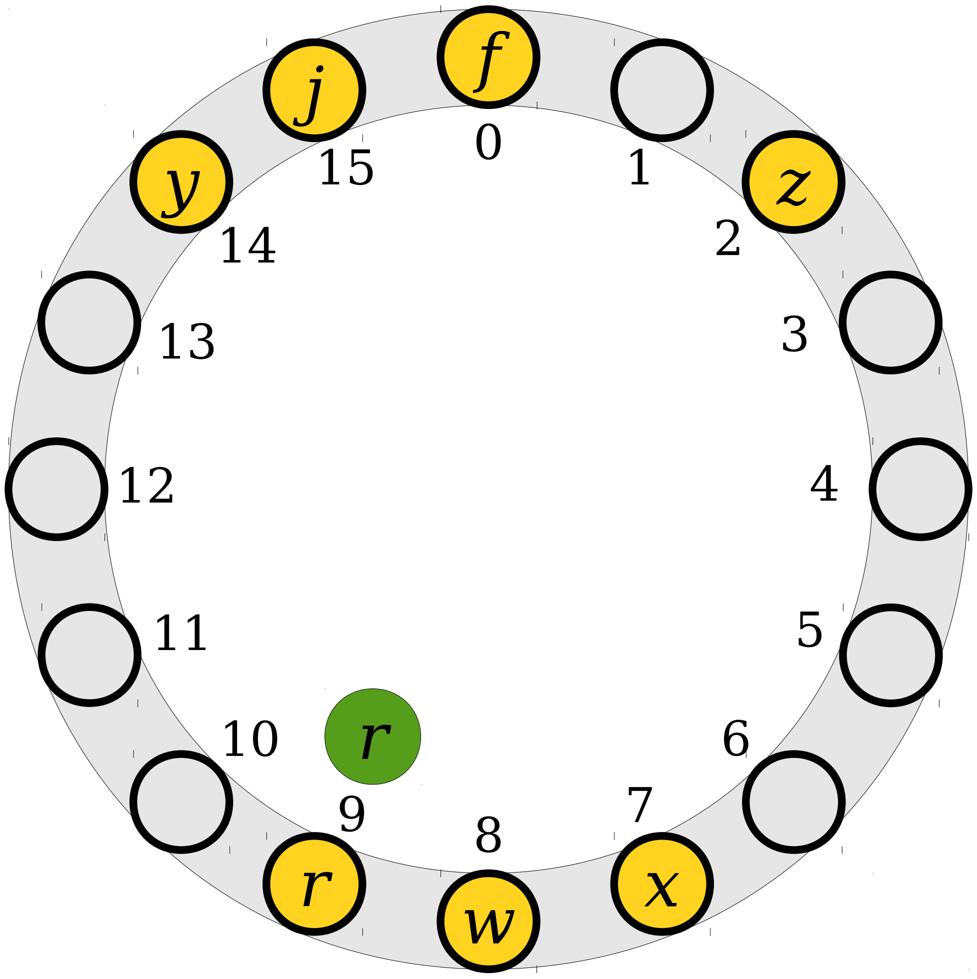

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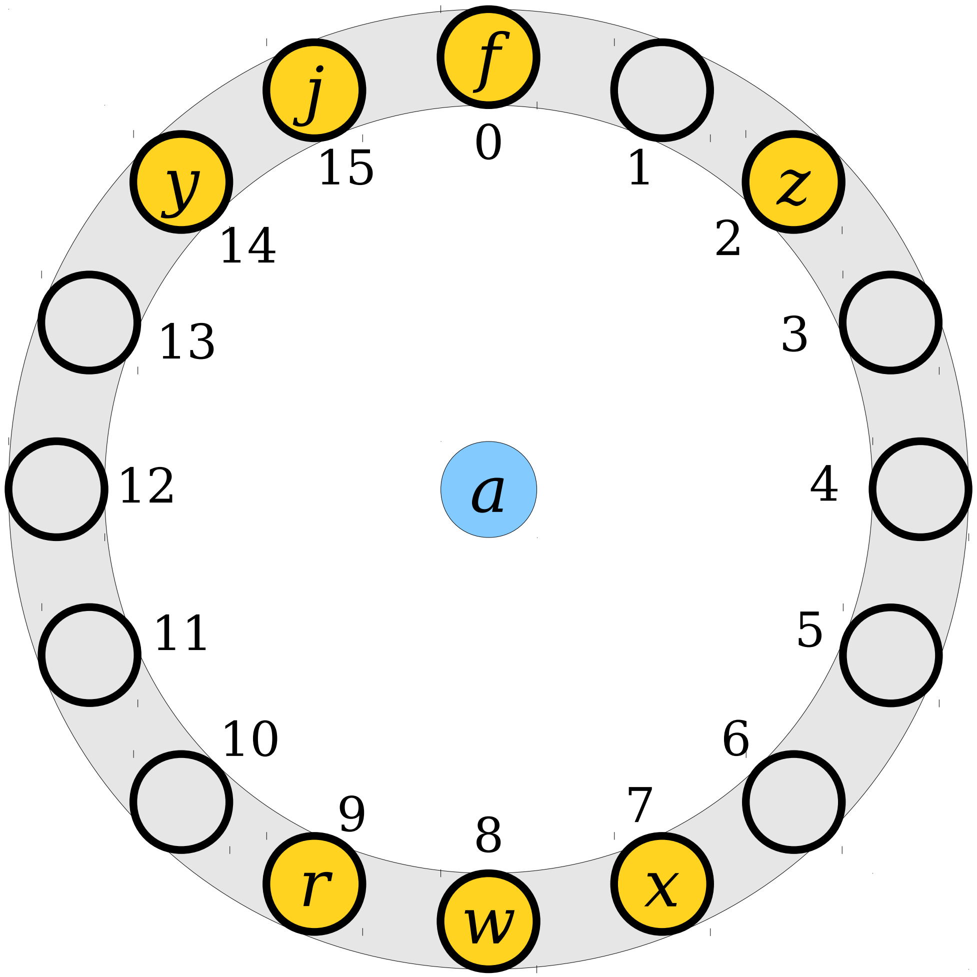

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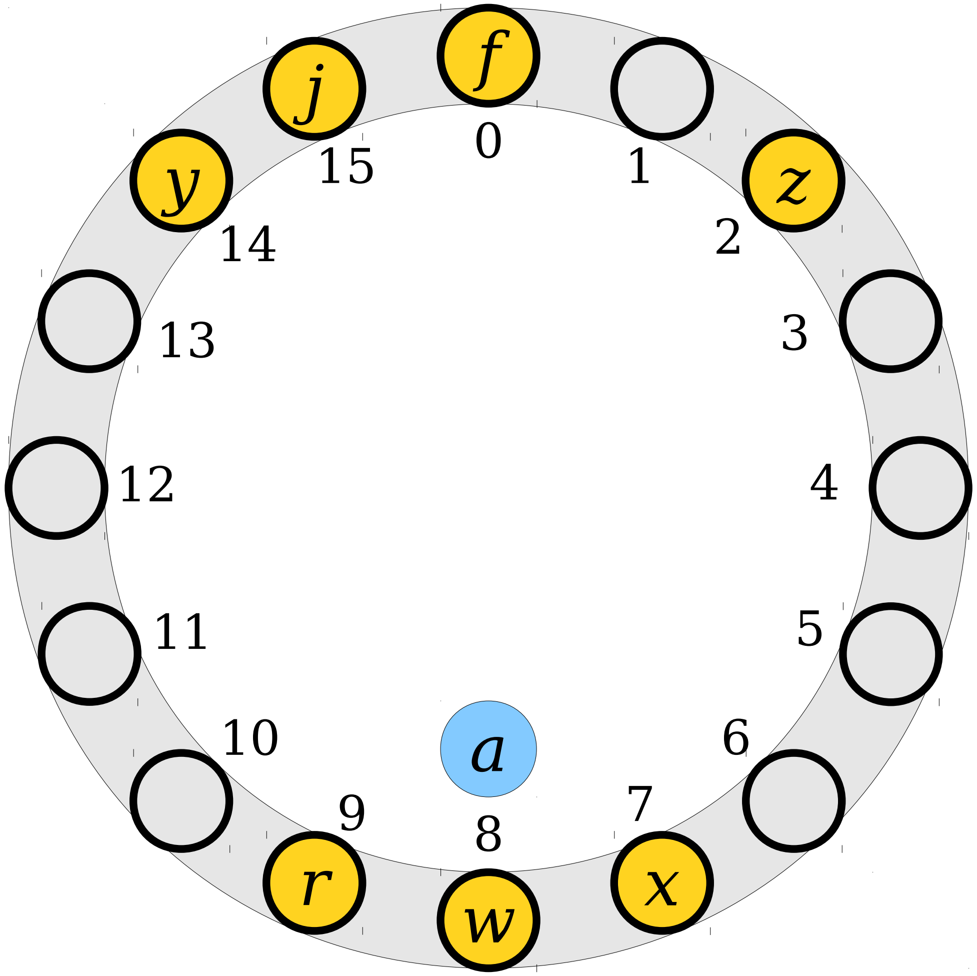

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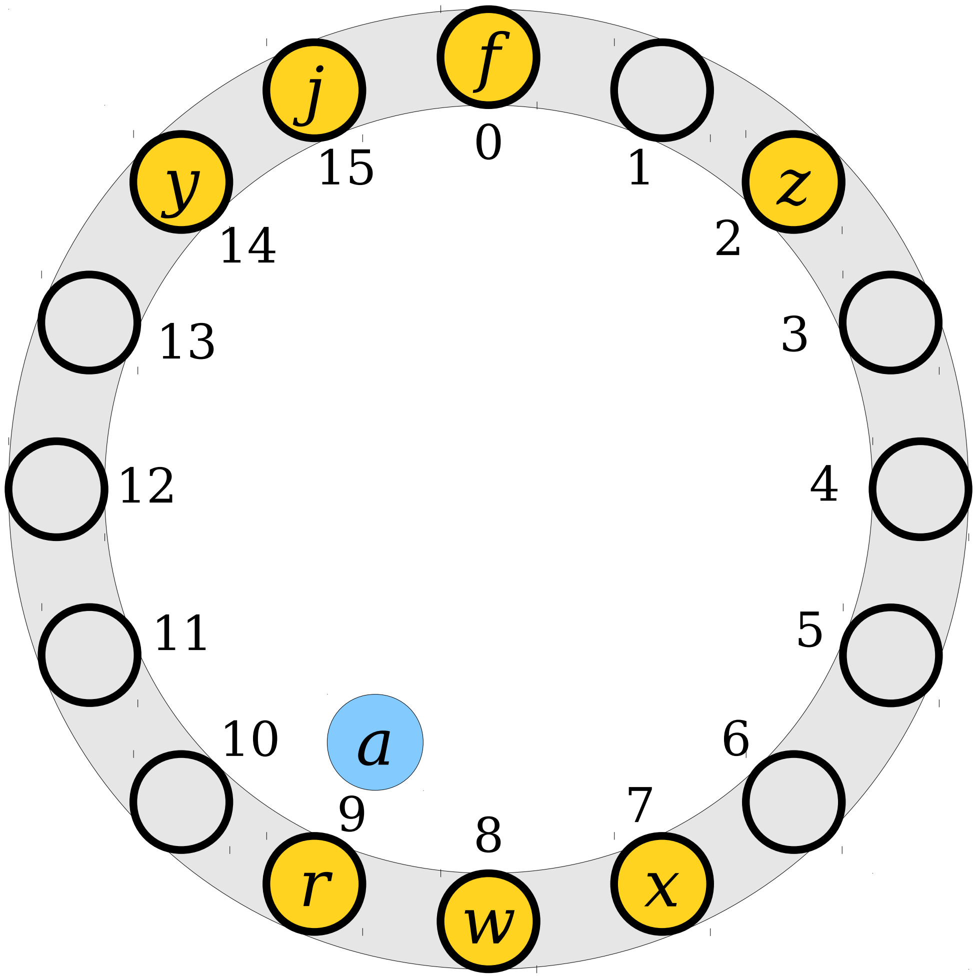

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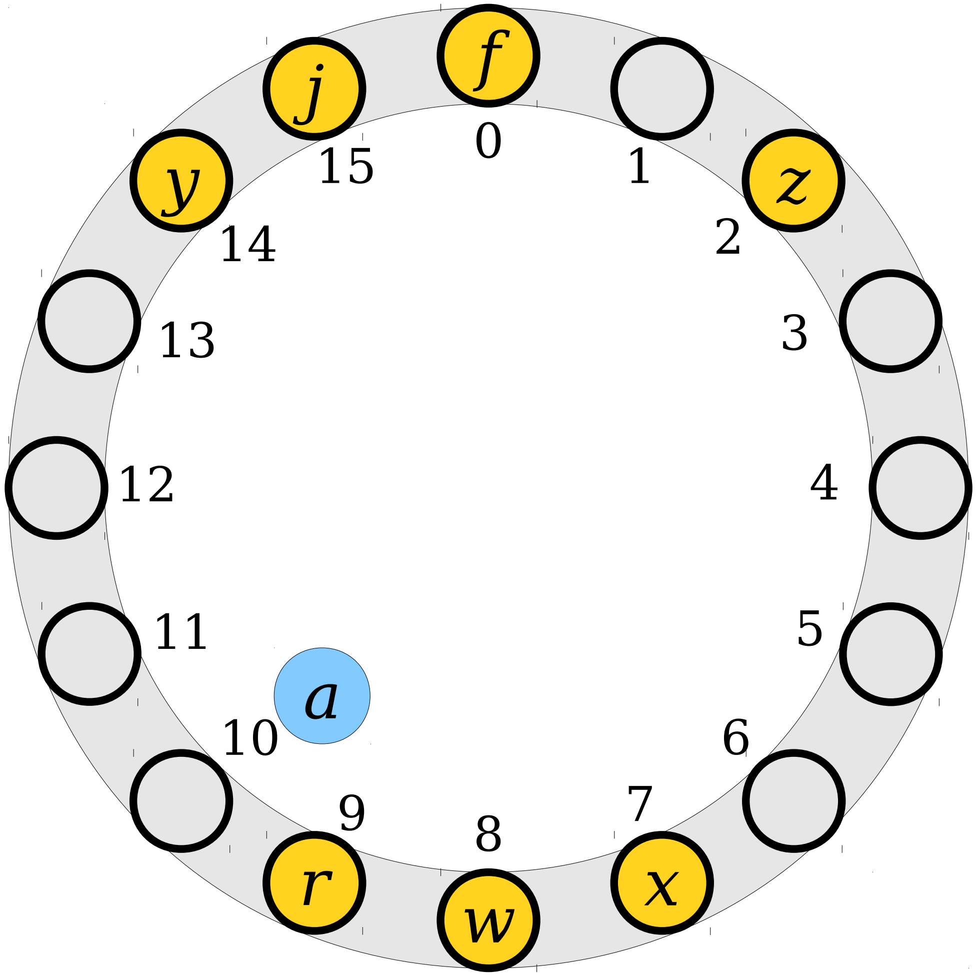

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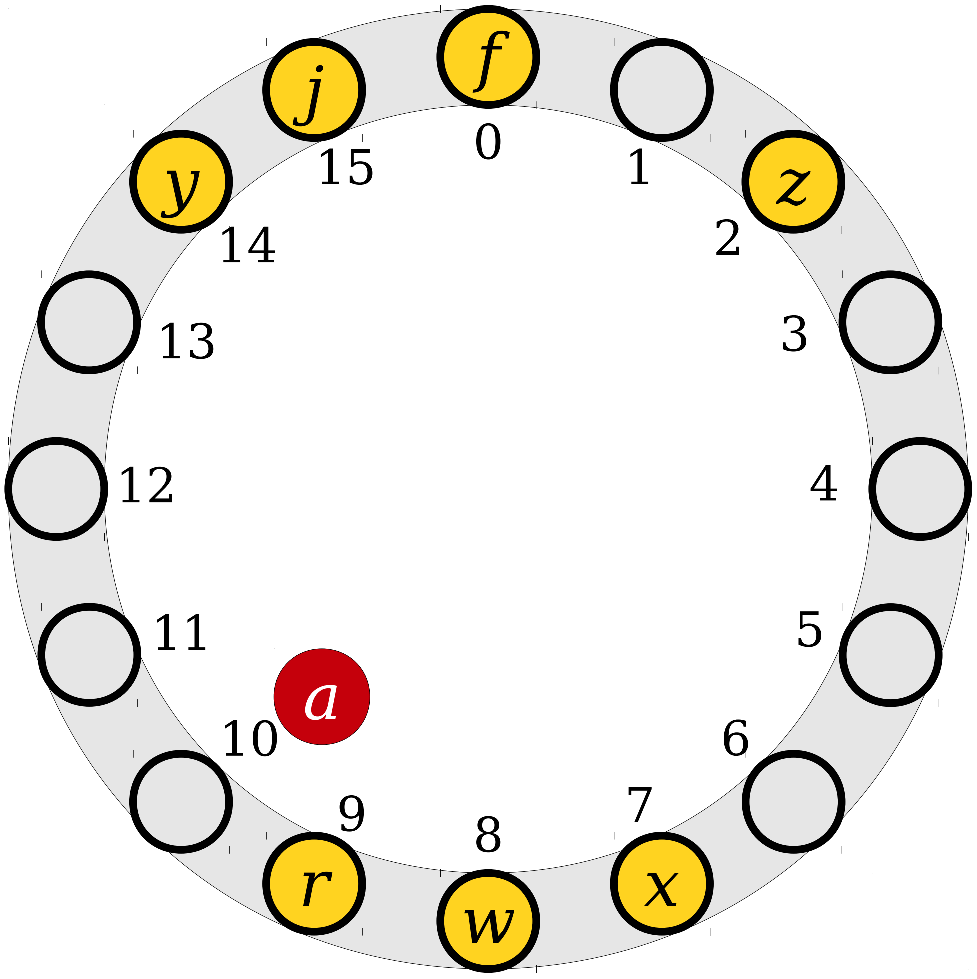
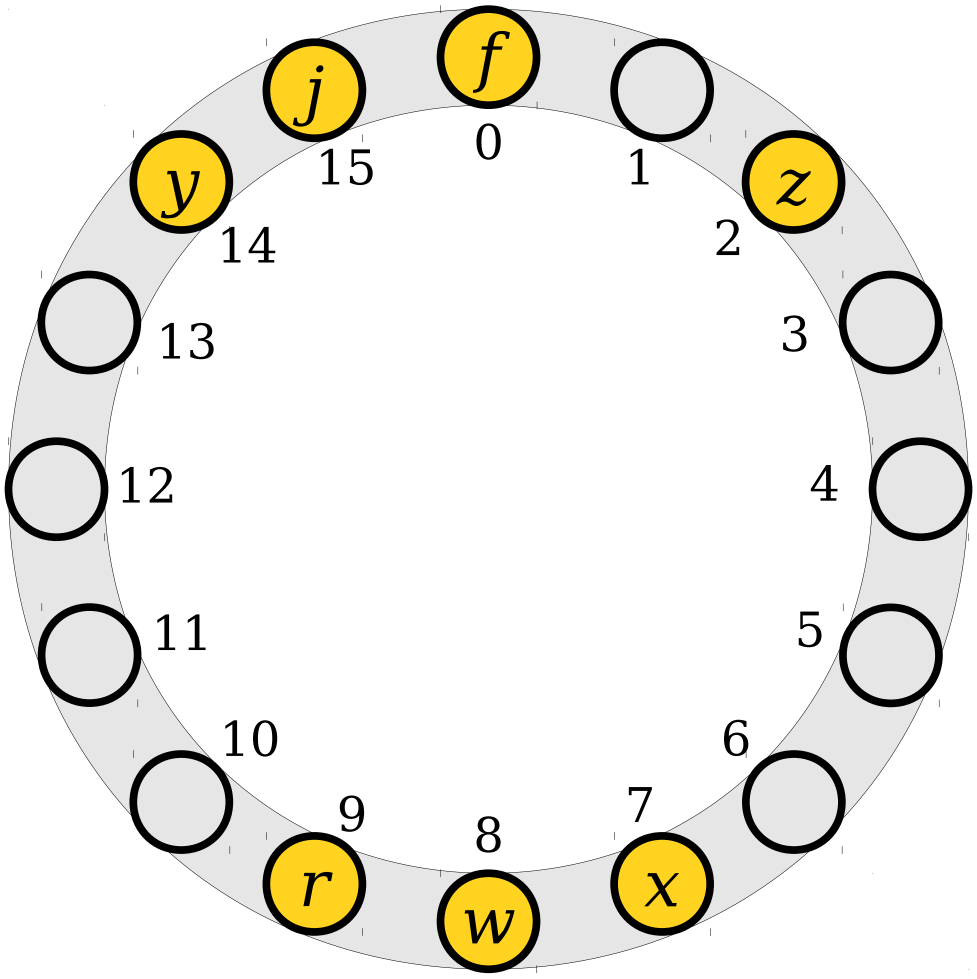


Linear Probing

- Deletions are a bit trickier than in chained hashing.
- We cannot just do a search and remove the element where we find it.
- Why?



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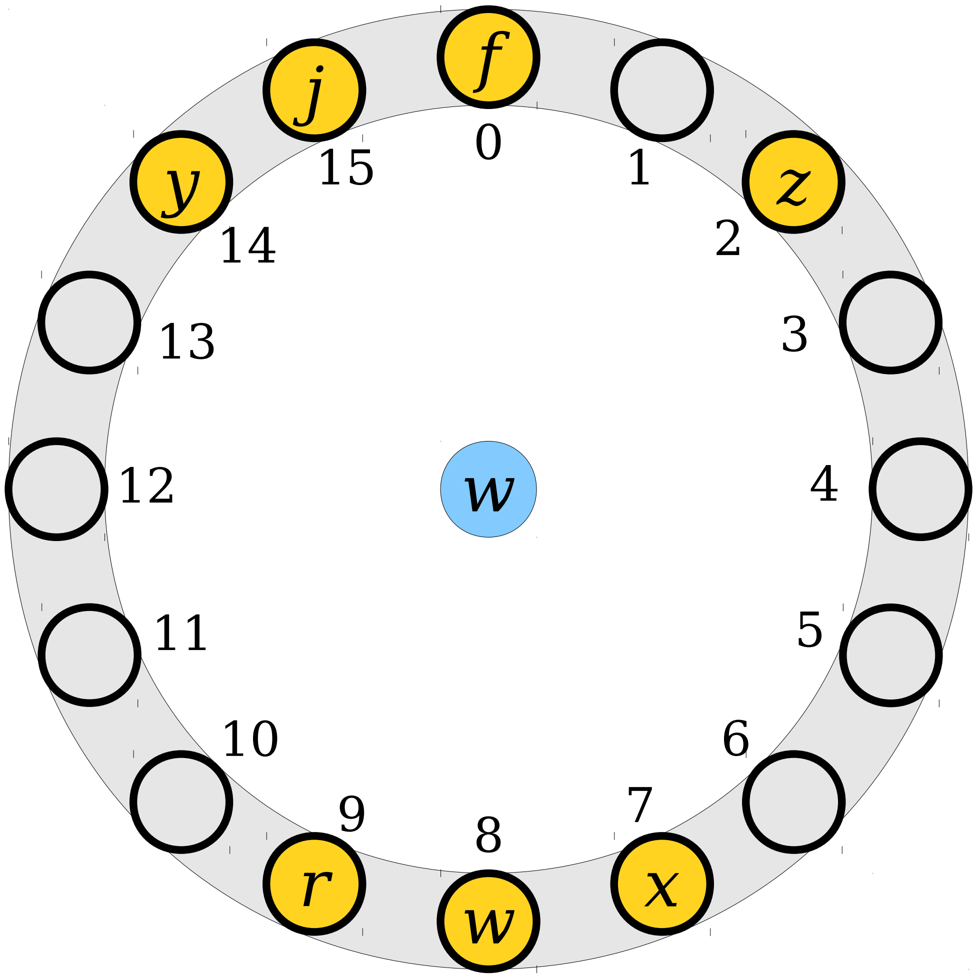

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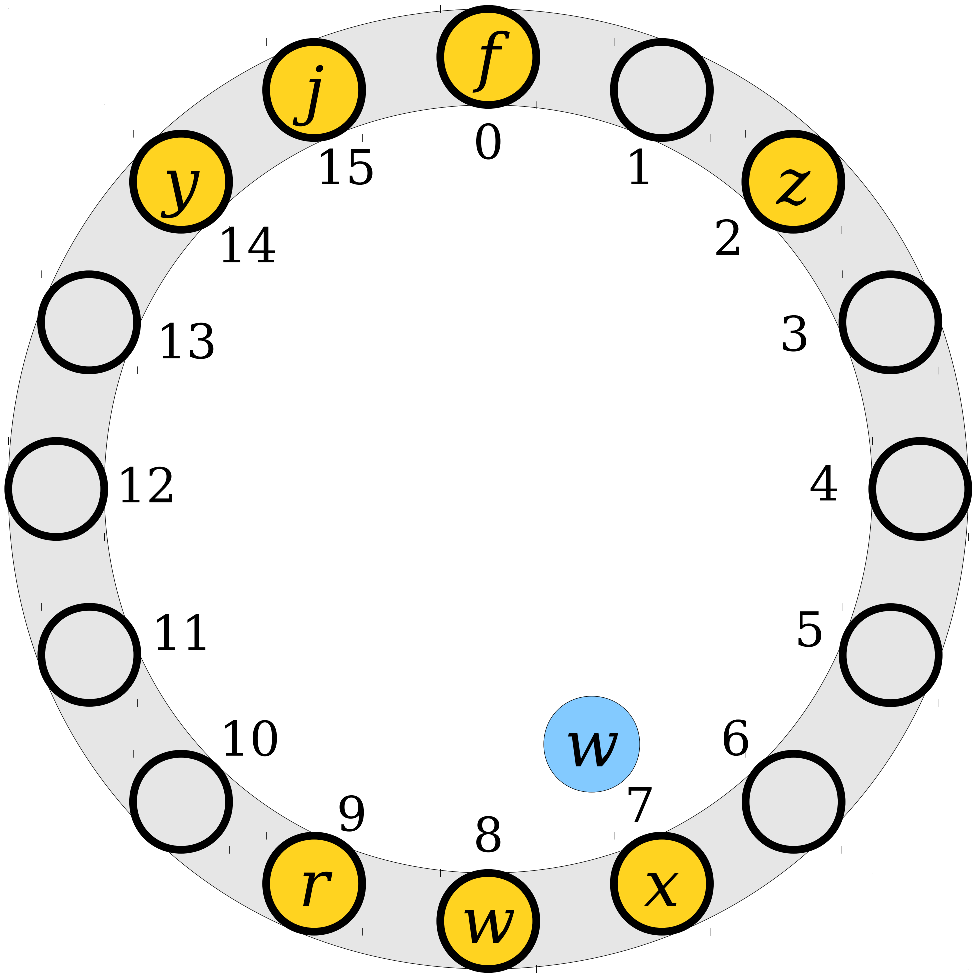

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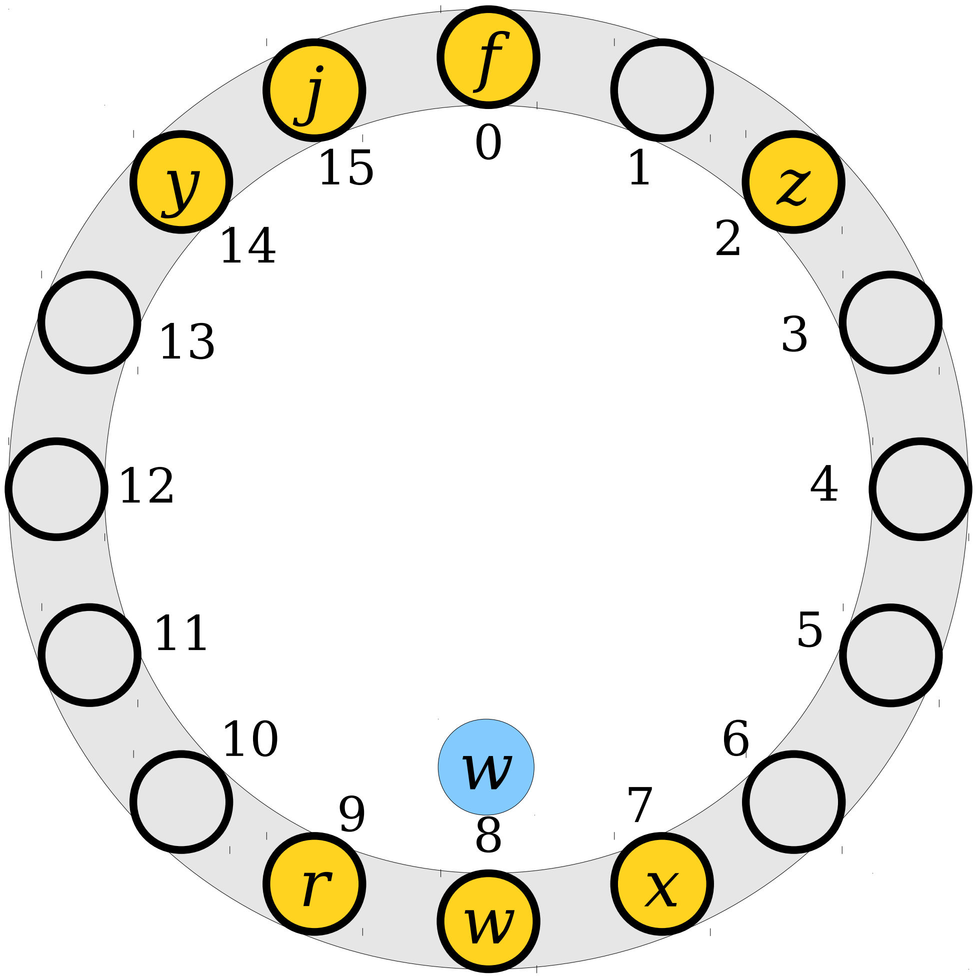

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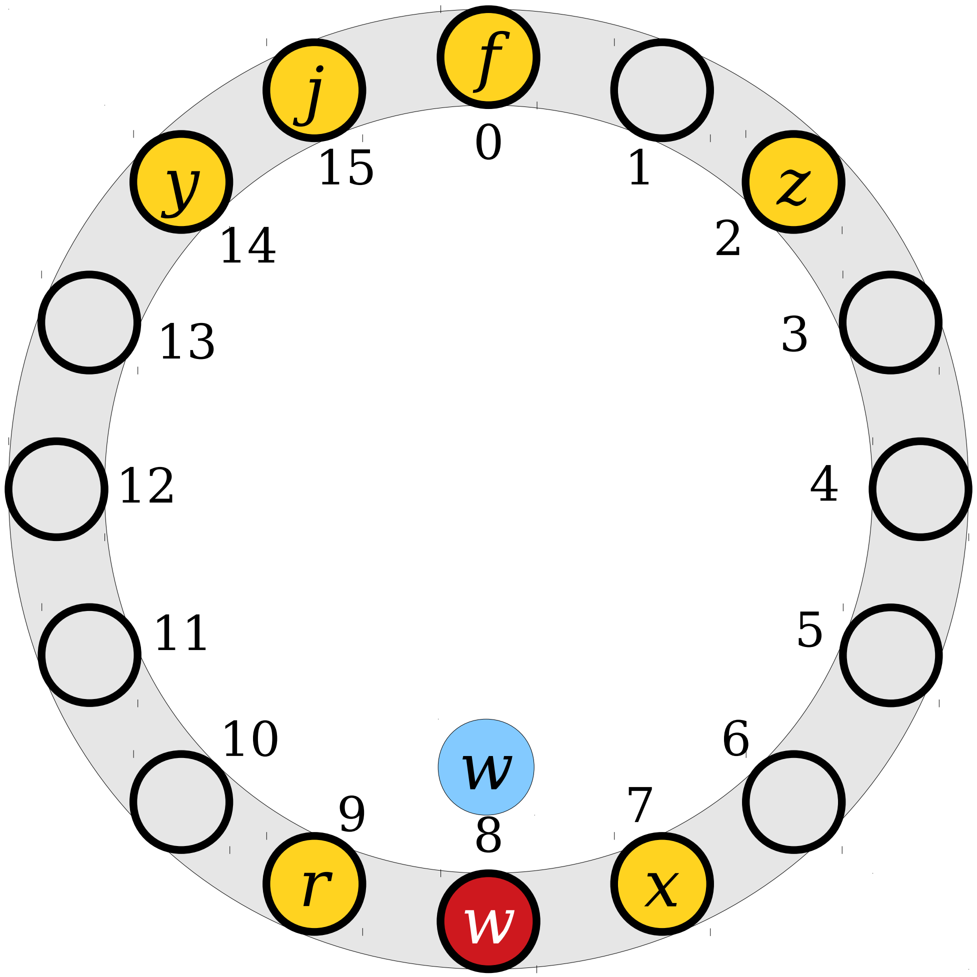

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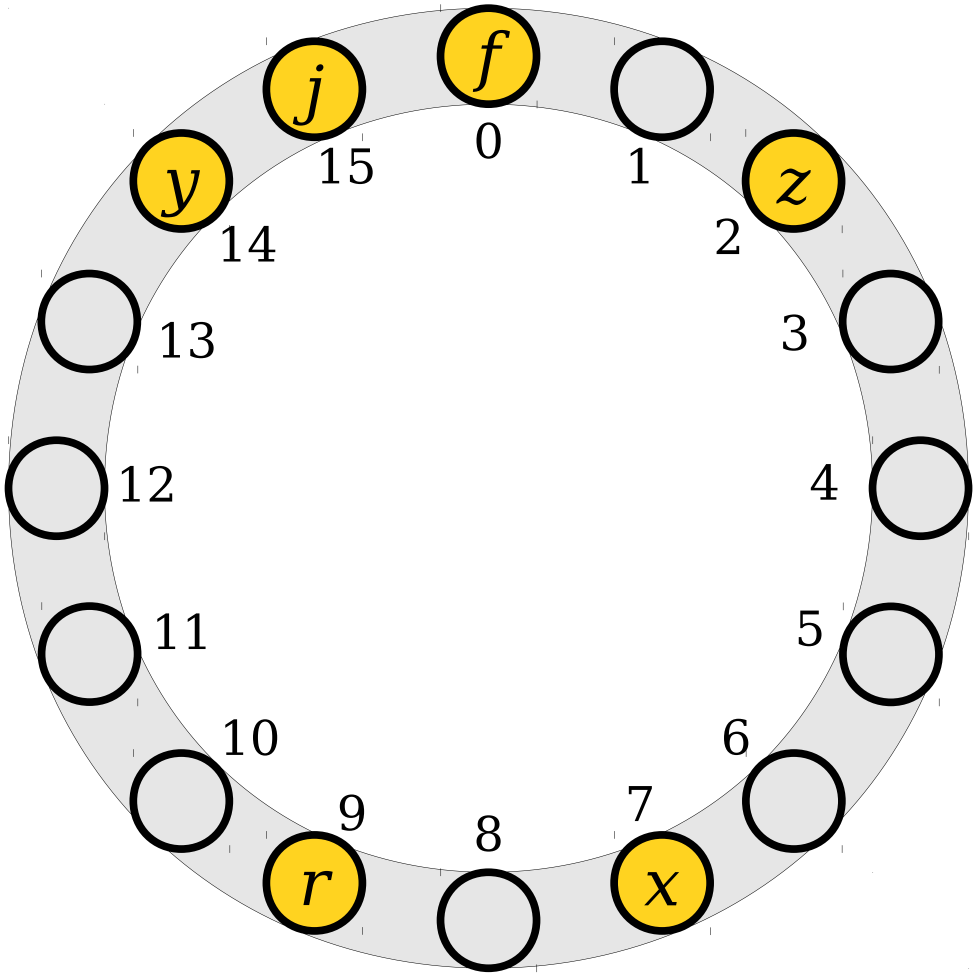

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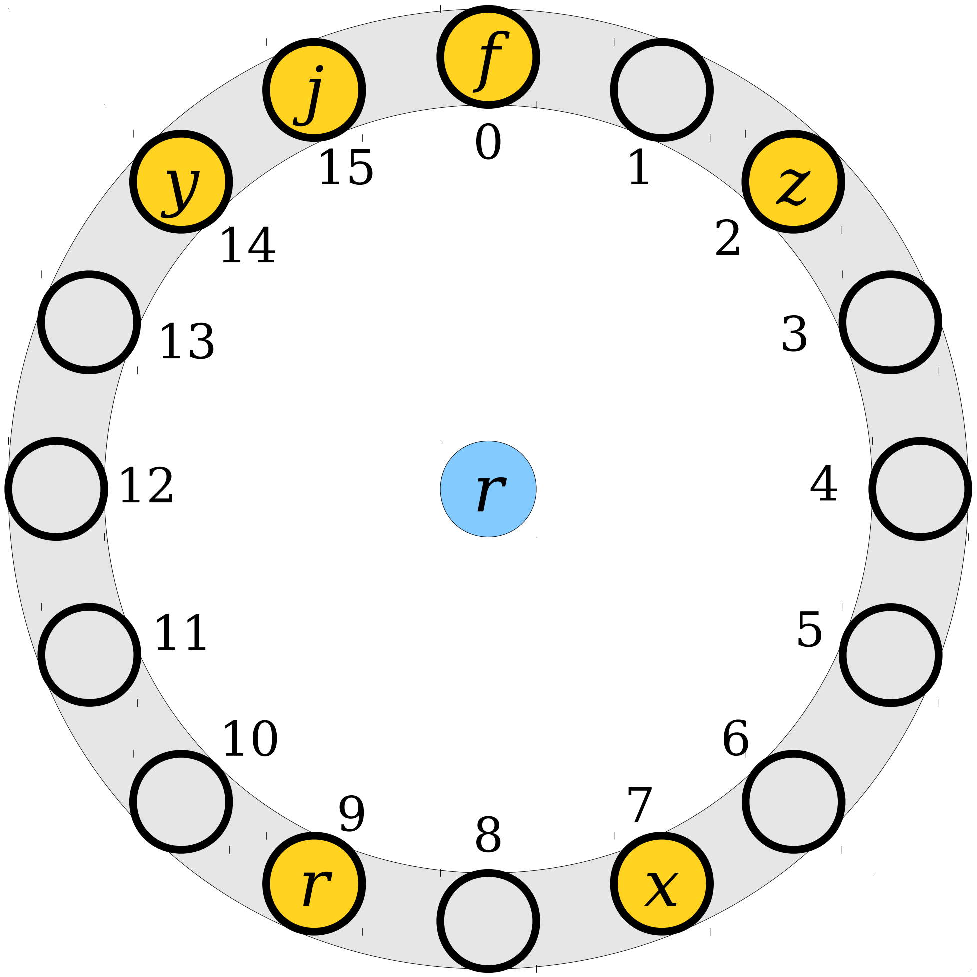

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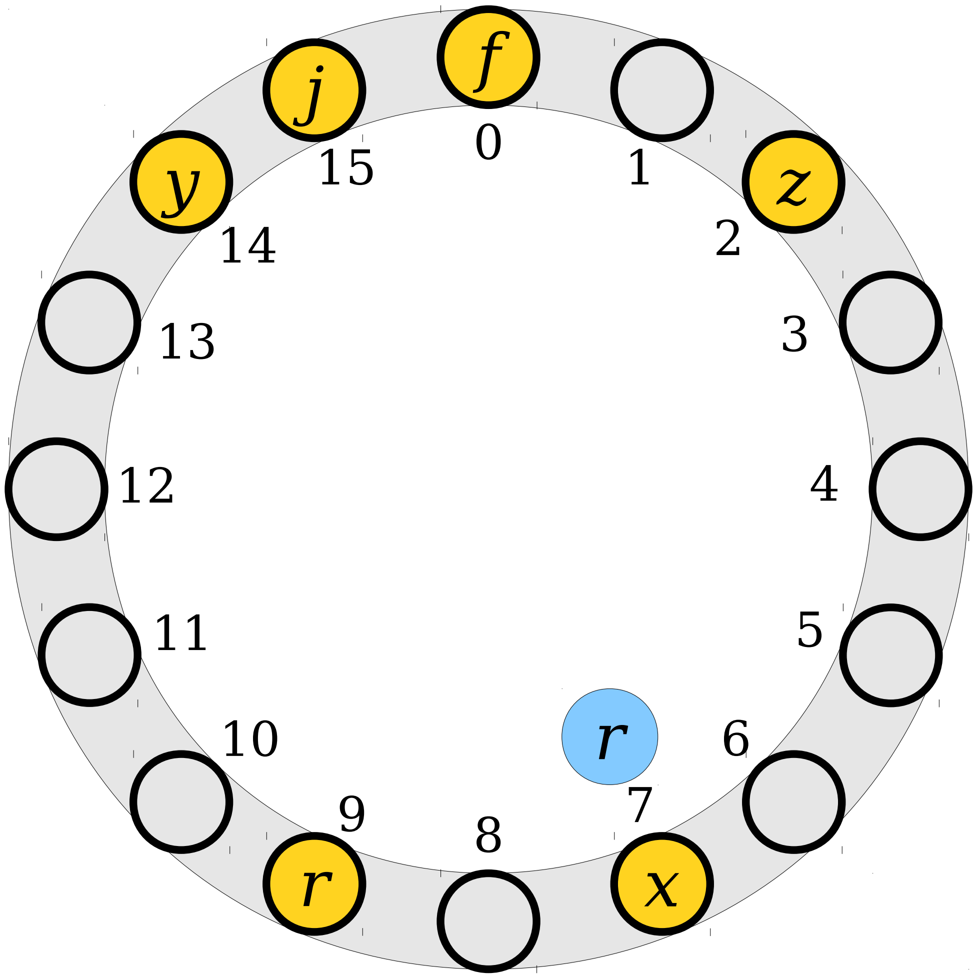

