Introduction: Machine Learning

Lecture Series "Machine Learning"

Niels Landwehr

Research Group "Data Science" Institute of Computer Science University of Hildesheim

Agenda For Lecture Today

- Organization and format of the lecture
- Motivation: why machine learning?
- Basic concepts in machine learning



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Welcome

 Lecture "Maschinelles Lernen"/"Machine Learning" given by "Data Science" group this year

Headed by Prof. Dr. Niels Landwehr

Research and teaching in machine learning and its applications

Contact: <landwehr@uni-hildesheim.de>
Building J, Room 103 on main campus



- Lecture is alternating between ISMLL and data science groups
- Lecture content is similar, but not identical

Organization: Degree Programmes

- Lecture "Maschinelles Lernen"/"Machine Learning": 2+2 SWS, 6 CPs
- Degree programmes in which the lecture appears:
 - Bsc degrees "Informationsmanagement und Informationstechnologie" and "Angewandte Informatik" (mandatory)
 - Bsc degree "Wirtschaftsinformatik" (elective)
 - Msc degree "Data Analytics" (mandatory)
 - Msc degrees "Informationsmanagenement und Informationstechnologie" and "Angewandte Informatik" (elective, if not taken during Bachelor studies)
- Other degrees: as given in degree regulations
- The lecture is a pre-requisitive for many follow-up courses in the area of machine learning offered by ISMLL and Data Science groups



Organization: Lecture

- Language: language of lecture and also exercises will generally be English, but ok to ask questions in German
- Format of lecture: we will (mostly) use an inverted-classroom format:
 - The main lecture content will be in a weekly 90-minute video lecture that is made available every Wednesday on the Learnweb page of the course
 - Additionally, we will meet every Friday at 10:15 in lecture hall H2 for a Q & A session to ask questions and discuss the lecture content
 - In order to follow the lecture, you must watch the weekly video lecture
 - Additionally, it is highly recommended to come to the on-campus Q & A, especially if you have questions about the current video lecture
 - To be able to ask meaningful questions, please watch the video lecture before coming to the Q & A on Friday



Organization: Lecture

Motivation for inverted-classroom format:

- In inverted-classroom format, we use the in-presence time for interactive discussions rather than just a frontal lecture which tends to not be very interactive
- Experience is that students often can best ask questions after having gone through a lecture in detail: difficult to ask questions "live" during the lecture
- Having a video lecture is helpful for reviewing the lecture content later and studying for exams

Timeline:

- Every Wednesday there will be a new video lecture
- This video lecture will be discussed Friday the same week
- Today we have an introductory regular lecture as there is nothing to discuss yet
- First video lecture will be available Wednesday next week (08.11.2023)
- First Q & A session will be on Friday 10.11.2023



Organization: Tutorials

- **Tutorials**: The lecture will be accompanied by weekly tutorial sessions
- There will be five groups:
 - G1: Monday 8:15-9:45, Room C 2.13, Samelson [Jafar Bakhshaliyev]
 - G2: Monday 8:15-9:45, Room B 1.48, Samelson [Muhammad Sameer Ali Khan]
 - G3: Wednesday 8:15-9:45, Room C 2.13, Samelson [Aseel Alhermi]
 - G4: Wednesday 8:15-9:45, Room B 0.26, Samelson [Can Shenol Berk]
 - G5: Wednesday 14:15-15:45, Room G 009, main campus [Ujjwal]
- You will be assigned to a tutorial group using a poll in the Learnweb:
 - On the learnweb page of the tutorials, you will find a link called "Choose your tutorial group"
 - Click on the link and enter three options for tutorial groups that you would like to attend (ordered by your preferences)
 - You have to enter your preferences until Monday, 06.11.2023 using that link
 - On Tuesday 07.11.2023, we publish the assignment of students to tutorial groups

Learnweb Page of Course

- Learnweb page of course:
 - https://www.uni-hildesheim.de/learnweb2023/course/view.php?id=2991

- Learnweb page contains much important information about organization of lecture
 - Dates, deadlines, announcements (also per email)
 - Lecture slides and video lecture
 - Weekly exercise sheets
- Learnweb course for the exercises:
 - https://www.uni-hildesheim.de/learnweb2023/course/view.php?id=2992
- Please enroll in the Learnweb courses for the lecture and for the exercises if you want to participate in the lecture and check the Learnweb frequently!



