Project Work
Combinatorial Decision Making and Optimization
2021/2022

Topic: Modelling & solving a combinatorial optimization problem
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This document describes the project work of the Combinatorial Decision Making and Optimization course for the academic year 2021/2022, which is about modelling and solving a combinatorial optimization problem. The students are free to choose one of the proposed problems, as well as to propose another problem taken from the literature provided that they get our approval before start working on it. The project work involves approaching the chosen problem using (i) Constraint Programming (CP), (ii) propositional SATisfiability (SAT) and/or its extension to Satisfiability Modulo Theories (SMT), and (iii) Mixed-Integer Linear Programming (MIP). The students can form a group of 3 members, and it suffices for point (ii) to develop a SAT or an SMT solution. If they do both, they obtain bonus points. They can also form a group of 4 members, but in that case they are required to develop both SAT and SMT solutions.

1 Problem 1: VLSI Design

1.1 Problem description

VLSI (Very Large Scale Integration) refers to the trend of integrating circuits into silicon chips. A typical example is the smartphone. The modern trend of shrinking transistor sizes, allowing engineers to fit more and more transistors into the same area of silicon, has pushed the integration of more and more functions of cellphone circuitry into a single silicon die (i.e. plate). This enabled the modern cellphone to mature into a powerful tool that shrank from the size of a large brick-sized unit to a device small enough to comfortably carry in a pocket or purse, with a video camera, touchscreen, and other advanced features.

As the combinatorial decision and optimization expert, the student is assigned to design the VLSI of the circuits defining their electrical device: given a fixed-width plate and a list of rectangular circuits, decide how to place them on the plate so that the length of the final device is minimized

(improving its portability). Consider two variants of the problem. In the first, each circuit must be placed in a fixed orientation with respect to the others. This means that, an $n \times m$ circuit cannot be positioned as an $m \times n$ circuit in the silicon plate. In the second case, the rotation is allowed, which means that an $n \times m$ circuit can be positioned either as it is or as $m \times n$.

1.2 Format of the Instances

This section describes the format in which the VLSI instances are written, as well as the expected format of the corresponding solutions.

Instance Format An instance of VLSI is a text file consisting of lines of integer values. The first line gives w, which is the width of the silicon plate. The following line gives n, which is the number of necessary circuits to place inside the plate. Then n lines follow, each with x_i and y_i, representing the horizontal and vertical dimensions of the i-th circuit. For example, a file with the following lines:

```
9
5
3 3
2 4
2 8
3 9
4 12
```

describes an instance in which the silicon plate has the width 9, and we need to place 5 circuits, with the dimensions 3×3, 2×4, 2×8, 3×9, and 4×12. Figure 1 shows the graphical representation of the instance.

Solution Format Where to place a circuit i can be described by the position of i in the silicon plate. The solution should indicate the length of the plate l, as well as the position of each i by its $\hat{x}_i$ and $\hat{y}_i$, which are the coordinates of the left-bottom corner i. This could be done by for instance adding l next to w, and adding $\hat{x}_i$ and $\hat{y}_i$ next to x_i and y_i in the instance file. To exemplify, the solution of the instance depicted in Figure 1 could look like:

```
9 12
5
```

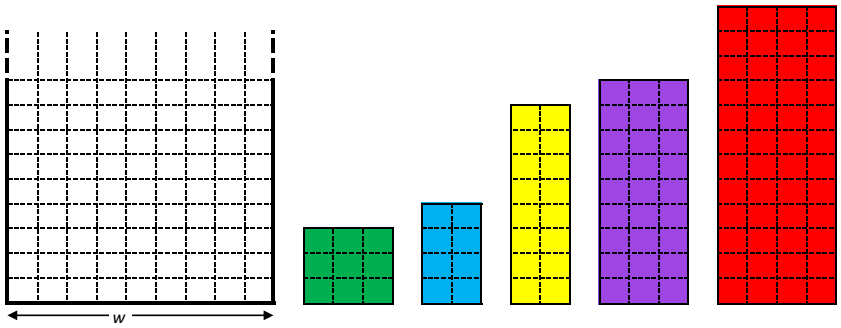

Figure 1: Graphical representation of a VLSI design instance.

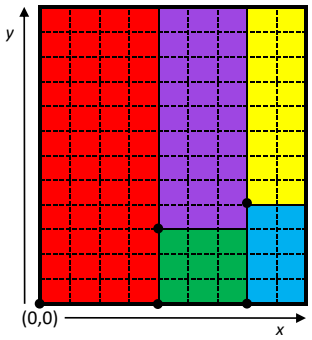

Figure 2: Graphical representation of a VLSI design solution.

```
3 3 4 0
2 4 7 0
2 8 7 4
3 9 4 3
4 12 0 0
```

which says for instance that the left-bottom corner of the 3×3 circuit is at $(4, 0)$. The solution can be represented graphically as in Figure 2.

For correctness check, the students are advised to visualize the solutions as in Figure 2. This can be done manually by hand, or automatically by a program which takes as input the solution of the problem.