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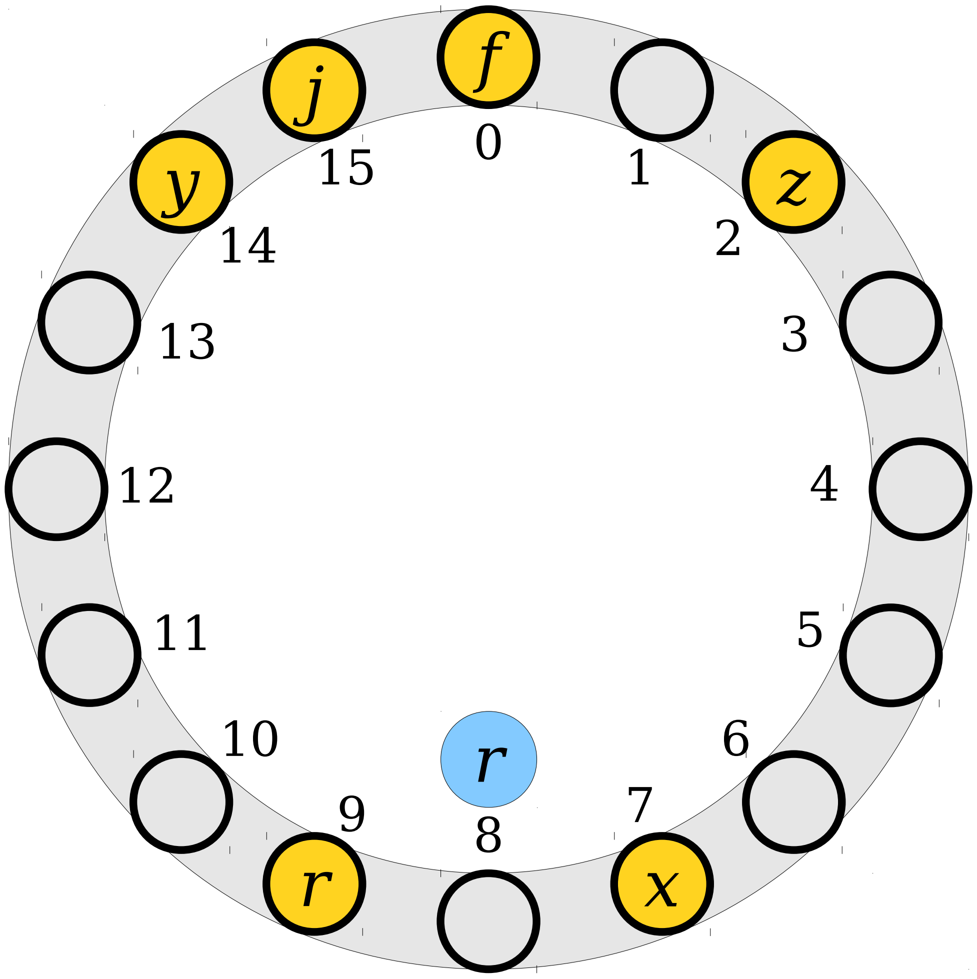

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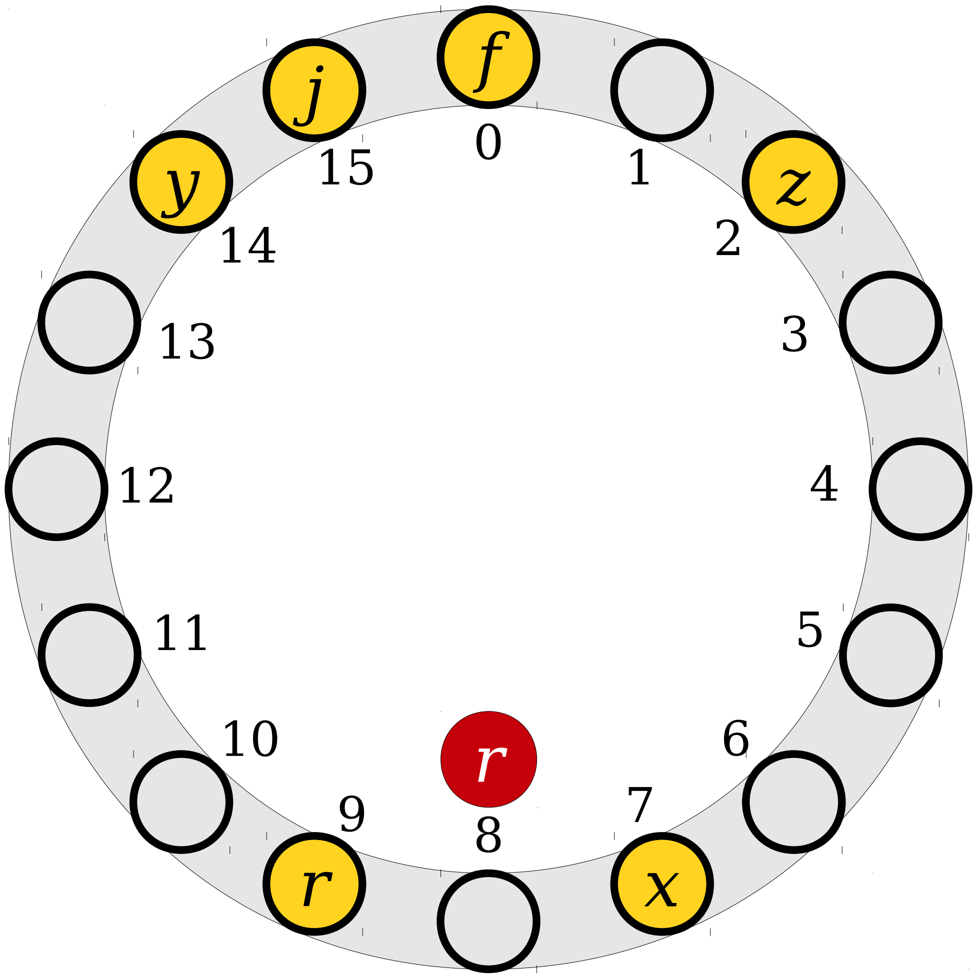

Linear Probing

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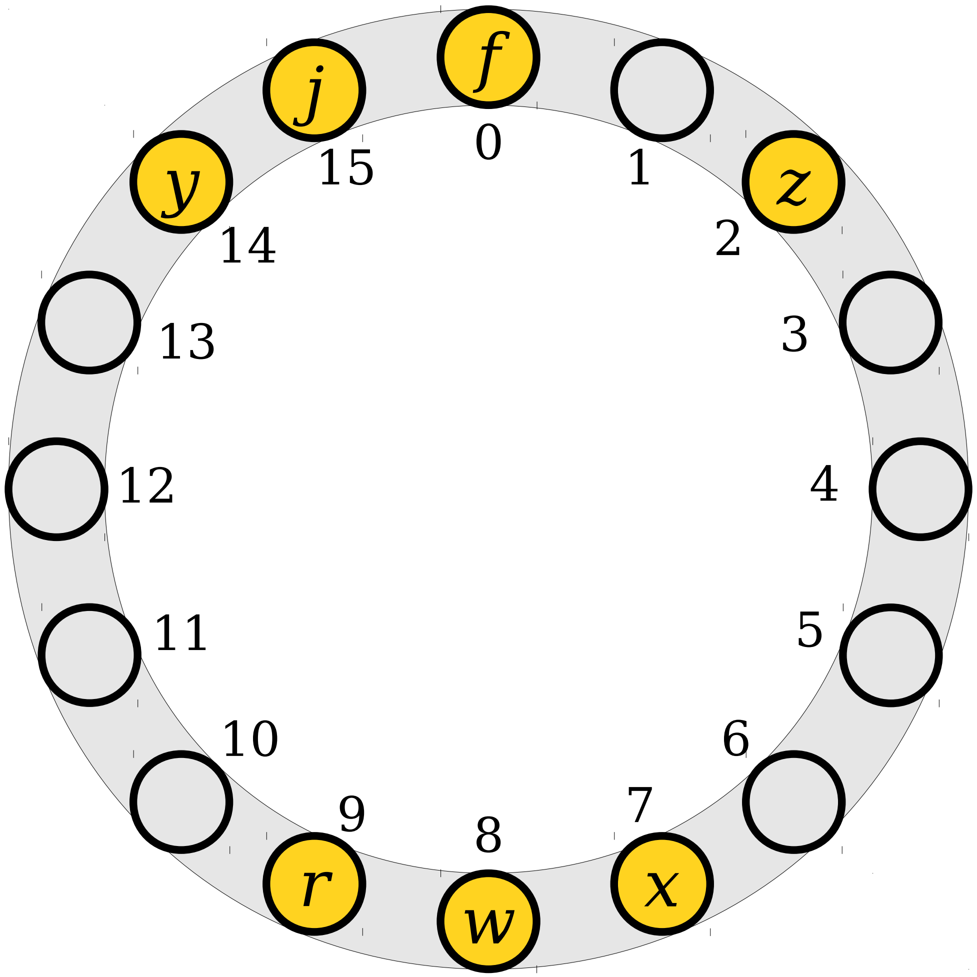

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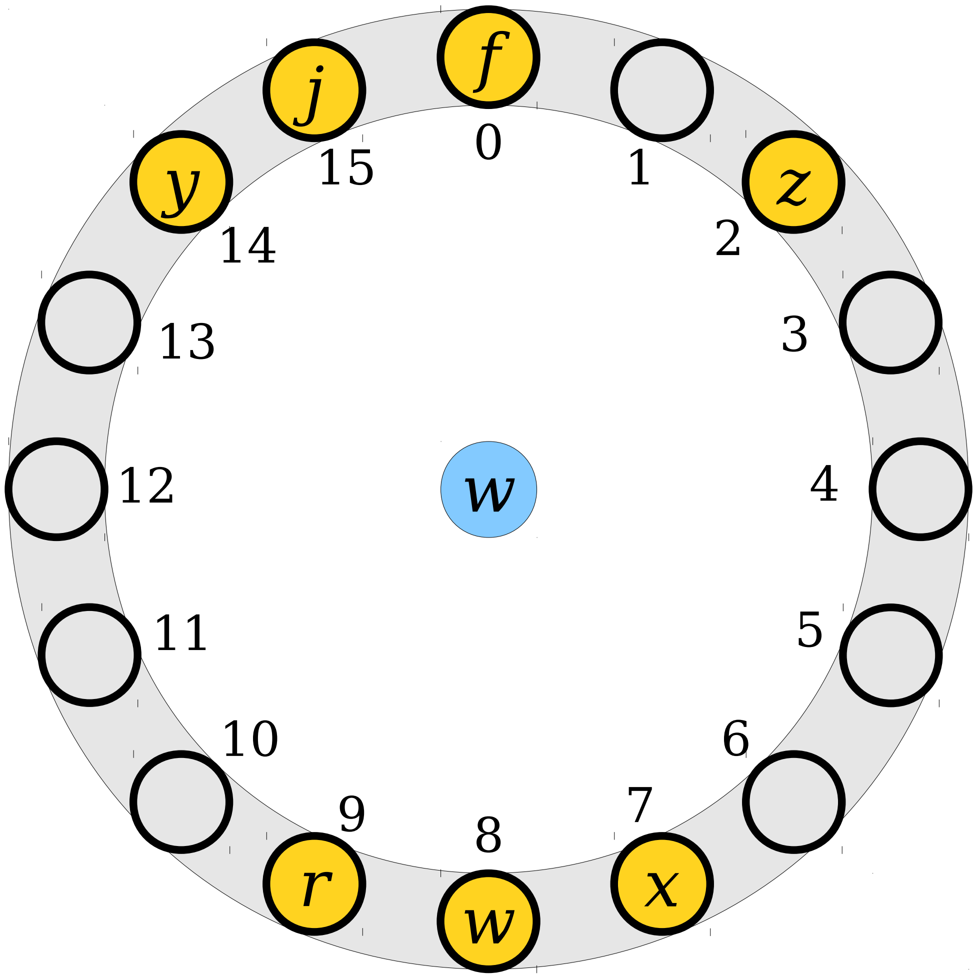

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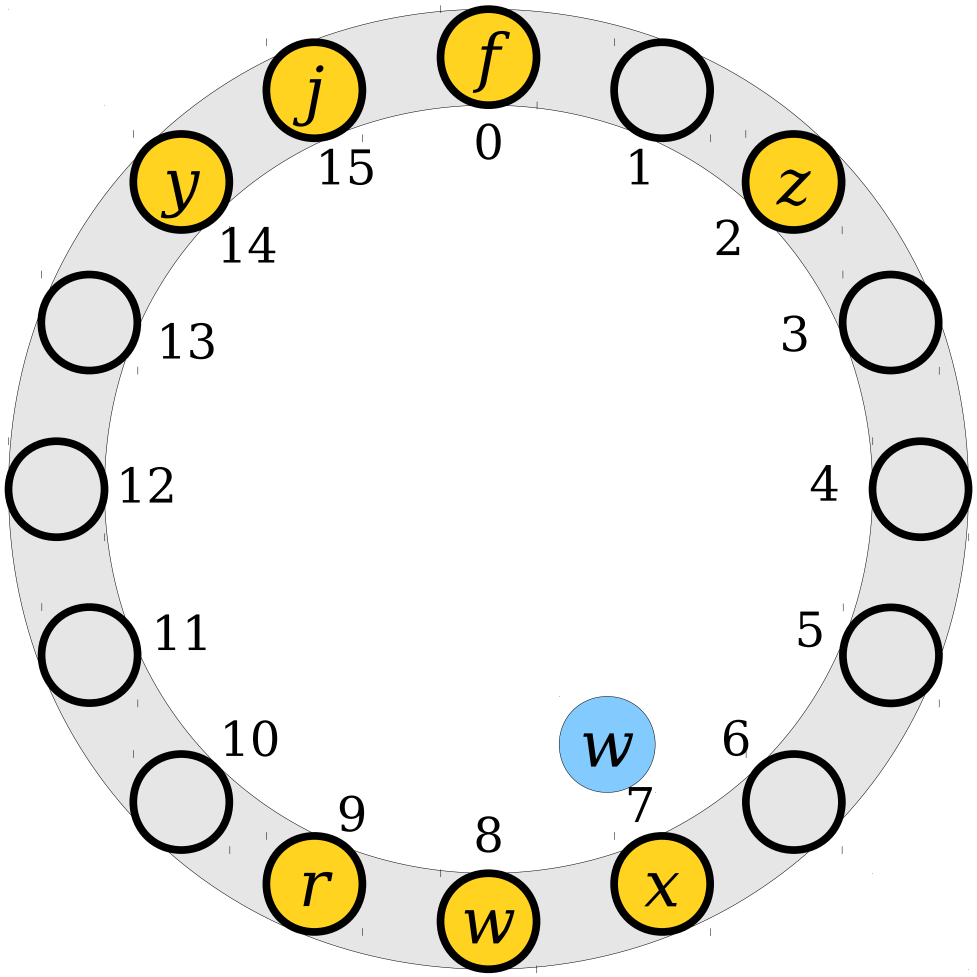

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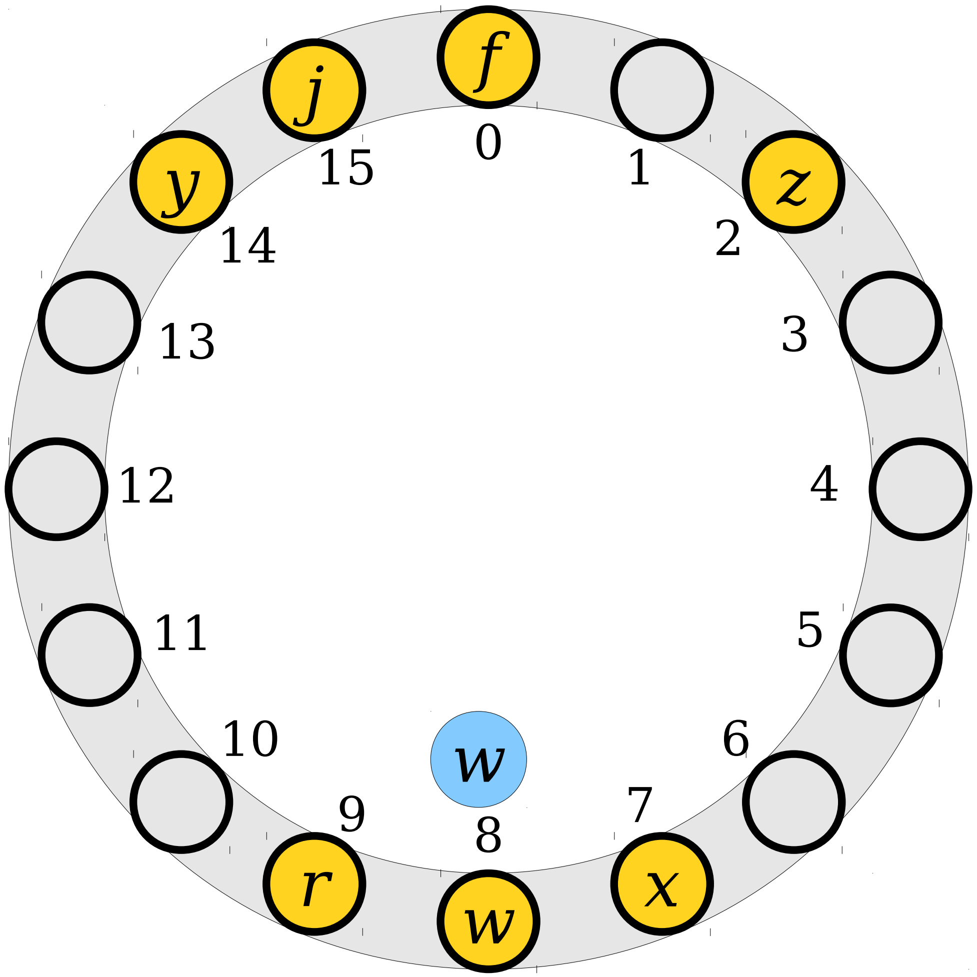

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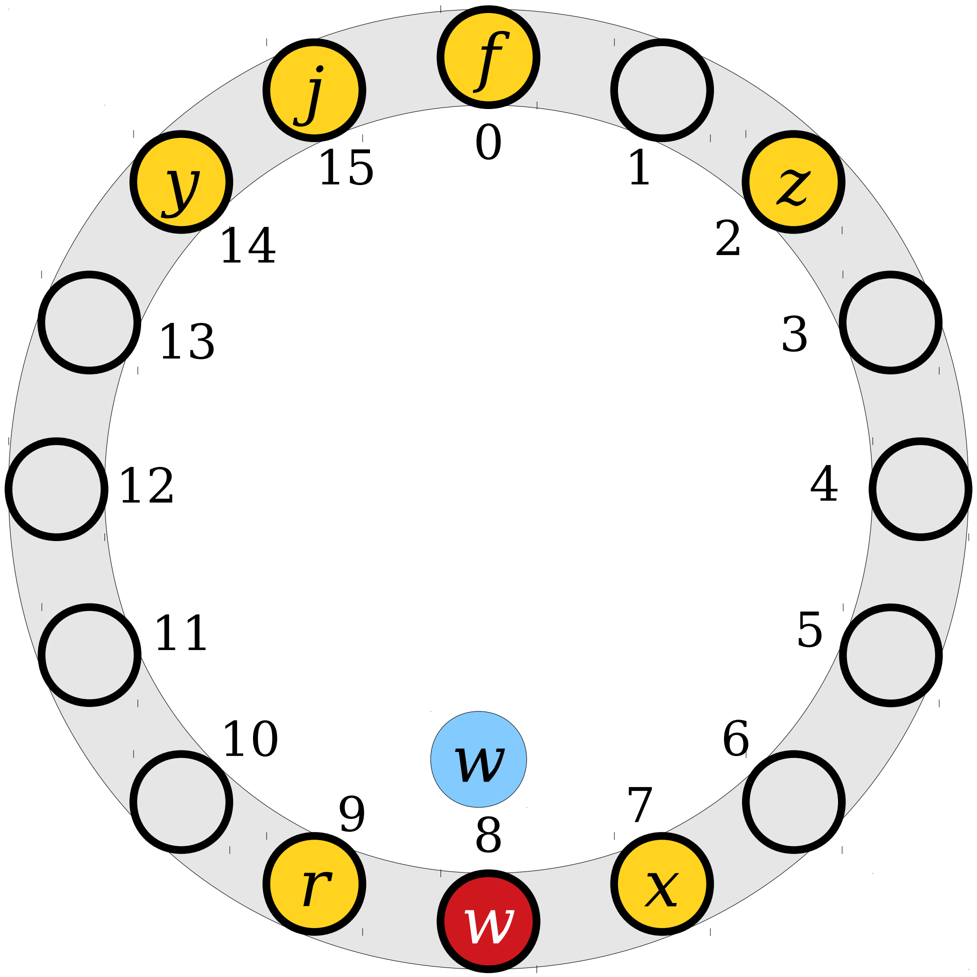

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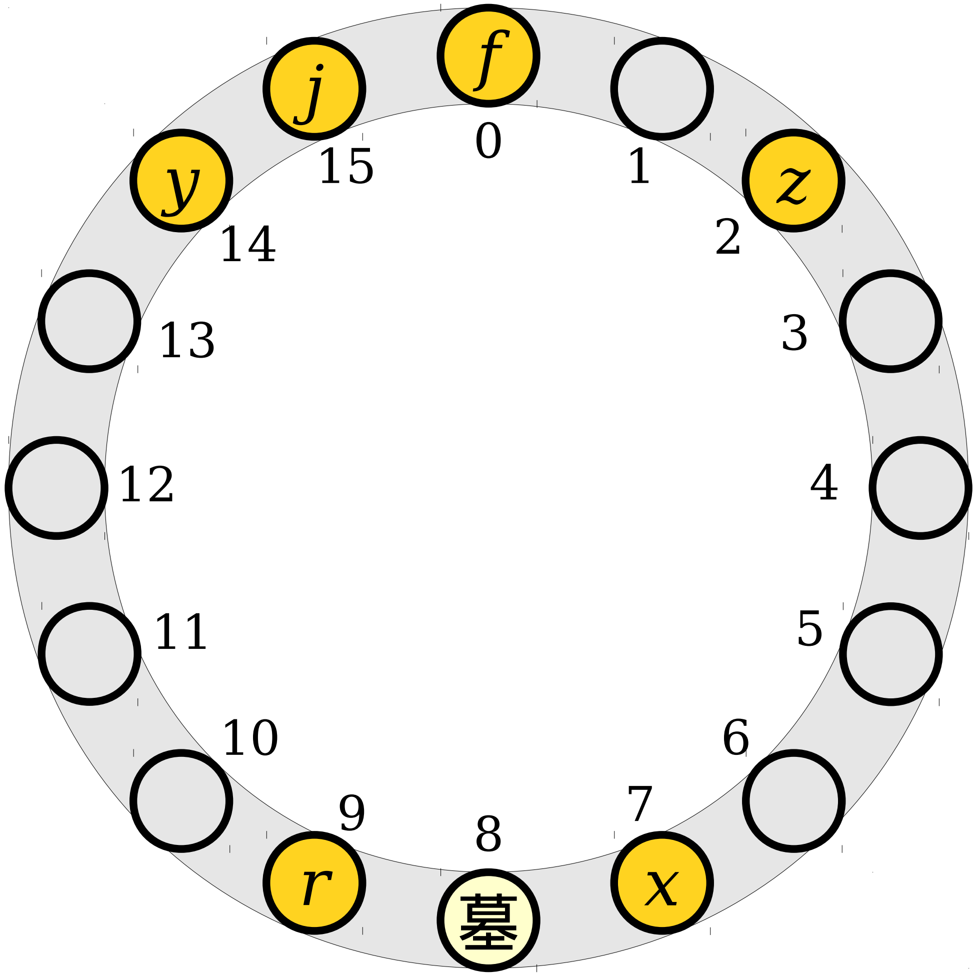

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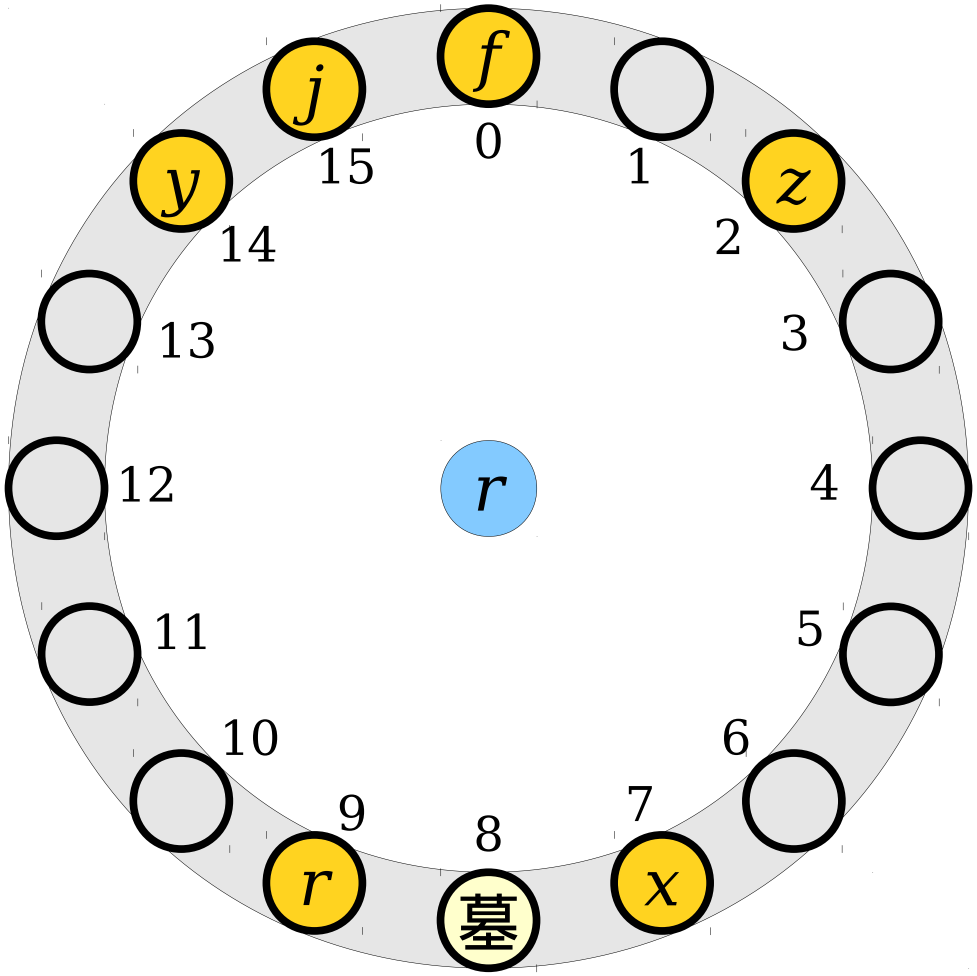

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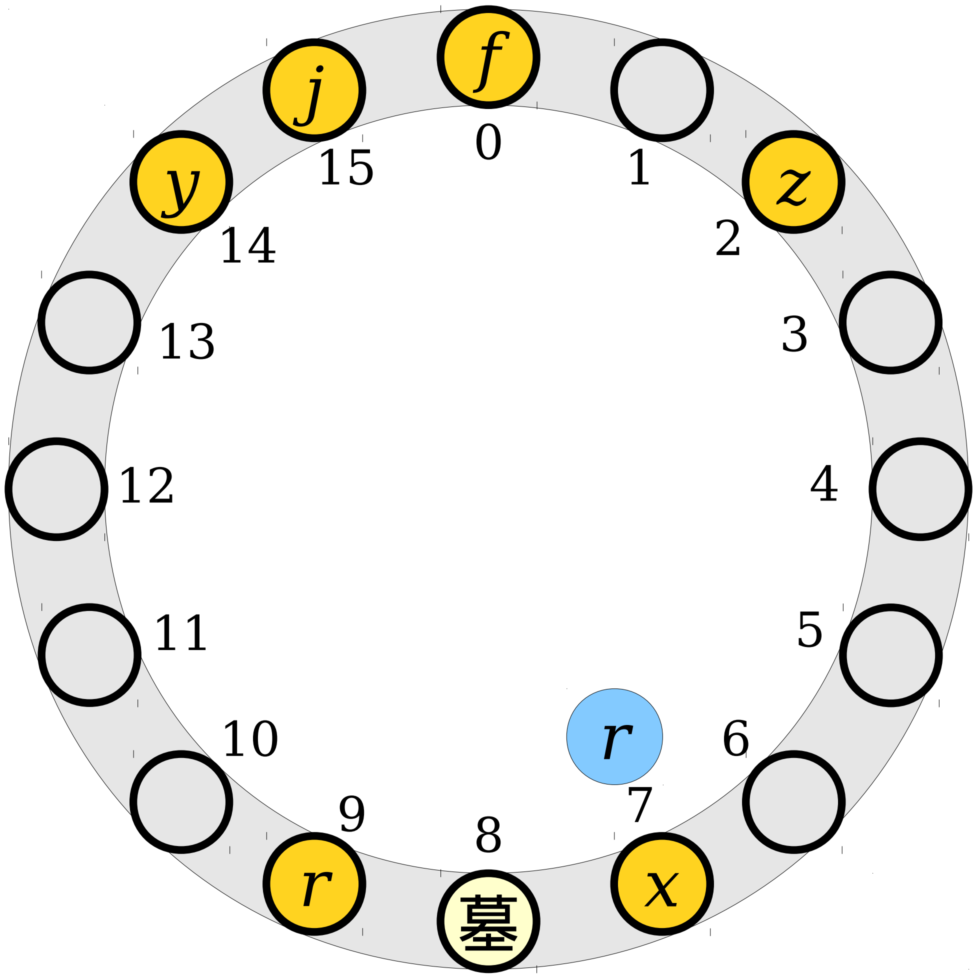

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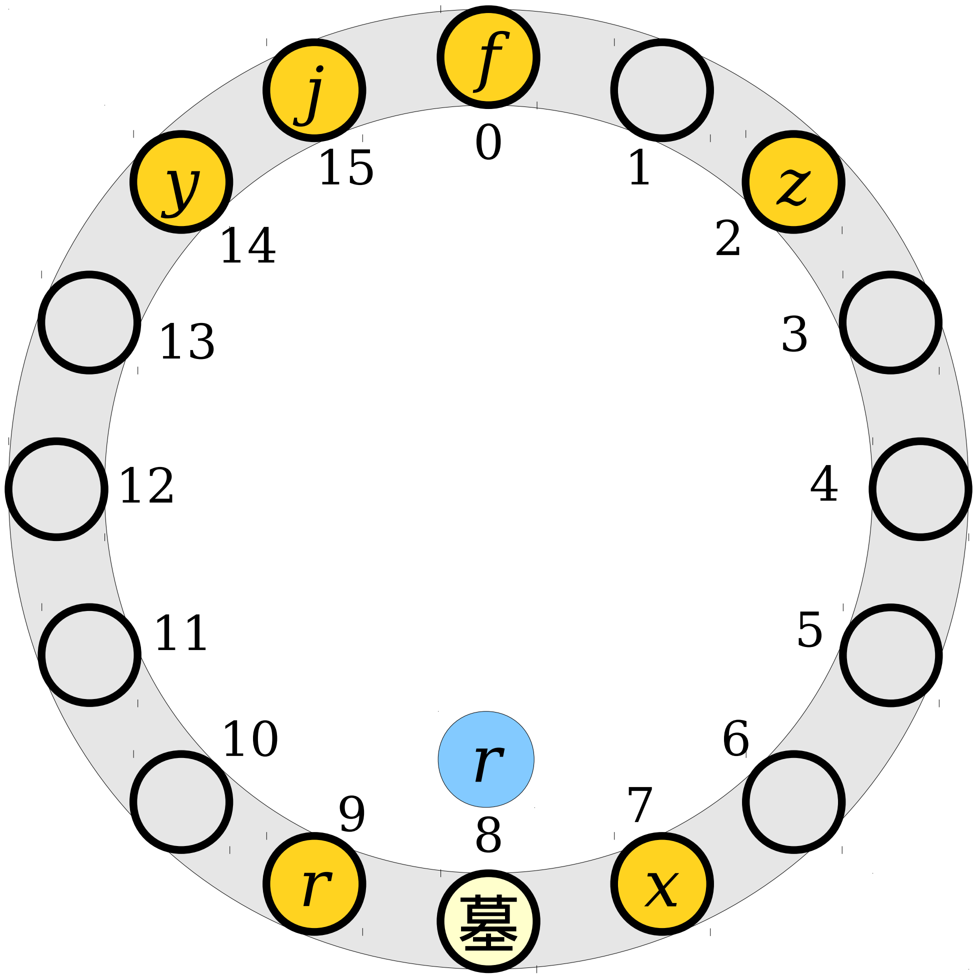

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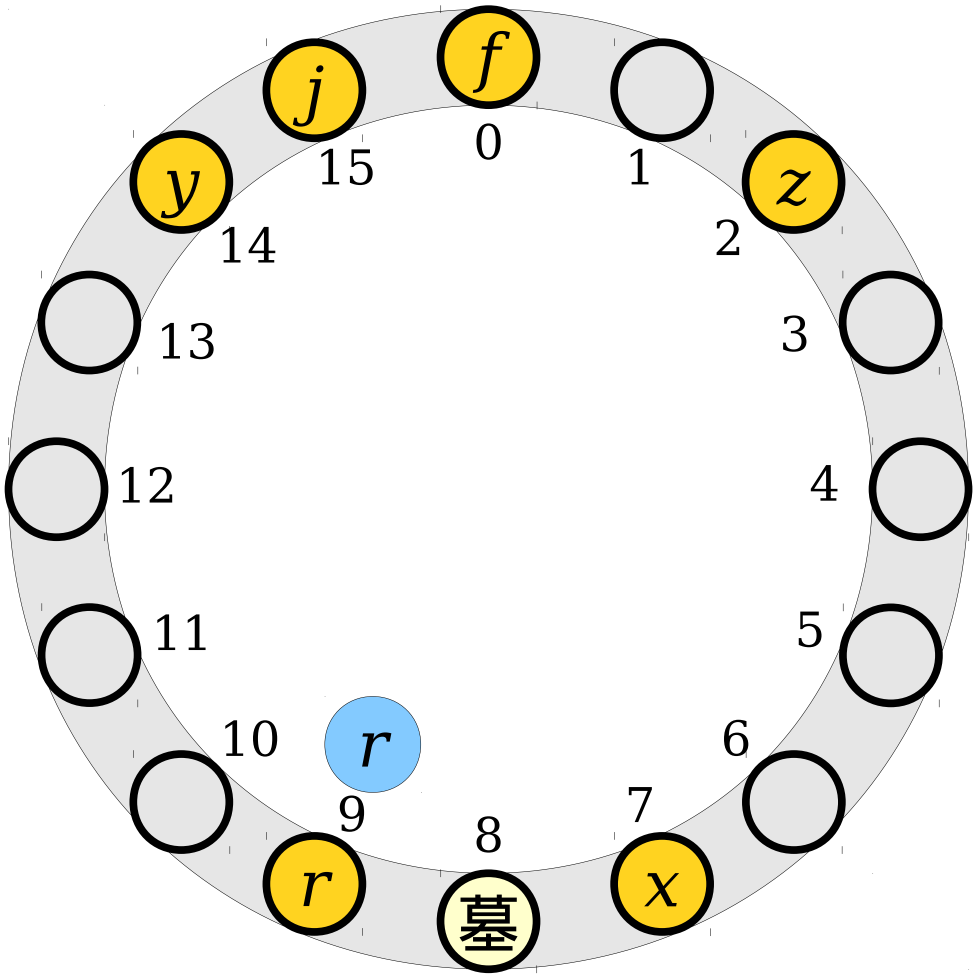

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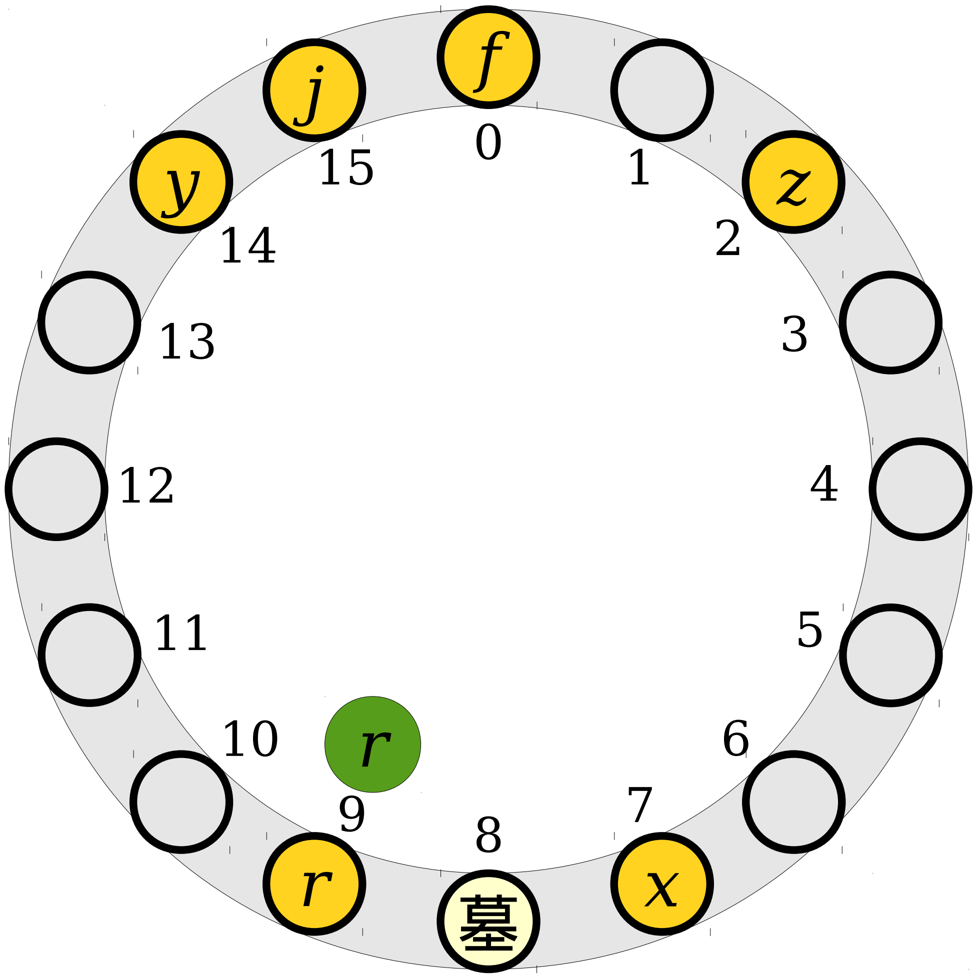

Linear Probing

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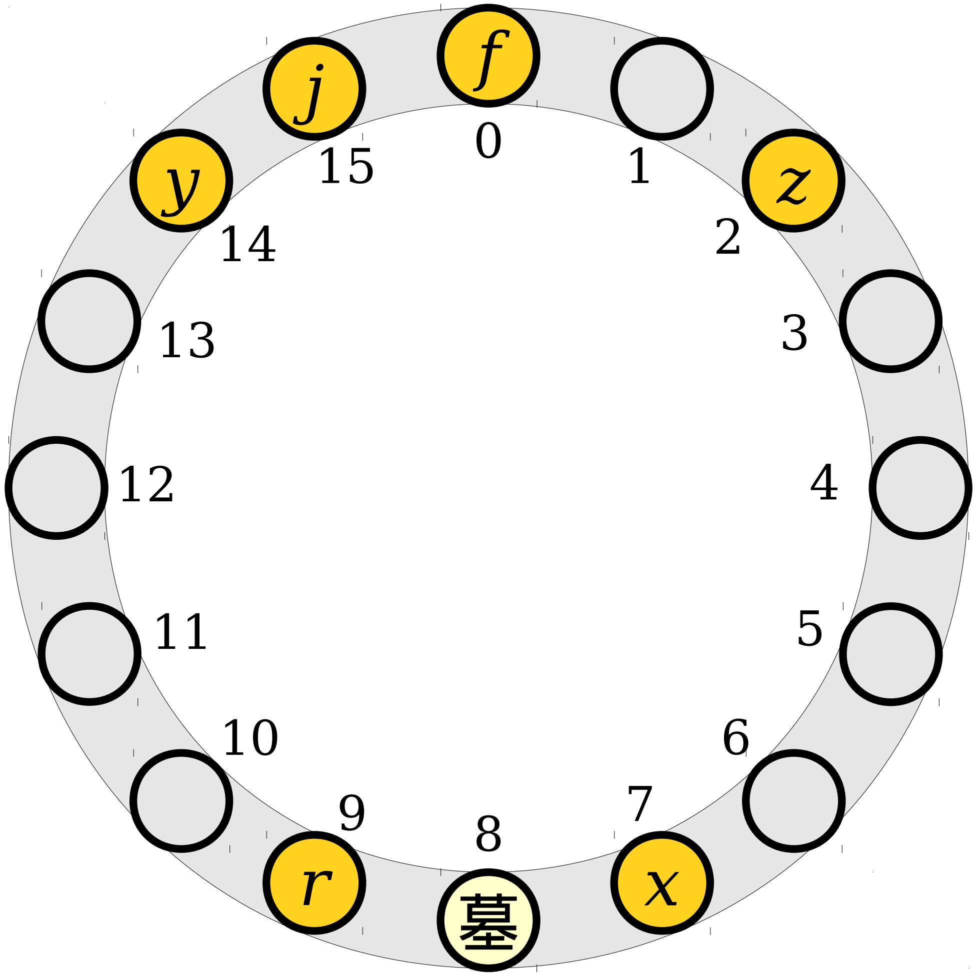
- Tombstones should be overwritten when new elements are inserted.