Organization: Exercise Sheets

- There will be weekly exercises that are made available on the Learnweb page of the course
 - Some exercises are theoretical, some practical programming tasks
 - Exercise sheets are made available every Thursday, and you have to solve them and hand them in until the following Thursday at noon (12:00)
 - Solutions have to be handed in through the exercise Learnweb: we will make separate upload links available for tutorial groups 1, 2, 3, 4, 5.
 - Please make sure you upload your solution to the right tutorial group (the one you were assigned to) to avoid confusion
- Exercises will be corrected by tutors and the points you get in the exercises count up to 10% in the final grade you achieve for the course
- First exercise sheet has just been made available, and is due 09.11.2023 at noon

Next Steps and Next Meetings

Next steps and next meetings

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Fri 03.11.2023: First introductory lecture (lecture hall H2)
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Fri 03.11.2023: First introductory exercise sheet is made available

Wed 08.11.2023: First video lecture is made available in the Learnweb

Thu 09.11.2023: You have to hand in solution to first exercise sheet

Thu 09.11.2023: Second exercise sheet is made available

Fri 10.11.2023: First Q & A (lecture hall H2, main campus)

Mon 13.11.2023/Wed 15.11.2023: First tutorials take place

Wed 15.11.2023: Second video lecture is made available in the Learnweb

Thu 16.11.2023: You have to hand in solution to second exercise sheet

Thu 16.11.2023: Third exercise sheet is made available

Fri 17.11.2023: Second Q & A (lecture hall H2, main campus)

Mon 20.11.2023/Wed 22.11.2023: Second tutorials take place

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Questions?

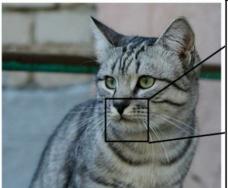
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Motivation: Computer Vision

- Motivation: how do we solve problems in computer science for which manually programming a solution is difficult?
- **Example image classification**: it is very difficult to manually write a program that can classify images from their pixel representation
 - Image analysis is difficult due to "semantic gap": pixel values determine semantic content of image, but the mapping from pixels to content is not straightforward
 - Manually writing code that analyzes pixel representation and determines, for example, if a cat is visible in an image is very difficult

In this example image, a human observer will immediately recognize the cat.



[165 112 108 111 104 99 106 99 96 103 112 119 104 97 93 87]
[91 98 102 106 104 79 98 103 99 105 123 136 110 105 94 85]
[76 85 90 105 128 105 87 96 95 99 115 112 106 103 99 85]
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[122 124 144 80 87 15 56 88 93 110 185 130 193 194 06 107 112 199]
[122 124 144 80 87 15 56 88 93 110 185 130 139 39 106 16 16 84]

Computer only "sees"
big grid of numbers
(pixel values).
Not easy to go from pixel
values to semantic category!

This image by Nikita is licensed under CC-BY 2.0

J. Johnson, 2019



Motivation: Natural Language Processing

- Motivation: how do we solve problems in computer science for which manually programming a solution is difficult?
- **Example automatic translation**: it is very difficult to manually write a program that can translate text e.g. from German to English at high quality
 - We know a lot about languages (world-level dictionaries, rules of grammar, etc.),
 but the complexity and ambiguity of natural language is a huge challenge
 - Manually designed rule-based systems can translate language up to a certain level of quality, but lack far behind human translators

Example for ambiguity in language:

"The dog liked to guard the house and the postman could not make it to the door because he was barking viciously"

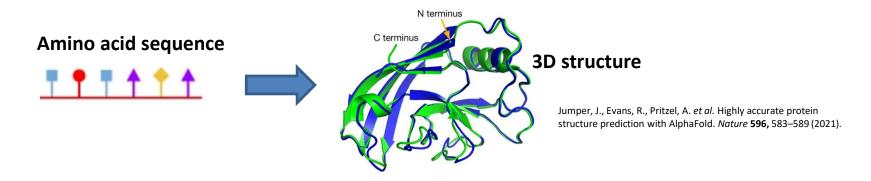


From grammatical perspective, unclear if the dog is barking or the postman



Motivation: Protein Folding

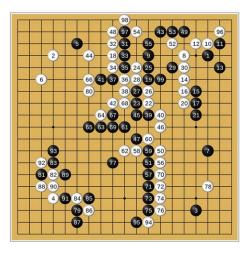
- Motivation: how do we solve problems in computer science for which manually programming a solution is difficult?
- Protein folding: it is very difficult to manually write a program that can predict how a
 protein will fold in 3D based on its amino acid sequence
 - long standing and important problem in biology: predict the 3D structure of a protein from its amino acid sequence



