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Ex11

Task 1) The null space is the solutions for the $Ax=0$.

$$\begin{pmatrix} 2 & 3 & 4 \\ -1 & 2 & 3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \begin{cases} 2x_1+3x_2+4x_3=0 & (I) \\ -x_1+2x_2+3x_3=0 & (II) \end{cases} \rightarrow x_1 = 2x_2+3x_3 \quad (III)$$

$(III)-(I) \rightarrow 7x_2+10x_3=0 \Rightarrow x_2=\frac{-10}{7}x_3$ $\xrightarrow{(III)}$ $x_1=\frac{1}{7}x_3$

if we set $z=7t \Rightarrow \text{null}(A)=\left\{ t\begin{pmatrix} 1 \\ -10 \\ 7 \end{pmatrix} : t\in\mathbb{R} \right\}$

for $t=1$: $\begin{pmatrix} 2 & 3 & 4 \\ -1 & 2 & 3 \end{pmatrix}\begin{pmatrix} \frac{1}{7} \\ \frac{-10}{7} \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Task 2) $x_1^2+x_2^2$
st $x_1-2x_2=2$
$\nabla f = \begin{pmatrix} 2x_1 \\ 2x_2 \end{pmatrix}$, $A=(1\ -2)$

$x^0=(2,0)$ – $\alpha=0.4$ – $\beta=0.5$, $\mu^0=1$
↳ feasible starting point

Gradient Projection method (min-gp-affeq) with backtracking

$$F = I_n - A^T(AA^T)^{-1}A = \begin{pmatrix} 0.8 & 0.4 \\ 0.4 & 0.2 \end{pmatrix}$$

epoch 1)

$$\Delta x_0 = -F^T\nabla f(x_0) = \begin{pmatrix} -3.2 \\ 1.6 \end{pmatrix}$$

Backtracking

$\Delta f = a\nabla f(x)^T\Delta x = -5.12$

$f(x_0+\mu_0\Delta x_0) \le 4$ True
$f(x_0)+\mu\Delta f = -0.12$ or
not $(Ax_0=2)$ not(True)
$\rightarrow \mu' = \beta\mu_0 = 0.5$

another iteration: $f(x_0+\mu_1\Delta x_0)=0.0$ $\le$ or >1.44 false
$f(x_0)+\mu_1\Delta f = 1.44$
not $(Ax_0=2)$ false
$\rightarrow$ final $\mu = \mu_1 = 0.5$

$$x_1 = x_0+\mu_1\Delta x_0 = \begin{pmatrix} 0.4 \\ -0.8 \end{pmatrix}$$

epoch 2)

$\nabla f(x_1) = \begin{pmatrix} 0.8 \\ -1.6 \end{pmatrix}$ $\Delta x_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ → Coverage → $x^* = \begin{pmatrix} 0.4 \\ -0.8 \end{pmatrix}$