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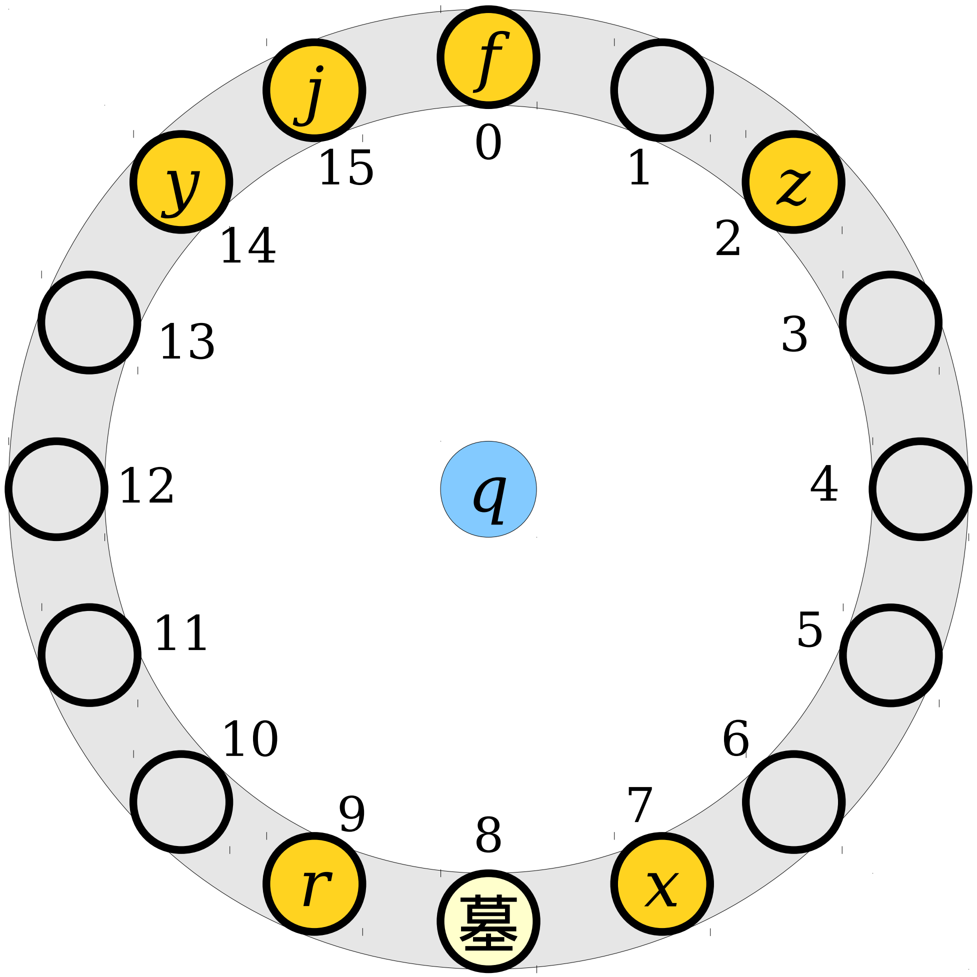

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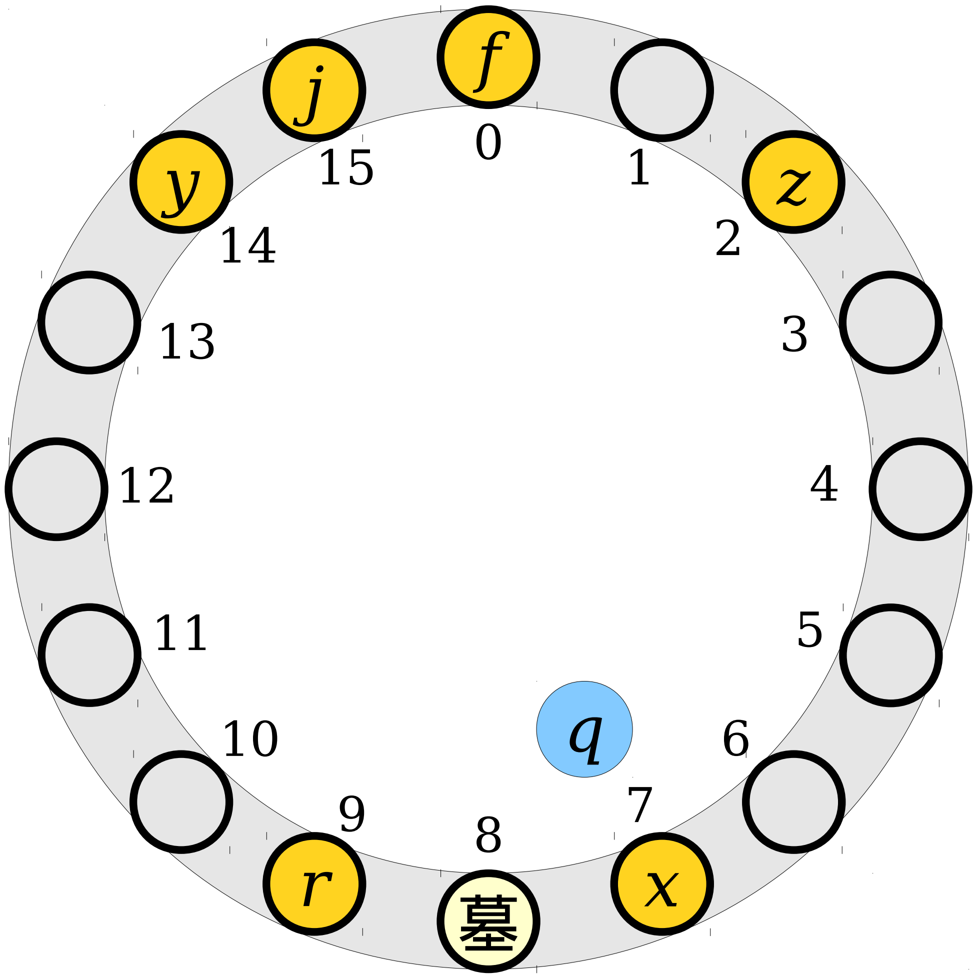

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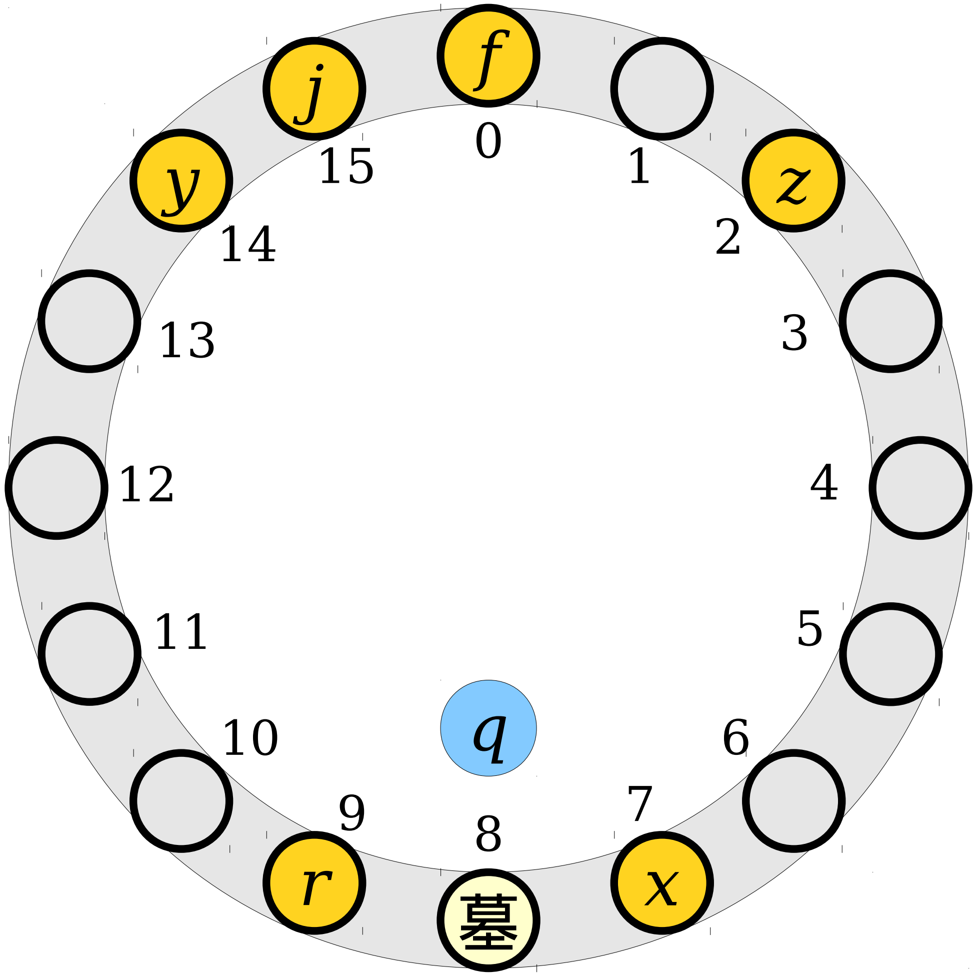

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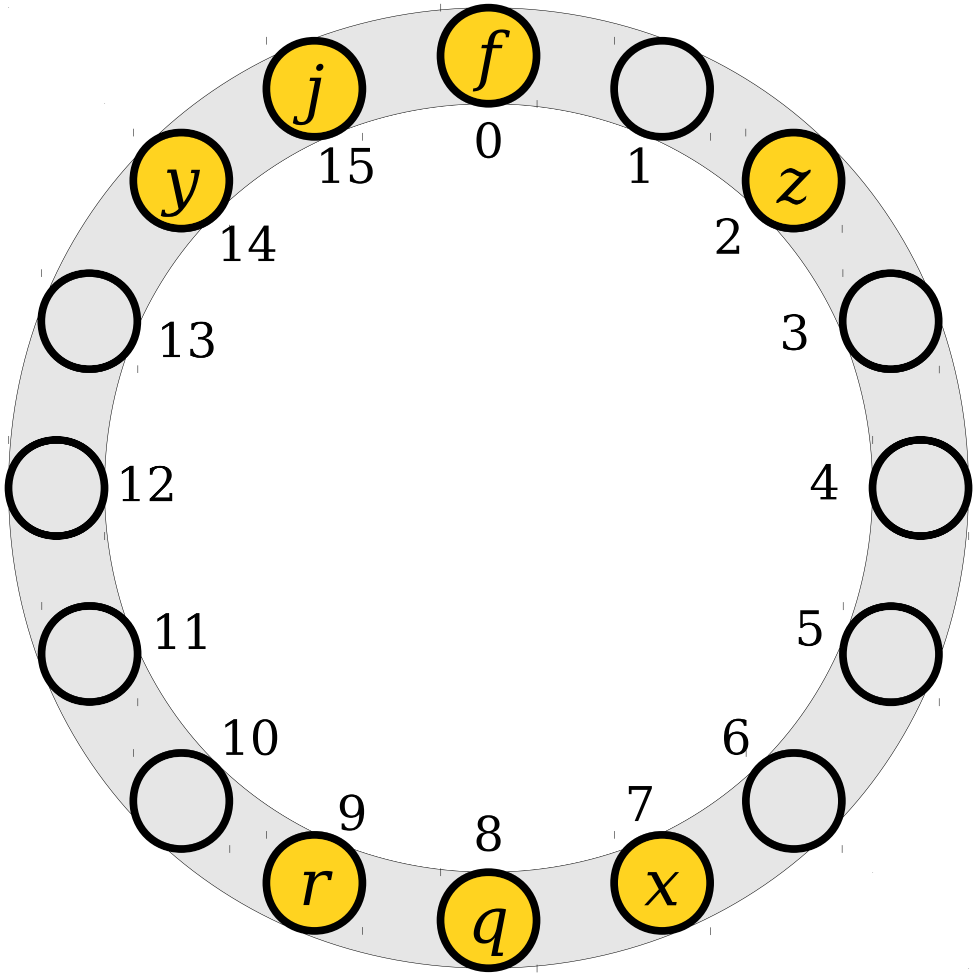

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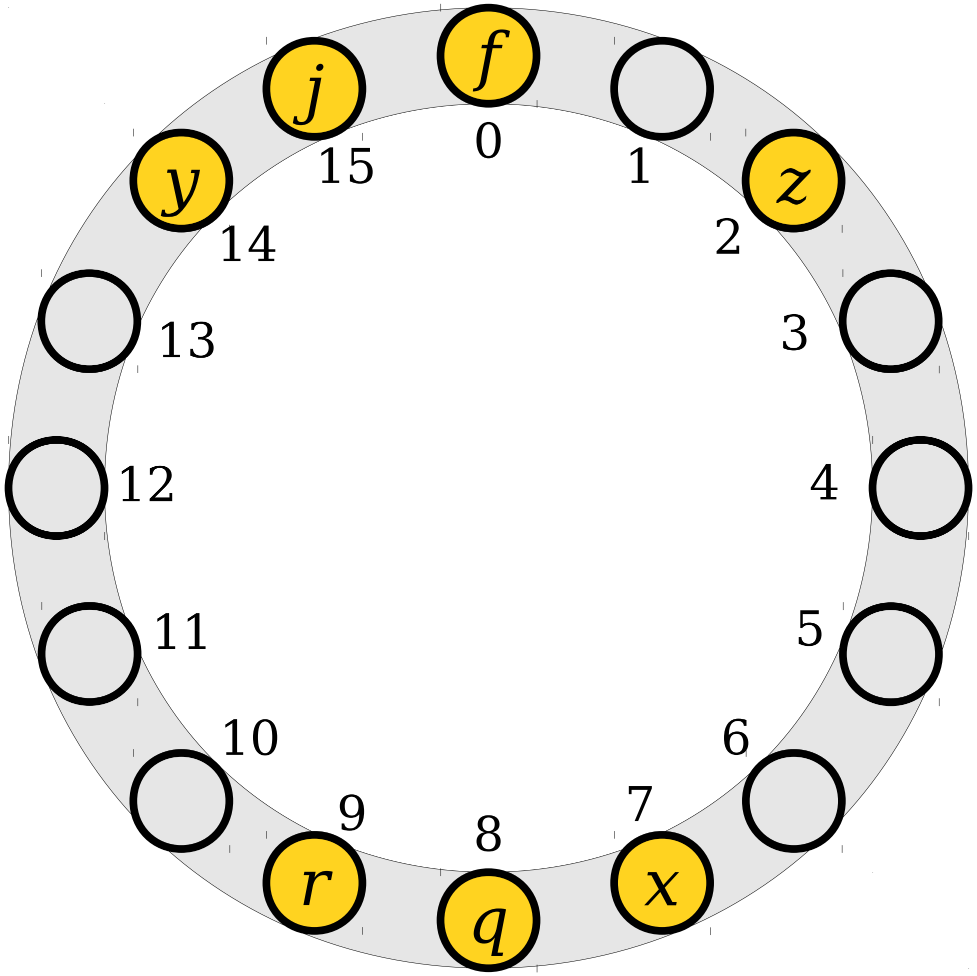

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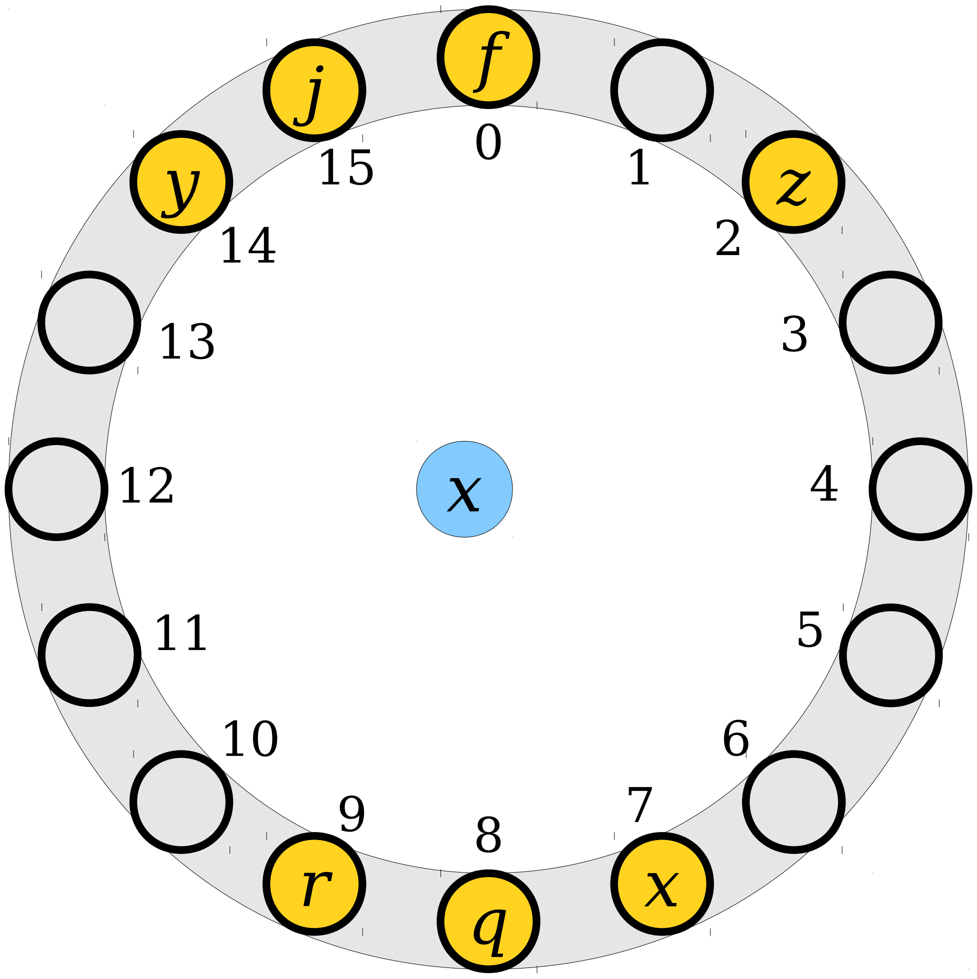

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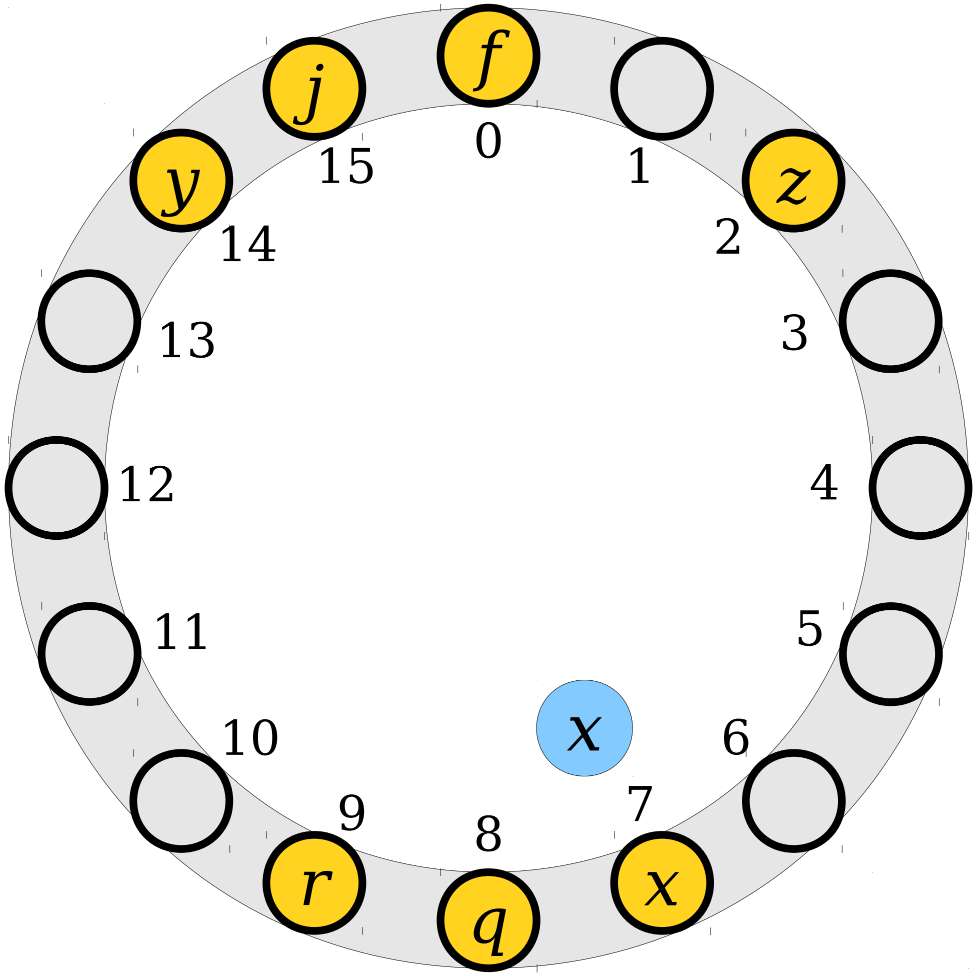

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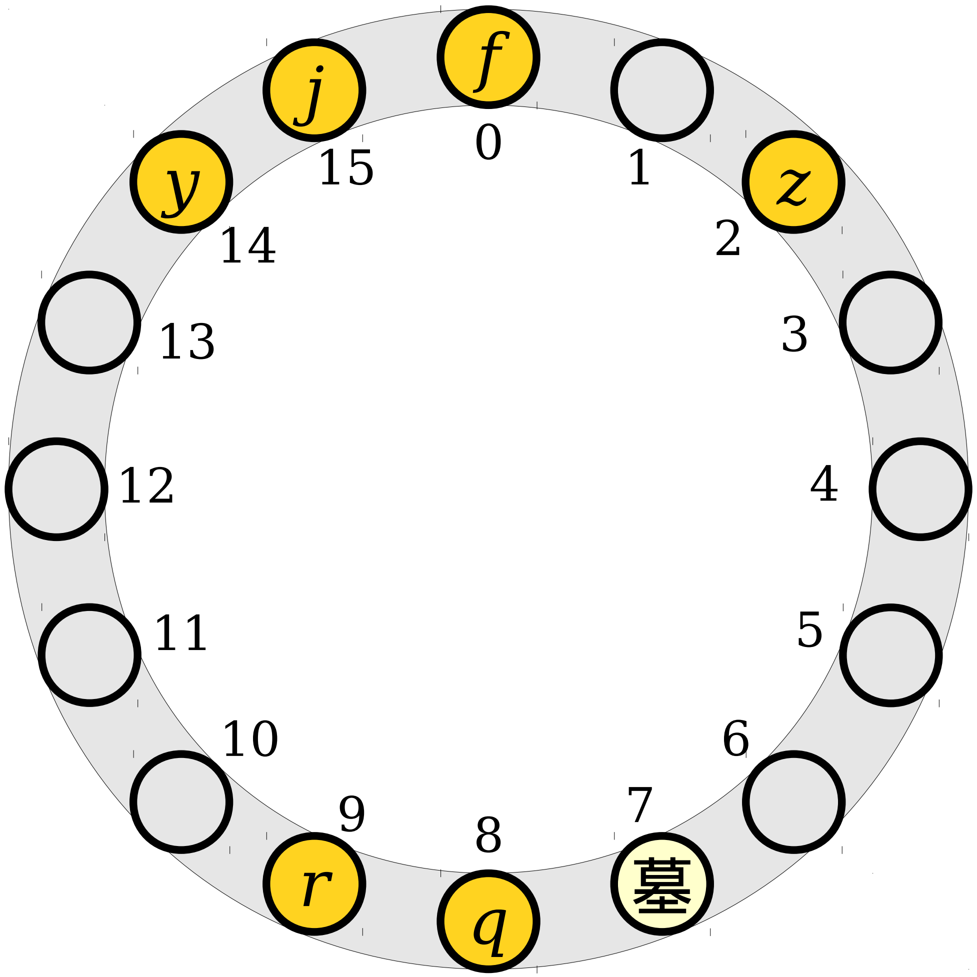

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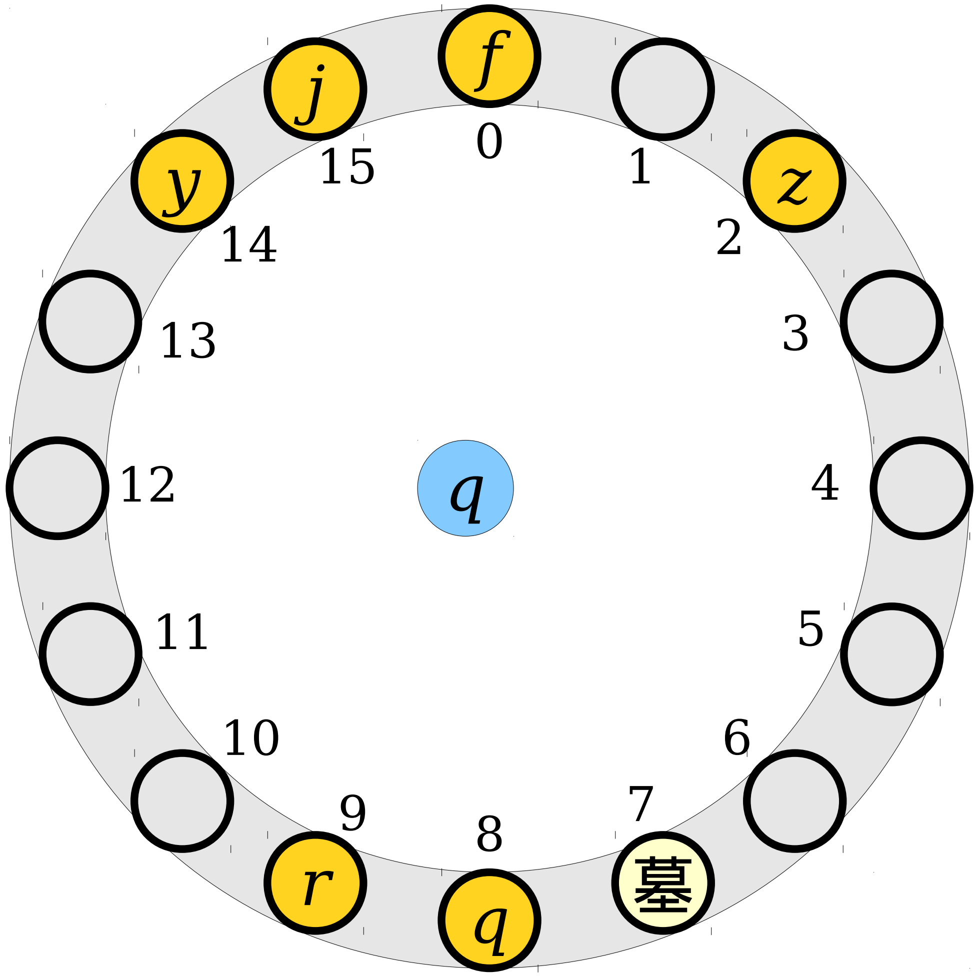

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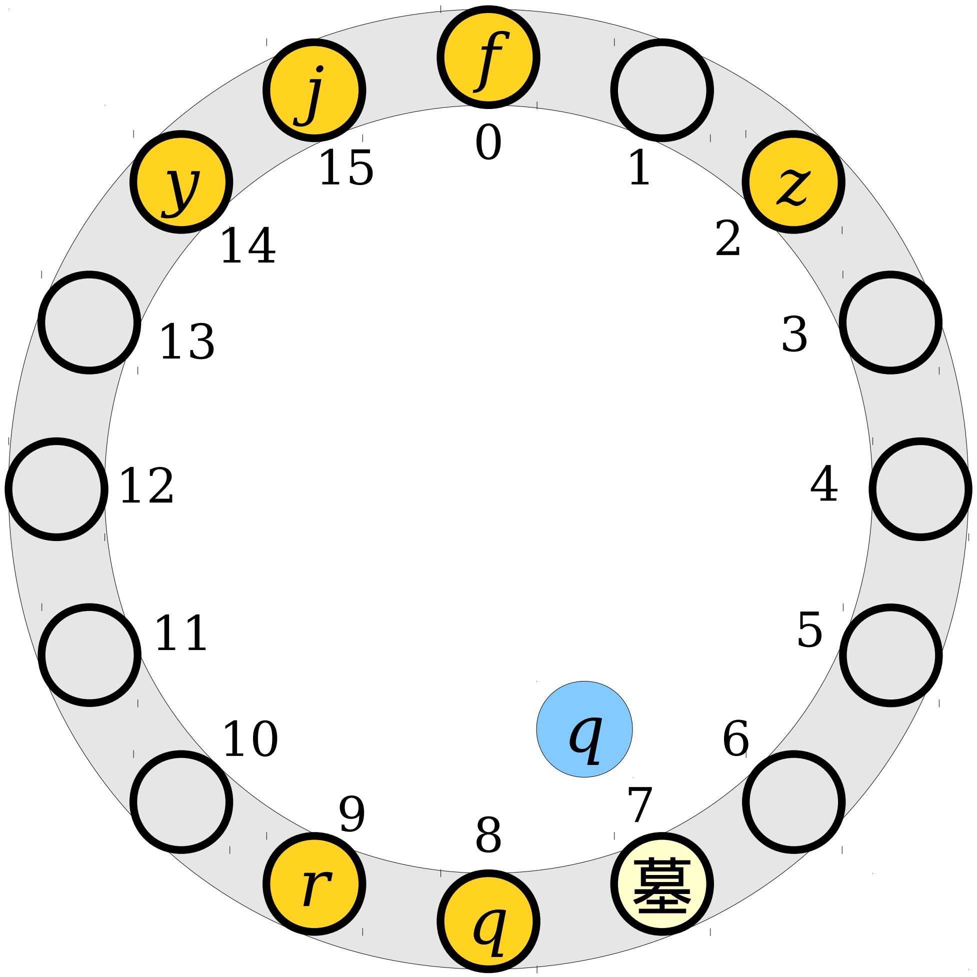

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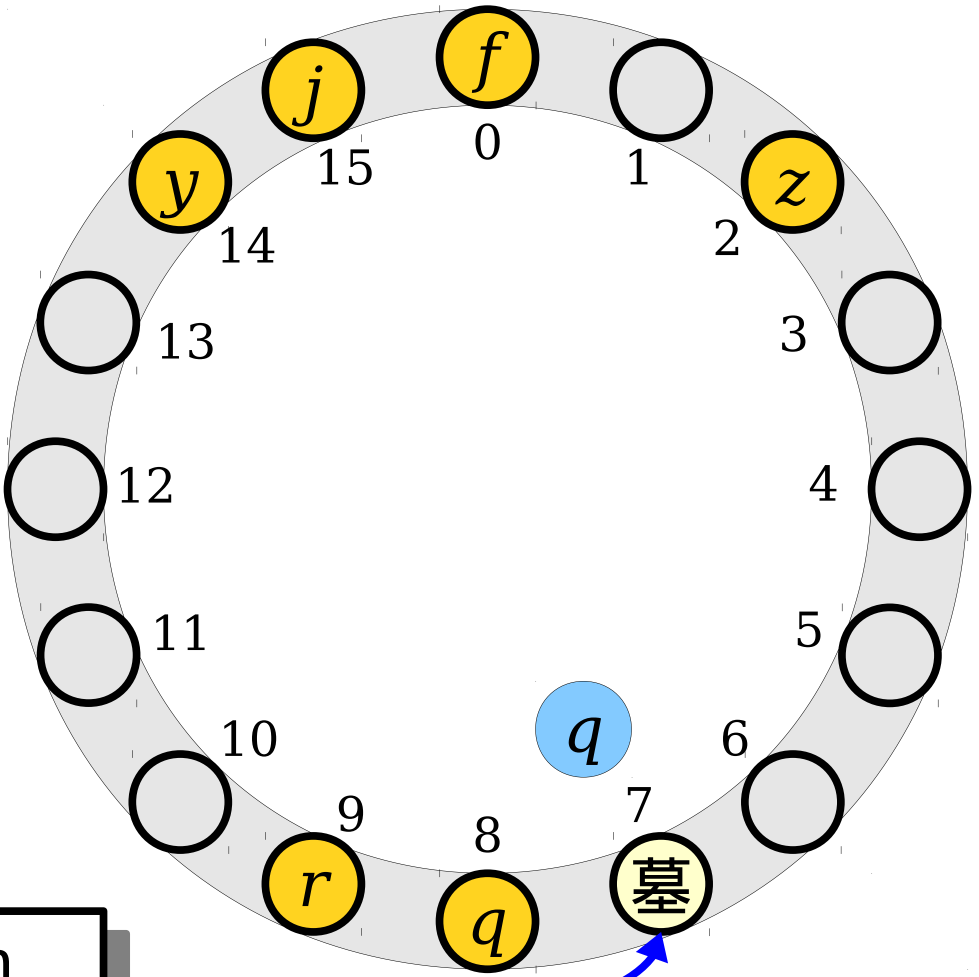

Don't put q in this slot – it already exists!

Linear Probing

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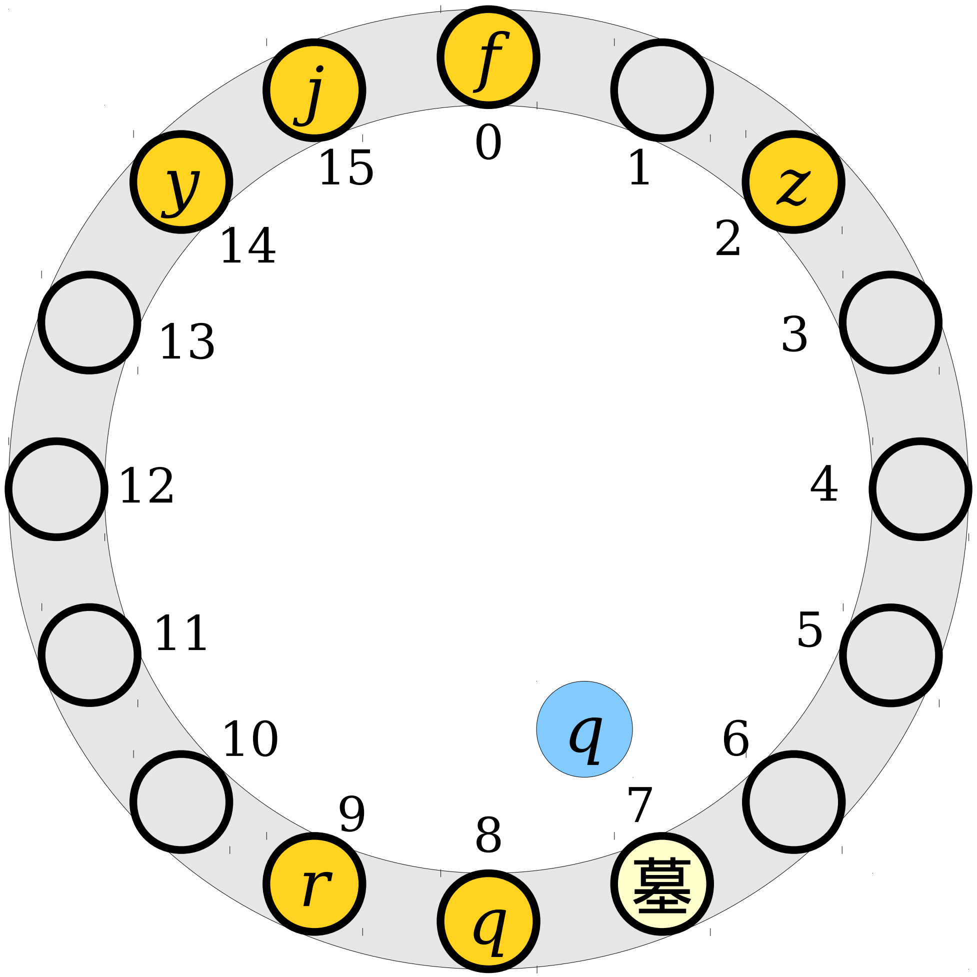

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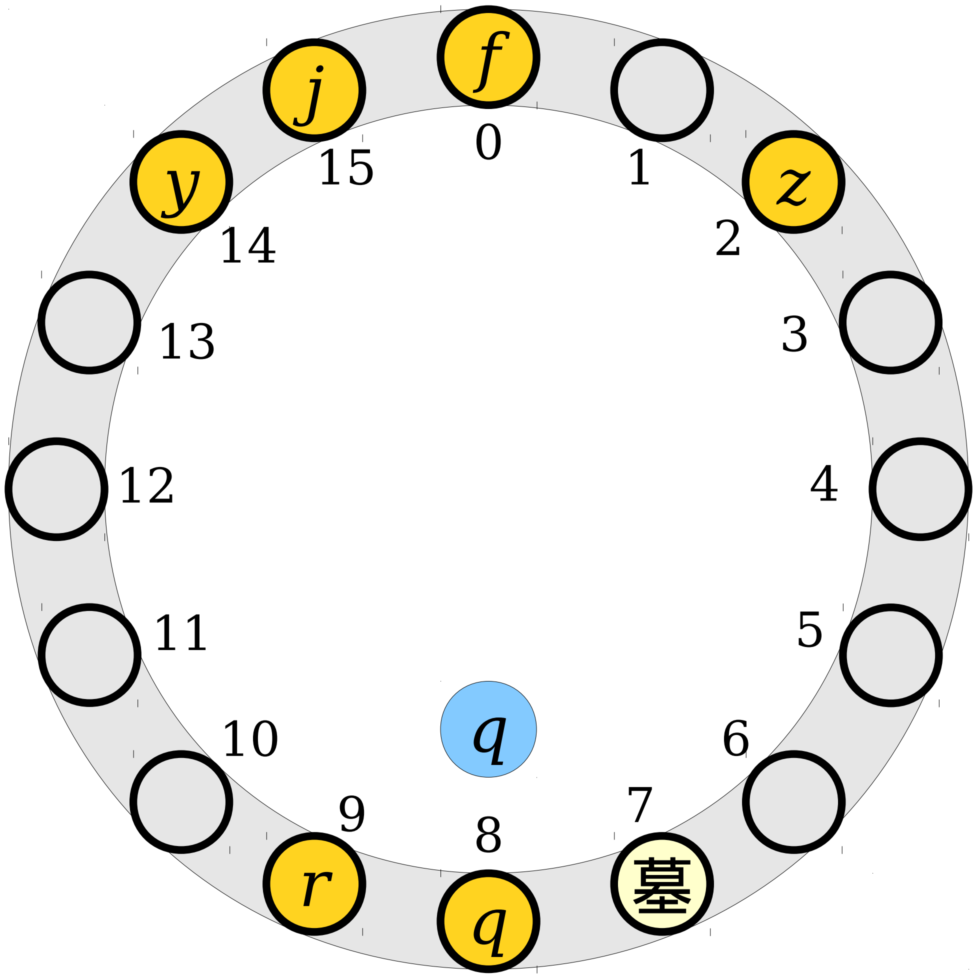

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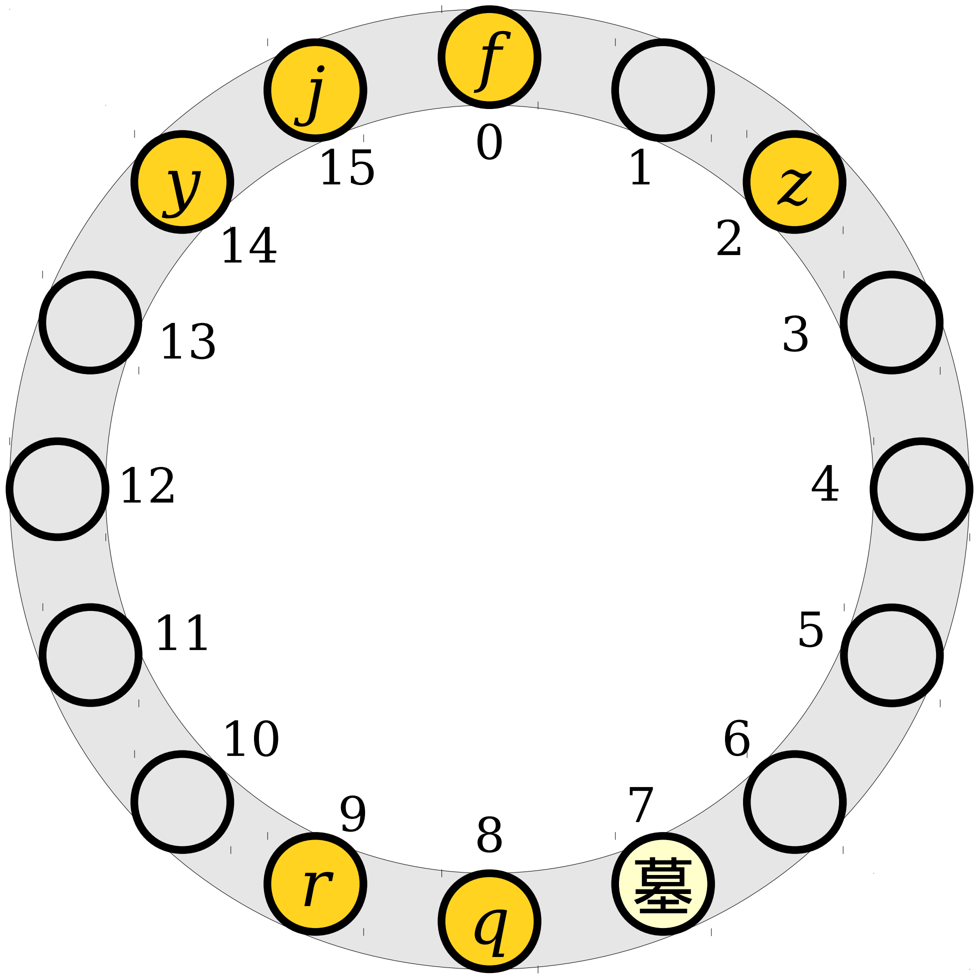

Linear Probing

- To search for an item, jump to the slot given by its hash code, then keep moving to the next slot, wrapping around if necessary, until you find the item or a blank slot.
- To insert an item that doesn’t already exist in the table, start at the slot for its hash code and move until you find a blank spot or tombstone.
- To remove an item, search for it and replace it with a tombstone.

How Fast is Linear Probing?

- ***Recall:*** The load factor of a hash table, denoted α, is the ratio of the number of items in the table to the number of slots.
- For any fixed value $\alpha < 1$, the expected cost of a lookup in a linear probing table is O(1), assuming you have a good hash function.
- This is the same big-O cost as a chained hash table, though with a totally different strategy!