- Folding is based on principles of physics, which can be simulated using software
- However, complexity is so high that predictions based on physical simulations are not always reliable

Motivation: Playing Go

- Motivation: how do we solve problems in computer science for which manually programming a solution is difficult?
- Game playing ("Go"): it is very difficult to write a computer program that can beat a
 professional human player at the game "Go"
 - Even though the rules of Go are simple and can be easily encoded in software,
 programming a computer to play Go at expert level is very difficult
 - Studied for decades as a difficult benchmark problem for game-tree search
 - Much more difficult than e.g. chess due to high branching factor and difficulty in evaluating board positions (determining which player is ahead)



Game state in "Go"

Machine Learning

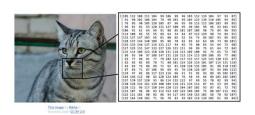
 Machine learning: solve such difficult problems by learning a solution from data, rather than manually programming a solution!

- 1. Collect **data** about the problem we are trying to solve: for example, images of cats and non-cats, known German-English translations, known protein structures, observed Go games, ...
- 2. From the observed data, learn a model that can solve the problem
- 3. Model can then be applied to new problem instances to solve them

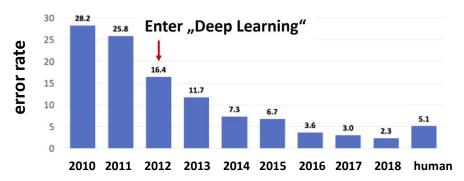
- Surprisingly powerful paradigm that can be applied to surprisingly many domains
- Machine learning is the driving force behind the "AI Revolution"

Machine Learning for Computer Vision

- Motivation: how do we solve problems in computer science for which manually programming a solution is difficult?
- Example machine learning for computer vision: instead of manually programming a solution, collect example images and learn a classifier
- Modern machine learning has dramatically pushed the state-of-the-art in almost all computer vision problems
 - In some domains, machine-learning approaches reach superhuman performance
 - E.g. ImageNet competition: error rates have fallen from approx. 30% in 2010 to
 2.3% in 2017, which is better than human performance



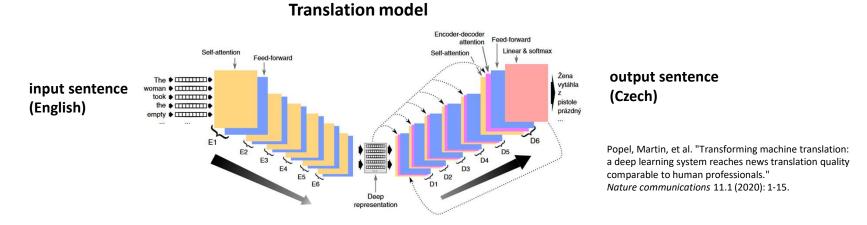
Results ImageNet competition





Machine Learning for Natural Language Processing

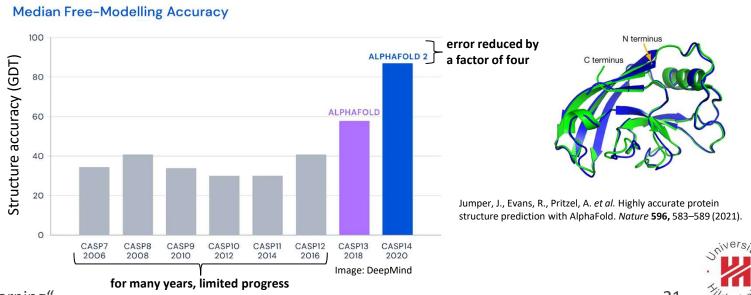
- Motivation: how do we solve problems in computer science for which manually programming a solution is difficult?
- Example machine learning for automatic translation: instead of manually programming a solution, collect example translations and train a machine learning model



 Dramatically improved the quality of machine translation compared to earlier systems, in some cases reaching or surpassing translation quality of human professionals

