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Ex-03

Task-1) $L = (f_\theta(X) - Y)^2$

$\frac{dL}{dw_1} = 2[f(x)-y] \cdot \frac{df_\theta}{dw_1} \overset{(I)}{\Rightarrow} \frac{dL}{dw_1} = 2[f(x)-y]\, w_2^T\, \delta'(w_1 x + b_1)\, x$

$\frac{dL}{dw_2} = 2[f(x)-y] \cdot \frac{df_\theta}{dw_2} \overset{(II)}{\Rightarrow} \frac{dL}{dw_2} = 2[f(x)-y]\, \left(\delta(w_1 x + b_1)\right)^T$

$\frac{dL}{db_1} = 2[f(x)-y] \cdot \frac{df_\theta}{db_1} \overset{(III)}{\Rightarrow} \frac{dL}{db_1} = 2[f(x)-y]\left(w_2^T \circ \delta'(w_1 x + b_1)\right)$

$\frac{dL}{db_2} = 2(f(x)-y) \cdot \frac{df_\theta}{db_2} \overset{(IV)}{\Rightarrow} \frac{dL}{db_2} = 2[f(x)-y]$

$f_\theta = w_2 \delta(w_1 x + b_1) + b_2$

$\frac{df_\theta}{dw_1} = w_2 \frac{d\,\delta(w_1 x + b_1)}{dw_1} = w_2 \left[\delta'(w_1 x + b_1)\right] \frac{d(w_1 x + b_1)}{dw_1}$

$= w_2\, \delta'(w_1 x + b_1)\, x$ (I)

$\frac{df_\theta}{dw_2} = \delta(w_1 x + b_1)$ (II)

$\frac{df_\theta}{db_1} = w_2 \frac{d\,\delta(w_1 x + b_1)}{db_1} = w_2\, \delta'(w_1 x + b_1) \frac{d(w_1 x + b_1)}{db_1} =$

$w_2\, \delta'(w_1 x + b_1)$ (III)

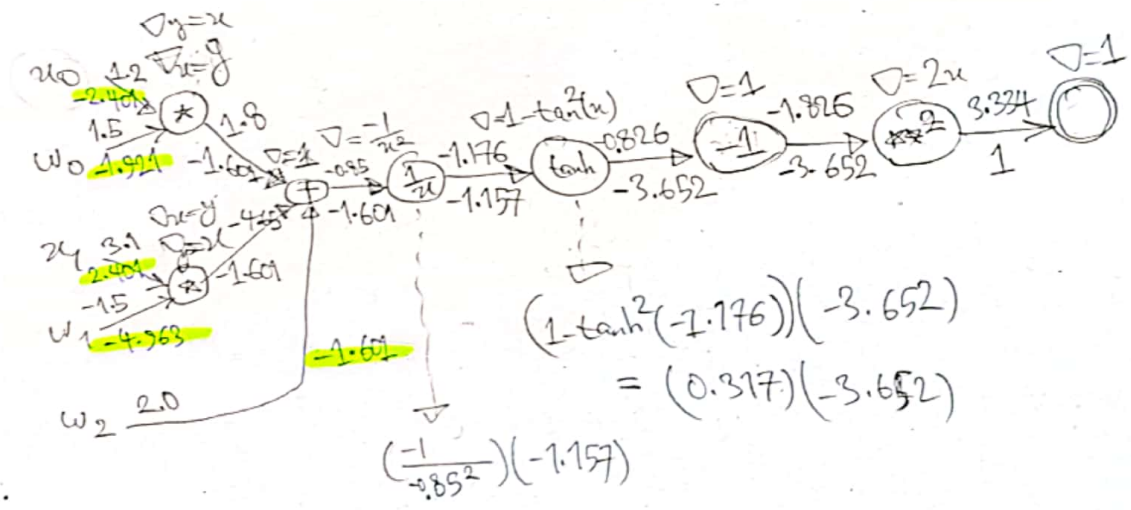
$\frac{df_\theta}{d2} = 1$ (IV)

Task 3

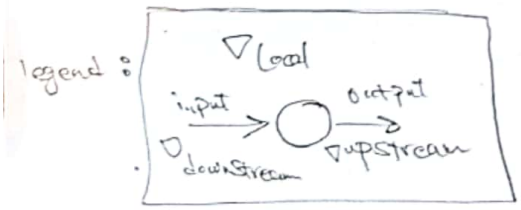
$$\text{Tanh}(x) = \frac{2}{1+e^{-2x}} - 1 = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\frac{d\,\text{Tanh}(x)}{dx} = \frac{(e^x+e^{-x})(e^x+e^{-x}) - (e^x-e^{-x})(e^x-e^{-x})}{(e^x+e^{-x})^2}$$

$$= \frac{(e^x+e^{-x})^2}{(e^x+e^{-x})^2} - \frac{(e^x-e^{-x})^2}{(e^x+e^{-x})^2} = 1 - \tanh^2(x)$$