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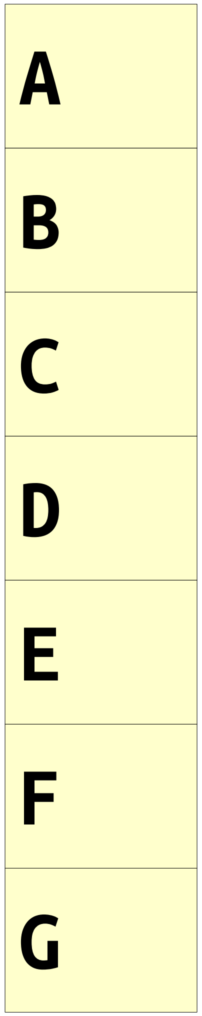

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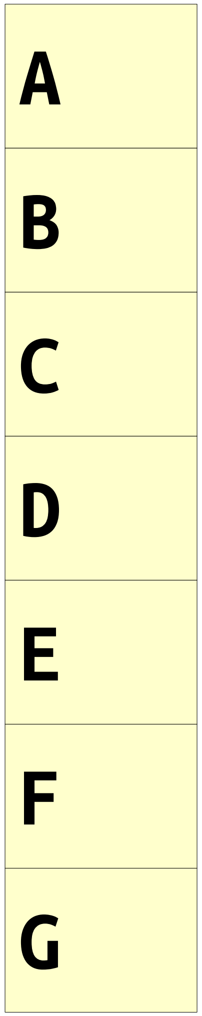

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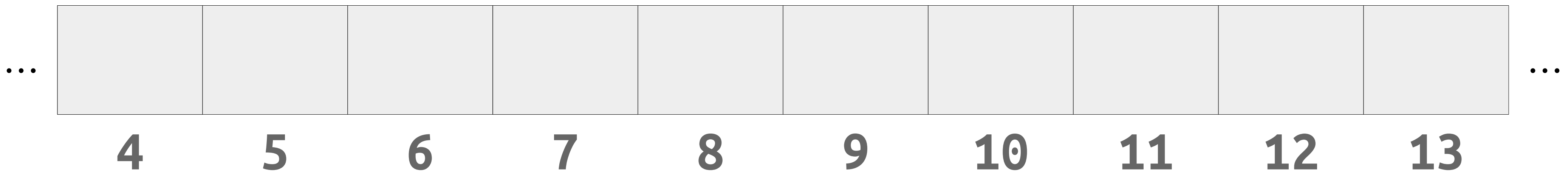

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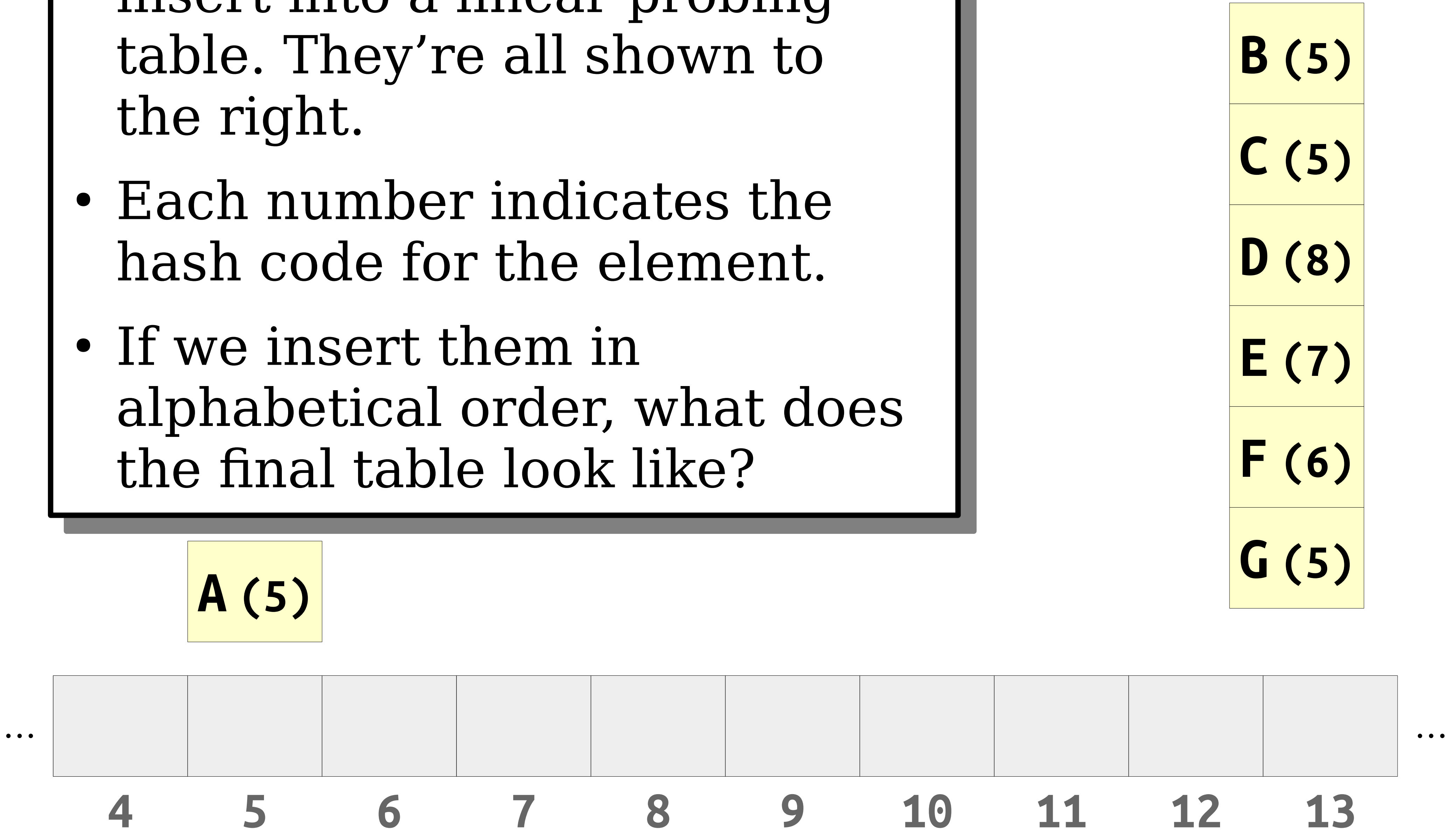

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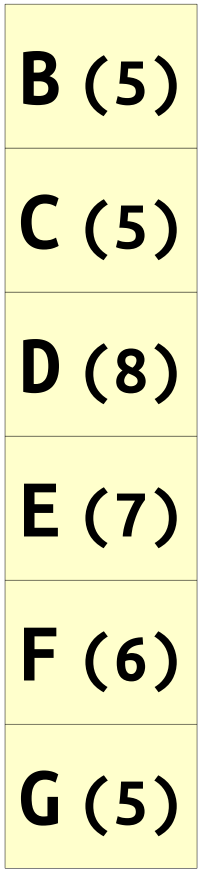

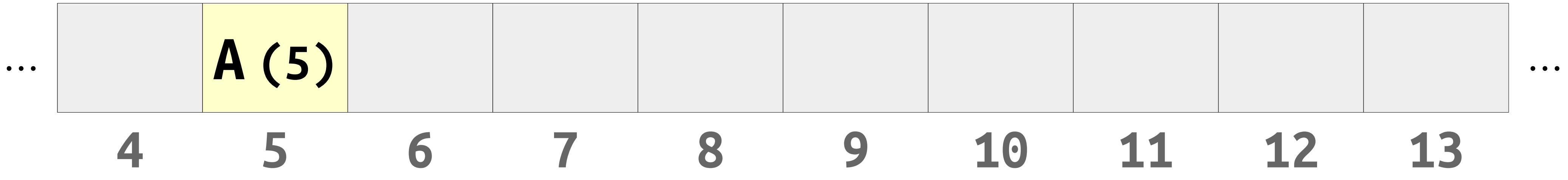

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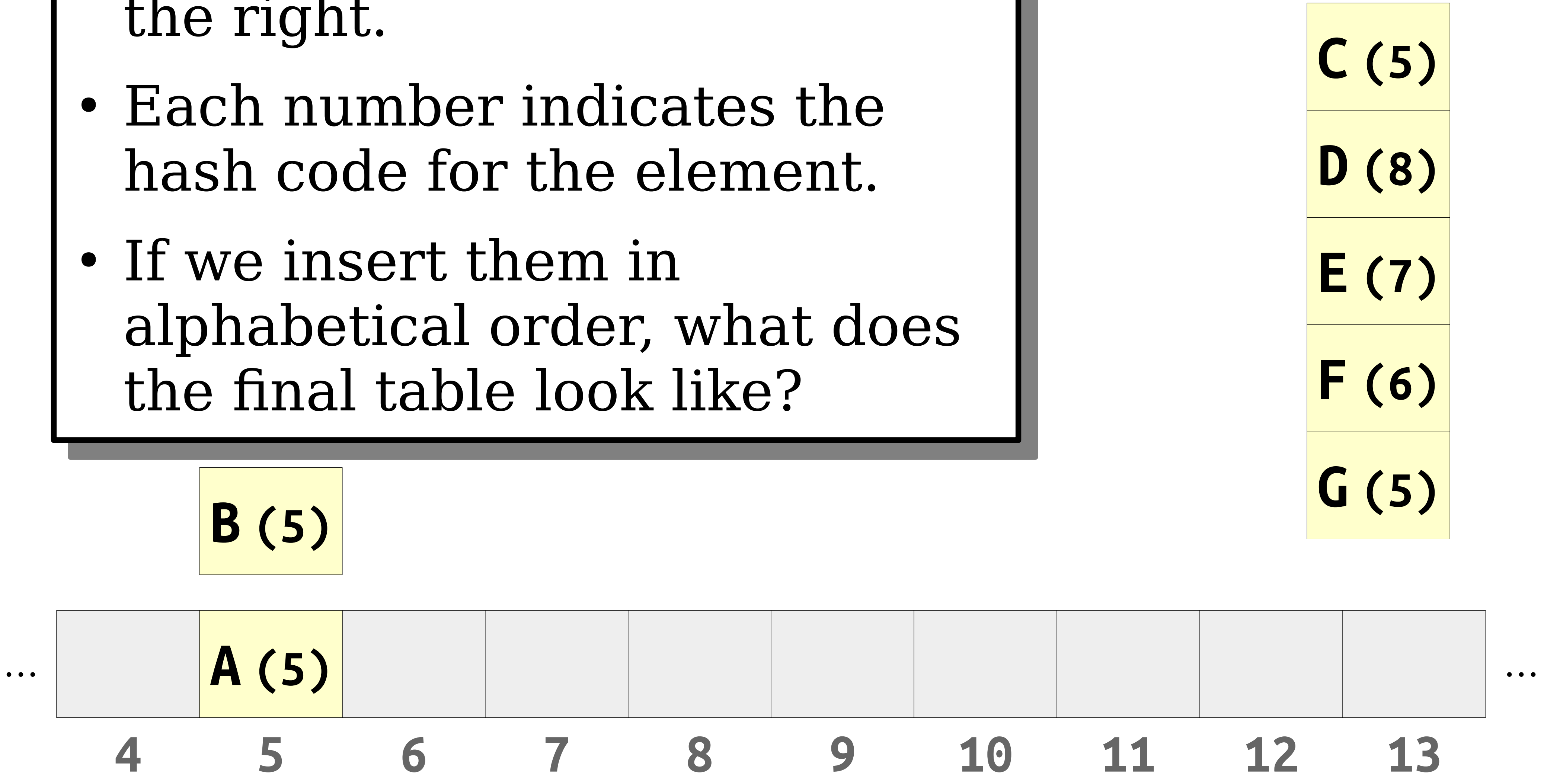

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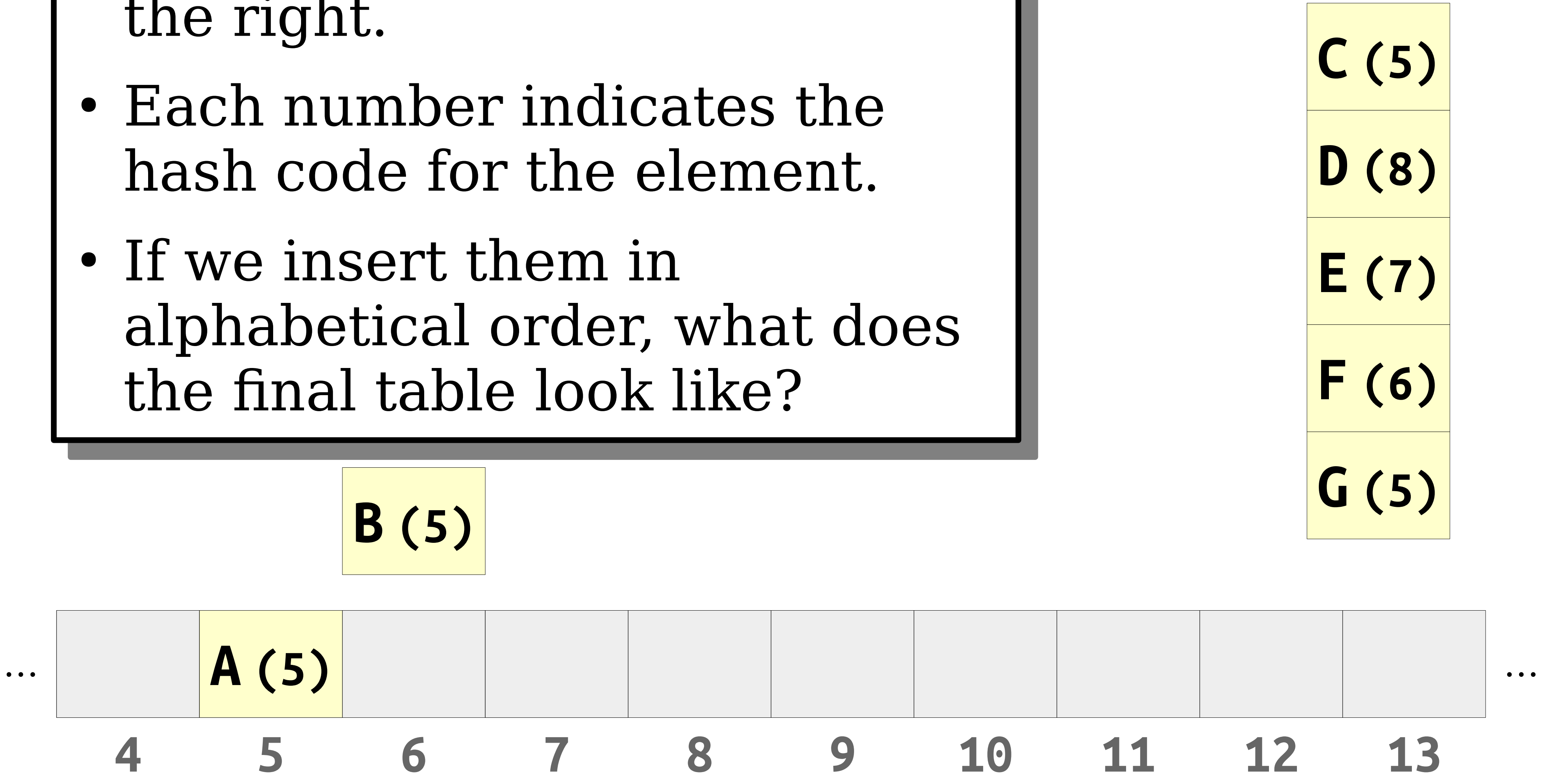

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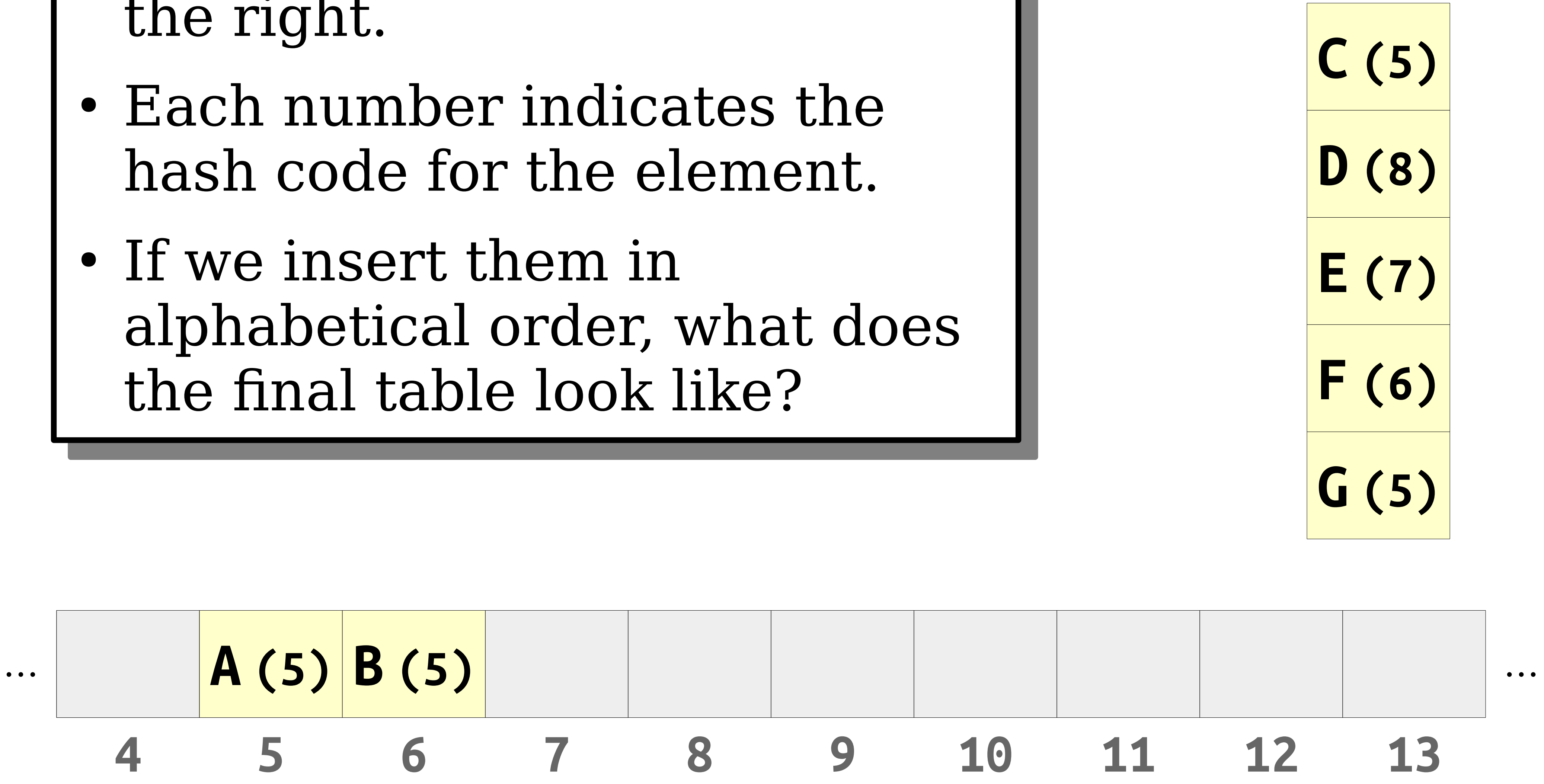

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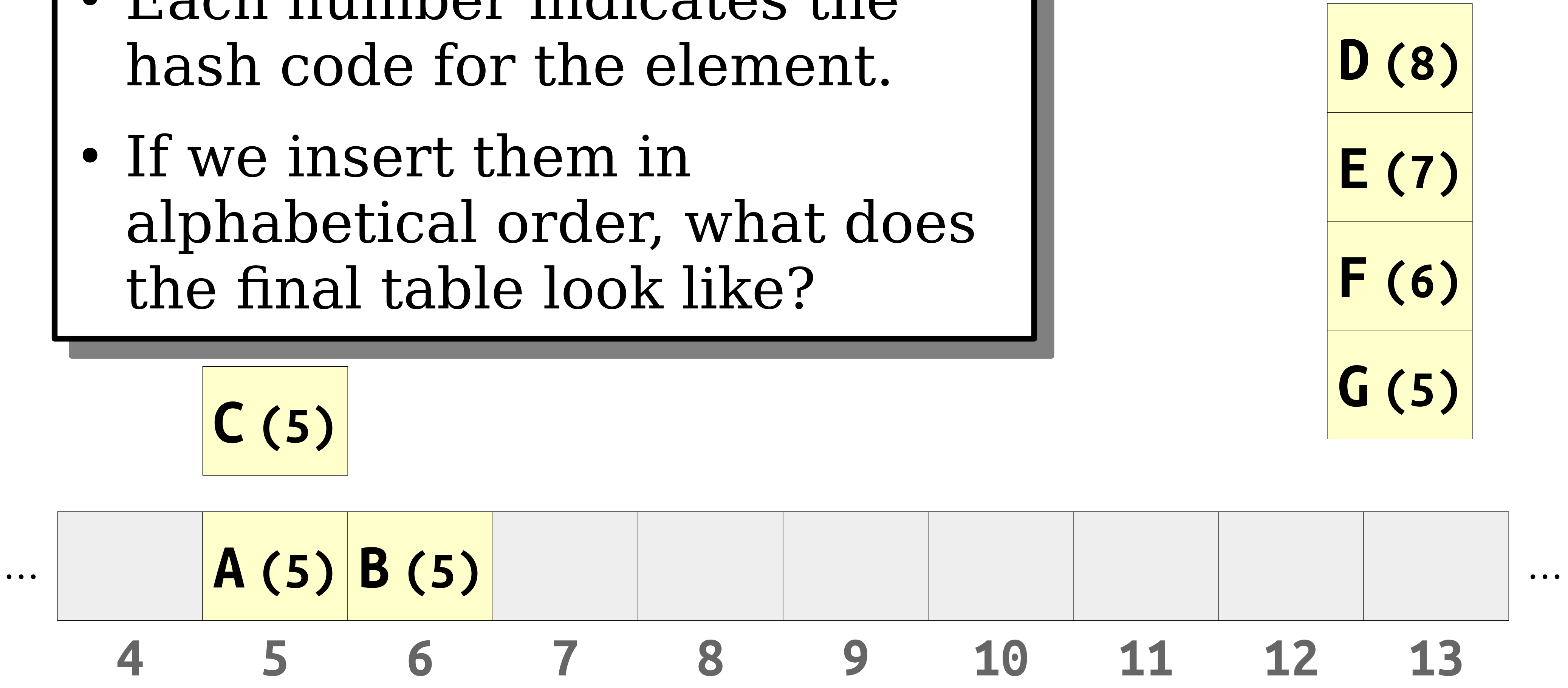

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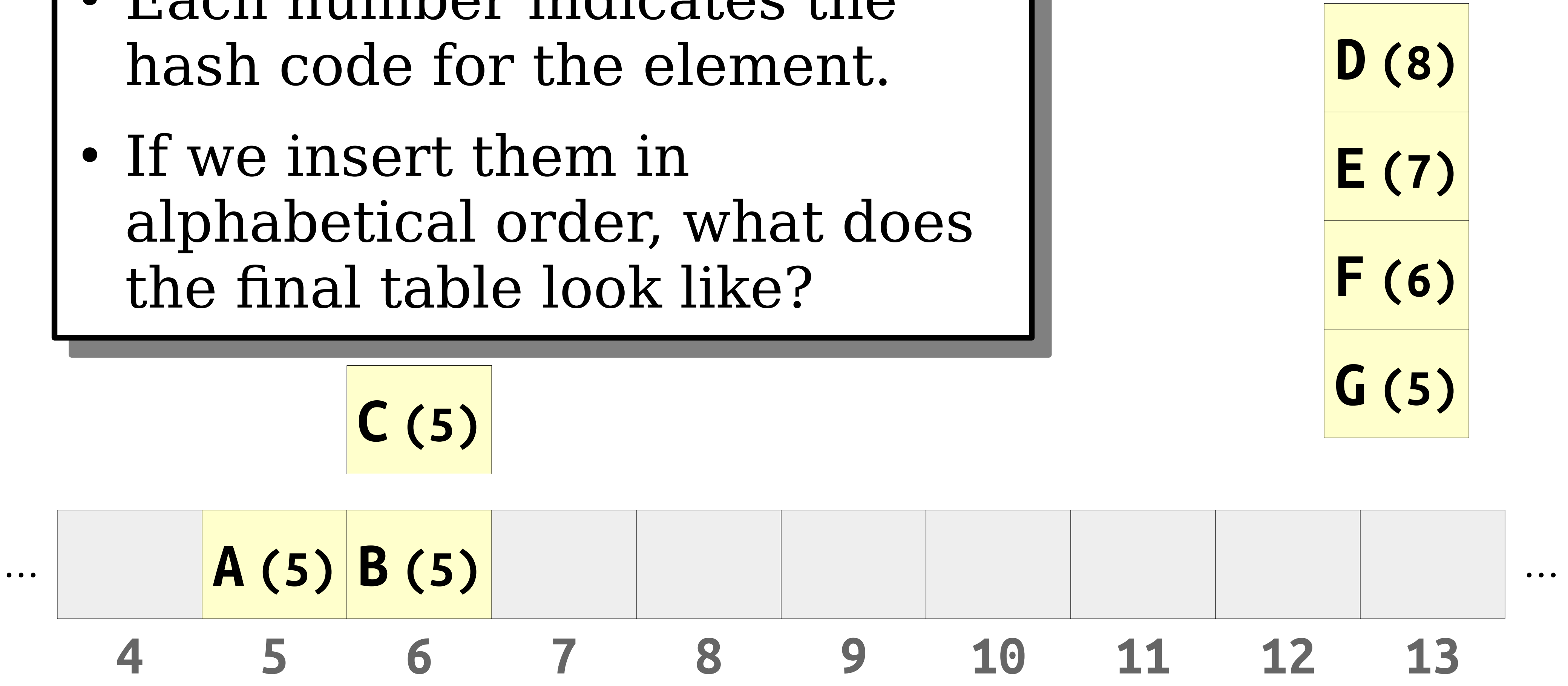

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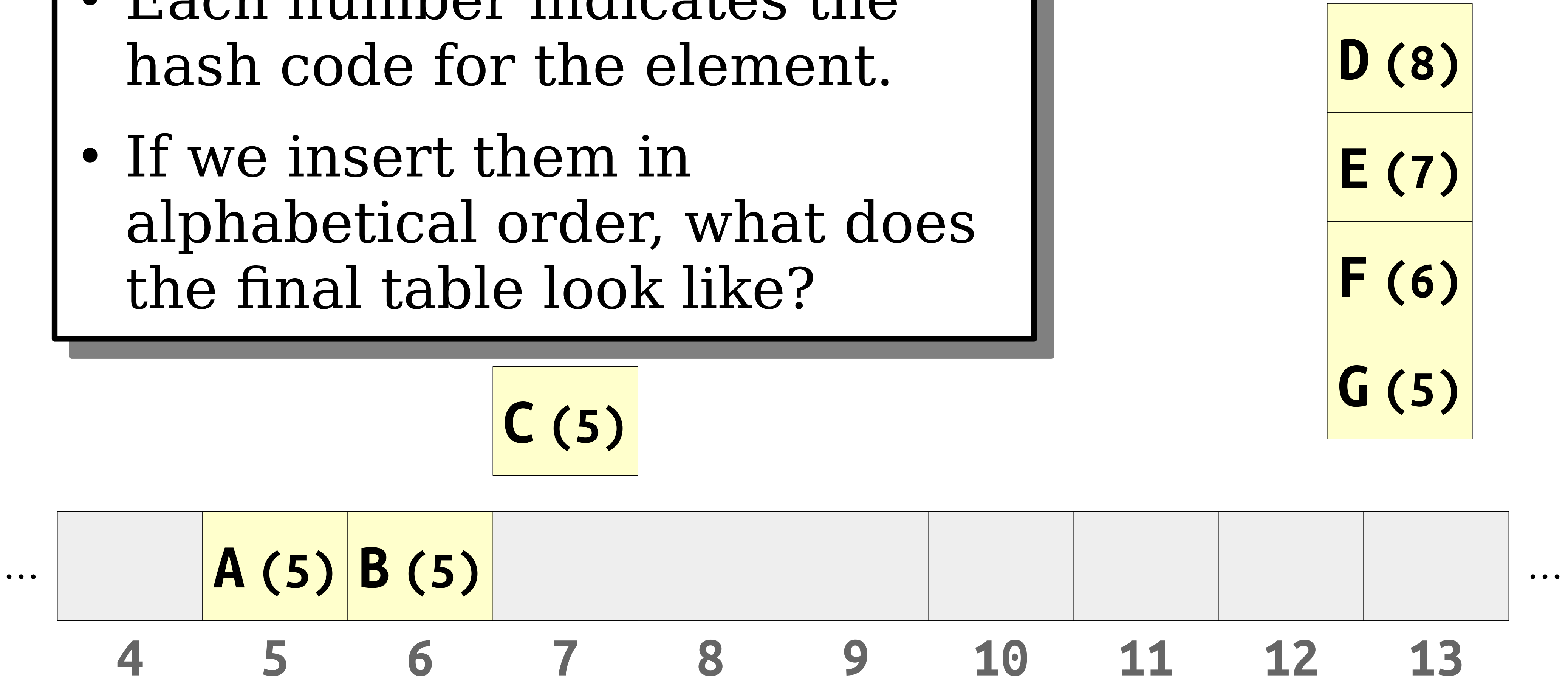

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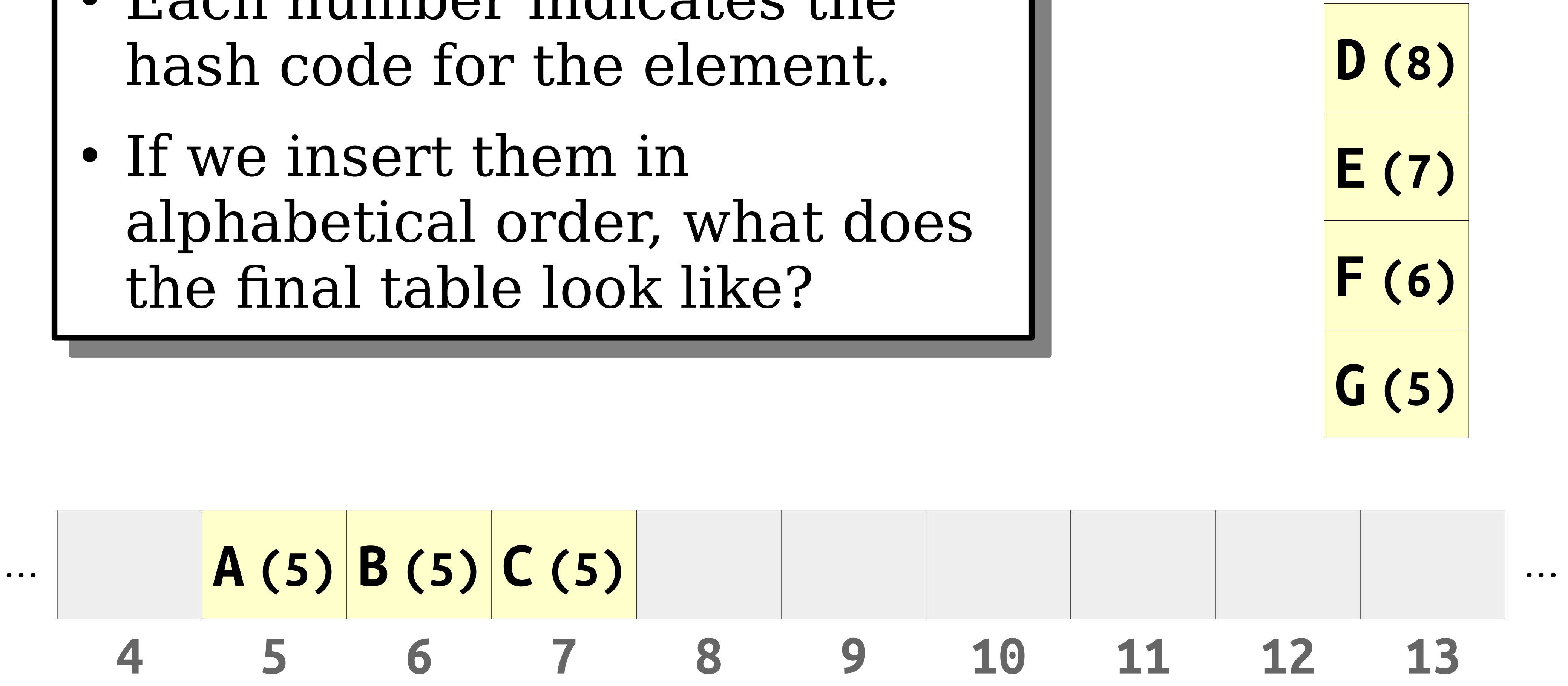

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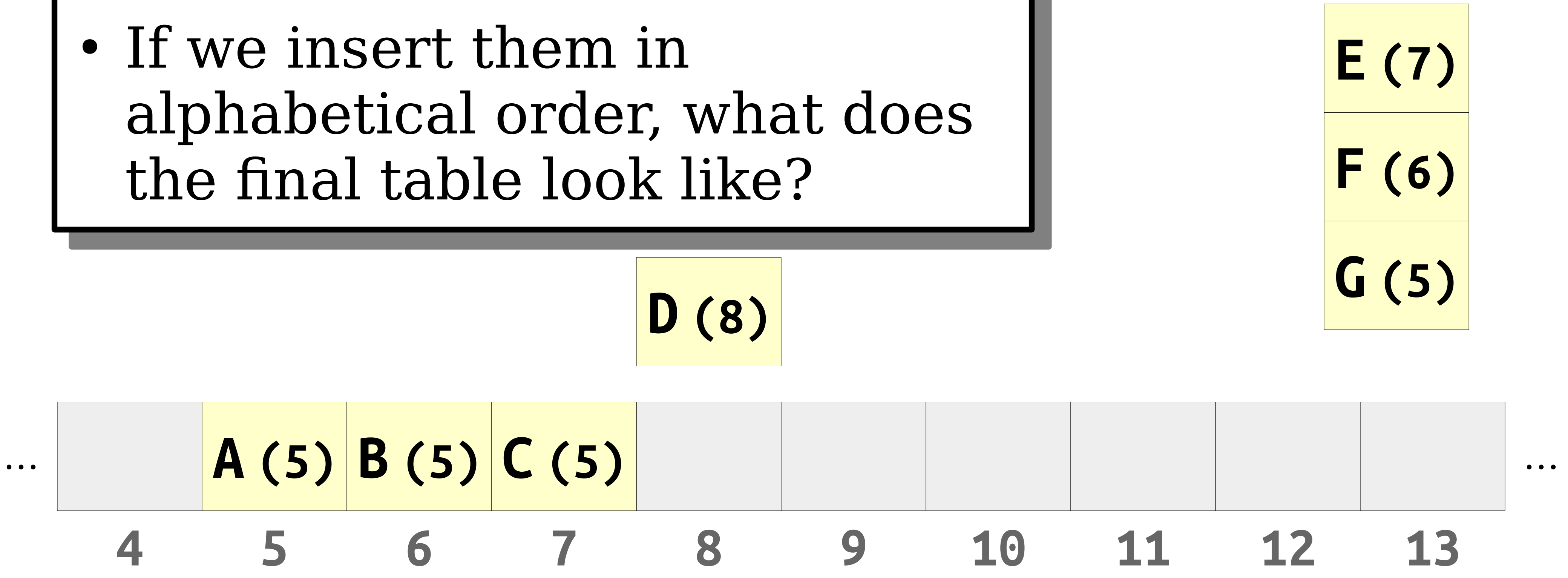

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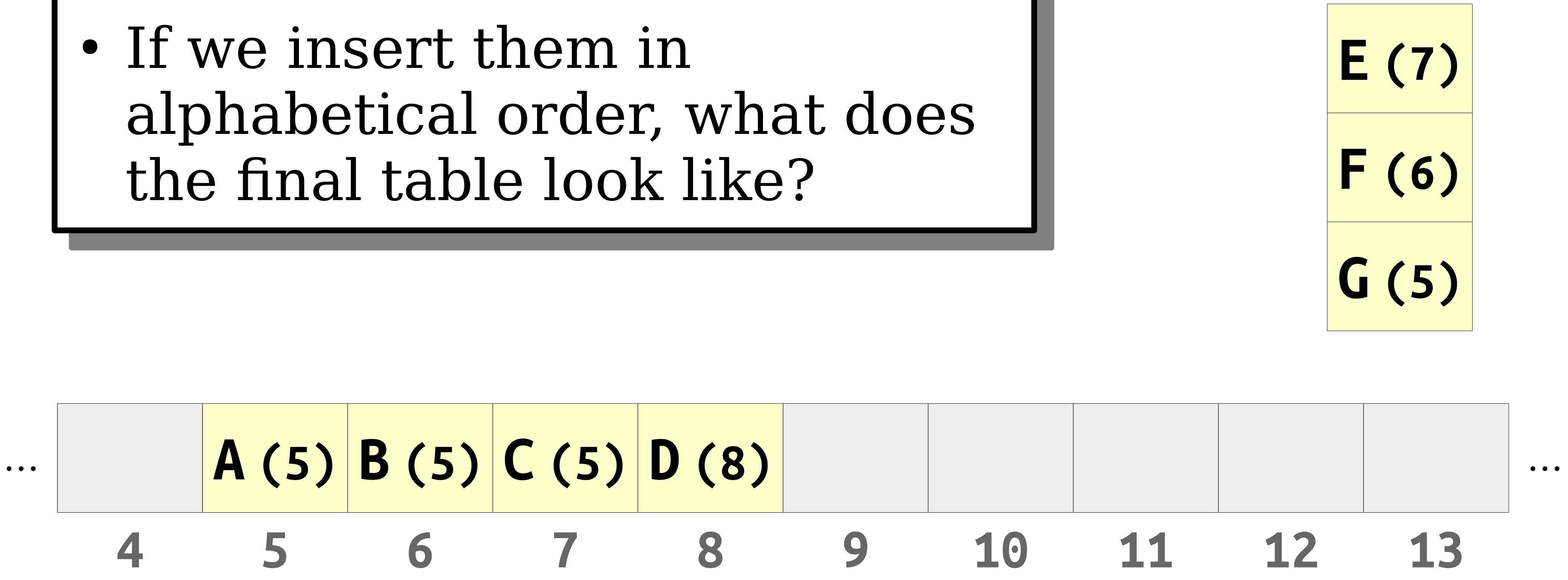

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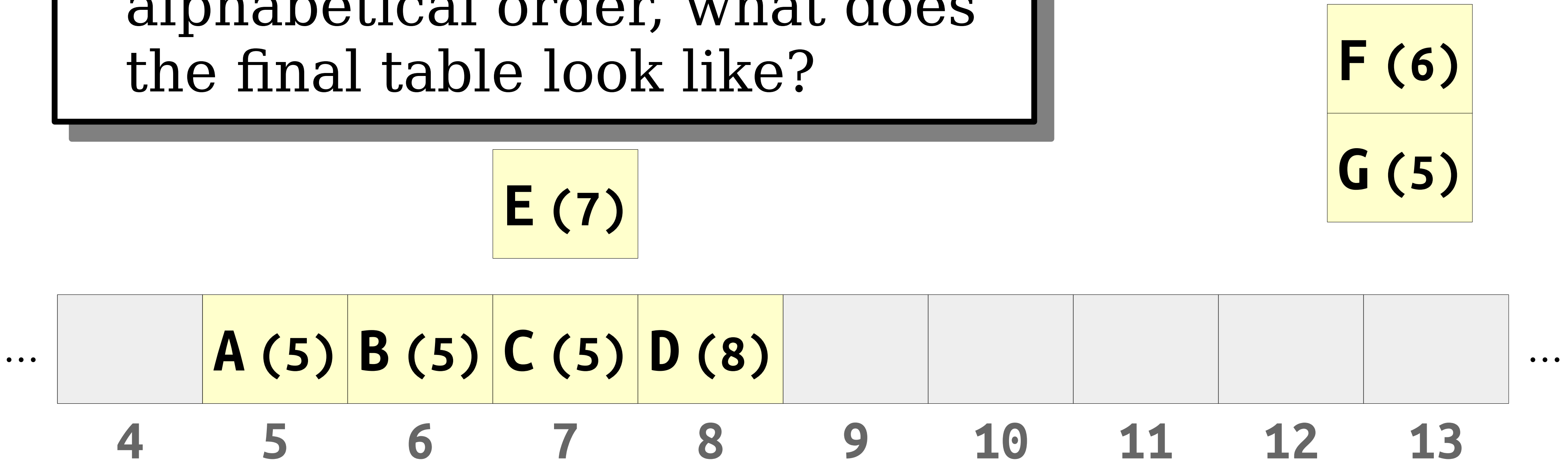

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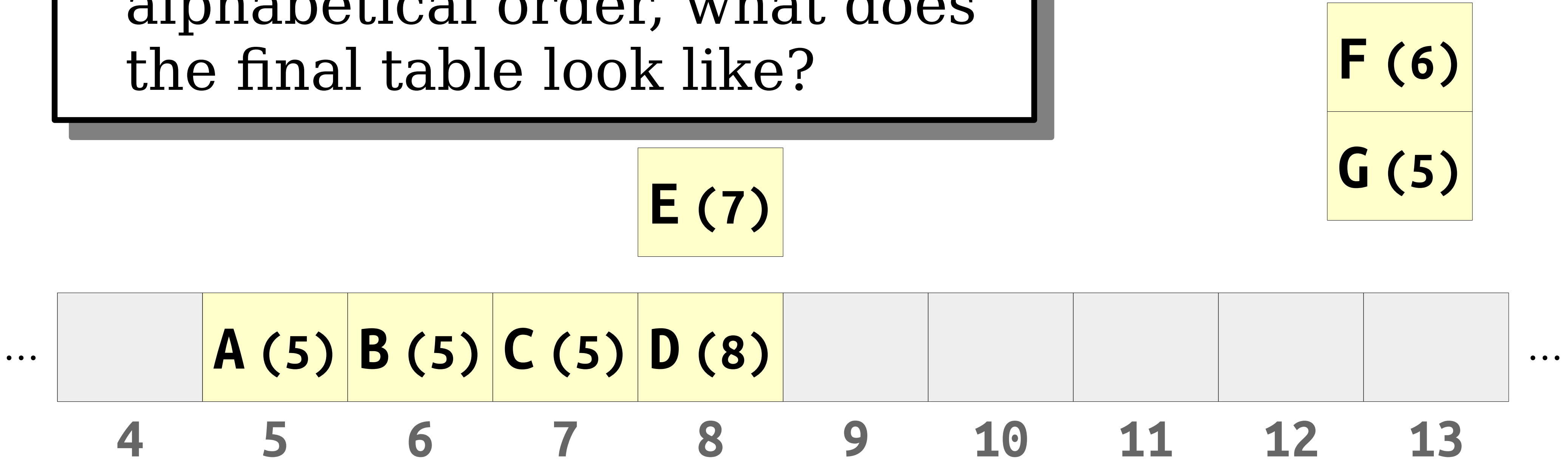

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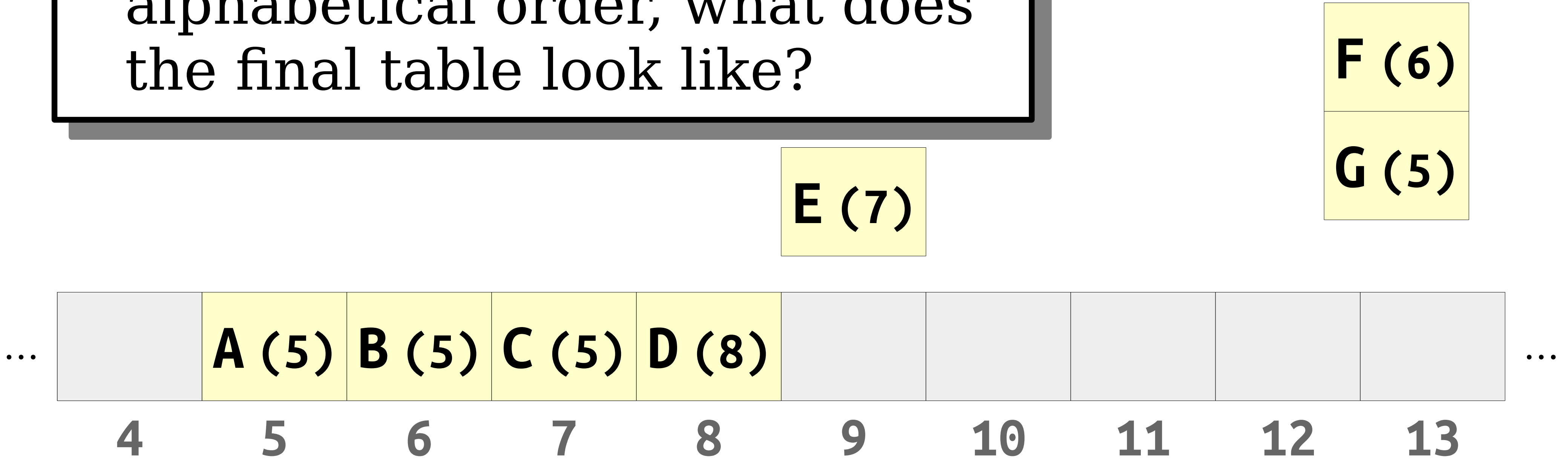