Machine Learning for Protein Folding

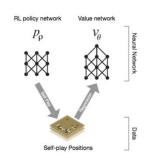
- Motivation: how do we solve problems in computer science for which manually programming a solution is difficult?
- Example machine learning for protein fold prediction: instead of relying on physical simulations, learn to predict structures based on a data set of known structures
 - Again, machine learning approaches have yielded dramatic improvements
 - E.g. "Critical Assessment of Structure Prediction" (CASP) competition: machinelearning system AlphaFold greatly reduces error compared to previous methods



Machine Learning for Playing Go

- Motivation: how do we solve problems in computer science for which manually programming a solution is difficult?
- Example machine learning for game playing ("Go"): instead of manually programming a Go playing engine, let a computer learn to play Go through playing games against itself ("reinforcement learning")
 - Until relatively recently, it was thought that Go programs that play at the level of professional human players were still decades away
 - Using deep reinforcement learning has quickly lead to Go programs that play far better than any human player

In 2016, in a widely televised match, the machine learning-based system AlphaGo beats Lee Sedol, the best Go player in the world









One Research Area, Many Names (and Aspects)

- Data-driven solutions in computer science are a broad field and are studied under different names, which each stress different aspects:
 - Machine learning: focus on the actual step of building models from data, using different kinds of models from decision trees to probababilistic models or deep neural networks
 - Pattern recognition: similar meaning as machine learning, often used in engineering, for example for computer vision and speech task
 - Data mining, big data: stresses the aspect of large data sets and complicated tasks
 - Knowledge discovery in data bases (KDD): stresses the embedding of machine learning tasks in applications, that is, aspects such as preprocessing and deployment
 - Data analysis: historically stresses multivariate regression and unsupervised tasks (see below)
 - Applied statistics: stresses underlying statistical models, testing and methodological rigor
 - Predictive analytics, business analytics, data analytics: stresses business applications
 - Data science: umbrella term encompassing aspects of machine learning, big data, data analysis etc



No fully standardized terminology, can be a bit confusing...

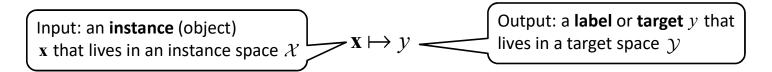


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Supervised Learning: Prediction

- In the following, we start to formalize the "learn a model from data" idea, starting with the most common machine learning setting called supervised learning
- In supervised learning, the goal is to make predictions y about some type of objects x:



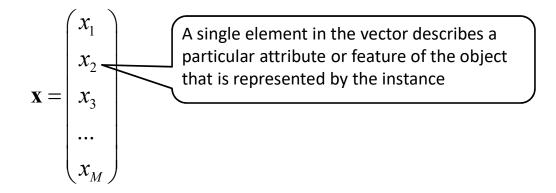
• To obtain predictions, we are looking for a **model** f that produces a prediction $f(\mathbf{x}) \in \mathcal{Y}$ for an input instance $\mathbf{x} \in \mathcal{X}$:

$$f: \mathcal{X} \to \mathcal{Y}$$
Input: instance $\mathbf{x} \mapsto f(\mathbf{x})$ Output: prediction $f(\mathbf{x})$

 Remark on notation: within formulas, we will usually use bold notation to indicate vectors or matrices and non-bold for scalar variables

Supervised Learning: Instances

- How can we represent objects as instances x?
- In the simplest case, instances are simply represented by vectors, that is, $\mathcal{X} = \mathbb{R}^M$:
- The elements x_m of the vector \mathbf{x} are called **features** or **attributes**



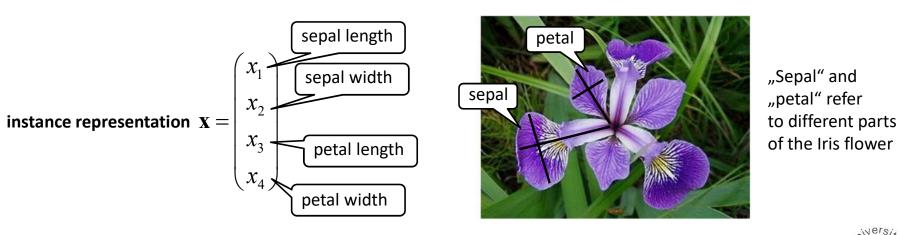
• The dimension M is given by the number of attributes or features that describe an object: can range from <10 to hundreds of thousands

Example Instance Representation: Iris Data Set

 As an example for representing objects as instances, assume we are trying to classify Iris flowers into different classes