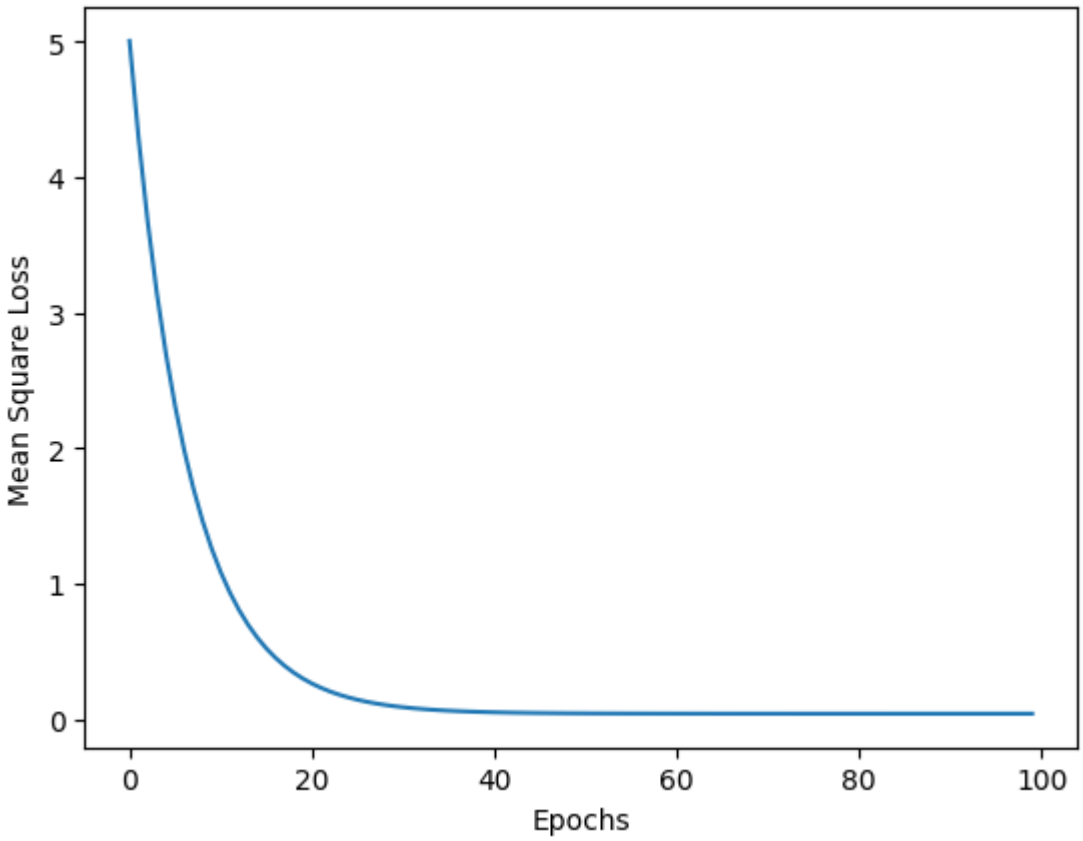
$$\nabla \text{downStream} = \nabla_{\text{local}}(\text{input}) * \nabla_{\text{upStream}}$$



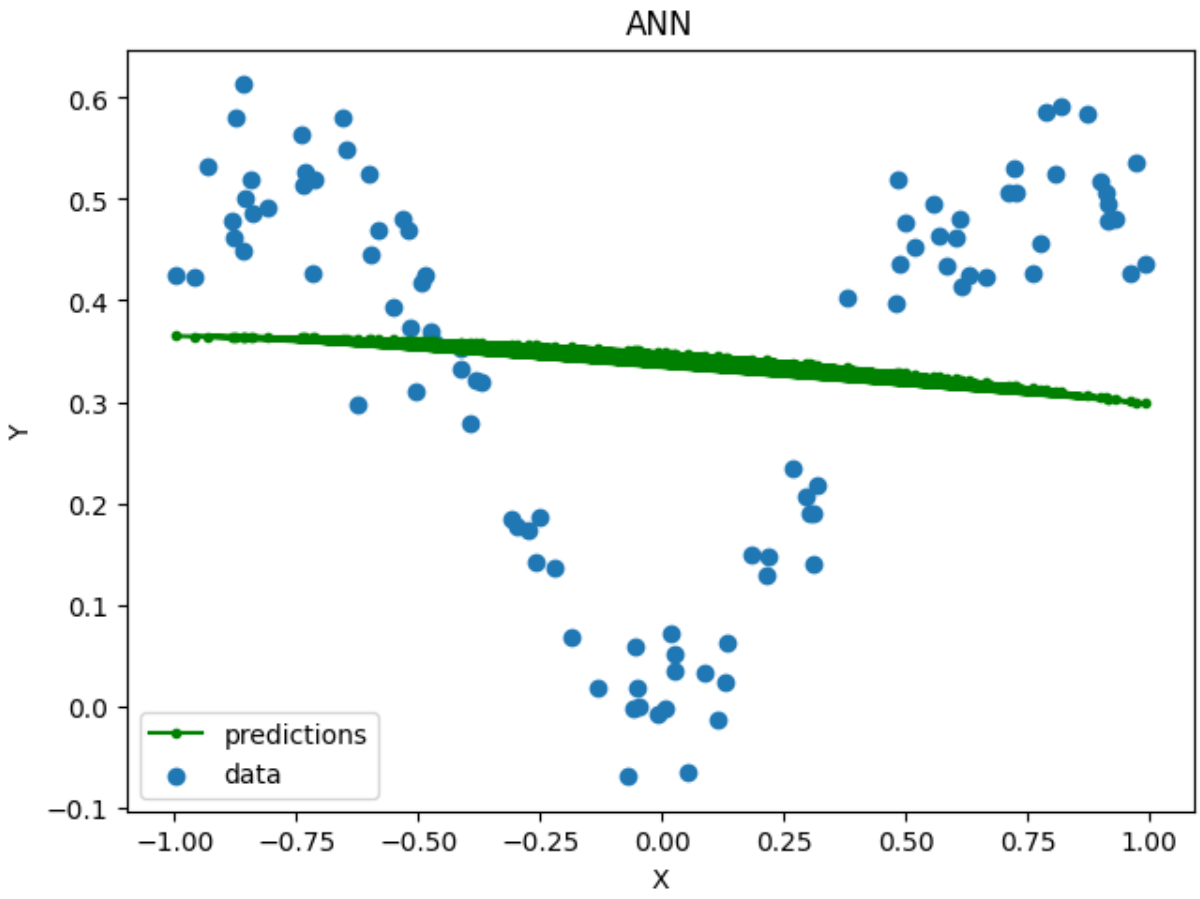
In [ ]:
```
import numpy as np
import matplotlib.pyplot as plt
```