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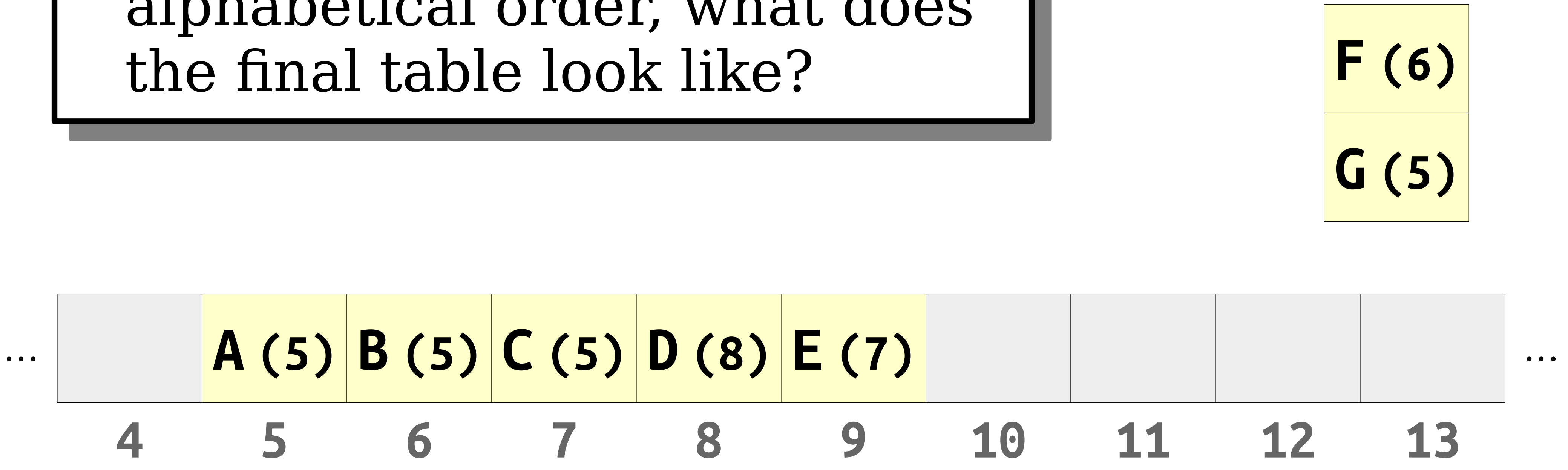

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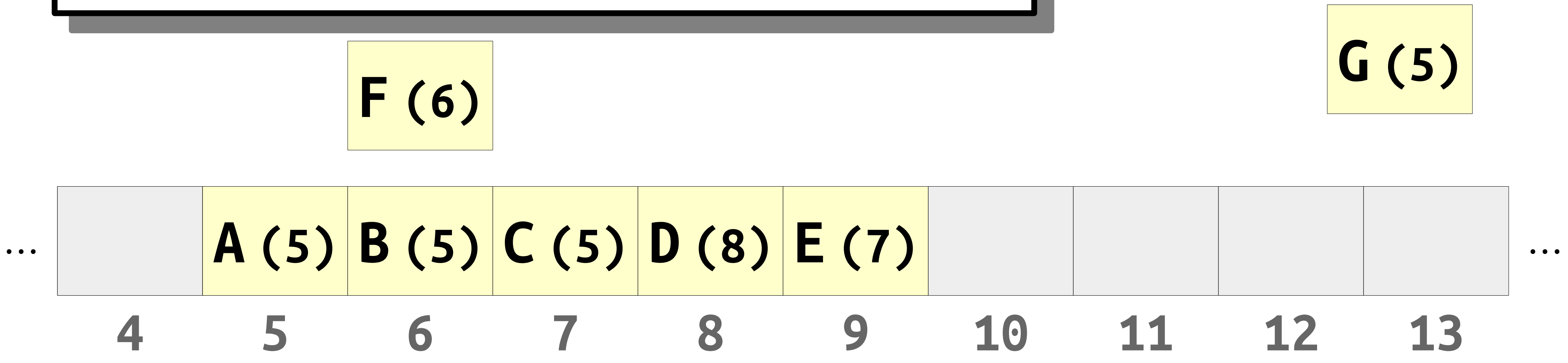

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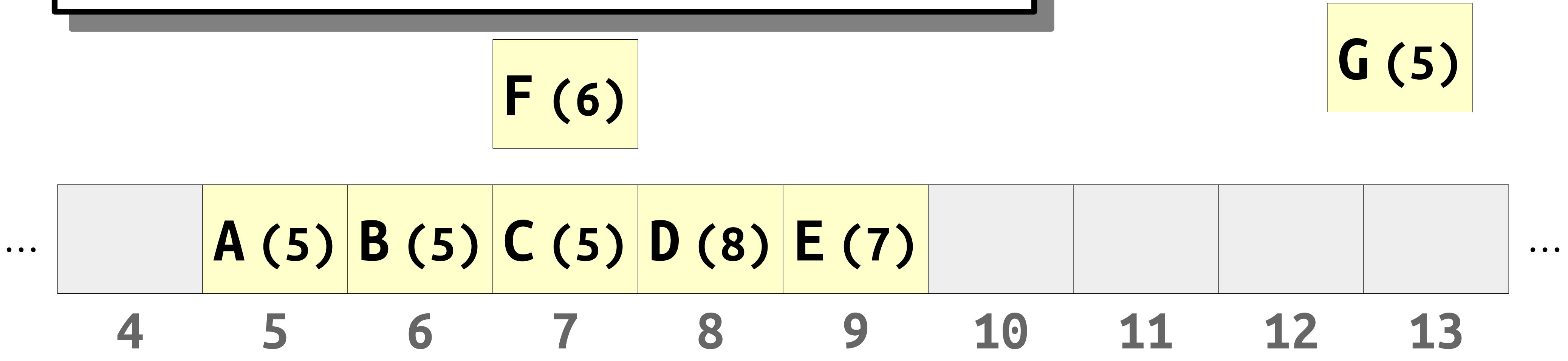

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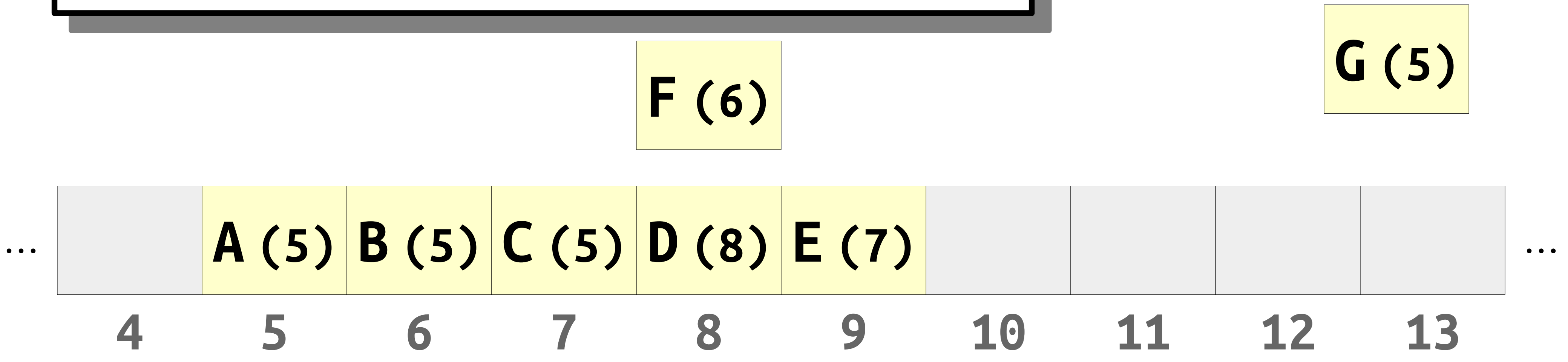

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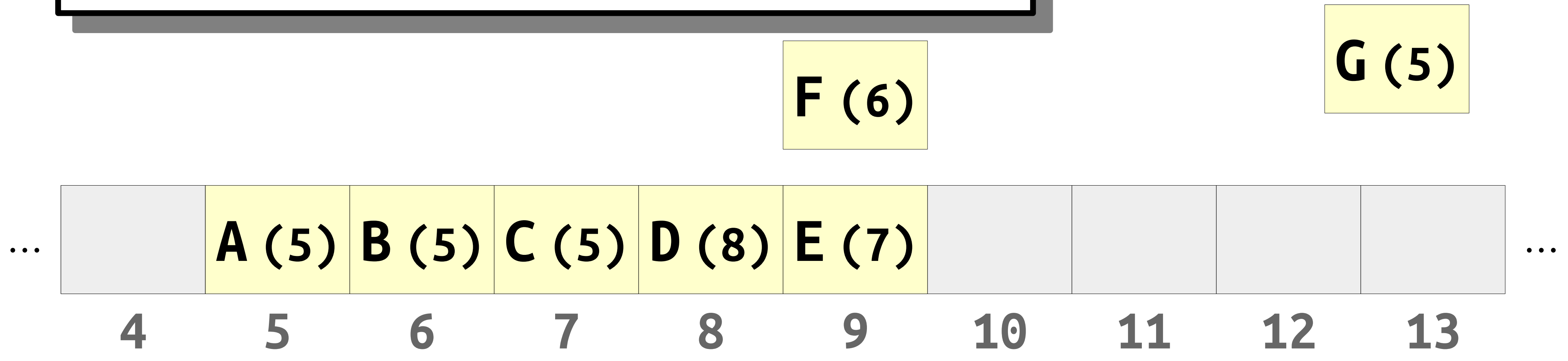

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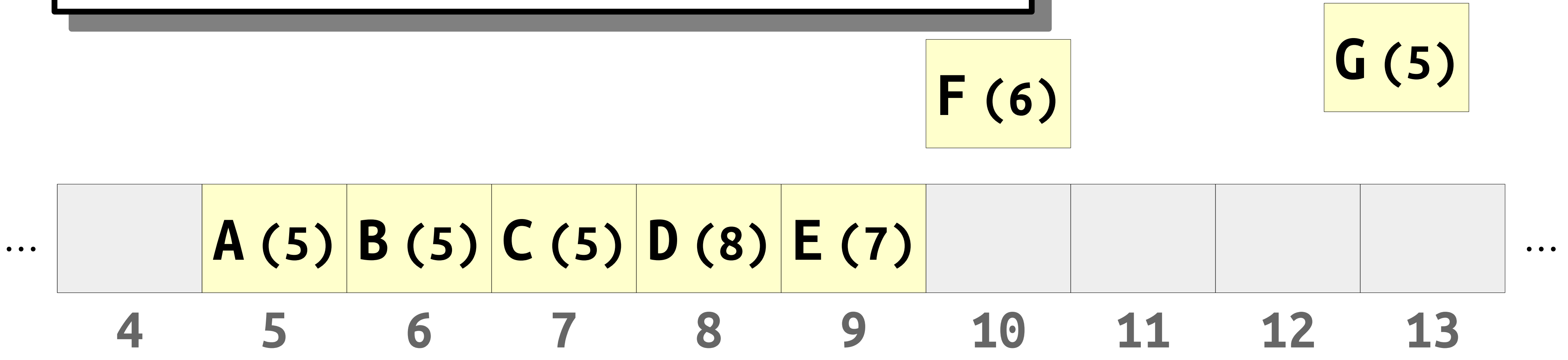

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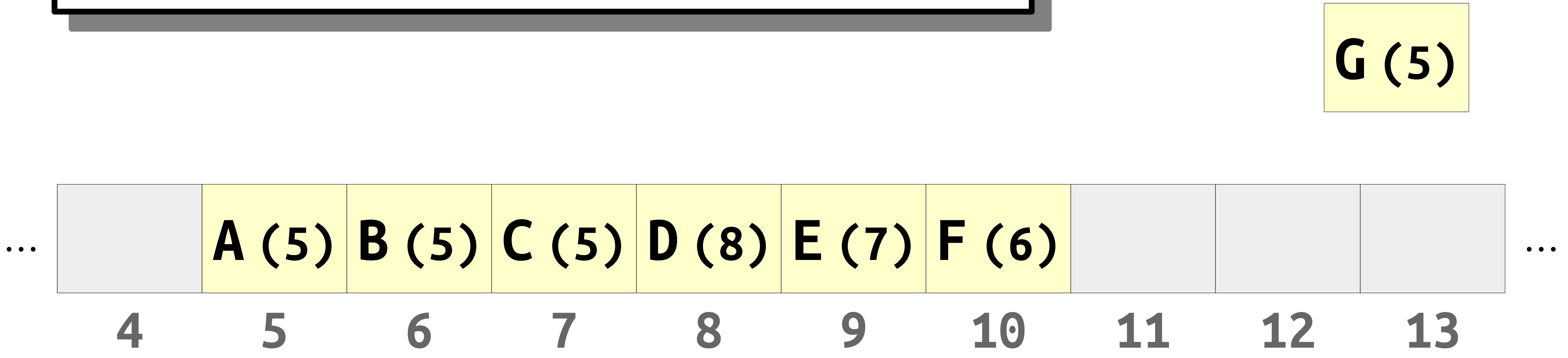

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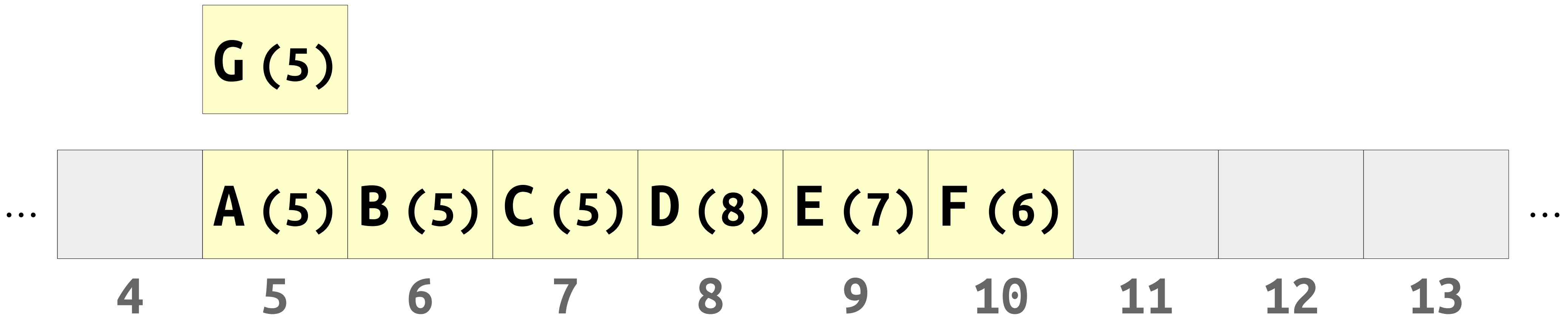

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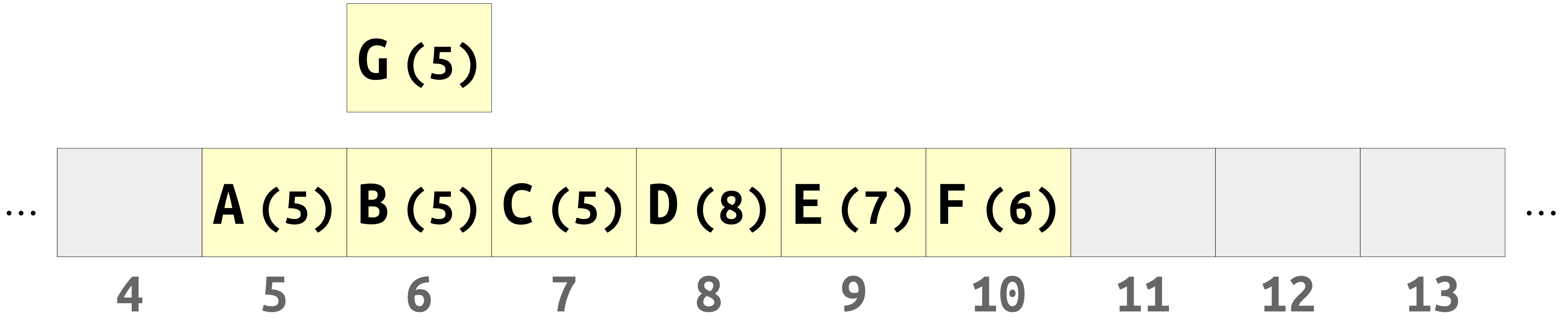

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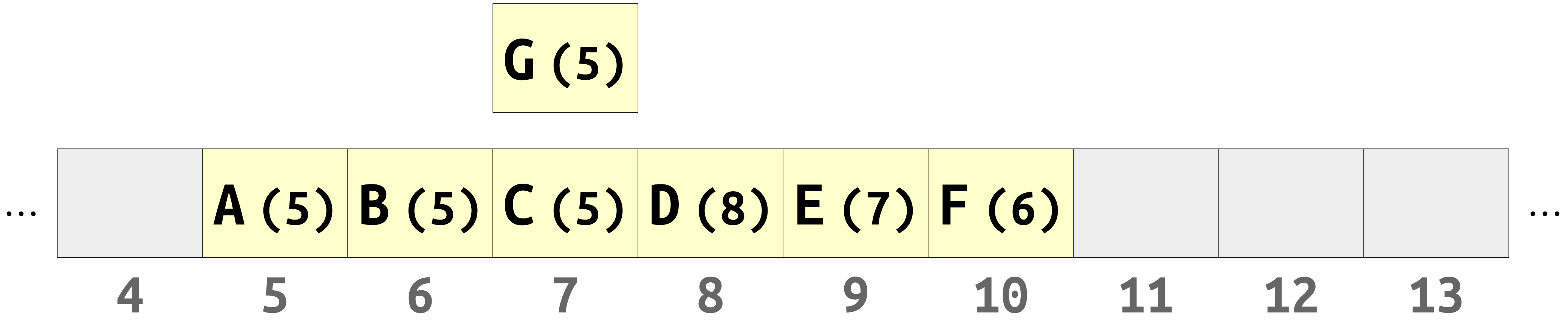

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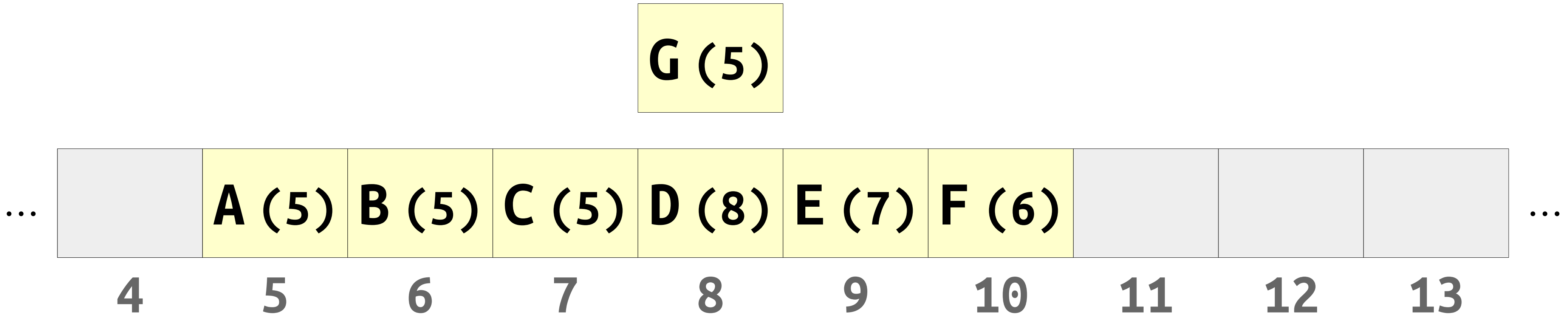

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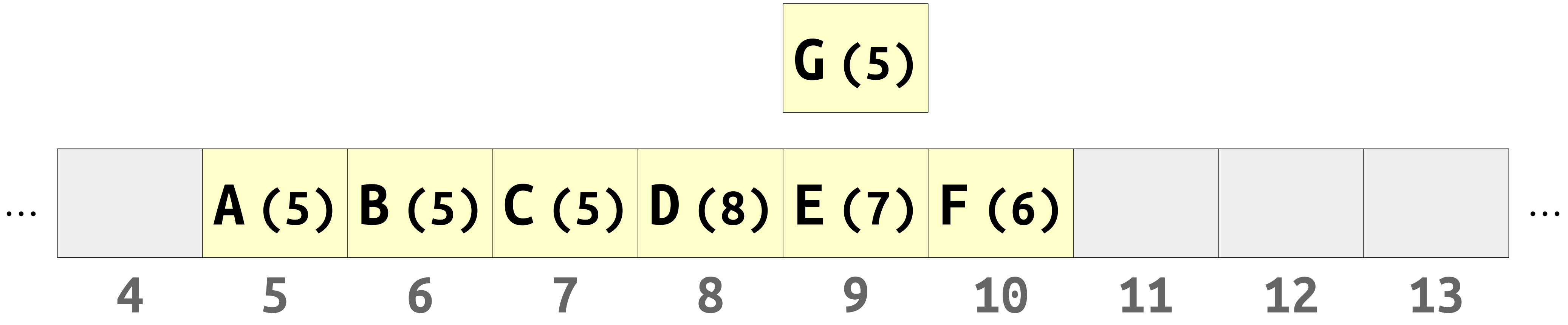

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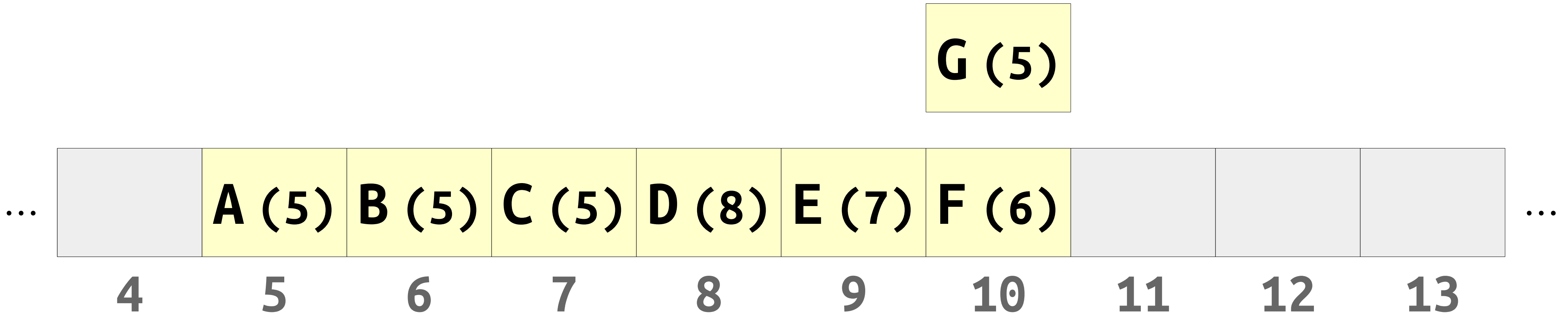

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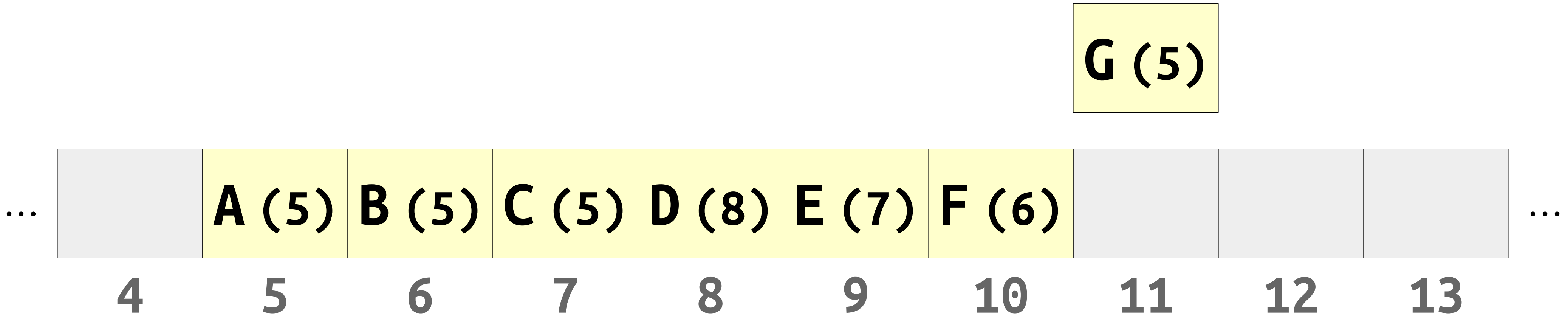

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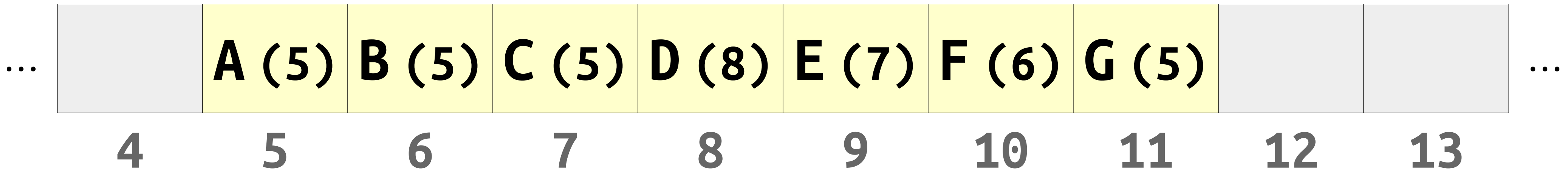
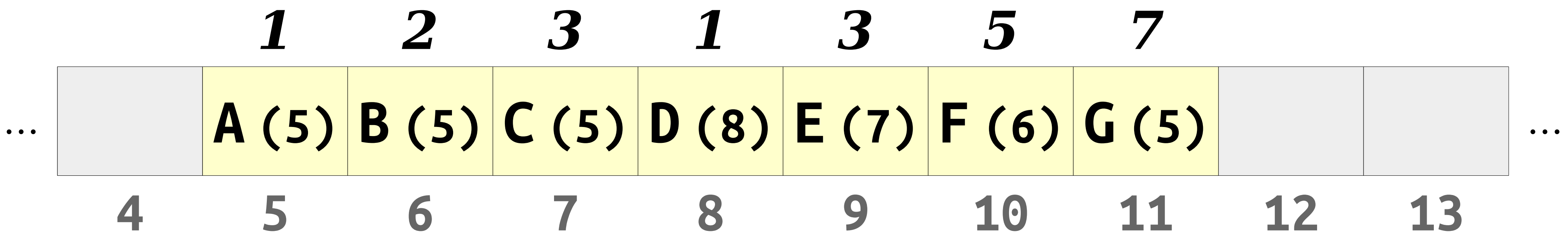


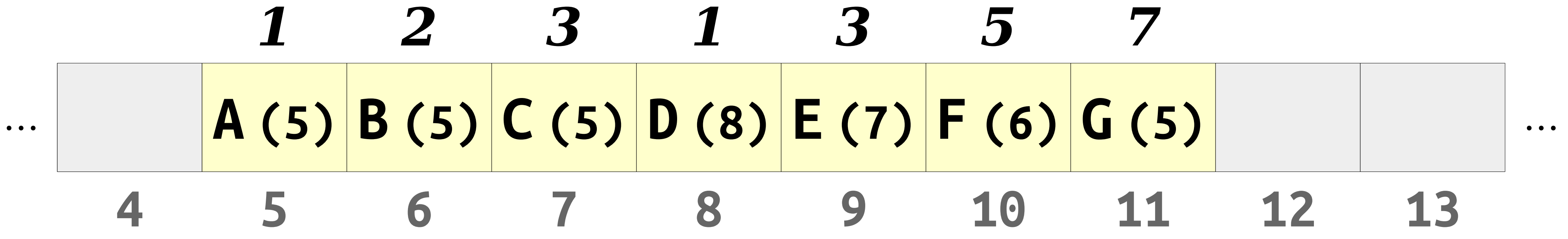
A Question of Fairness

- Suppose we look up each of these elements. How many slots would we need to look at to find each of them?



A Question of Fairness

- Suppose we look up each of these elements. How many slots would we need to look at to find each of them?
- There's a large *variance* in how long it's going to take to find things.
- How can we fix this?



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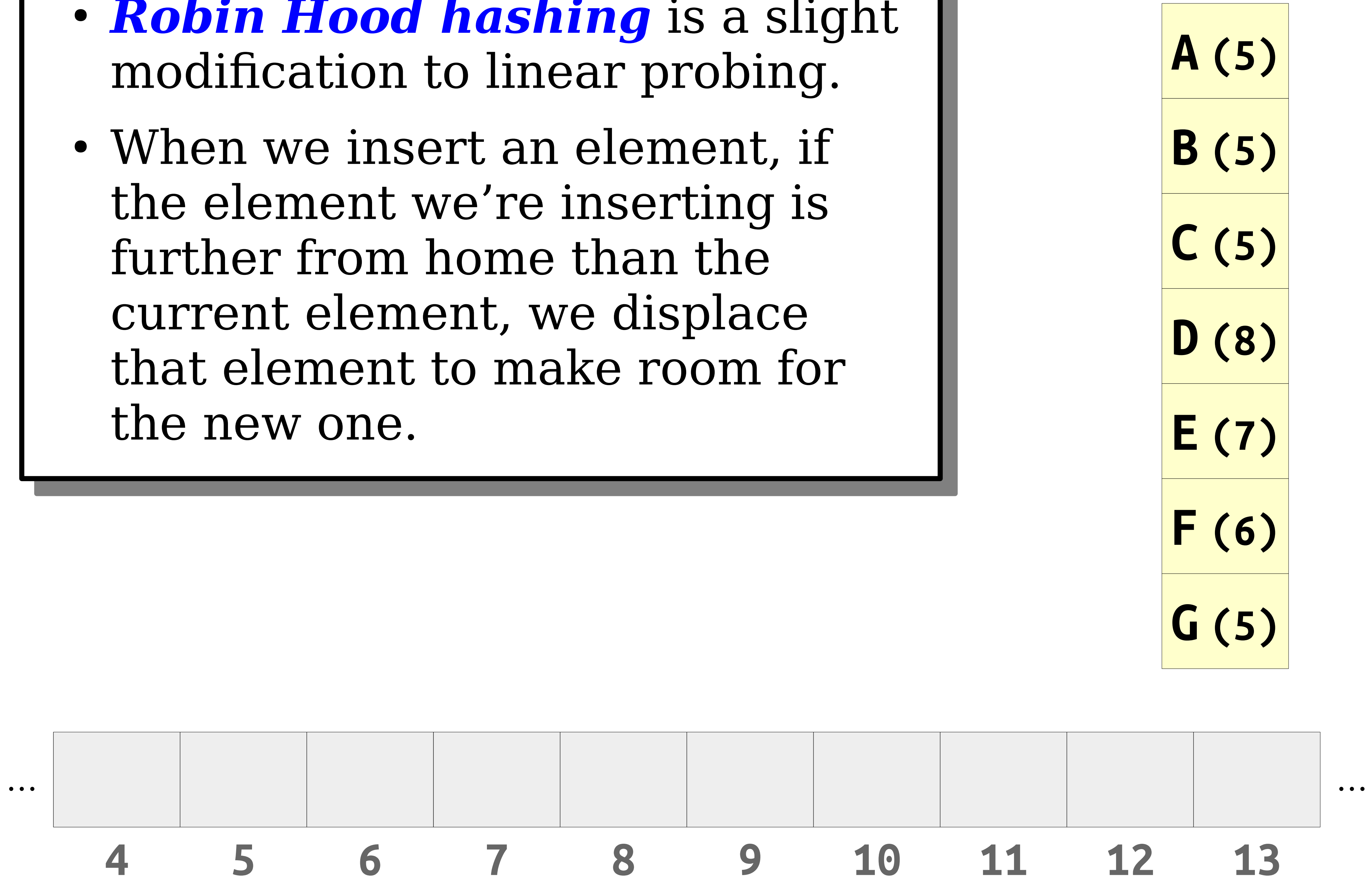
- When we insert an element, if the element we're inserting is further from home than the current element, we displace that element to make room for the new one.



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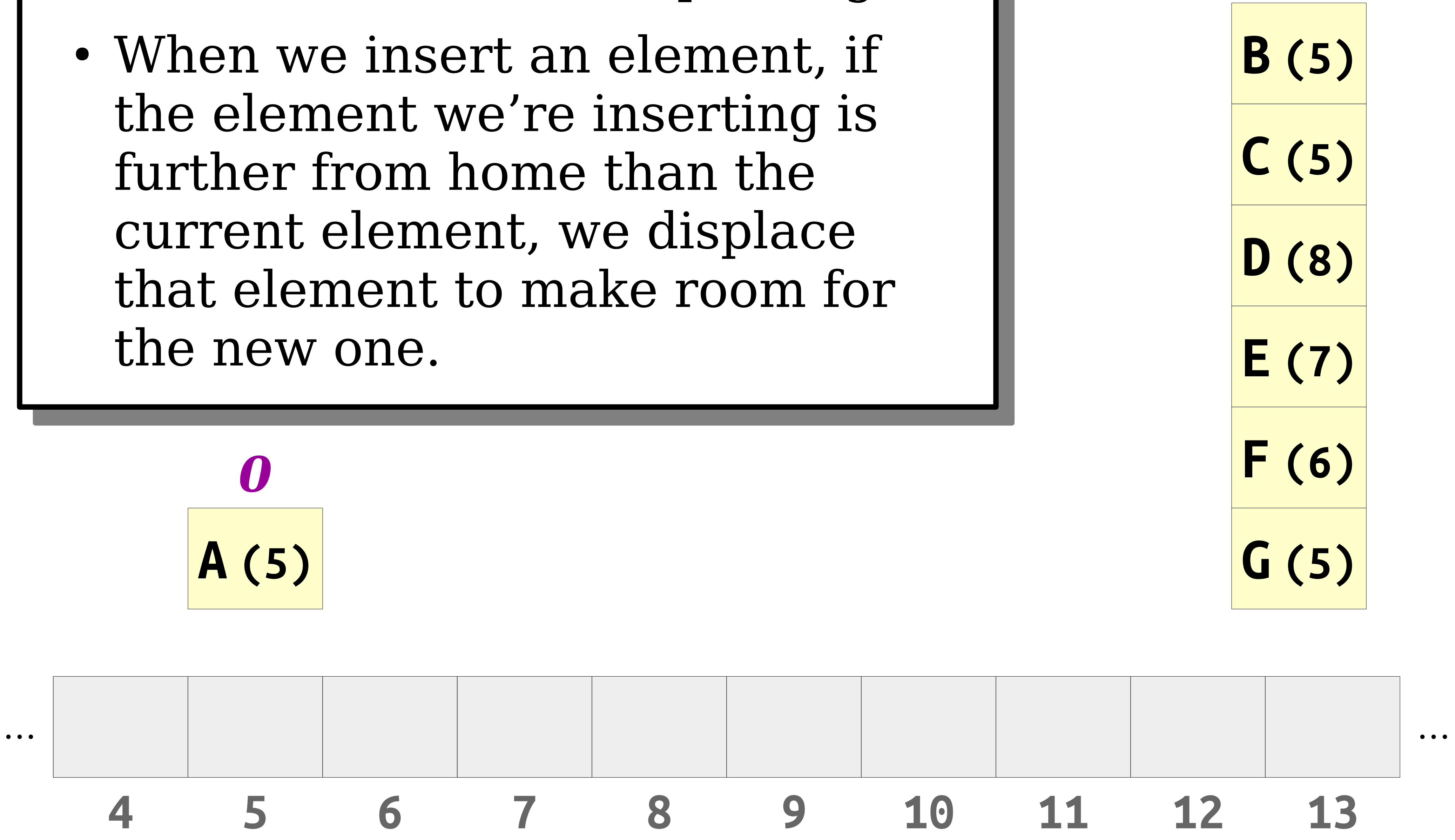

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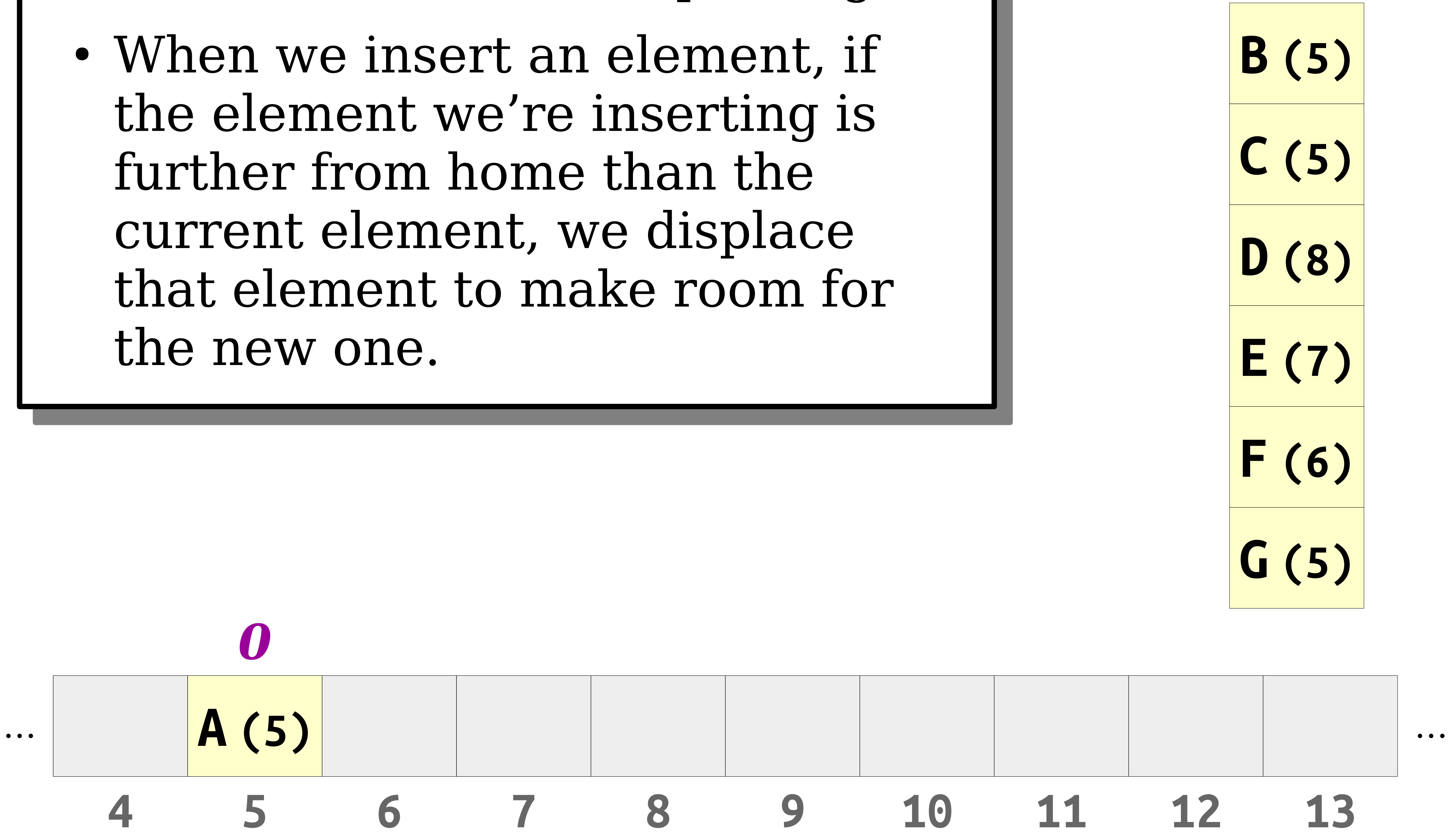

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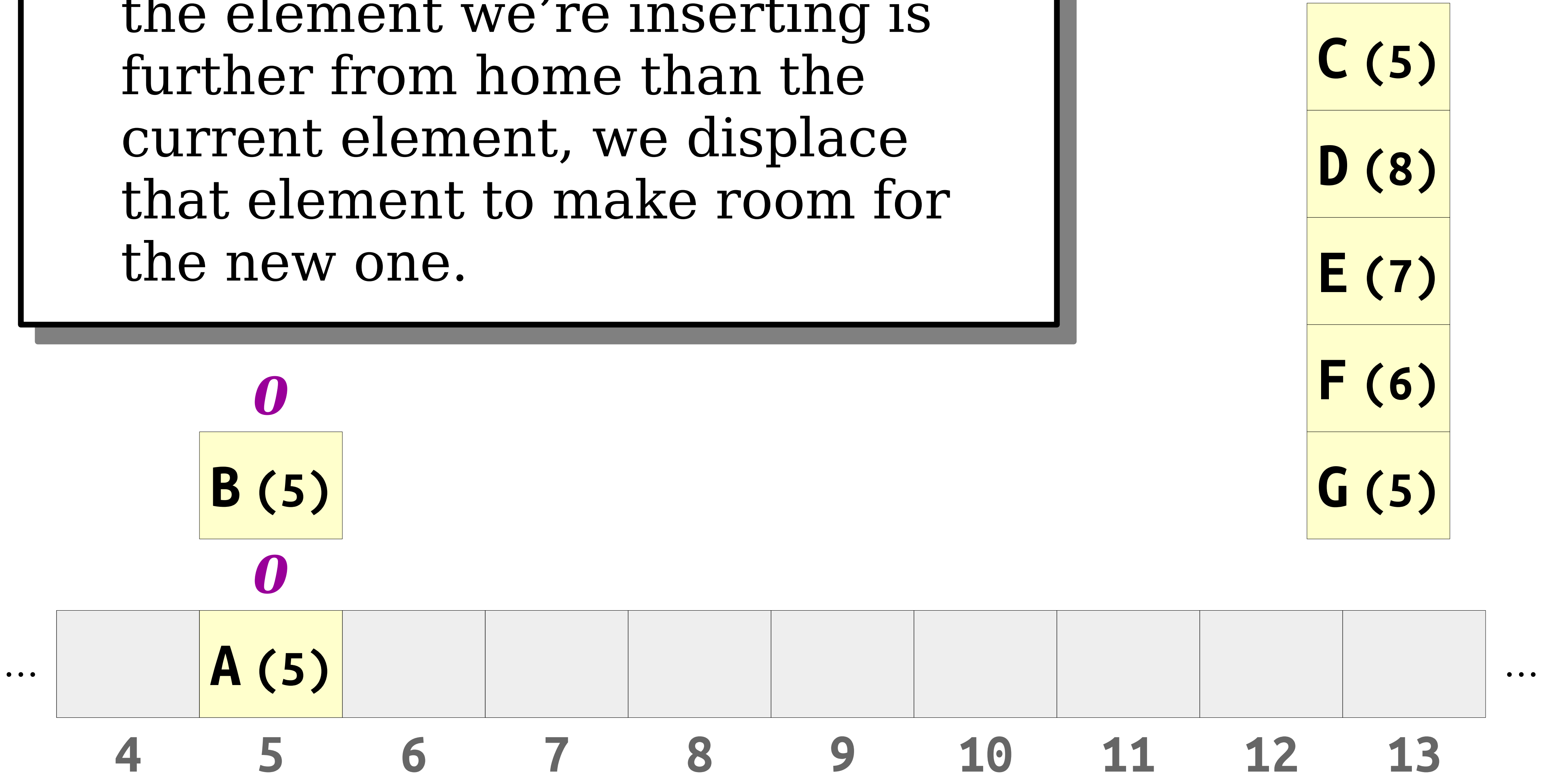