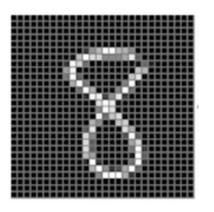
 A particular flower could be characterized by measuring its petal and sepal width and height, resulting in four attributes that describe an object/instance:



Example Instance Representation: Digit Recognition

- As another example for representing objects, consider the problem of recognizing scanned handwritten digits, which are represented as 28 x 28 pixel grayscale images
- These images can be represented by a $28 \cdot 28 = 784$ dimensional vector:

Scanned handwritten digit: 28 x 28 pixel grayscale image



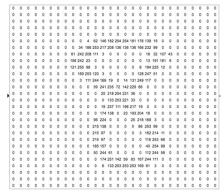
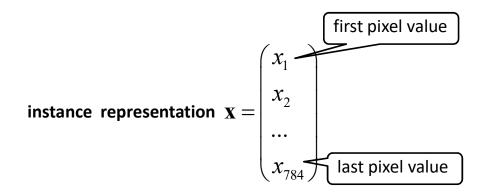


Image information: $28 \cdot 28 = 784$ pixel values between 0 and 255.

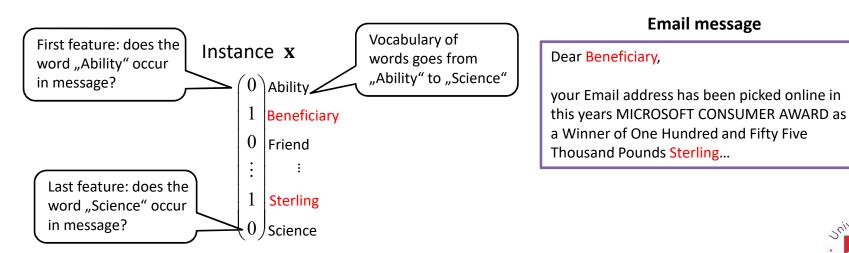


Note: representing images as vectors is not the best idea, better to use a matrix or tensor representation(see e.g. lecture "Advanced Computer Vision")



Example Instance Representation: Spam Filtering

- As another example for representing objects as instances, assume we are trying to classify email messages into legitimate and spam messages
- An email message can be represented by a binary word occurence vector
 - There is a vocabulary of words that can occur in messages
 - An instance is represented by a vector $\mathbf{x} \in \mathbb{R}^M$, where M is the size of the vocabulary. That is, there is one element in the vector for every word
 - The word-specific features x_m are zero or one depending on whether that word appears in the message or not



Supervised Learning: Targets

- We have talked about representing instances $x \in \mathcal{X}$ (using feature vectors)
- What are possible outputs or targets $y \in \mathcal{Y}$?
- Depending on the outputs $y \in \mathcal{Y}$, different settings can be distinguished
- In **classification**, the output is one of a fixed set of possible classes
 - Formally, the target space is therefore $\mathcal{Y} = \{c_1, ..., c_T\}$
 - Without loss of generality and to simplify notation, we often assume $\mathcal{Y}=\{1,...,T\}$: can simply rename the classes to 1, ..., T
- In regression, the output is a continuous value (e.g. temperature, price, ...)
 - Formally, the target space is therefore $\mathcal{Y}=\mathbb{R}$
- More complex settings also exist: for example, in **structured output prediction** the target space y can contain structured outputs such as sequences, trees, graphs etc.

Learning Models From Data

- Recap: we want to have a model $f: \mathcal{X} \to \mathcal{Y}$ that predicts targets for instances
- Idea of machine learning: the model $f: \mathcal{X} \to \mathcal{Y}$ will be inferred from training data
- The training data is a set

$$\mathcal{D} = \{(\mathbf{x}_1, y_1), ..., (\mathbf{x}_N, y_N)\}$$

Training instances $\mathbf{x}_n \in \mathcal{X}$: observed objects in training data, for example flowers, images of digits, or emails

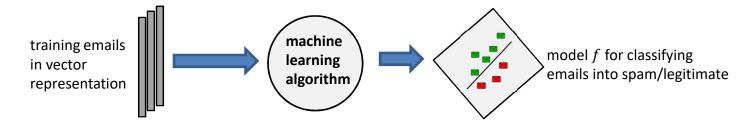
Observed labels or targets $y_n \in \mathcal{Y}$ in training data, for example classes of flowers, digits 0...9, or spam/legitimate classifications

