Deadline: Sun Dec 12, 2021, 8:00 am Submit single unzipped PDF file on learn-web course "SoSe 2021: 3104 Modern Optimization Techniques

Instructions

Please following these instructions for solving and submitting the exercise sheet.

1. Student should clearly write his/her name, matriculation number and tutorial group number (i.e. "Group 1: Tuesday Tutorial", "Group 2: Wednesday Tutorial").
2. The submission should be made before the deadline, only through learnweb to your group submission link.
3. Should be submitted as a single unzipped PDF file on learn-web course "SoSe 2021: 3104 Modern Optimization Techniques".
4. Each student must submit an individual solution in-order to be eligible for bonus points.
5. Group submission are acceptable but will not contribute towards bonus points.

1 Exact Newton Method (10P) (10 points)

Let us (theoretically) optimize the following function using a Newton Descent approach:

$$f(x, y) = ln(1 + x + 2y)$$

a) Compute the gradient $\nabla f(x, y)$ and the Hessian $\nabla^2 f(x, y)$!

b) Compute $\nabla^2 f(x, y)^{-1}$, using Cramer's Rule:

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \implies A^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

c) Compute the update step of the Newton Algorithm:

$$\Delta_{x,y} = -\nabla^2 f(x, y)^{-1} \nabla f(x, y)$$

2 Quasi-Newton Method: BFGS (10P) (10 points)

Optimize the following function using the BFGS Quasi-Newton method.

$$f(x_1, x_2) = x_1^2 + 0.5x_2^2 + 3$$
$$x_0 = [1, 2]^T$$

Start with a random initialization of step size = 0.001 and show 2 iterations of the algorithm! Also start with a diagonal Hessian in the first iteration!

Important Notes:

- You have to show ALL the steps in detail using the BFGS Quasi-Newton method algorithm mentioned in the lecture.
- You are not permitted to code this question; otherwise, you will receive a zero.