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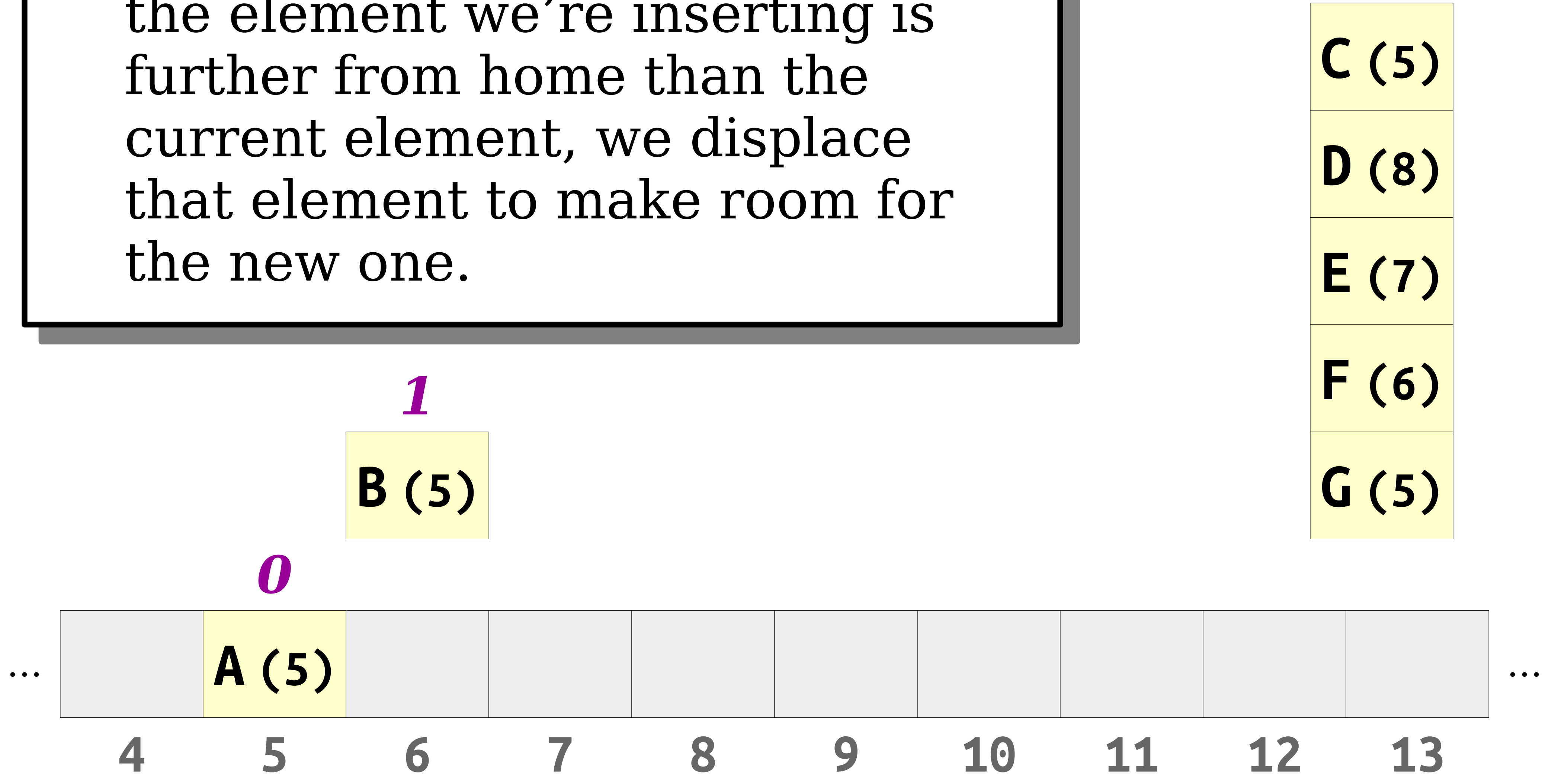

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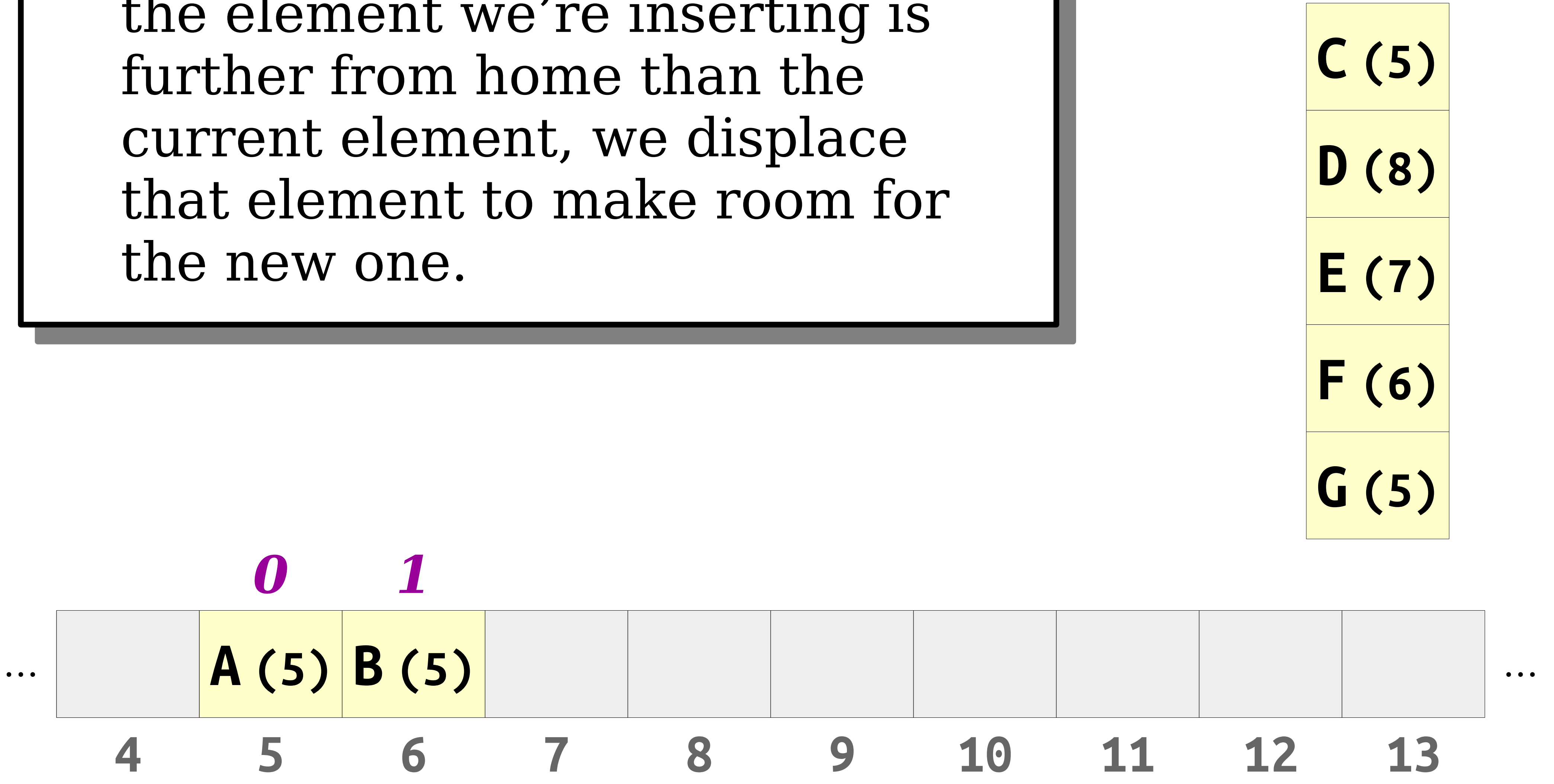

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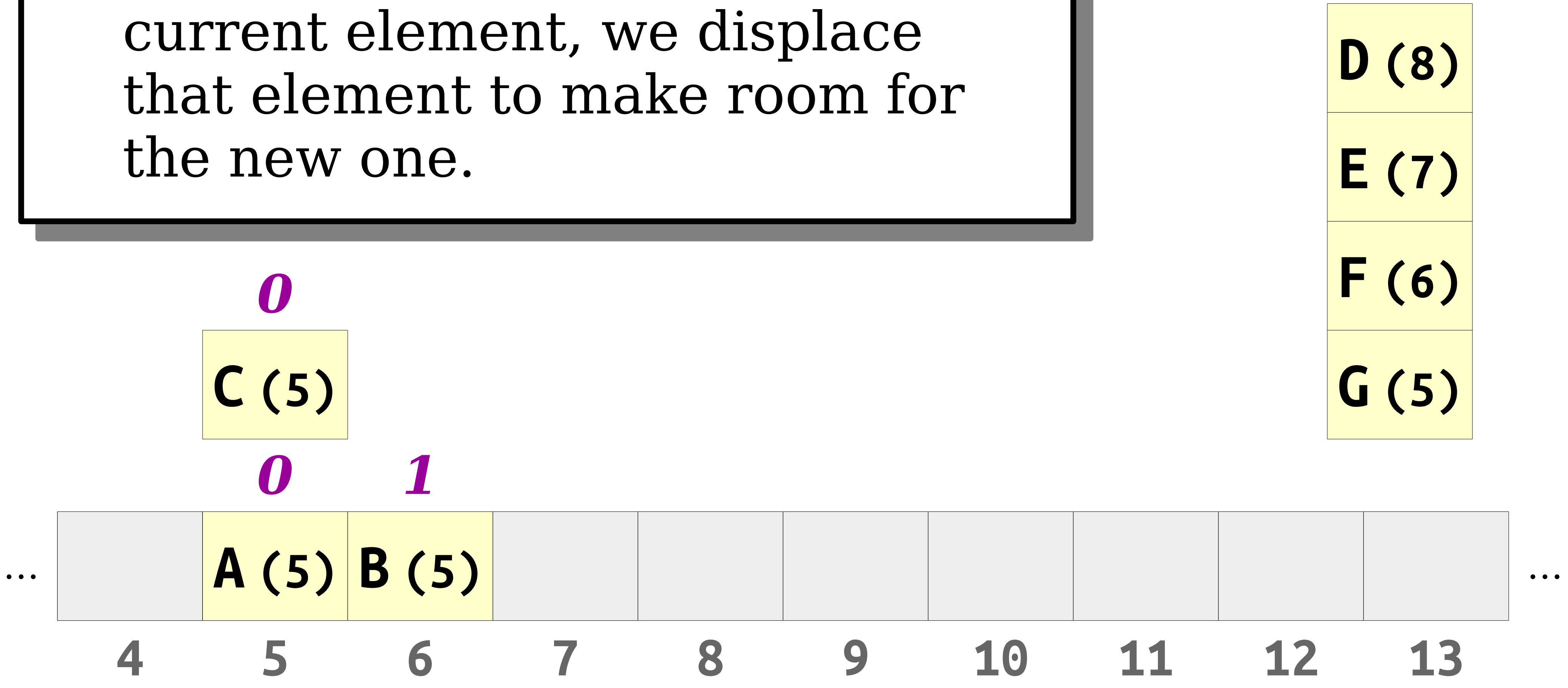

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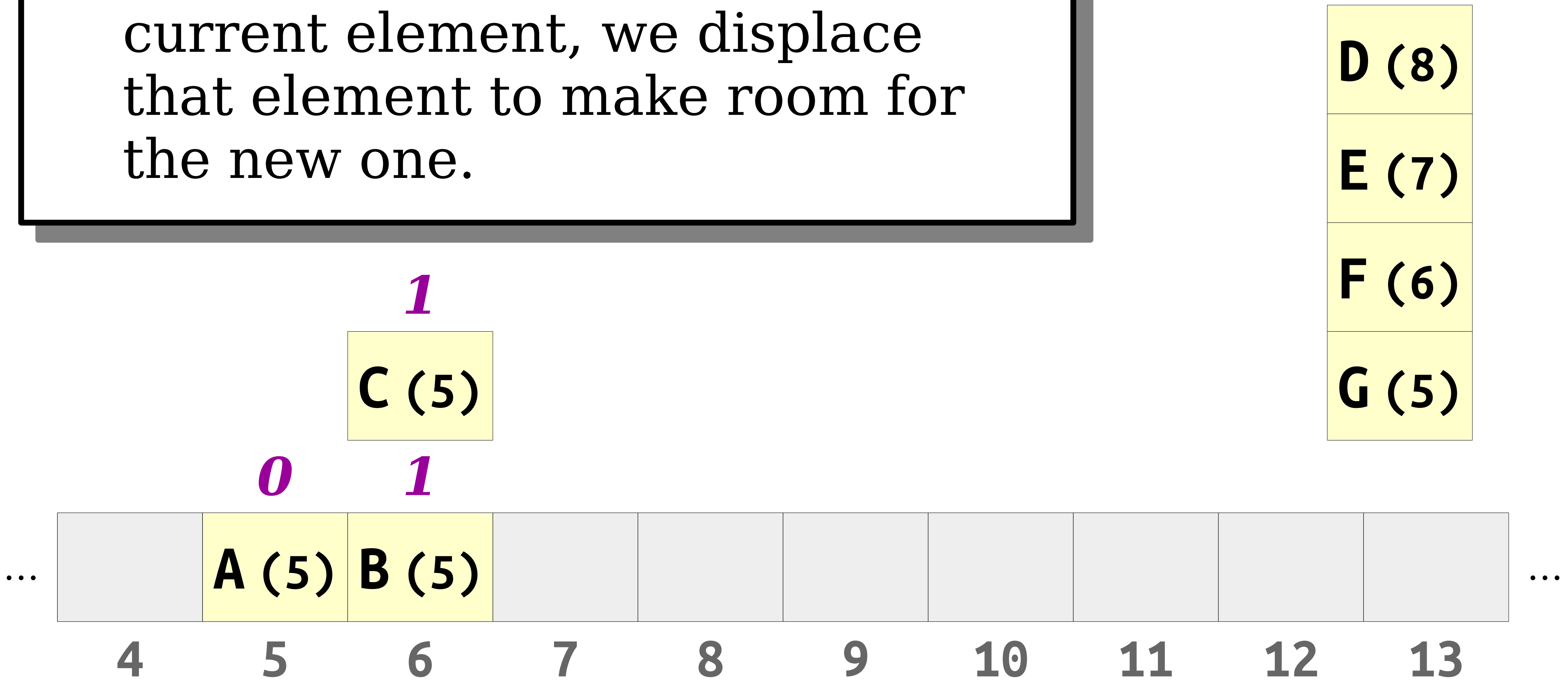

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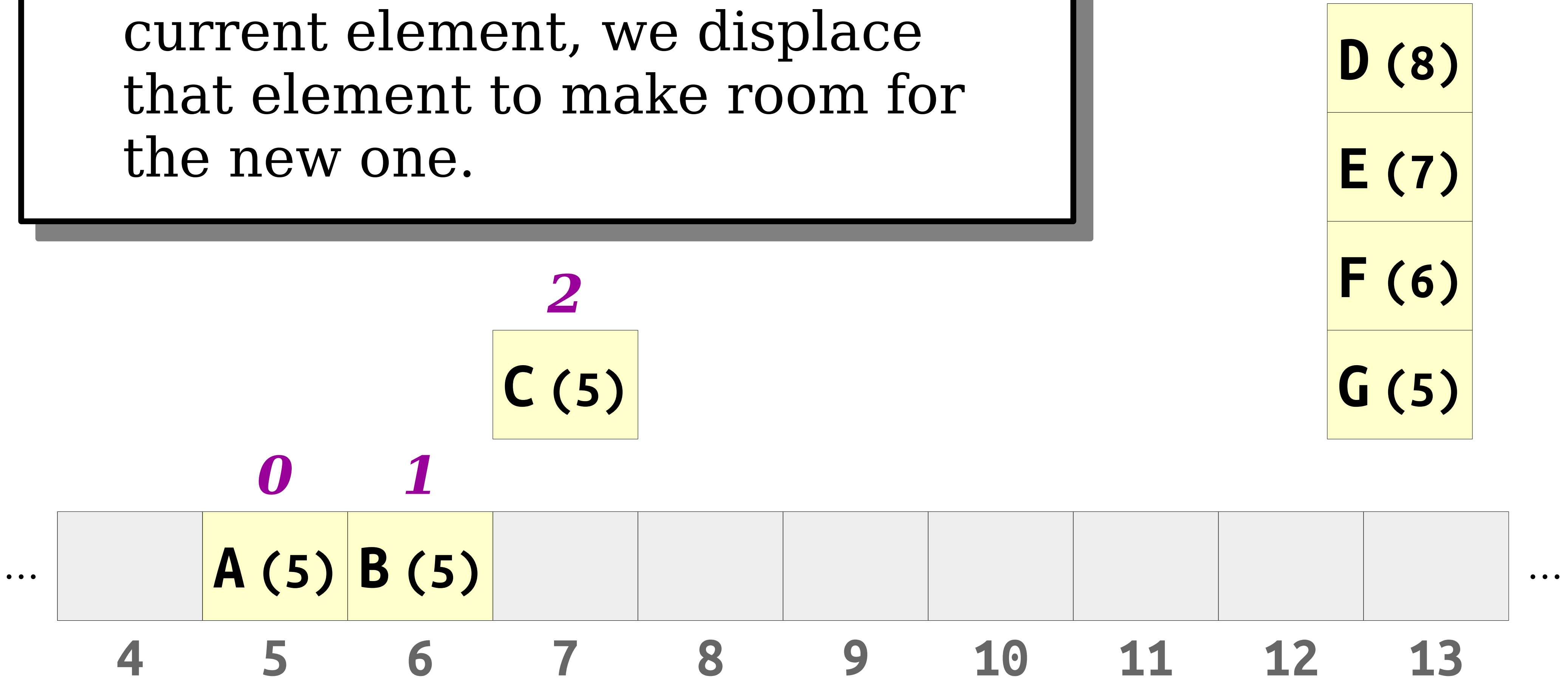

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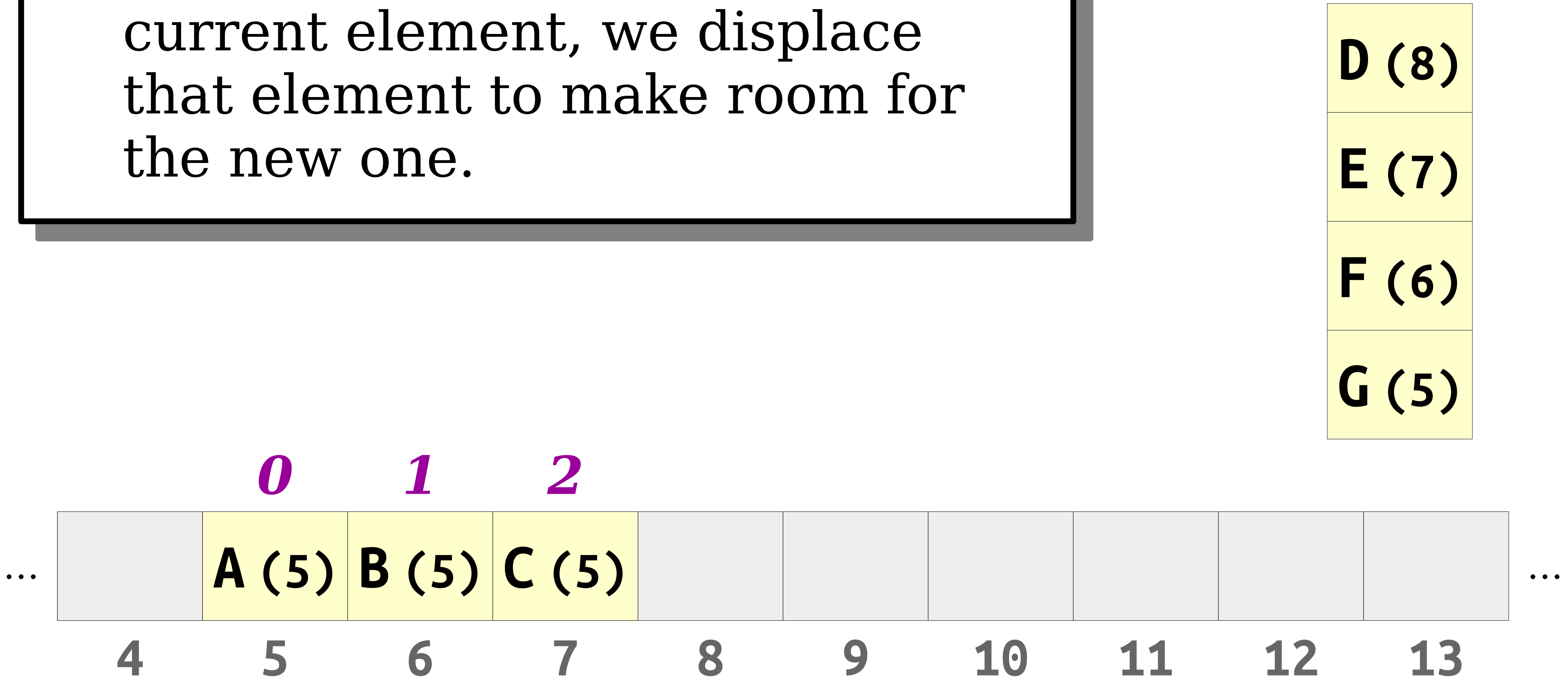

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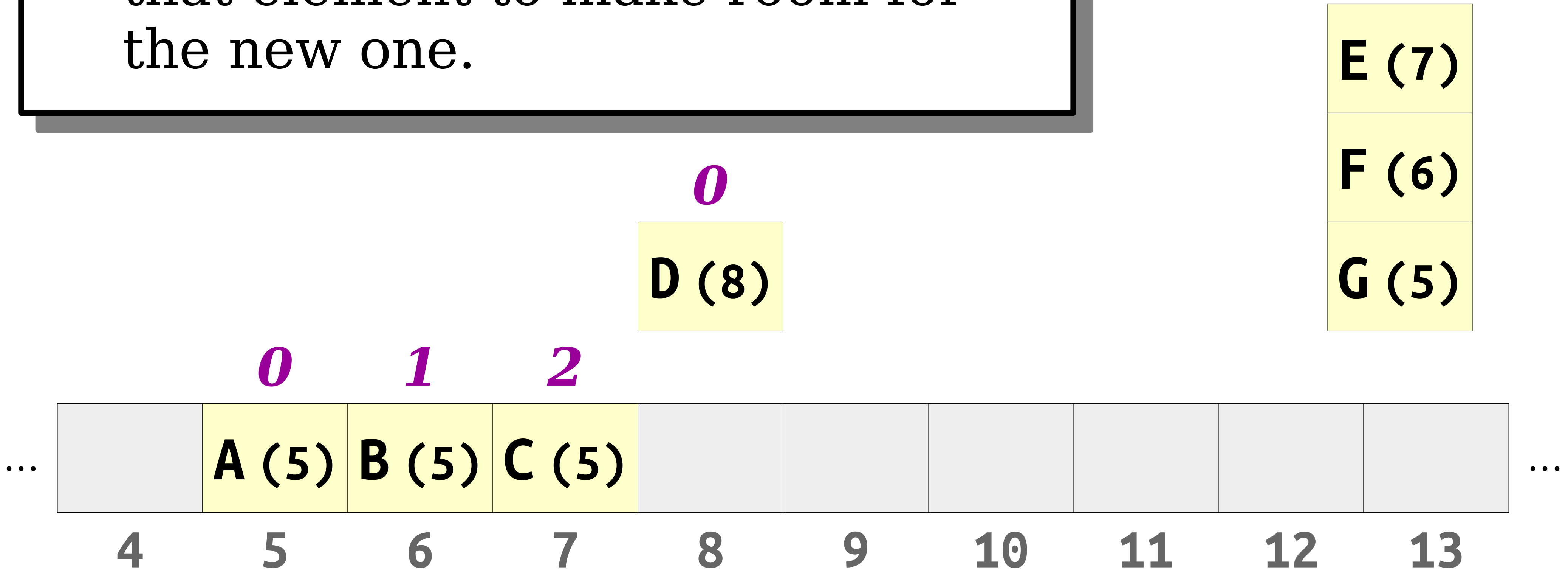

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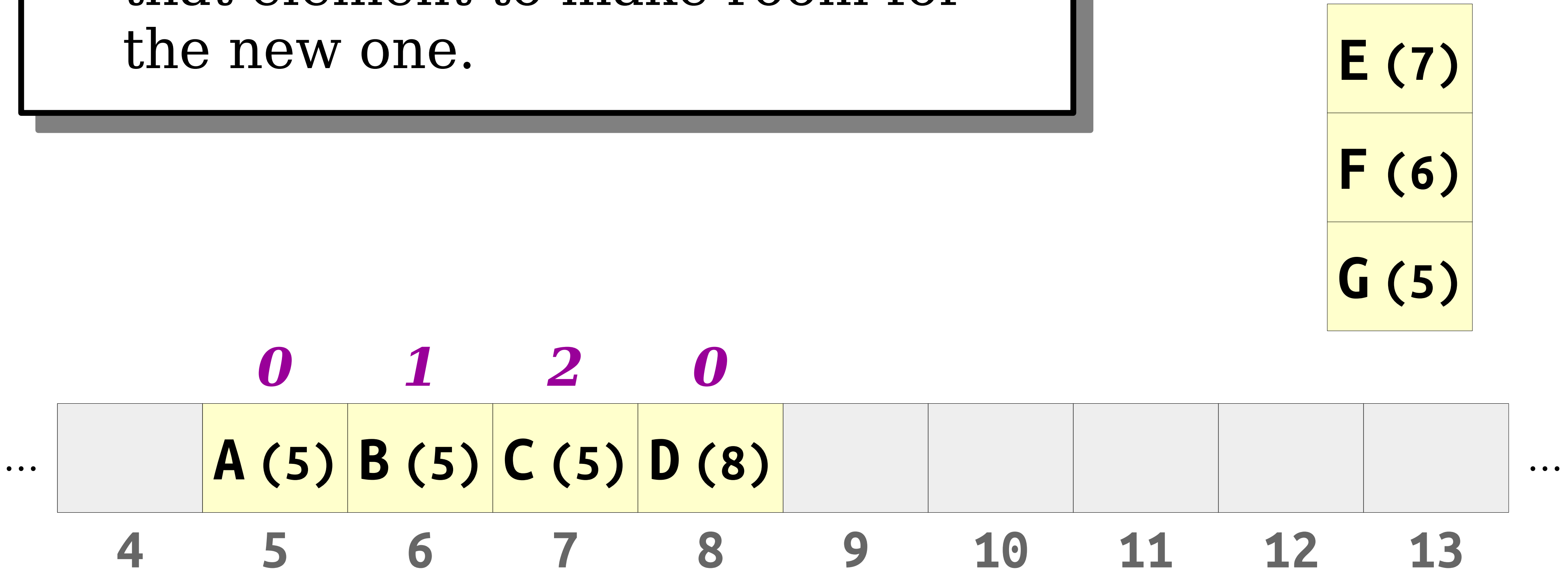

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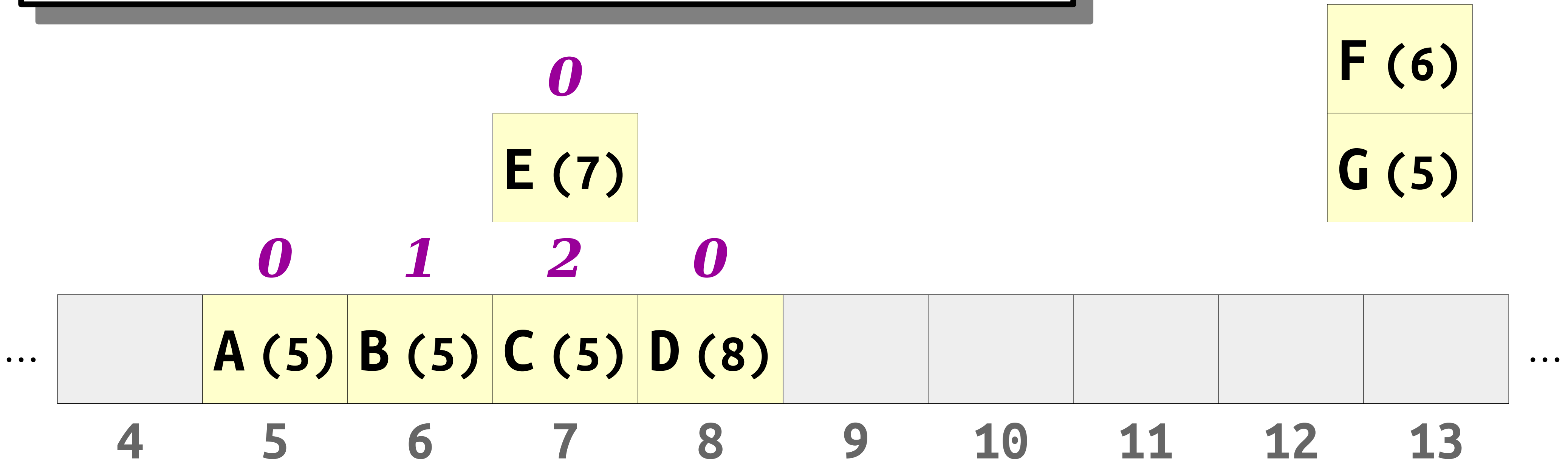

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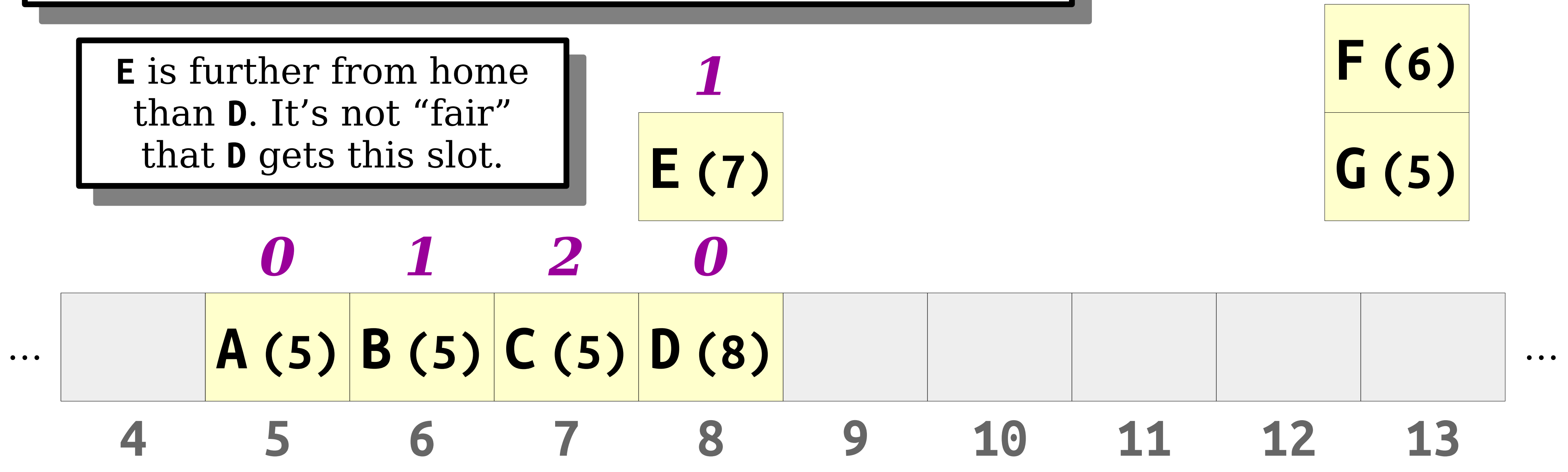
E is further from home than **D**. It's not "fair" that **D** gets this slot.



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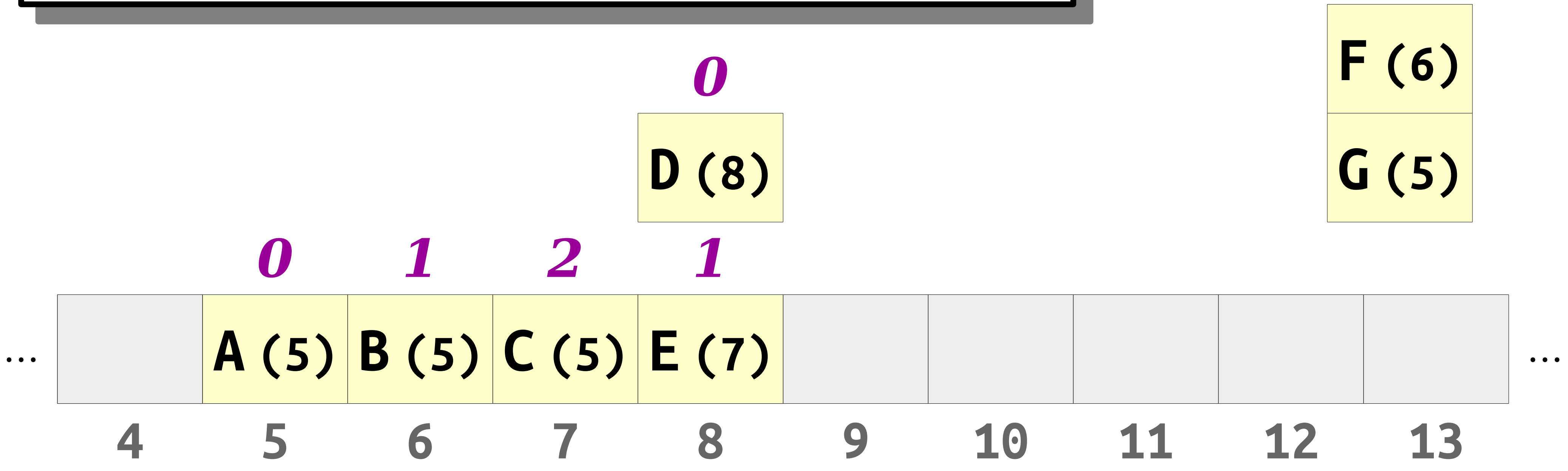

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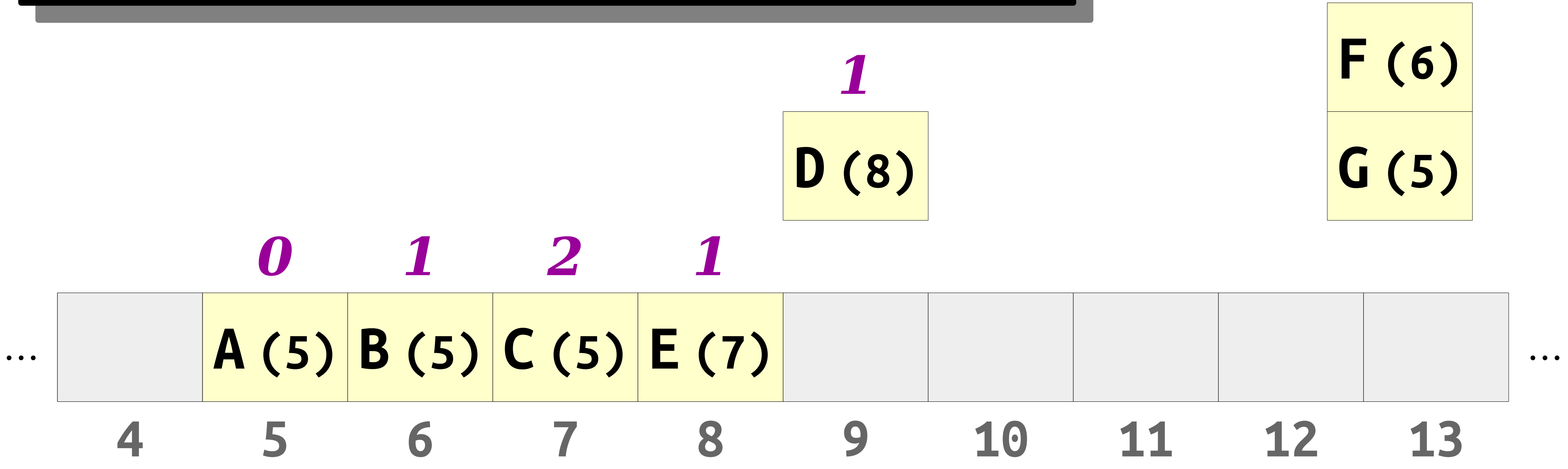

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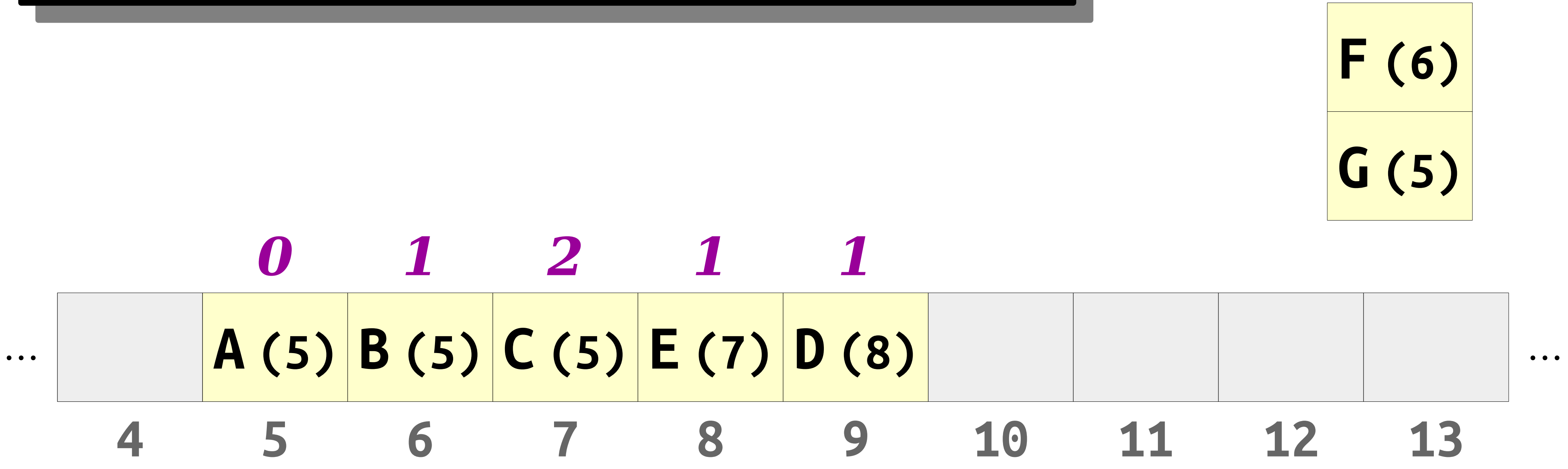

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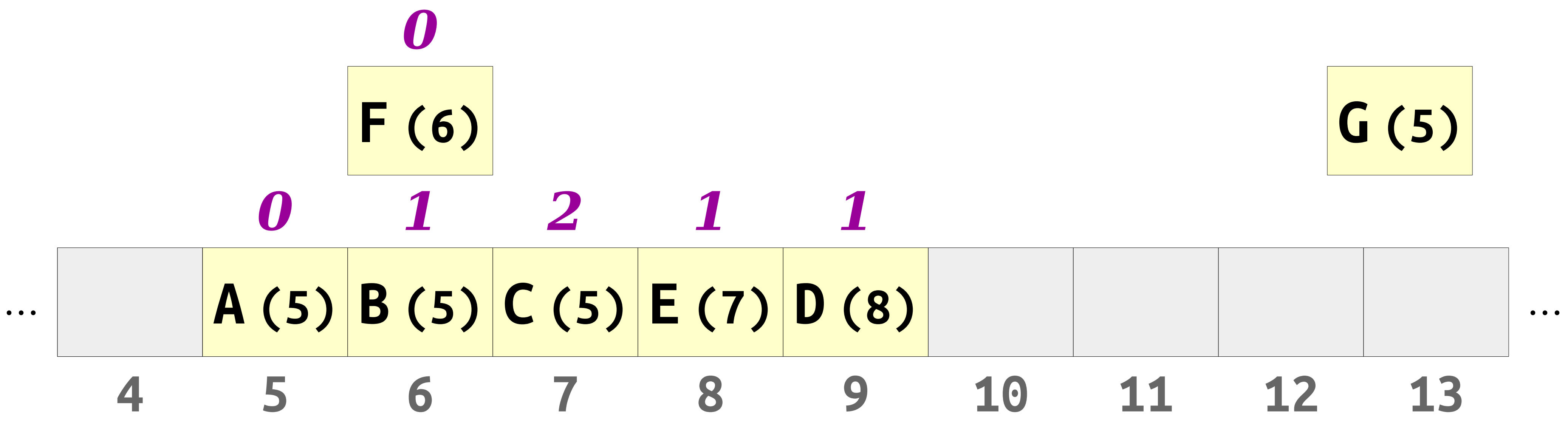

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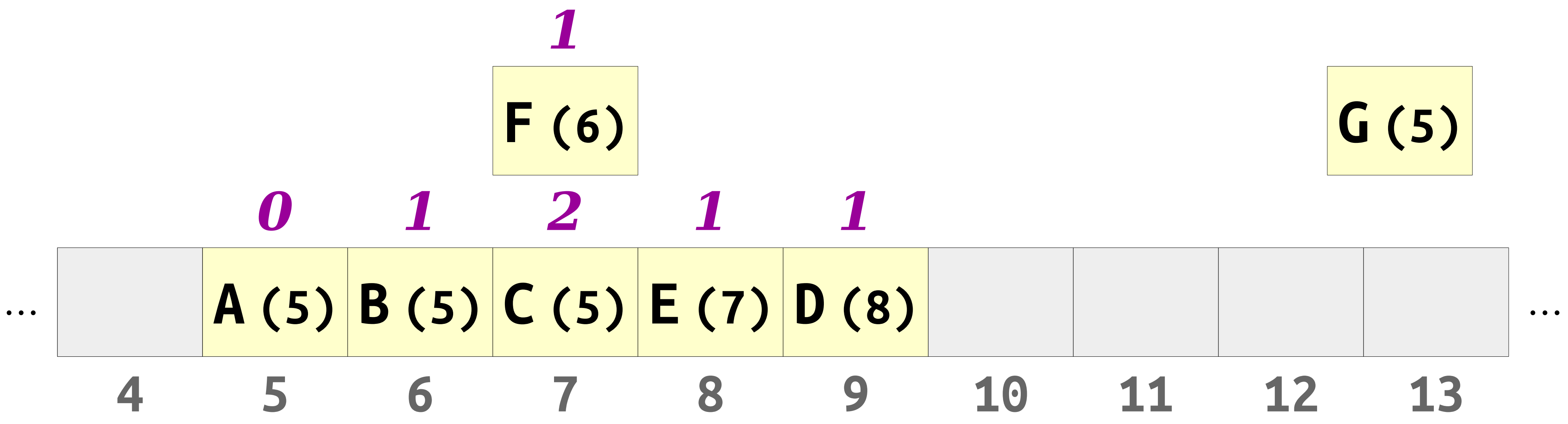

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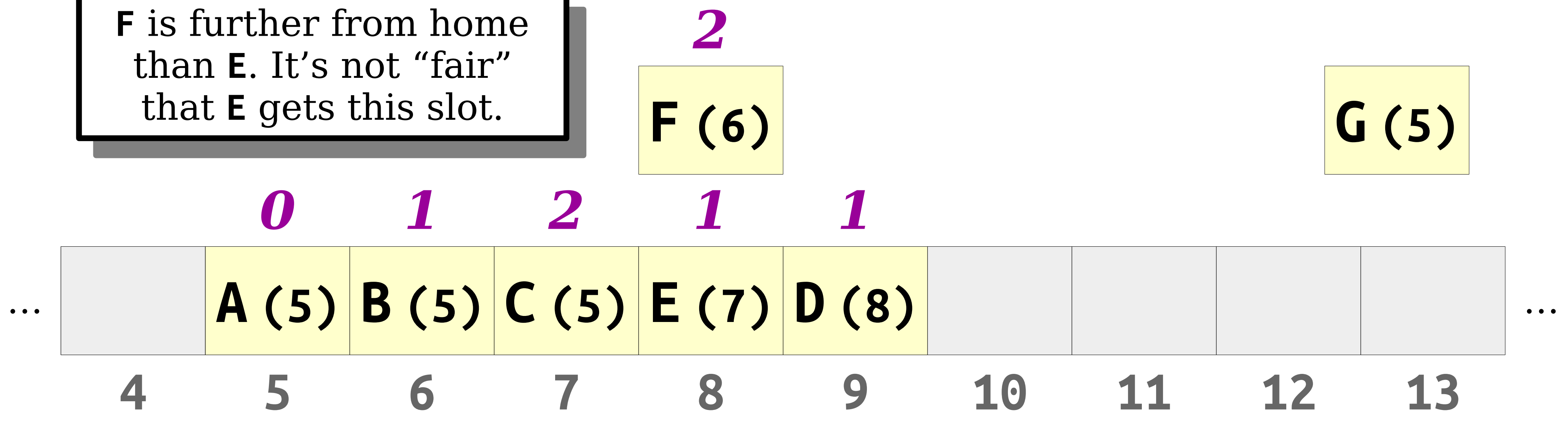
F is further from home than **E**. It's not "fair" that **E** gets this slot.