- Idea: from the training data we can indirectly observe the relationship between inputs and targets
 - Are there any patterns in instances that make a certain class more likely?
 - E.g. pixel patterns that indicate cats, or words that indicate spam messages
- **Learning**: constructing a model $f: \mathcal{X} \to \mathcal{Y}$ that extracts these relationships from the data and thereby makes them explicit and applicable to predictions

Learning Algorithms

- The main task in machine learning is to construct the model $f: \mathcal{X} \to \mathcal{Y}$ from data \mathcal{D}
- A **learning algorithm** takes as input the training data $\,\mathcal{D}\,$ and produces a model $\,f\,$

Example spam filter:



- Often, learning is formulated as an optimization problem:
 - There is a space of possible models f
 - The learning algorithm has to search in this space for a model that captures the (\mathbf{x}, y) -relationship observed in training data well
 - Many possible choices for models, settings, and optimization approaches: see following lectures during the semester

Training Data: Assumptions

- For learning to work, we have to assume that there is some reasonably stable relationship between inputs and outputs that can be captured by a model
- Typical assumption: training examples are independently drawn from a (constant) joint distribution over inputs and outputs:

$$(\mathbf{x}_n, y_n) \sim p(\mathbf{x}, y)$$

- Because $p(\mathbf{x}, y) = p(\mathbf{x})p(y | \mathbf{x})$, the assumption can be reformulated as
 - The instances \mathbf{x}_n are sampled from a probability distribution over instances
 - p(x) describes distribution over population of objects
 - For example, certain flowers, digits, or email texts are encountered with a certain probability

$$\mathbf{x}_n \sim p(\mathbf{x})$$
$$\mathbf{y}_n \sim p(\mathbf{y} \,|\, \mathbf{x}_n)$$

- Given an instance \mathbf{x}_n , its label is drawn from a distribution $p(y | \mathbf{x}_n)$ that represents the relationship between input and output
- The relationship could be deterministic (probabilities 0 or 1) but this formulation also allows for randomness or noise in data

Assumptions About Data at Application Time

- Crucially (and somewhat optimistically) we assume that at application time, that is, when the trained model is deployed in the real world, it will encounter instances from the same distribution $p(\mathbf{x}, y)$ that the training set has been drawn from
- At application time, the model will encounter novel instances: e.g. novel measurements of Iris flowers, novel images of digits, or novel email texts
 - We assume that such a novel instance is drawn from the same distribution:

$$\mathbf{x}_{new} \sim p(\mathbf{x})$$

The model produces a prediction for the novel instance:

$$\hat{y} = f(\mathbf{x}_{new})$$

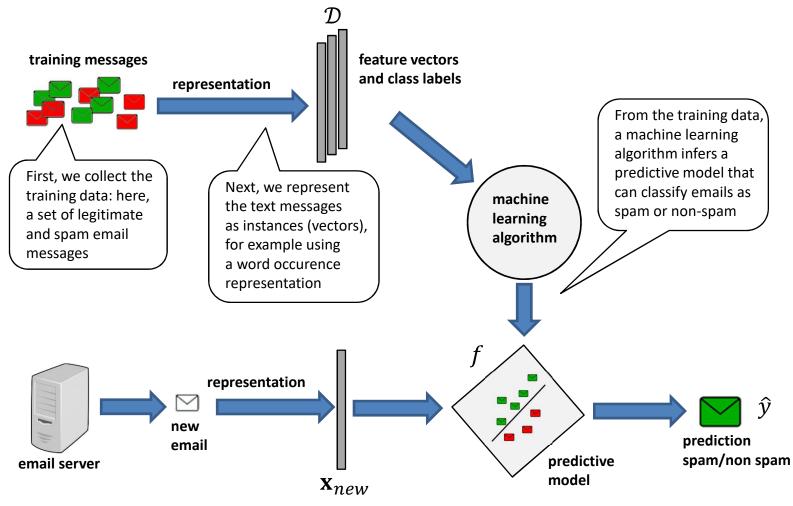
 There is also a true target for the new instance, which is unknown (for example, the true class of the flower, true digit, or true legitimate/spam status of email):

$$y_{new} \sim p(y \mid \mathbf{x}_{new})$$

- If learning has worked, we expect that prediction \hat{y} and true target y_{new} are the same or close (more formal treatment later...)

