a) Outer loop: since n is being subtracted by $\sqrt{n}$ for every iteration $\rightarrow \Theta(n - k\sqrt{n})$

k	i	n
1	1	$n-\sqrt{n}$
2	2	$2-2\sqrt{n}$
3	3	$3-3\sqrt{n}$

i & k are growing at the same rate: $i = k$

$i = n - k\sqrt{n}$

$k = n - k\sqrt{n}$

$k + k\sqrt{n} = n$

$k(1+\sqrt{n}) = n$

$k = \frac{n}{1+\sqrt{n}}$

$$\sum_{k=1}^{\frac{n}{1+\sqrt{n}}} \Theta(1)\Theta(n-k\sqrt{n})$$

$$\sum_{k=1}^{\frac{n}{1+\sqrt{n}}} \Theta(n) - \sum_{k=1}^{\frac{n}{1+\sqrt{n}}} k\sqrt{n} \rightarrow \sqrt{n}\sum_{k=1}^{\frac{n}{1+\sqrt{n}}} k$$

$\sqrt{n}\left(\frac{n}{1+\sqrt{n}}\right)^2$

$n\left(\frac{n}{1+\sqrt{n}}\right)$ Arithmetic Series $\sqrt{n}\,\Theta(n^{\frac{3}{2}})$

$$\Theta\left(\frac{n^2}{1+\sqrt{n}}\right) = \Theta\left(\frac{n^2}{\sqrt{n}}\right) = \Theta(n^{\frac{3}{2}})$$

$$\Theta(n^{\frac{3}{2}}) - \Theta(n^{\frac{3}{2}})$$

$$\Theta(n^{\frac{3}{2}})$$

B) Outer loop: $\theta(n)$
Inner loop: $\theta(n)$) goes from 1 to n

if: worst case

case1 : all 1

[1, 1, 1, 1, 1]

+n

case 2: (1, ... n): has one for every n to make if run

+ n

third loop: m+m
log(n)

So log(n) runs n times

$\theta(n) \cdot \theta(n) + n\log(n)$

$n^2 + n\log(n)$

$\theta(n^2)$

C) first loop : $\Theta(n)$

make_pair: $\Theta(1)$

insert: $\log(1)+\log(2)+\ldots+\log(N) = \sum_{i=1}^{N} \log(i)$

$\Theta(n)\,\Theta(\log(N)) = \Theta(N\log(N))$

Second loop : $\Theta(n)$

find : $\log(n)$ runtime

k /= 2 = $\log(N)$

int k = max will be n since (x%n) max is n-1

$\Theta(\log(n))$ of while

push_back(x) : $\Theta(1)$

t1 [*]-> $\log(n)$ size of map

$(\log(n))^2$

$N\log N + n\log^2 n$

$\Theta(n\log n(\log n))$

D)

outer loop: n

if: max once

first: $\frac{3}{2}(10) = 15$

$\frac{3}{2}(15) = \frac{3}{2} \cdot \frac{3}{2} \cdot 10$

$\frac{3}{2} \cdot \frac{3}{2} \cdot \frac{3}{2} \cdot 10$

$\frac{3}{2}^K 10$ exp K in n

$n == 5120$

$\frac{3}{2} K 10 = n$

$\frac{3}{2}^K 10 = n$

$\frac{3}{2}^K = \frac{n}{10}$

$n \log_{\frac{3}{2}} \frac{n}{10}$

$\log_{\frac{3}{2}} \frac{n}{10} = \log\left(\frac{n}{10}\right)$

$K = \log\left(\frac{n}{10} \cdot \frac{3}{2}\right)$

$\log_{\frac{3}{2}} \frac{n}{10}$

$\sum_{i=0}^{\log_{\frac{3}{2}} \frac{n}{10}} 10\left(\frac{3}{2}\right)^i$

$\left[n - \log_{\frac{3}{2}}\left(\frac{n}{10}\right) + \sum_{i=1}^{\log_{\frac{3}{2}} \frac{n}{10}} 20\left(\frac{3}{2}\right)^{(i-1)} \right.$

num times else executes

$\Theta(N)$