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Ex 4

Task 1) A python jupyter notebook is uploaded for this and also is attached at the end of this pdf.

Task 2)

(a) $L(\theta) = \frac{1}{N}\sum_{n=1}^{N}\left(f_\theta(x_n) - y_n\right)^2 + \lambda\|\theta\|_2^2$

$\frac{dL(\theta)}{d\theta} = \frac{1}{N}(-x)^T 2(y - x\theta) + 2\lambda\theta$: from slides

we put θ^* → $\frac{dL(\theta^*)}{d\theta} = \frac{-2}{N}X^T\left(y - x\left(X^TX + N\lambda I\right)^{-1}X^Ty\right) + 2\lambda\left(X^TX + N\lambda I\right)^{-1}X^Ty$

$= \frac{-2}{N}X^Ty + \frac{2}{N}\left(X^TX + \lambda I_N\right)\left(X^TX + \lambda NI\right)^{-1}\left(X^Ty\right) = \frac{-2}{N}\left(X^Ty - X^Ty\right)$

$= 0$

(b) $\nabla^2 L = \frac{2}{N}X^TX + \underbrace{2\lambda}_{\text{we had } \lambda > 0 \text{ (II)} \Rightarrow 2\lambda > 0}$

X^TX is positive indefinite if there is $\theta \in R^n$ that

$\theta^T X^T X \theta > 0$

so, $\theta^T X^T X \theta = (\theta X)^T(\theta X) = \|\theta X\|_2^2 > 0 \Rightarrow X^TX$ is positive indefinite (I)

(I) + (II) → Hessian is positive indefinite.

Task 3) A Python notebook and a pdf is attached.