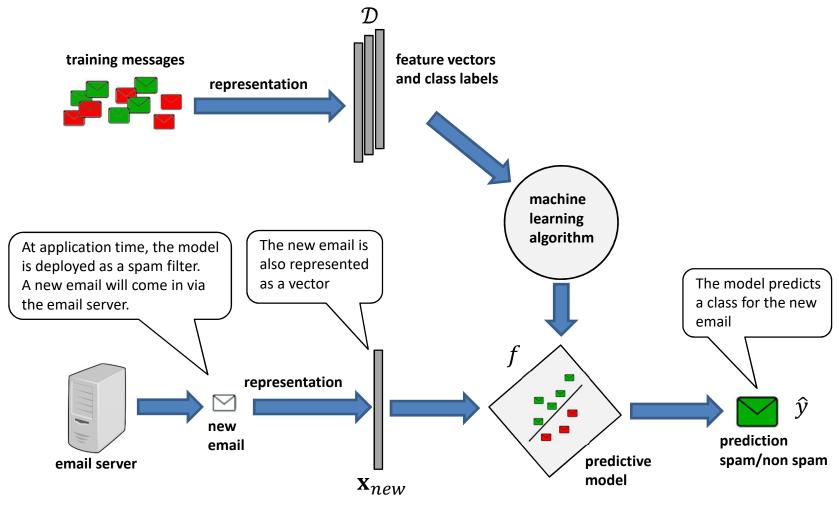
Overall Machine Learning Pipeline

• Typical overall machine learning pipeline, for the example of spam filtering



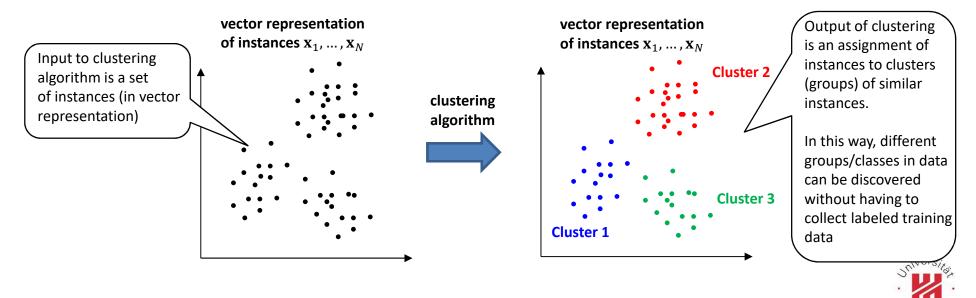
Overall Machine Learning Pipeline

• Typical overall machine learning pipeline, for the example of spam filtering



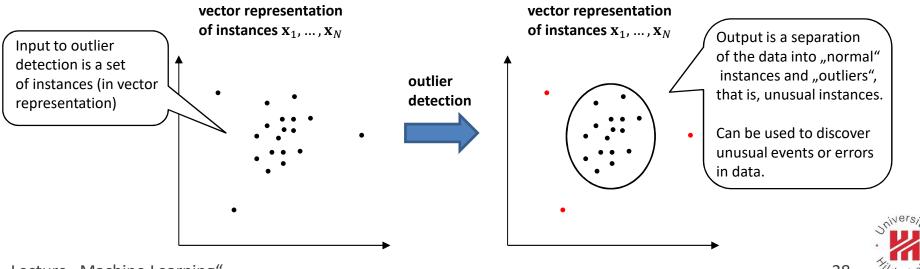
Unsupervised Learning

- So far we talked about supervised learning, which is charaterized by having inputoutput pairs of the form $(\mathbf{x}_1, y_1), ..., (\mathbf{x}_N, y_N)$ as training data
- Unsupervised learning is an alternative scenario for machine learning, in which only instances $\mathbf{x}_1,...,\mathbf{x}_N$ but no labels $y_1,...,y_N$ are available
- Within unsupervised learning, different problem settings can be studied, for example:
 - Clustering: find groups of similar instances



Unsupervised Learning

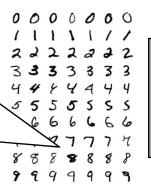
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 - Clustering: find groups of similar instances
 - Outlier detection



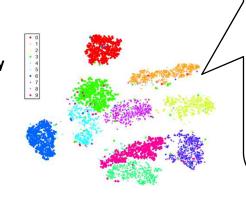
Unsupervised Learning

- So far we talked about supervised learning, which is charaterized by having inputoutput pairs of the form $(\mathbf{x}_1, y_1), ..., (\mathbf{x}_N, y_N)$ as training data
- **Unsupervised learning** is an alternative scenario for machine learning, in which only instances $\mathbf{x}_1,...,\mathbf{x}_N$ but no labels $y_1,...,y_N$ are available
- Within unsupervised learning, different problem settings can be studied, for example:
 - Clustering: find groups of similar instances
 - Outlier detection
 - Visualization/ dimensionality reduction

