Quiz: Bayesian Networks

Lecture series „Machine Learning“

Niels Landwehr

Research Group „Data Science“
Institute of Computer Science
University of Hildesheim

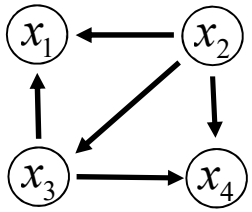
Quiz: Joint Distributions and Graph Structures

- Assume random variables $x_1,\ldots,x_4$ and a joint distribution of the form

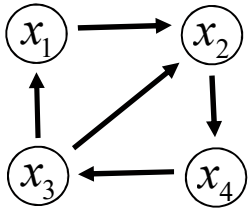
$$p(x_1,\ldots,x_4) = p(x_3)\,p(x_2 \mid x_3)\,p(x_1 \mid x_2,x_3)\,p(x_4 \mid x_2,x_3)$$

- **Question**: Which graph is correct for a Bayesian network representation for this distribution?

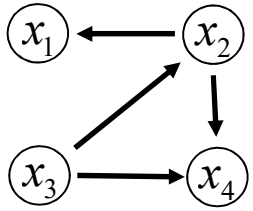
A



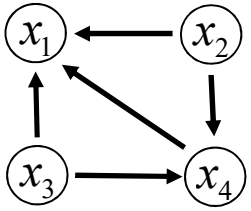
B



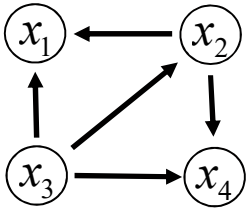
C



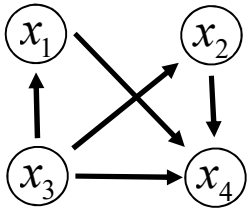
D



E

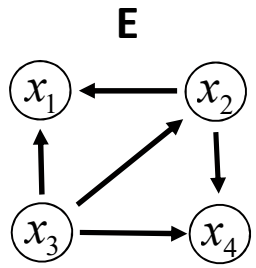


F



Solution: Joint Distributions and Graph Structures

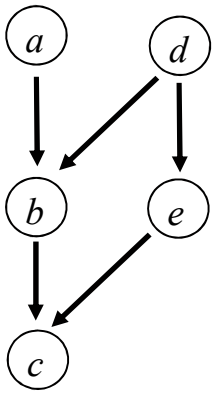
- **Solution**: graph E



$$p(x_1,...,x_4) = p(x_3)\,p(x_2 \mid x_3)\,p(x_1 \mid x_2,x_3)\,p(x_4 \mid x_2,x_3)$$

Quiz: D-Separation

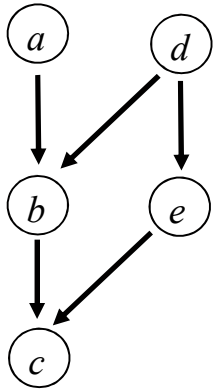
- Assume the following Bayesian network over random variables a, b, c, d, e:



- Which of the following statements is correct according to the graph structure of the network:
  - A) $a \perp c \mid b$
  - B) $a \perp c \mid \varnothing$
  - C) $d \perp c \mid b, e$
  - D) $a \perp d \mid b$
  - E) More than one of the statements is true

Solution: D-Separation

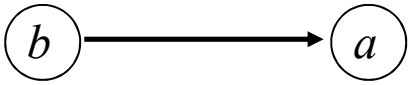
- Solution: C)



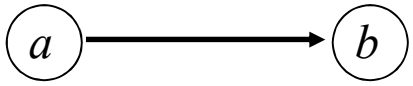
- Statement $a \perp c \mid b$ is false: path a,b,d,e,c is open (converging connection at b unblocked by evidence)
- Statement $a \perp c \mid \varnothing$ is false: path a,b,c is open (serial connection)
- Statement $d \perp c \mid b,e$ is true: pathes d,b,c and d,e,c are each blocked at serial connection
- Statement $a \perp d \mid b$ is false: path a,b,d is open (converging connection at b unblocked by evidence)

Quiz: Variable Orderings

- Assume the following simplified model for the "Alarm" domain discussed in the lecture. There are binary variables "Alarm" (a) and "Burglary" (b), and the Bayesian network structure is given by



- **Question**: Consider the following alternative Bayesian network structure:



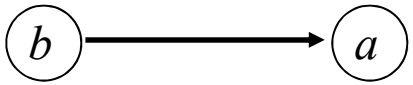
Is this network structure equivalent in the sense that it can represent exactly the same joint distributions over random variables a, b as the previous network structure?

  - Yes, it can represent the same distributions and encodes the same independence assumptions
  - It can only represent the same distribution as the network above for certain choices of the (conditional) probabilities $p(b), p(a \mid b)$ in the network above
  - No, it can never represent the same distribution

Solution: Variable Ordering

- **Solution**: Yes, it can represent exactly the same joint distributions

- Reason:
  - The original network does not make any independence assumptions, because the factorization $p(a,b) = p(b)\,p(a \mid b)$ is valid for any joint distribution $p(a,b)$. Therefore the original network can represent any joint distribution



  - The same holds for the alternative network: $p(a,b) = p(a)\,p(b \mid a)$ is valid for any joint distribution