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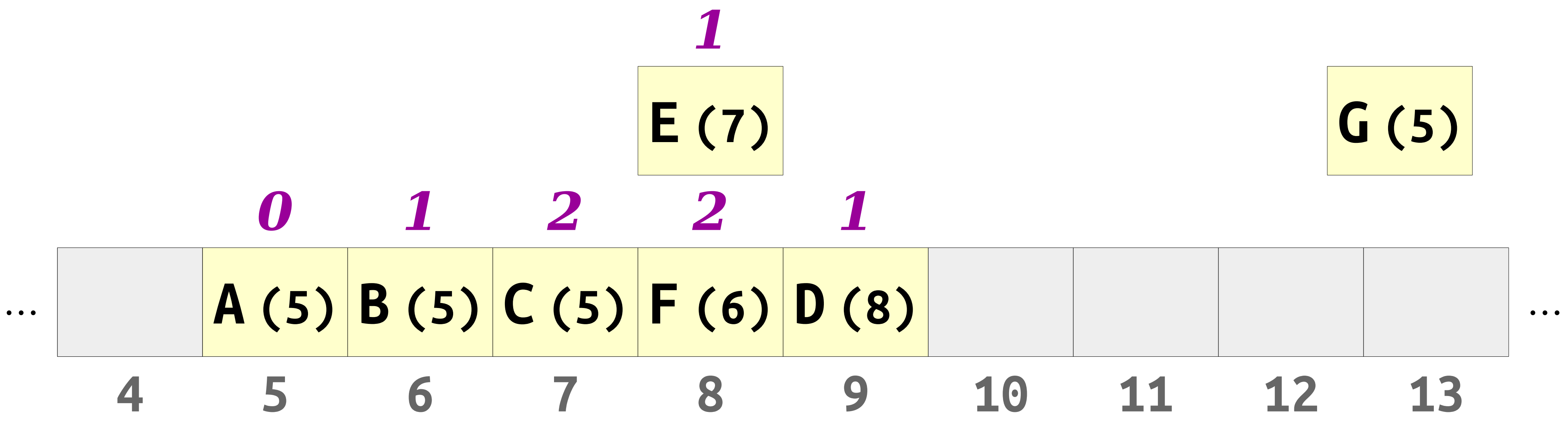

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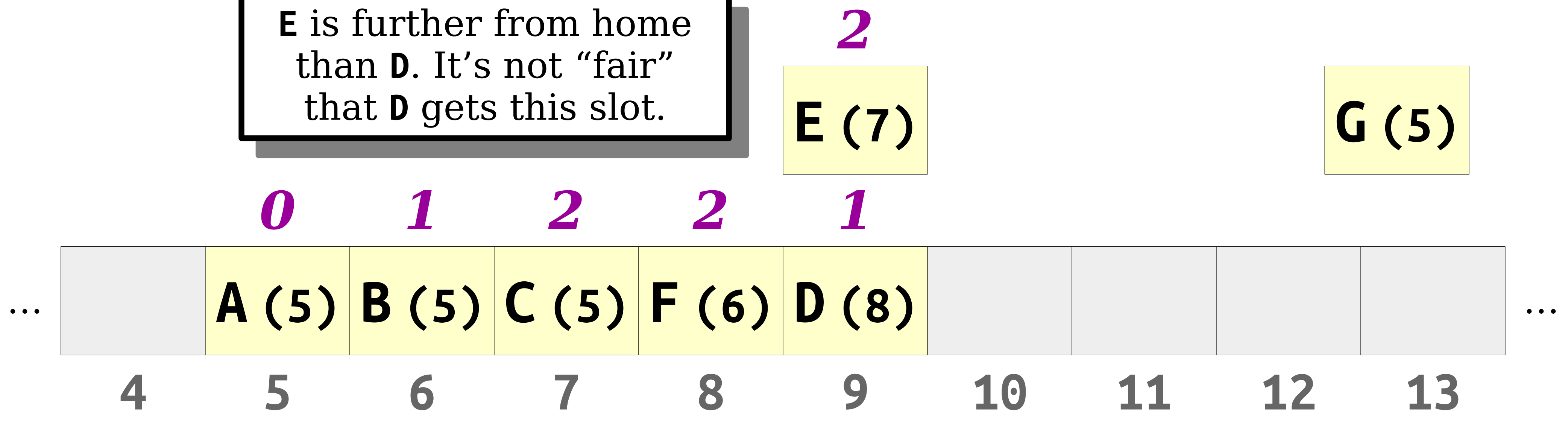
E is further from home than **D**. It's not "fair" that **D** gets this slot.



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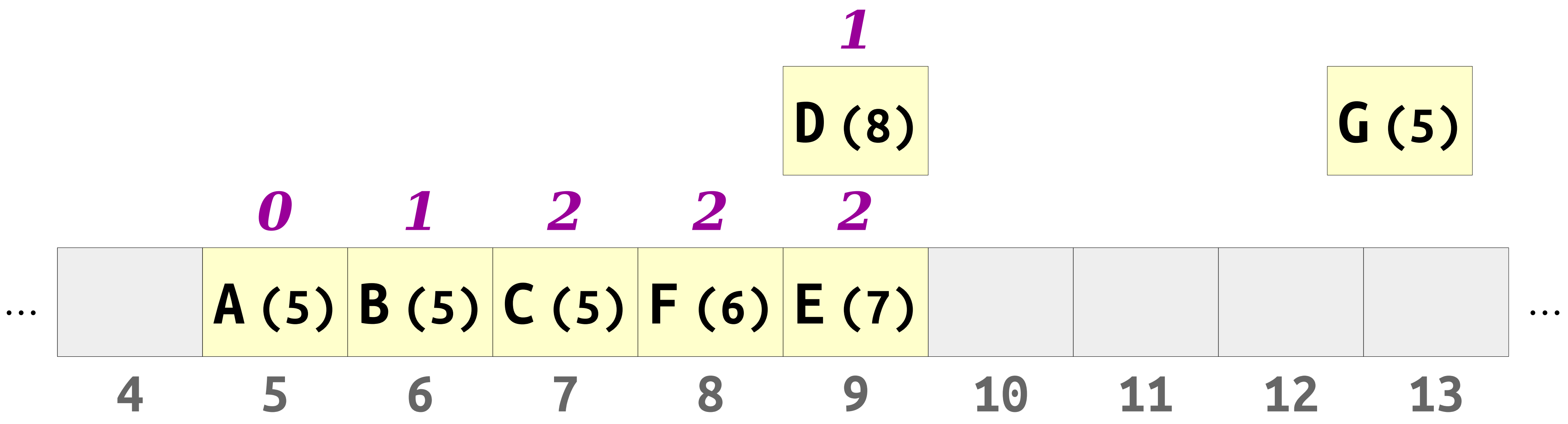

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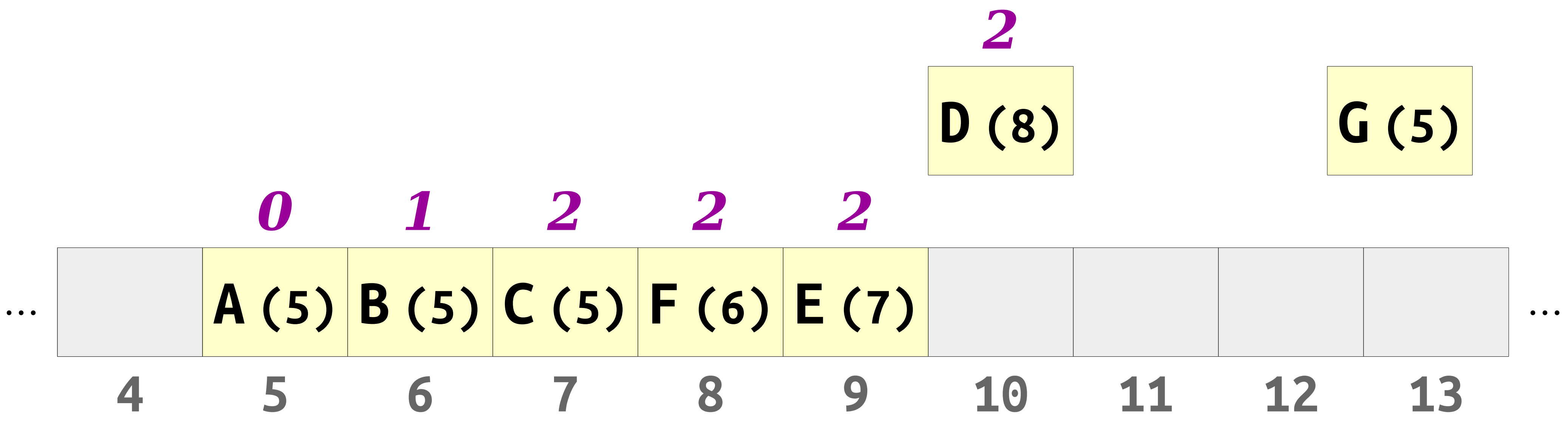

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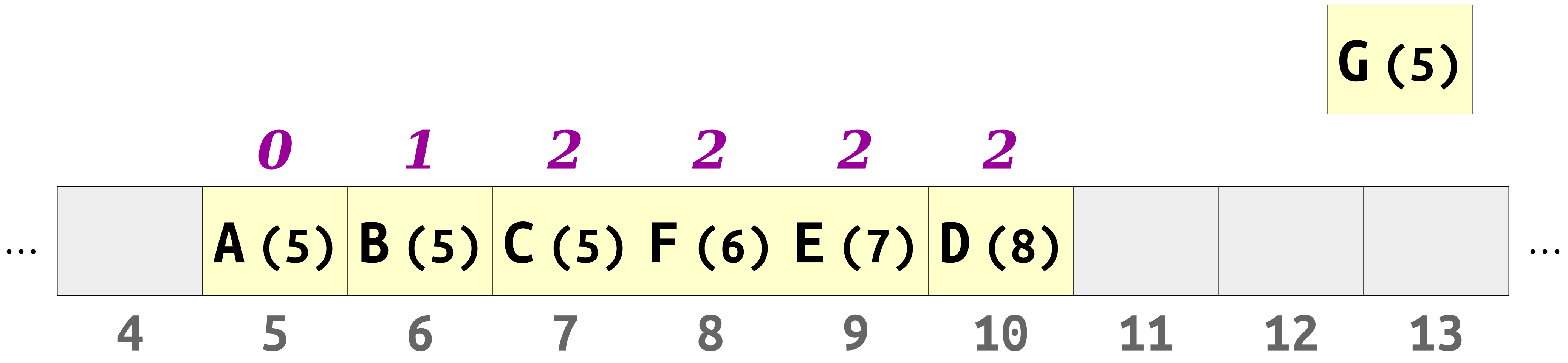

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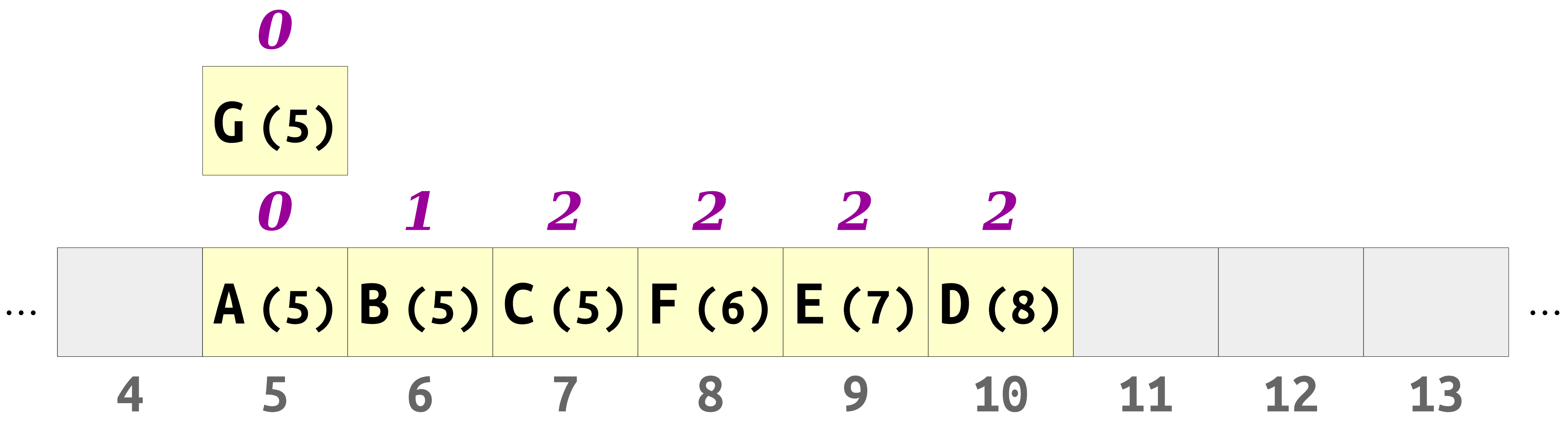

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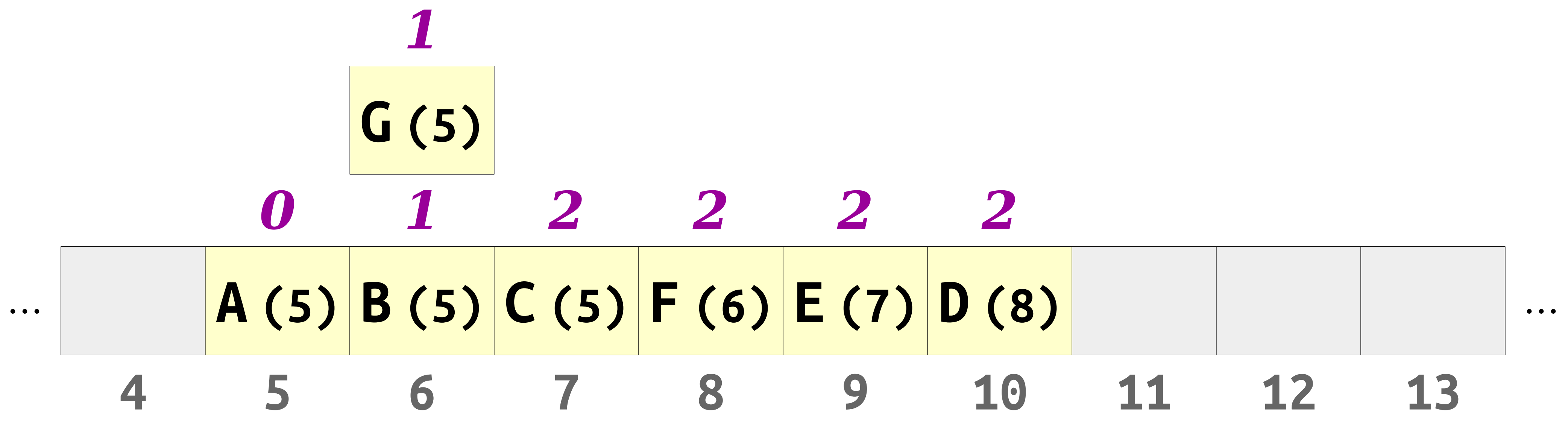

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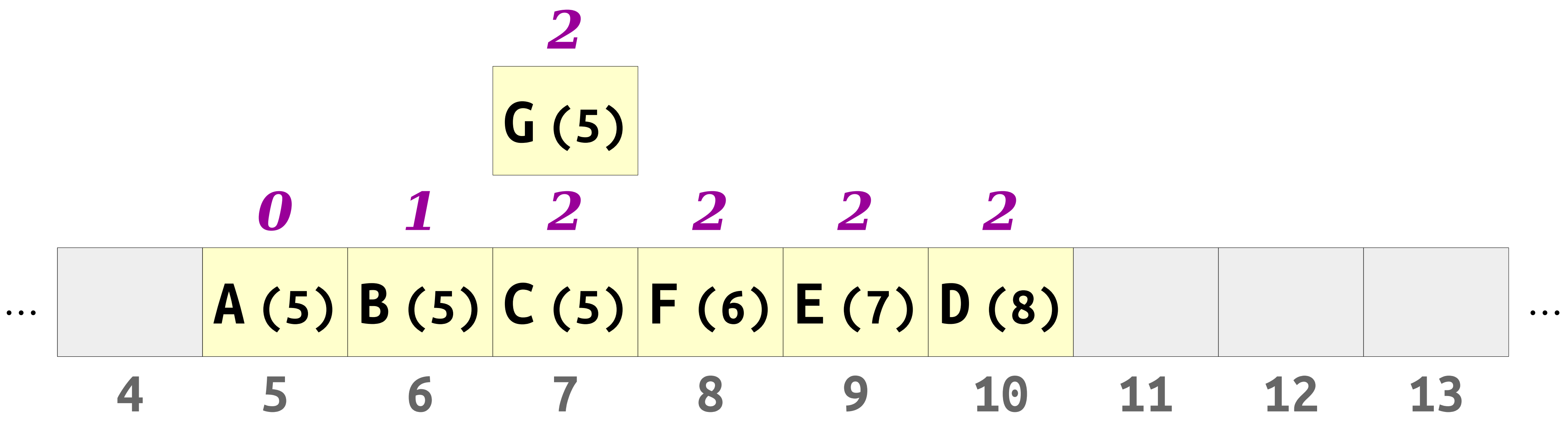

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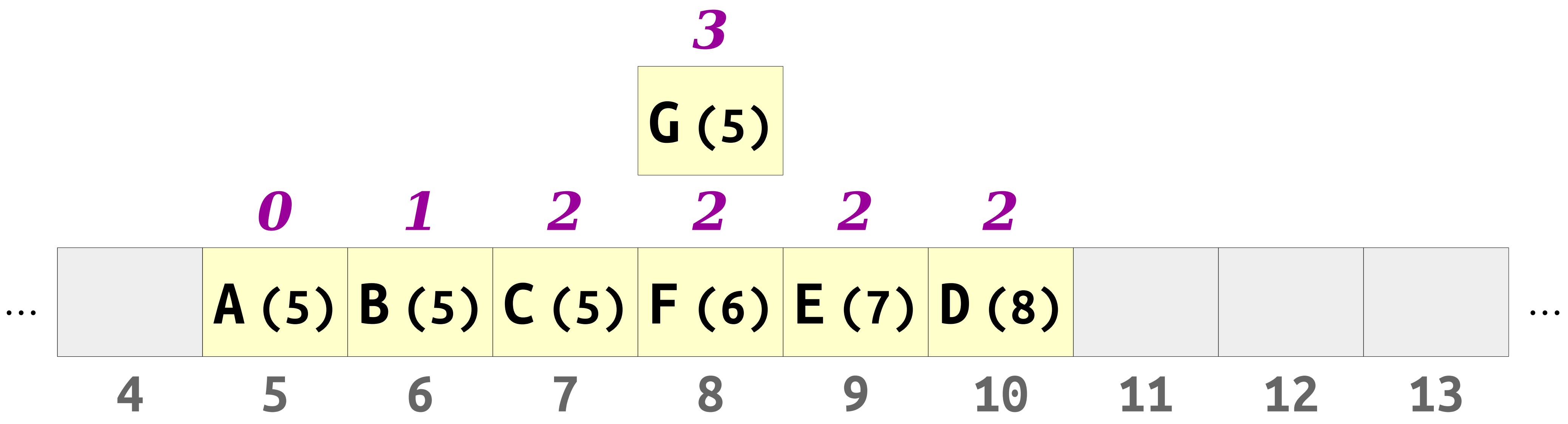

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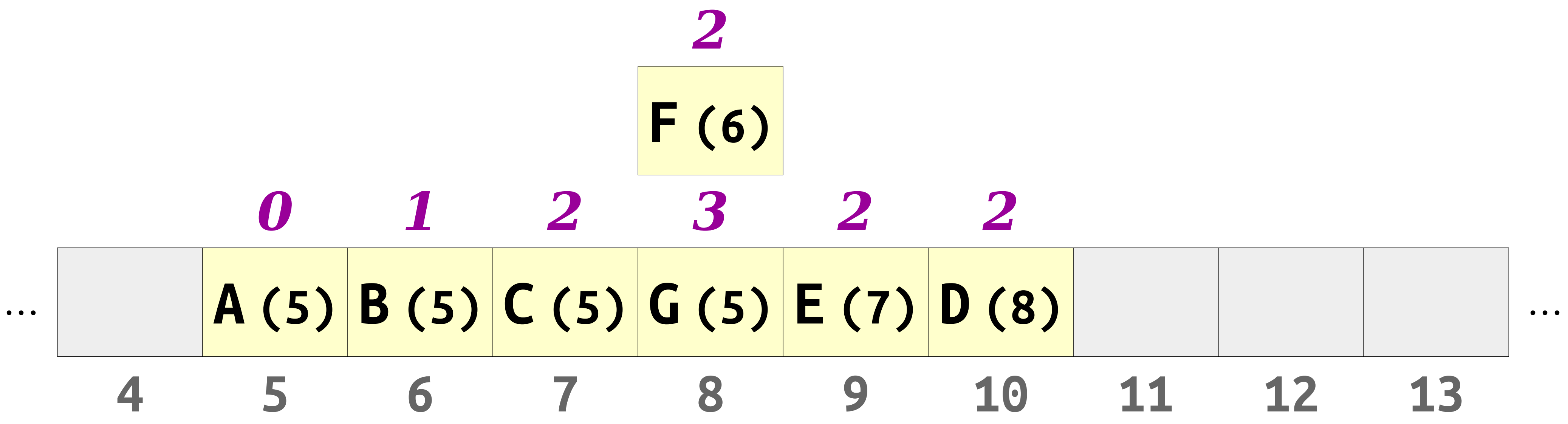

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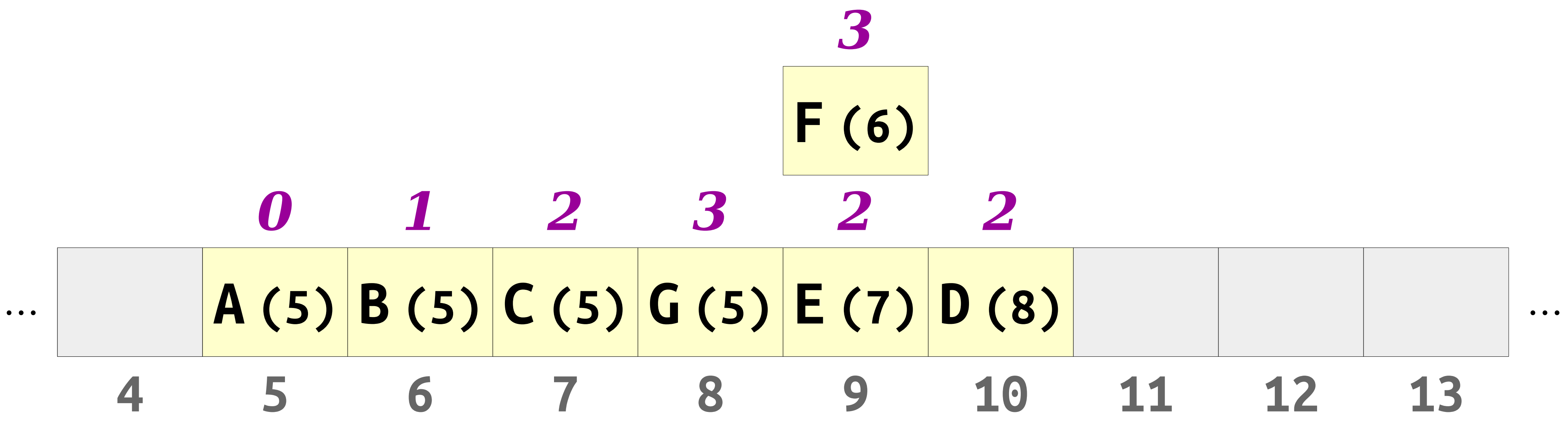

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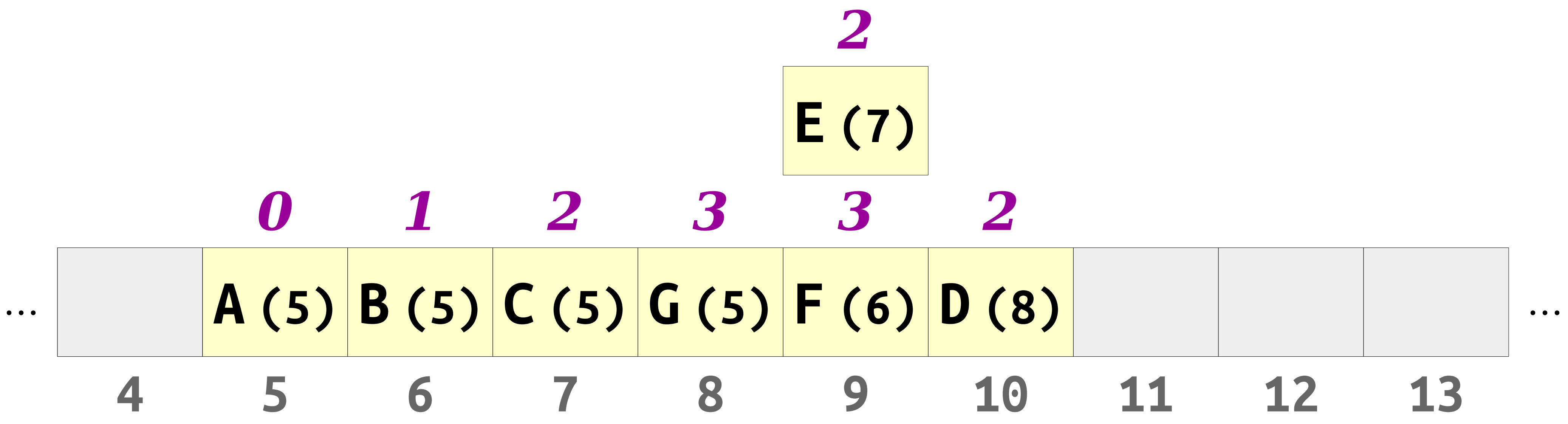

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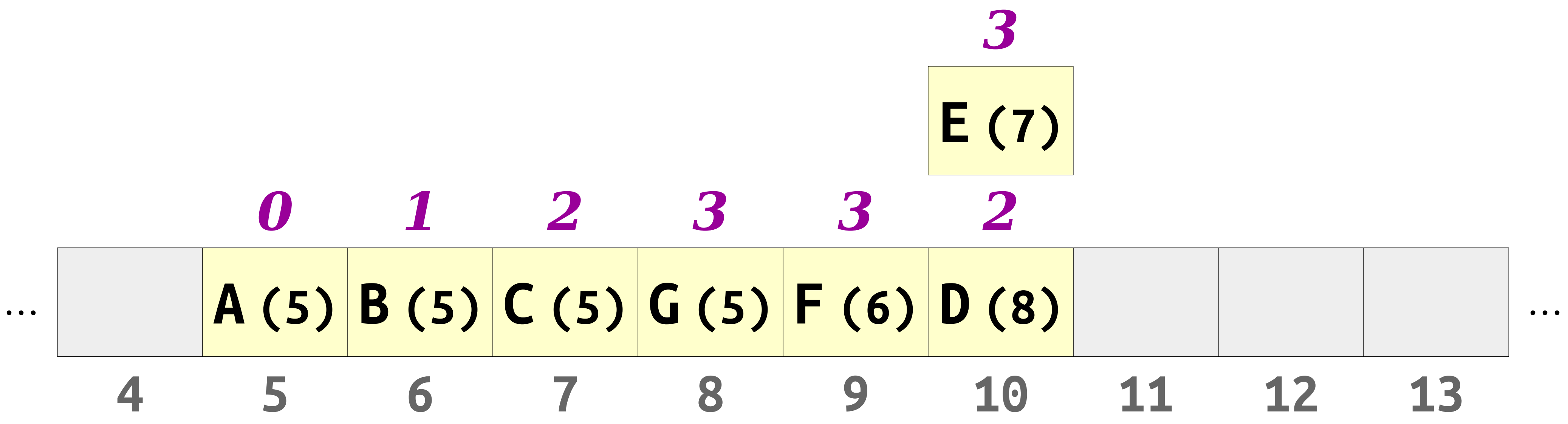

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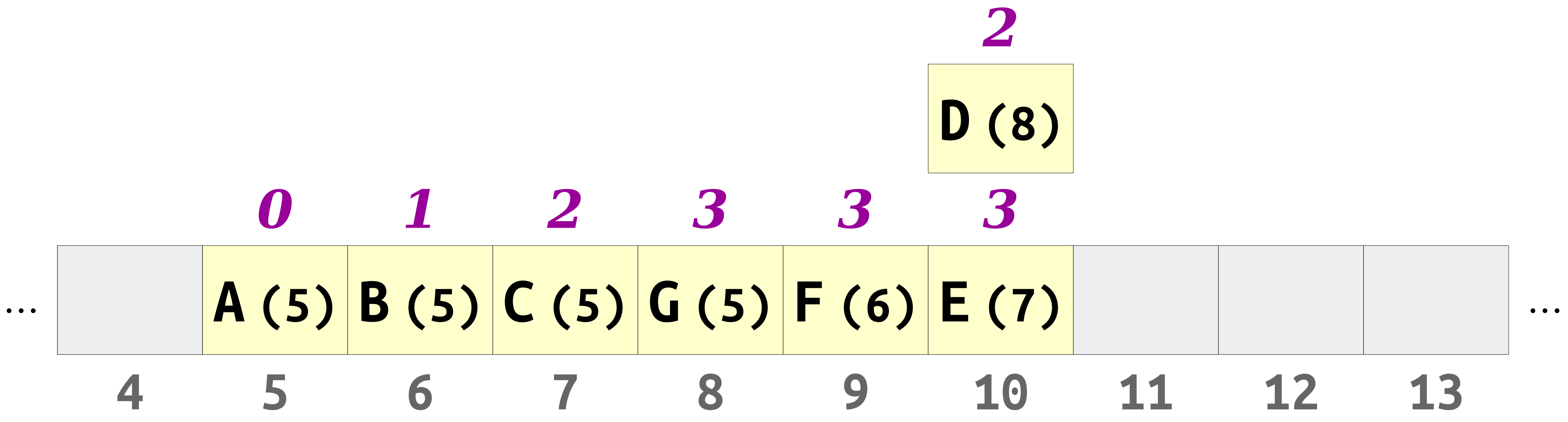

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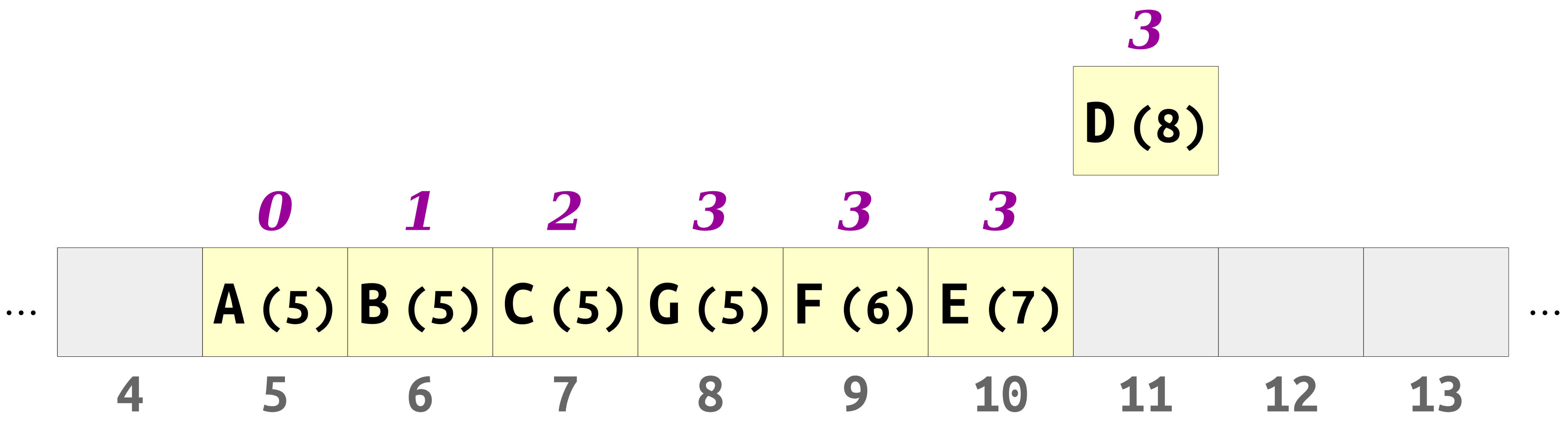

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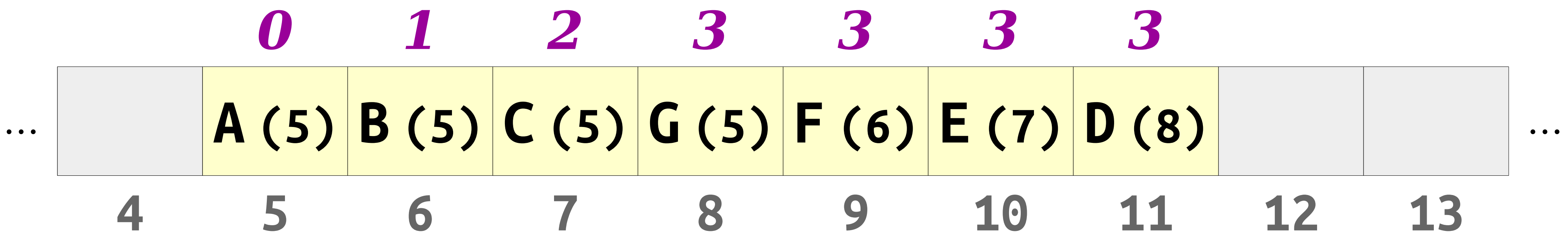

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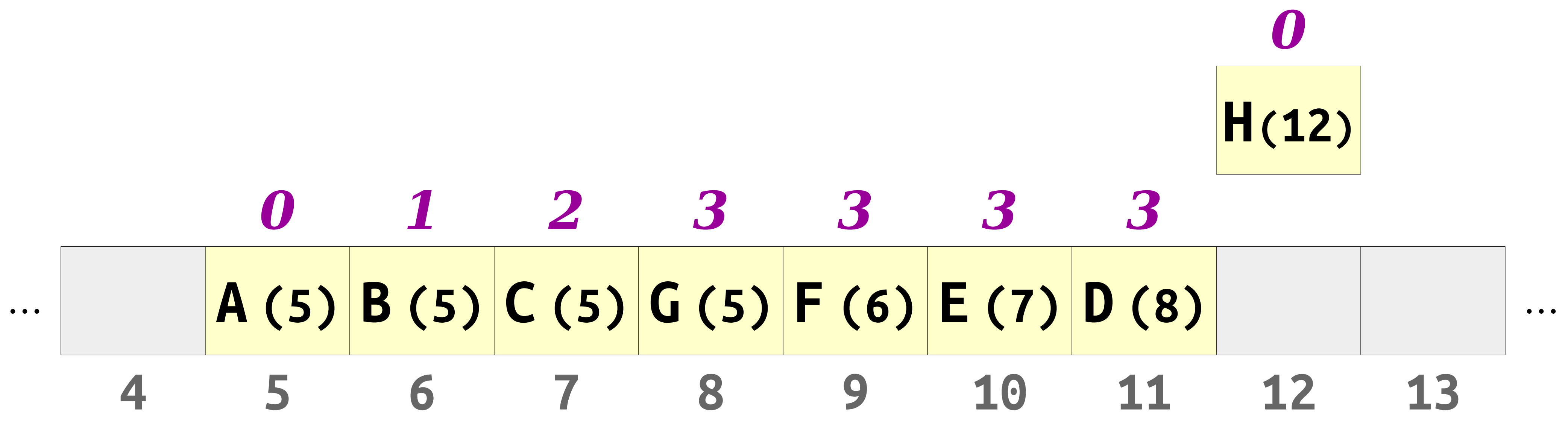

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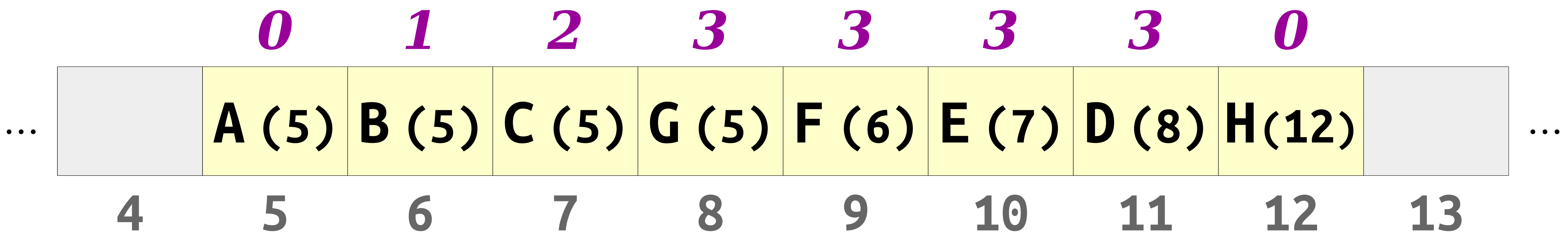

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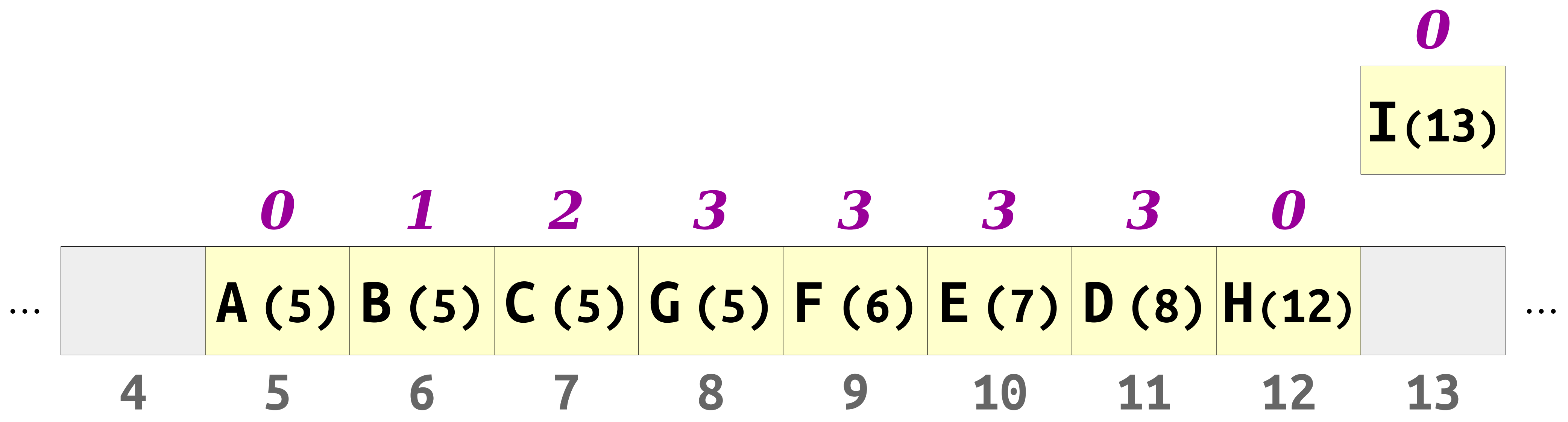

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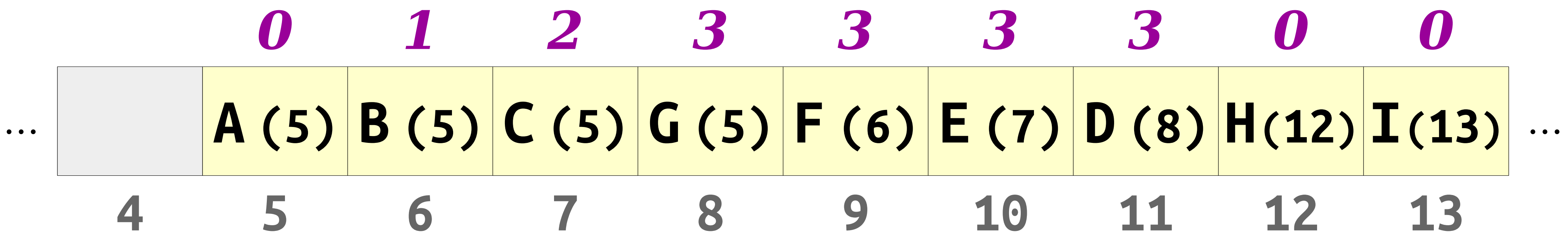
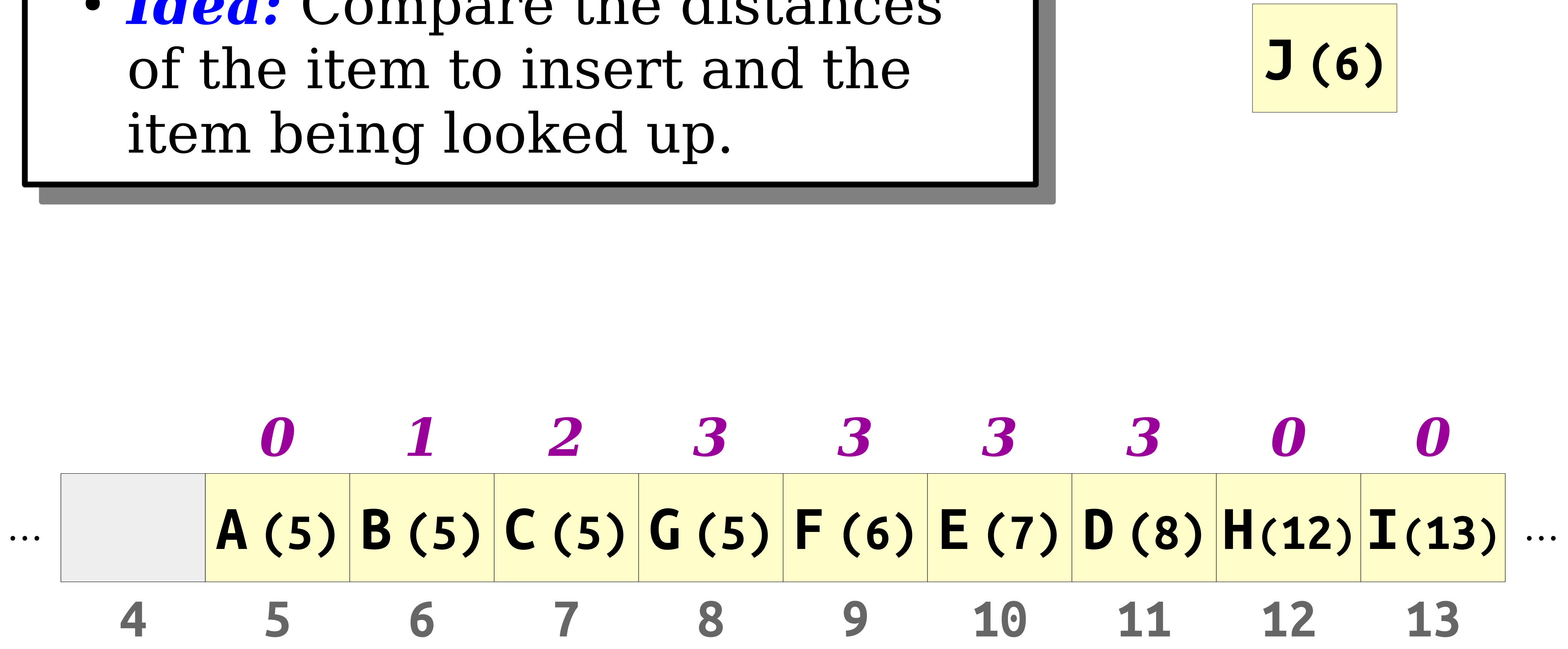


- ***Neat trick:*** We can make unsuccessful lookups in a Robin Hood hashing table faster than in a linear probing table.
- ***Idea:*** Compare the distances of the item to insert and the item being looked up.