In [ ]:
```
s = 0.5
k = 10
# number of points
n = 100
def f_org(x):
    return (1/(1+np.exp(-10*x**2))) - 0.5

def f_noisy(x):
    return f_org(x) + s*np.random.normal(0, 0.1, n)

x = np.random.uniform(-1, 1, n)
y = f_noisy(x)

class ANN:
    def __init__(self, x, y, epochs = 100, lr = 0.01):
        self.x = x
        self.y = y
        self.w1 = np.random.uniform(-1, 1, k).reshape(k, 1)
        self.w2 = np.random.uniform(-1, 1, k).reshape(1, k)
        self.b1 = np.random.uniform(-1, 1, k)
        self.b2 = np.random.uniform(-1, 1, 1)
        self.epochs = epochs
        self.lr = lr

    def sigmoid(self, z):
        # Sigmoid activation function
        return 1 / (1 + np.exp(-z))

    def sigmoid_d(self, z):
        return self.sigmoid(z)*(1 - self.sigmoid(z))

    def f(self, x):
        return (self.w2@self.sigmoid(np.dot(self.w1, x) + self.b1.reshape(k,1)) + s

    def grad_w1(self, x, y):
        return (2*(self.f(x) - y)*(self.w2.T*np.dot(
                self.sigmoid_d(np.dot(self.w1, x) + self.b1.reshape(k,1)), x
            ))).reshape(k, 1)

    def grad_w2(self, x, y):
        return (2*(self.f(x) - y)*(self.sigmoid(np.dot(self.w1, x) + self.b1.reshap

    def grad_b1(self, x, y):
        return (2*(self.f(x) - y)*(self.w2.T*(
                self.sigmoid_d(np.dot(self.w1, x) + self.b1.reshape(k,1))
            ))).reshape(k)

    def grad_b2(self, x, y):
        return (2*(self.f(x) - y)).reshape(1)

    def loss(self, x, y):
```

```
        return (self.f(x) - y)**2

    def mean_func(self, func):
        res = 0
        for i in range(n):
            res += func(self.x[i], self.y[i])
        return res / n

    def fit(self):
        self.history_loss = []
        for epoch in range(self.epochs):
            self.w1 -= self.lr*self.mean_func(self.grad_w1)
            self.w2 -= self.lr*self.mean_func(self.grad_w2)
            self.b1 -= self.lr*self.mean_func(self.grad_b1)
            self.b2 -= self.lr*self.mean_func(self.grad_b2)
            loss = self.mean_func(self.loss)
            self.history_loss.append(loss)

    def predict(self, x):
        return self.f(x)


model = ANN(x, y)
model.fit()

plt.plot(model.history_loss, label='Training Loss')
plt.xlabel('Epochs')
plt.ylabel('Mean Square Loss')
plt.show()
```



In [ ]:
```
fig, ax = plt.subplots(layout='constrained')
y_predicts = np.array([model.predict(d) for d in x])
# plot
ax.set_title("ANN")
ax.set_xlabel('X')
ax.set_ylabel('Y')
ax.plot(x, y_predicts, ".-", label='predictions', c='g')
ax.scatter(x, y,  label='data')
ax.legend()
plt.show()
```



In [ ]: