Deadline: **Sun Nov 12, 2023, 8:00 am** Submit single unzipped PDF file on learn-web course "SoSe 2021: 3104 Modern Optimization Techniques

Instructions

Please following these instructions for solving and submitting the exercise sheet.

1. Student should clearly write his/her name, matriculation number and tutorial group number (i.e. "Group 1: Tuesday Tutorial", "Group 2: Wednesday Tutorial").
2. The submission should be made before the deadline, only through learnweb to your group submission link.
3. Should be submitted as a single unzipped PDF file on learn-web course "SoSe 2023: 3104 Modern Optimization Techniques".
4. Each student must submit an individual solution in-order to be eligible for bonus points.
5. Group submission are acceptable but will not contribute towards bonus points.

1 Convex Sets and Convex Functions theory (10 points)

(a) What is convexity and why is important in optimization? (2pt)

(b) What is a convex set and what is a convex function? (2pt)

(c) Let

$$D = \left\{x \in \mathbb{R}^2 | 0 \leq x_1 \leq 2 \wedge 0 \leq x_2 \leq 3\right\}$$

be a subset of $\mathbb{R}^2$. Draw a sketch of D. Show, by using the definition, that D is a convex set.(3pt)

(d) In the lecture were presented three different ways to determine the convexity of a function. What represents each of those definitions? You can use a sketch. (3pt)

(i) Jensen's inequality: $f(\theta x + (1-\theta)y) \leq \theta f(x) + (1-\theta)f(y); \quad \forall x, y \in \text{Dom}(f), \theta \in [0,1]$

(ii) First-order Condition $f(y) \geq f(x) + (\nabla_x f(x))^T (y - x); \quad \forall x, y \in \text{Dom}(f),\ f$ differentiable

(iii) Second-order Condition: $\nabla_x^2 \geq 0; \quad \forall x, y \in \text{Dom}(f),\ f$ twice differentiable

2 Second Order Convexity Condition (10 points)

(a) Determine the values of the parameters a and b so that the following functions are convex in their domains.

(i) $f(x, y, z) = ax^2 + y^2 + 2z^2 - 4axy + 2yz$ (2pt)

(ii) $f(x, y) = 4ax^2 + 8xy + by^2$ (2pt)

(b) Determine the domains of the plane where the following functions are convex or concave by using the second order convexity condition.

(i) $f_1(x, y) = ln(\sqrt[2]{xy})$ (3pt)

(ii) $f_2(x, y) = (x-1)^2 + xy^2$ (3pt)