1) $T(n) = a(T(\frac{n}{b})) + f(n)$ rest of work done

$T(n)$: Time for f call

a: # of recursive calls

$\frac{n}{b}$: input to recursive call

$f(n) = 1$
B.C

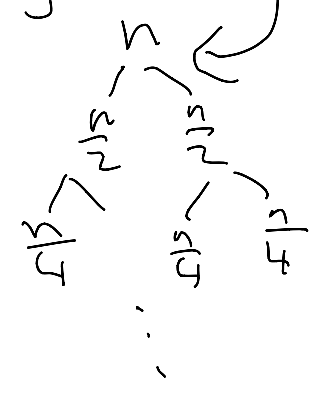
$T(n) = 2(T(\frac{n}{2})) + 1$

$= 1 + 2(1 + T(\frac{n}{4}))$

$= 1 + 2(1 + 2(\dots T(\frac{n}{2^n})))$

$= 2^0 + 2^1 + 2^2 + \dots + 2^{\log n}$

Height of the tree is $\log(n)$



Geometric summation: $\frac{2^{\log n + 1} - 1}{2 - 1}$

$= 2^{\log n + 1} - 1$

$= 2^{\log n} \cdot 2 - 1$; $2^{\log_2 n} = n$

$T(n) = (n \cdot 2) - 1$

2) $T^{rf2}(m) = a(T(\frac{n}{b})) + T(0)$

$T(0) = 1$

$= 1 + 1\,T_{rf2}(m-1)$

$= 1 + (1 + T_{rf2}(m-2))$

$= \underbrace{1 + 1 + (1 \ldots + T_{rf2}(0))}_{m} = m+1$

$n-1+1$

$T_{rf1}(n) = 1 + T_{rf2}(n-1) + T_{rf1}(n-1) \quad T(0)=1$

$(1+n) + (1 + T_{rf2}(n-2) + T_{rf1}(n-2))$

$1+n-3+1 = n-1$

$(1+n) + (n) + (1 + T_{rf2}(n-3) + T_{rf1}(n-3))$

$(1+n) + n + (n-1) + \ldots + 1$

Arithmetic summation

$$\sum_{i=1}^{n+1} i = \frac{(n+1)(n+1+1)}{2} = \frac{(n+1)(n+2)}{2}$$

Trace:

data1

n=5, data1=(9, 3, 1, 7, 4) $\xrightarrow{rf1}$ (3, 9, 1, 7, 4) *data

↳ (3, 1, 9, 7, 4)

↳ (3, 1, 7, 9, 4)

↳ (3, 1, 7, 4, 9)

↳ return

n=4, data1=(3, 1, 7, 4, 9) $\xrightarrow{rf1}$ After all recursive calls data*

= (1, 3, 4, 7, 9)

n=3, data1=(1, 3, 4, 7, 9) $\xrightarrow{rf1}$ = (1, 3, 4, 7, 9)

3 < 4

4 < 7

n=2 data1(1, 3, 4, 7, 9) → (1, 3, 4, 7, 9)

3 < 4

n=1 data1(1, 3, 4, 7, 9) → (1, 3, 4, 7, 9)

n=0 return

Each n runs (n-1) + 1 (return)) times

5 + ~~4~~ 4 3 + 2 + 1 + 0 = 15 runs

there was 4 swaps in rf1: n=5 and 2 in rf1: n=4

15 + 2 + 4 = 21 $\quad T_{rf1} = \frac{(5+1)(5+2)}{2} = 21$