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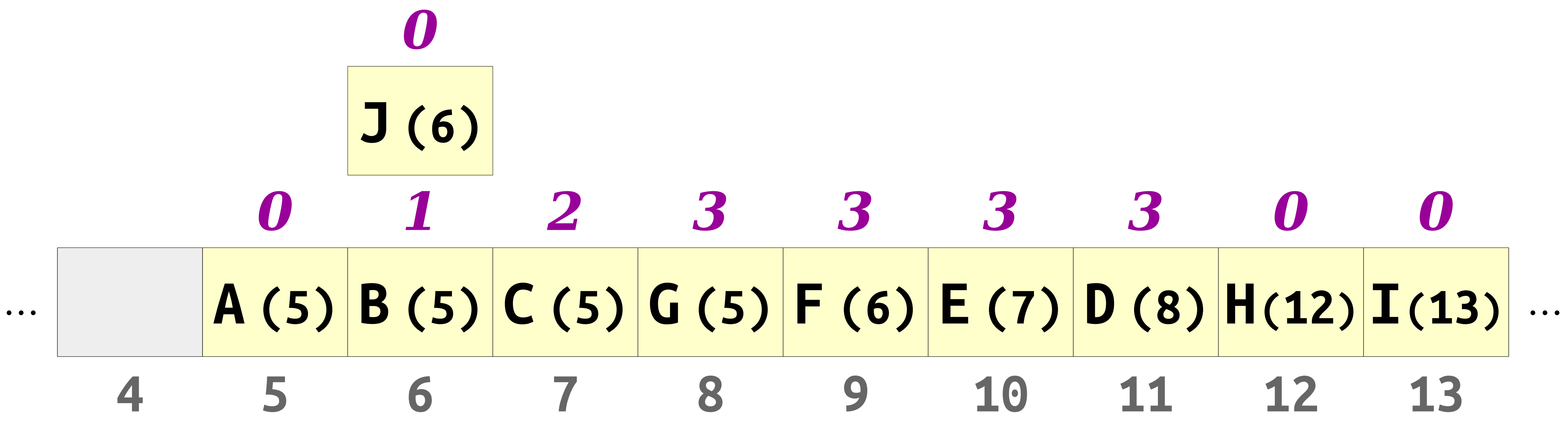

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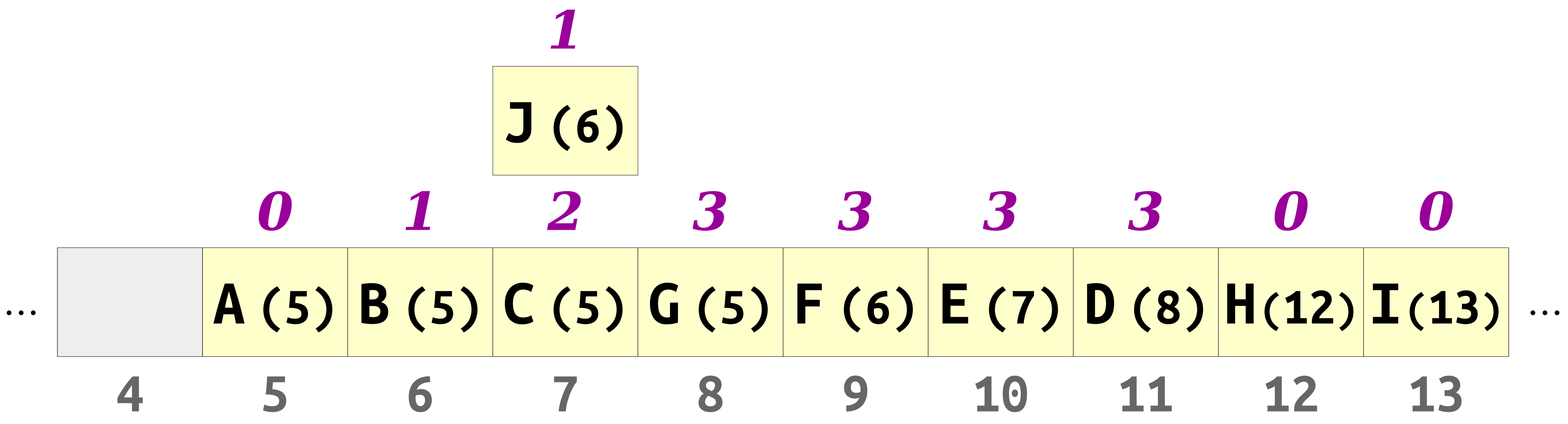

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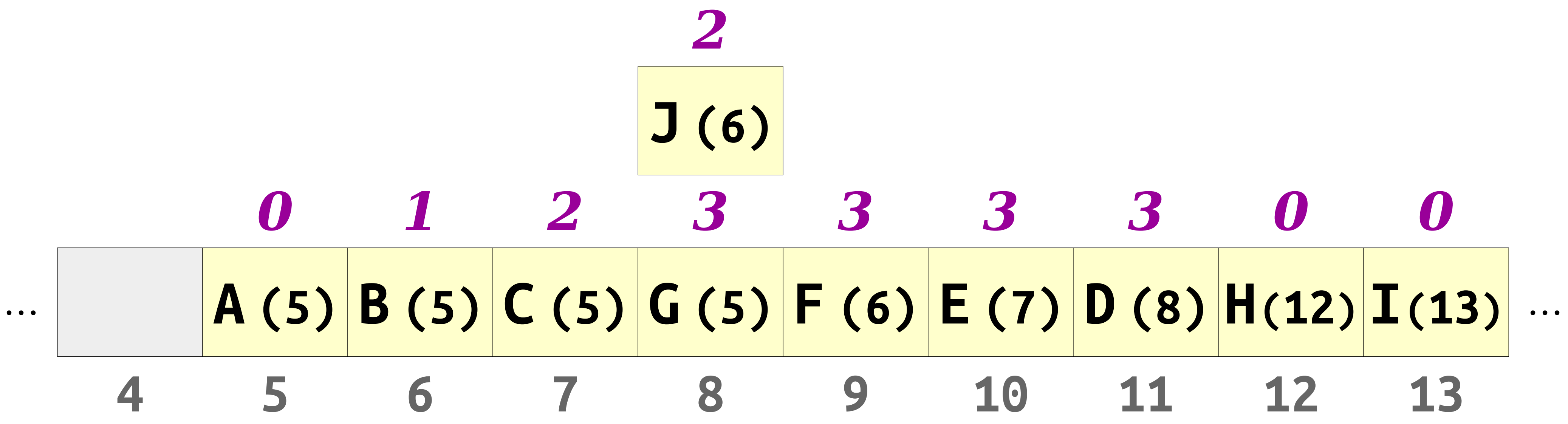

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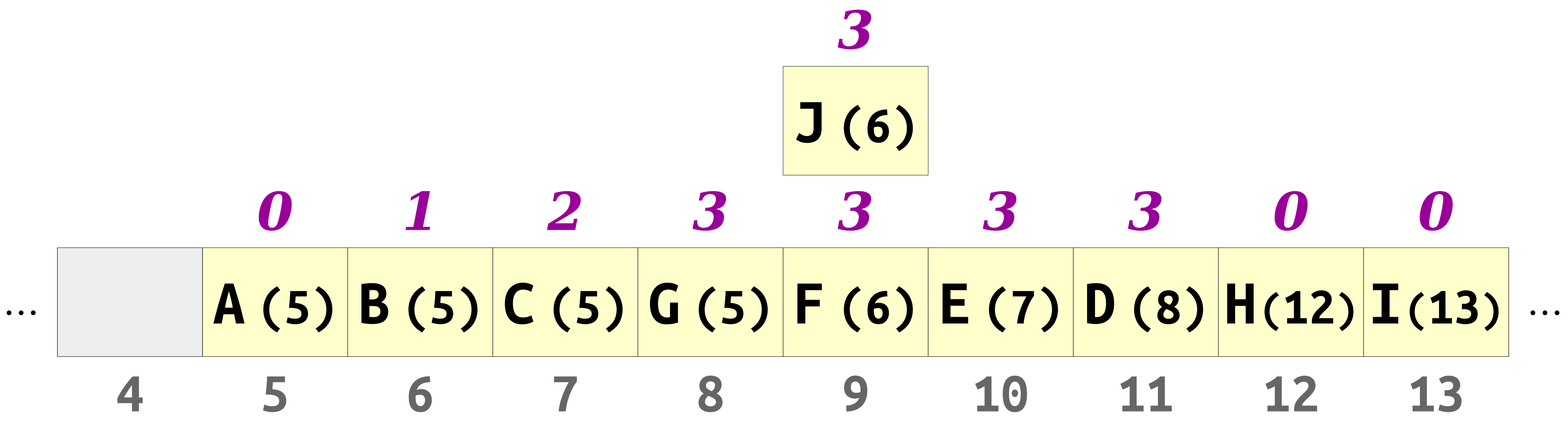

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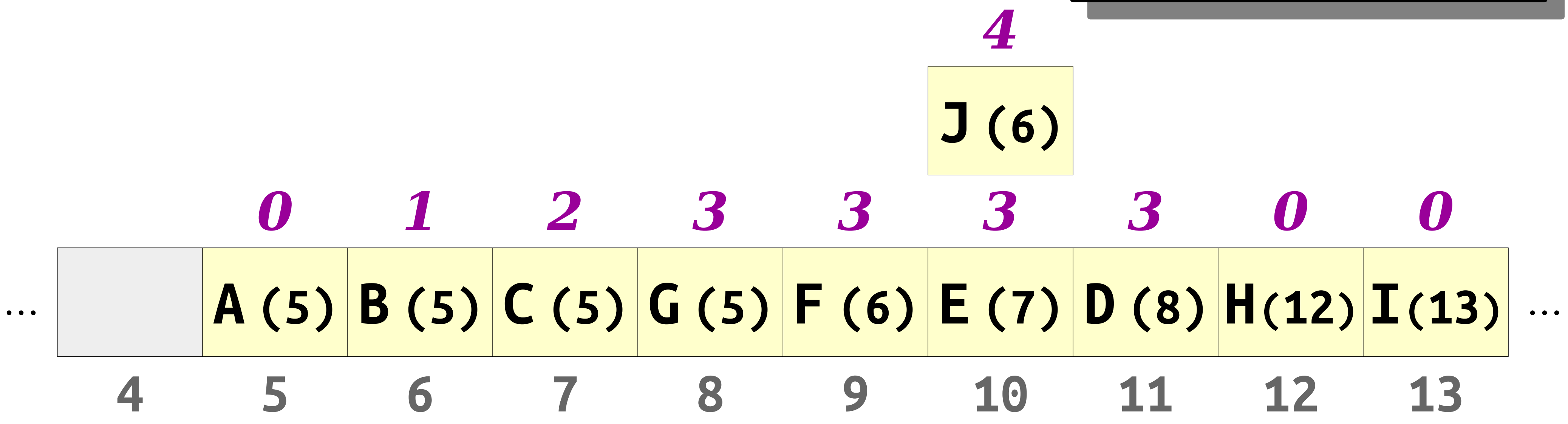
If **J** were in this table, it would have displaced the **E**. So **J** can't be in the table!



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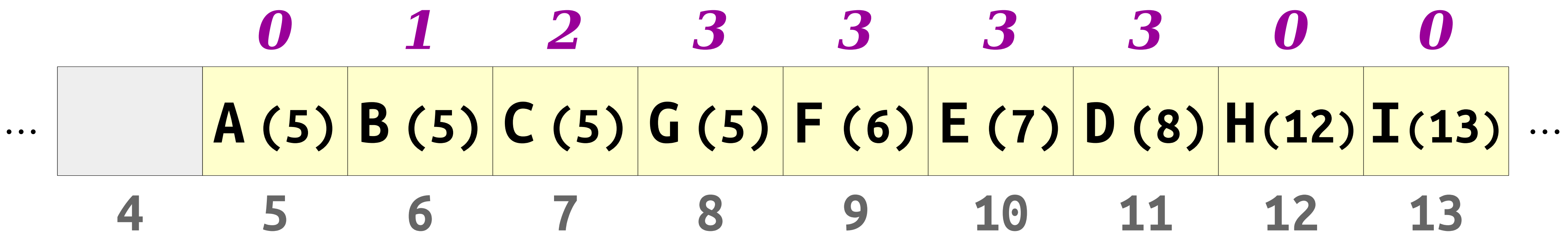
- We can use a technique called ***backward-shift deletion*** instead.



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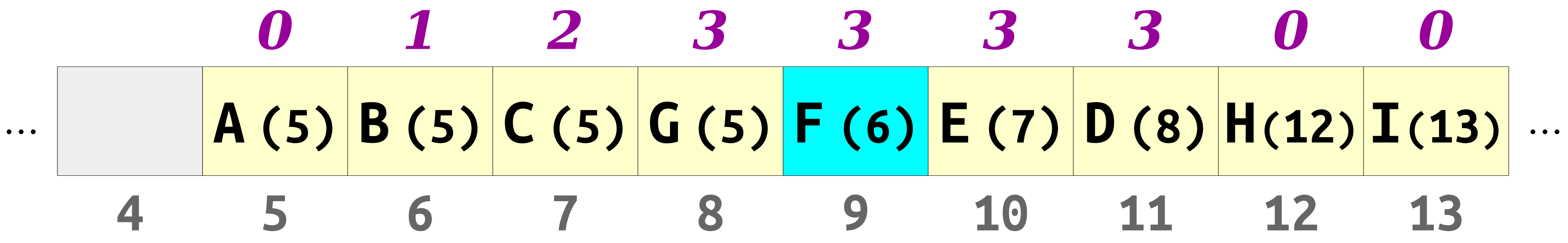

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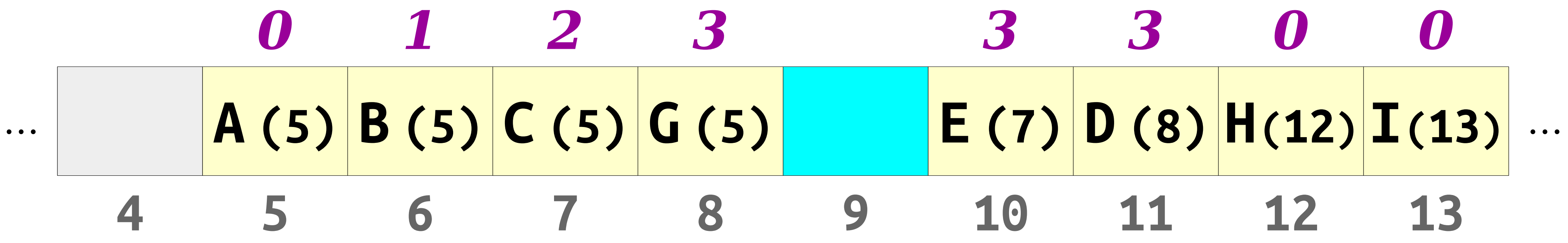

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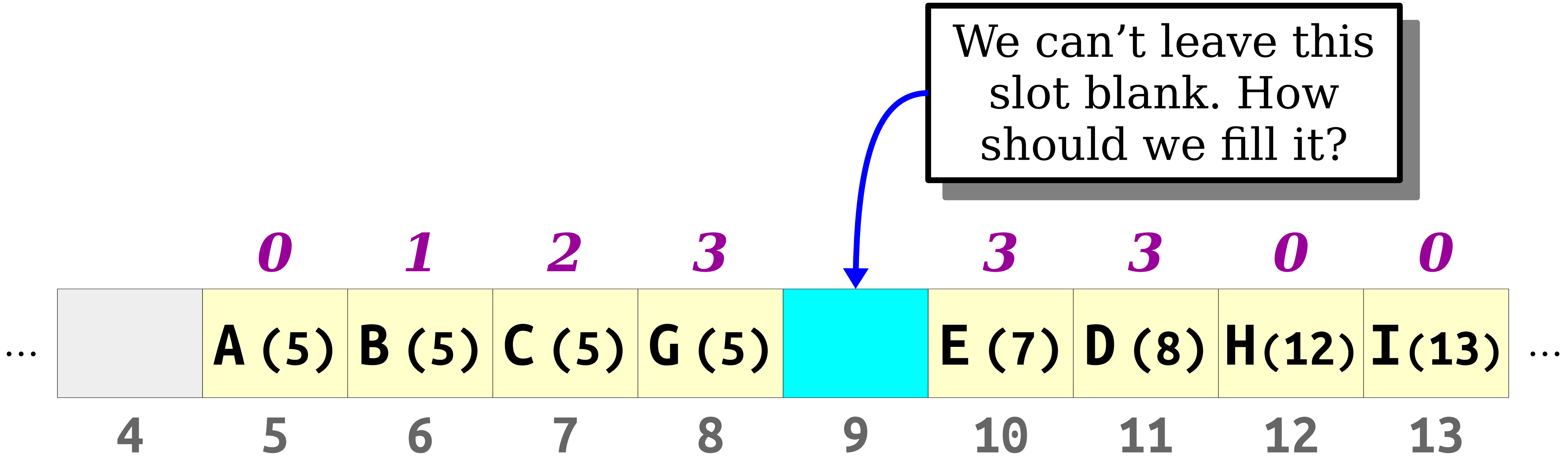

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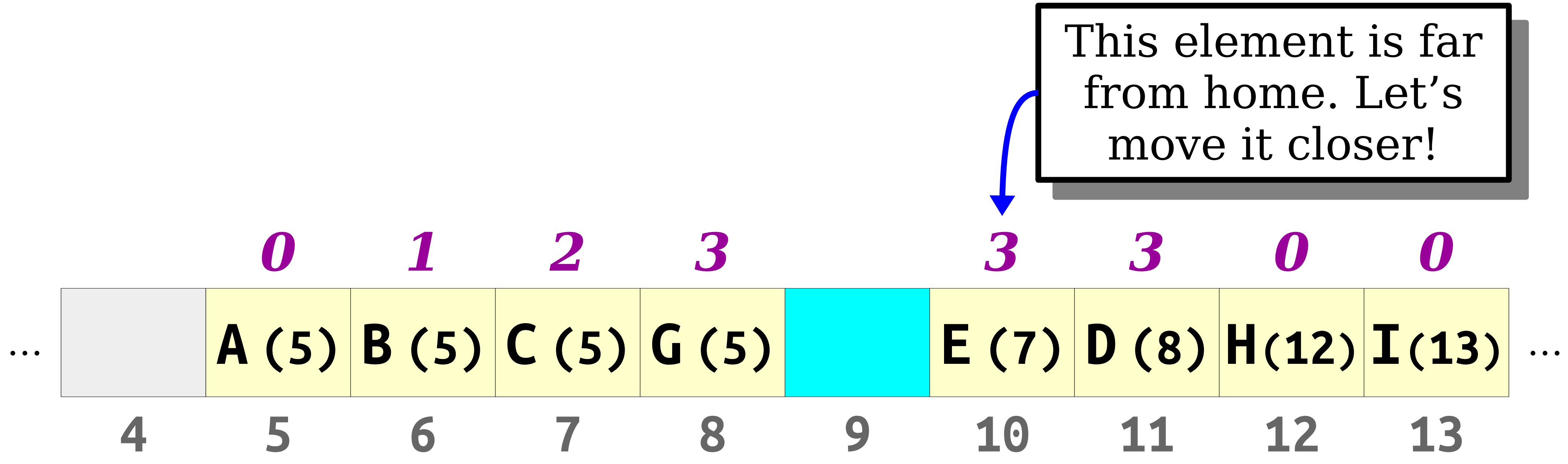

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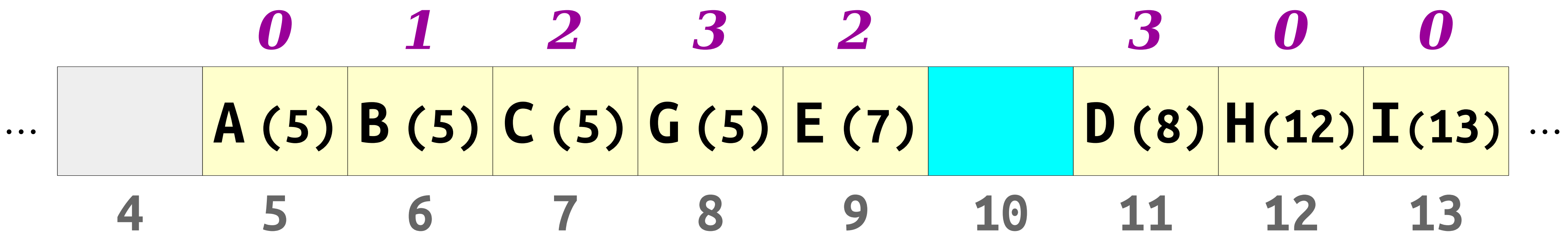

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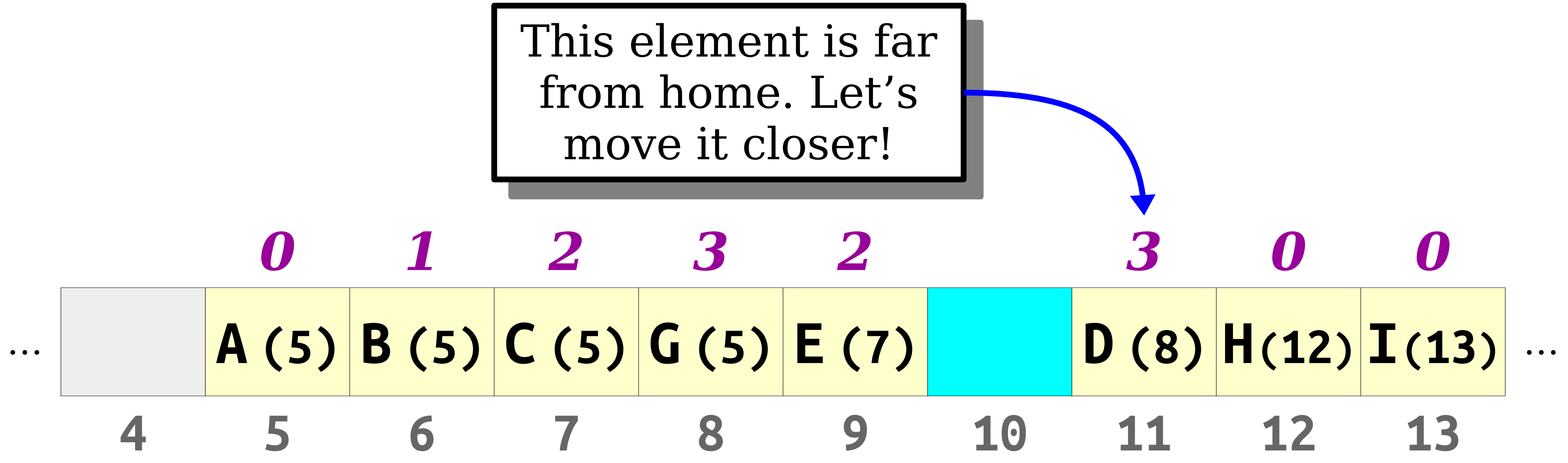

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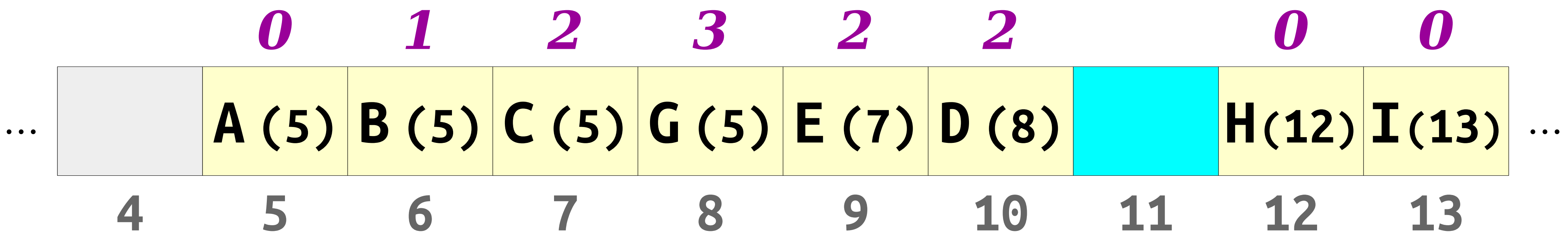

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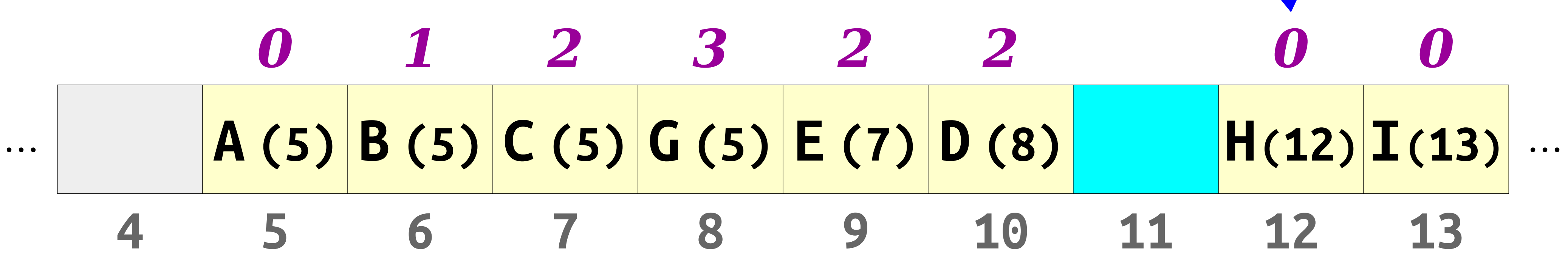
This element is already home. We're done!



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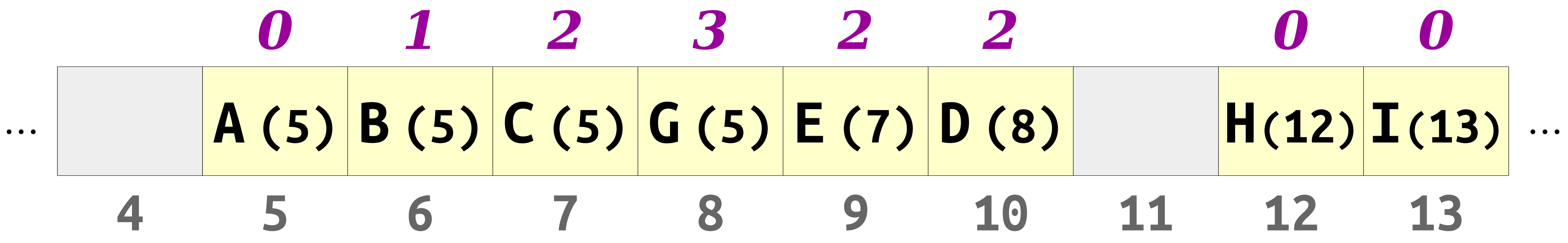

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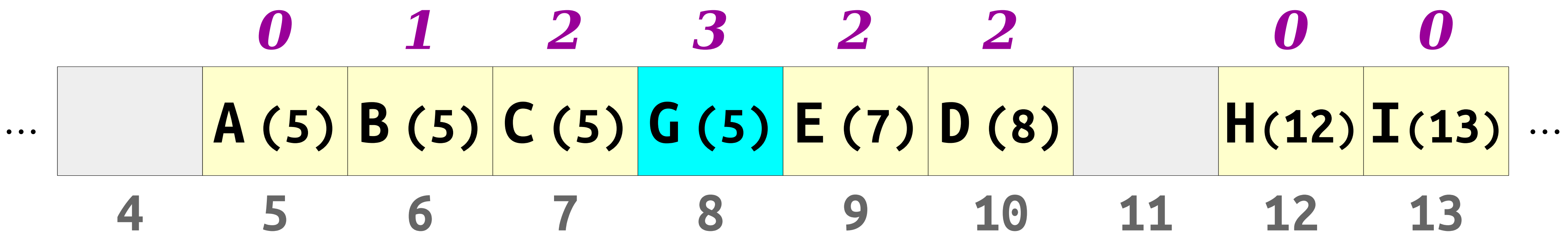

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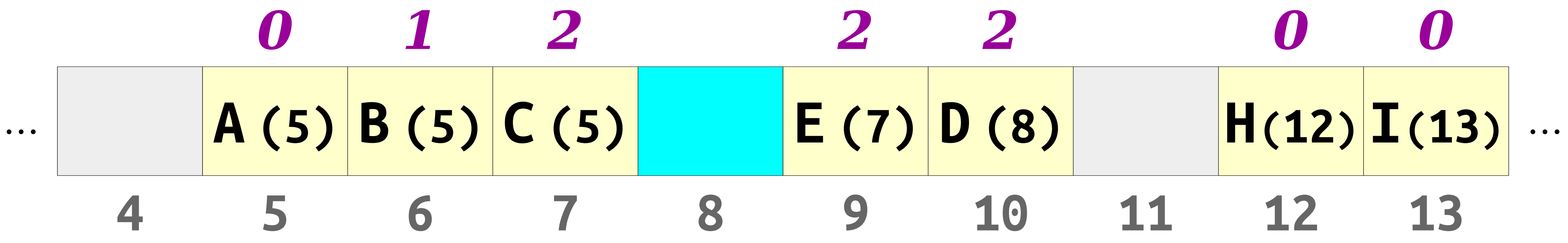

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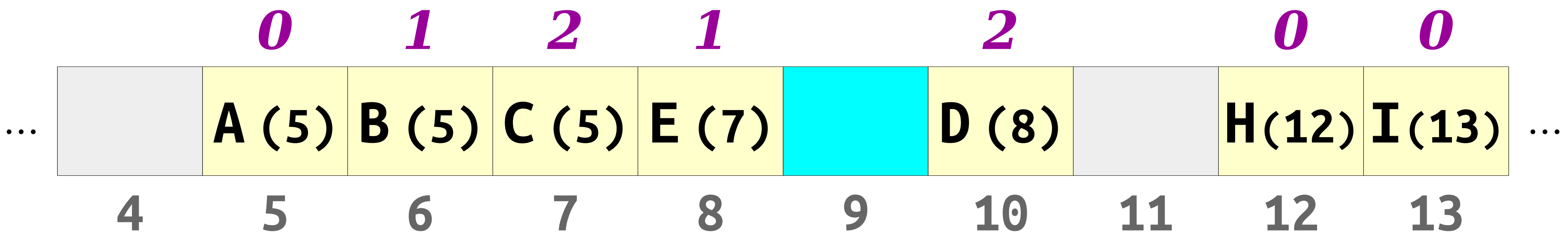

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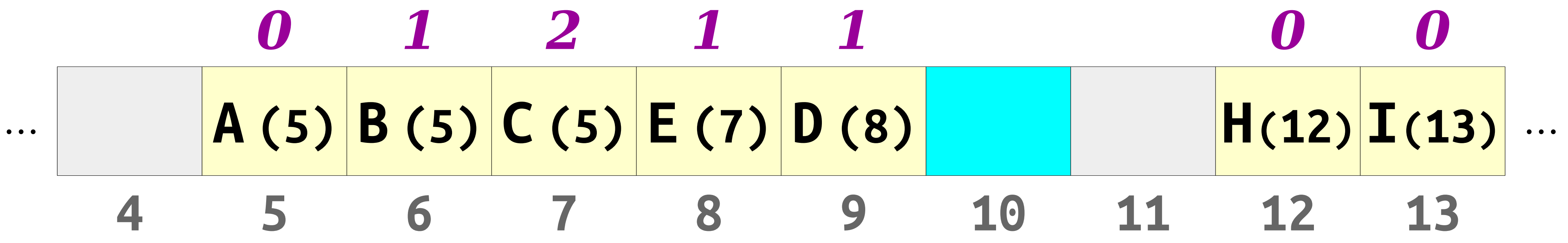

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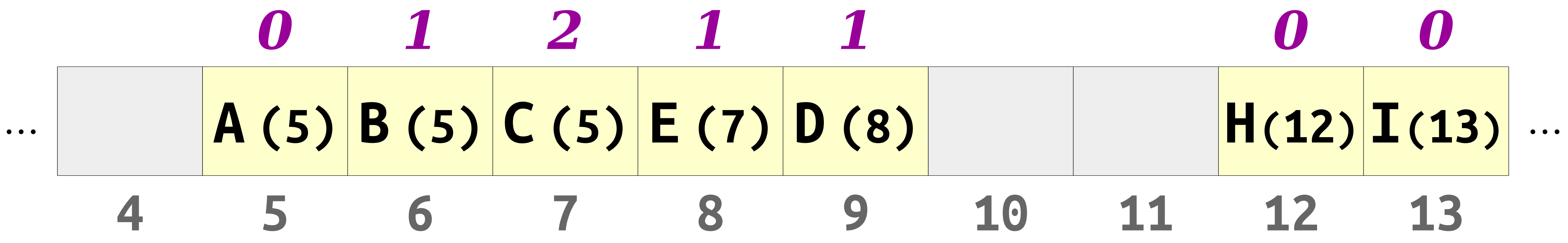

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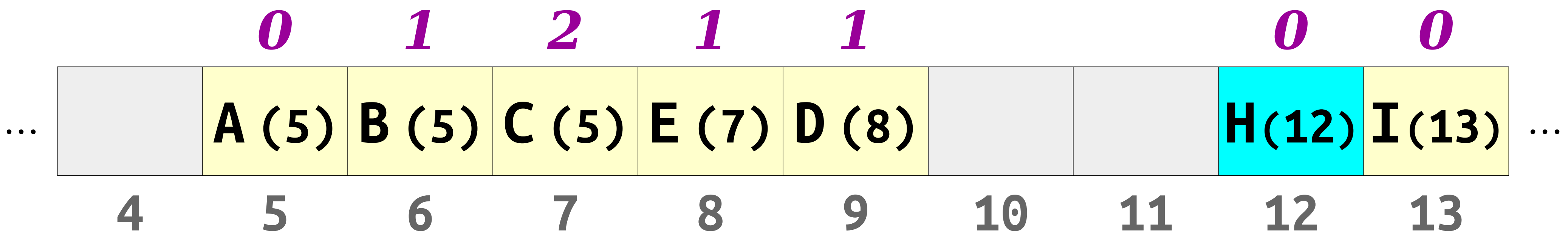

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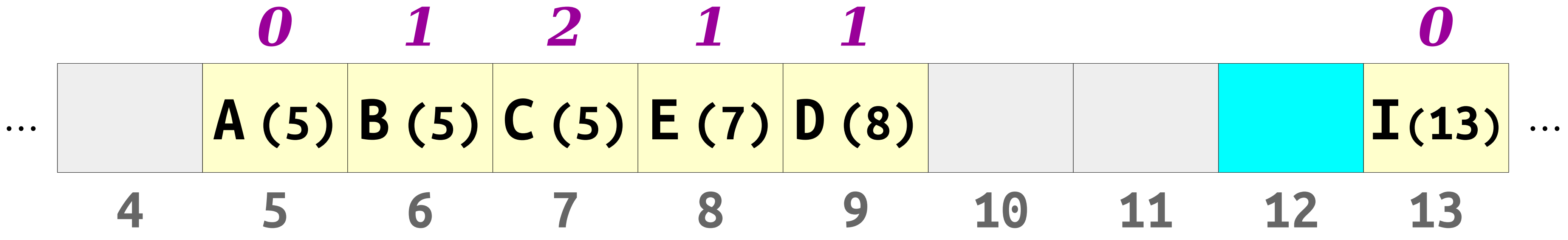
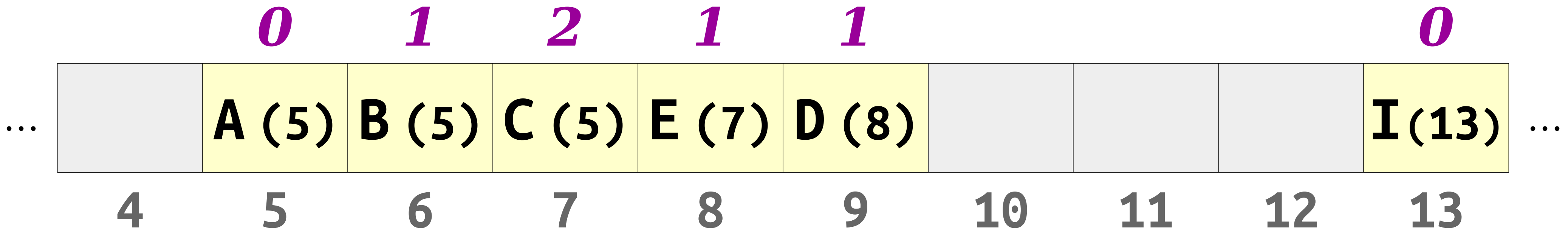


- ***Neat trick:*** Robin Hood hashing doesn't need tombstones.
- We can use a technique called ***backward-shift deletion*** instead.



Robin Hood Hashing

- Annotate each table slot with its distance from home.
- During insertions, if the element to insert is further from home than the current table element, “displace” the element in the table and insert that element instead.
- During lookups, stop searching if the element to search for is further from home than the current element.
- During deletions, pull elements backward until we hit an empty slot or find an element that’s already home.

References

References

- Some slides get from slides prepared by Kevin Wayne and Some gets from slides prepared in Stanford University.