Input to dimensionalty reduction is a set of instances in high-dimensional vector representation, for example images of digits



dimensionality reduction

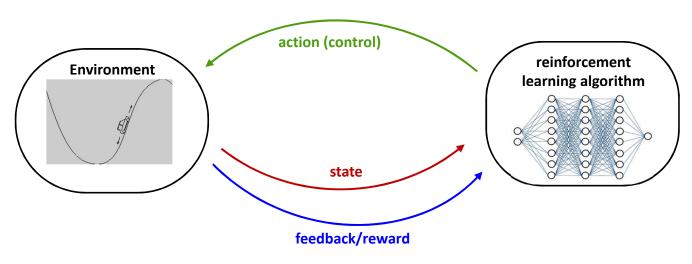


Output of dimensionality reduction is the same set of instances in a lower-dimensional representation (here: 2D), such that similarities between instances are preserved. Here, digits 0-9 form visible clusters in 2D.



Reinforcement Learning

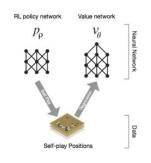
- Besides supervised and unsupervised settings, another important setting in machine learning is reinforcement learning
- In reinforcement learning, the goal is to solve a control problem:
 - An agent (controlled by the reinforcement learning algorithm) can interact with its environment and perceive the state of the environment
 - At any point in time, the agent can take one of several actions
 - The correct action is never shown, however, the agent gets delayed feedback: at certain points in time, it gets a positive or negative "reward" that indicates whether the executed sequence of actions was successful or not



Reinforcement Learning: Example

- Example for reinforcement learning: learn to play the boardgame "Go" (also see slides above)
- Observable is the current board state
- Possible actions: place a stone in one of the 381 grid cells of the board (minus illegal moves...)
- Reward: +1 if in the end game is won, -1 if game is lost
- Could train by playing against human players, or (more effective) a copy of itself

Leads to superhuman Go playing abilities (in combination with tree search techniques)



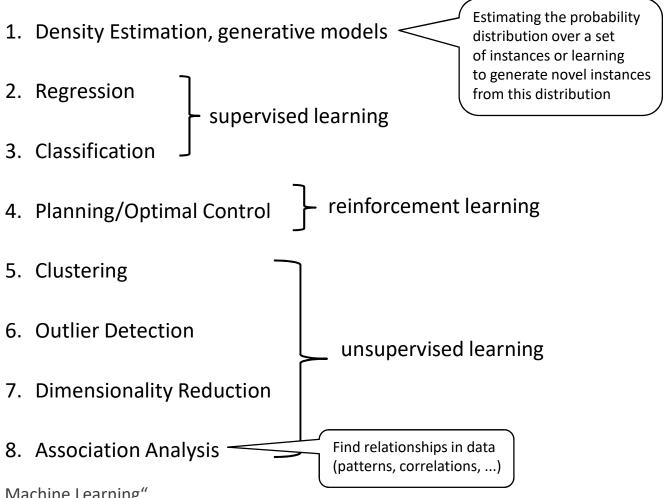






Summary: Machine Learning Problems

• Summary: There are many different problem settings within machine learning



Literature

- Some books about machine learning:
 - Gareth James, Daniela Witten, Trevor Hastie, R. Tibshirani (2013): An Introduction to Statistical Learning with Applications in R, Springer.
 - Kevin P. Murphy (2012): Machine Learning, A Probabilistic Perspective, MIT Press.
 - Trevor Hastie, Robert Tibshirani, Jerome Friedman (2009): The Elements of Statistical Learning, Springer. Also available online as PDF at http://www-stat.stanford.edu/~tibs/ElemStatLearn/
 - Christopher M. Bishop (2007): Pattern Recognition and Machine Learning,
 Springer.
 - Richard O. Duda, Peter E. Hart, David G. Stork (2001): Pattern Classication,
 Springer.
 - Ian Goodfellow and Yoshua Bengio and Aaron Courville (2016): Deep Learning,
 MIT Press. Also available online at www.deeplearningbook.org