Deadline: **Sun Nov 05, 2023, 8:00 am** Submit single unzipped PDF file on learn-web course "SoSe 2021: 3104 Modern Optimization Techniques

Instructions

Please following these instructions for solving and submitting the exercise sheet.

1. Student should clearly write his/her name, matriculation number and tutorial group number (i.e. "Group 1: Tuesday Tutorial", "Group 2: Wednesday Tutorial").
2. The submission should be made before the deadline, only through learnweb to your group submission link.
3. Should be submitted as a single unzipped PDF file on learn-web course "SoSe 2023: 3104 Modern Optimization Techniques".
4. Each student must submit an individual solution in-order to be eligible for bonus points.
5. Group submission are acceptable but will not contribute towards bonus points.

1 Linear Programming (7.5 points)

A bakery produces two types of pastries, Croissants and Danish, for a local cafe. The production process involves two key ingredients: flour and butter. The table below details the amount of each ingredient required to make one unit of each pastry:

Pastry	Flour	Butter
Croissants	250g	75g
Danish	200g	100g

The bakery has 30 kilograms of flour and 15 kilograms of butter available for the next production cycle. Croissants are sold for 2$ each, while Danish pastries are sold for 3$ each. The bakery aims to maximize its profit.

1. Formulate the problem as a linear program to determine how many Croissants and Danish pastries the bakery should produce to maximize profit.
2. Solve this linear program graphically to find the optimal production quantities of Croissants and Danish pastries.
3. Calculate the maximum profit achievable.

2 Optimization Problems (12.5 points)

For the following real valued functions, find their associated minimum $x^\star$ and the minimal value $p^\star$, if they exist. Is $x^\star$ unique?

1. $f_1 : (a, b) \longrightarrow \mathbb{R}$ with $f_1(x) = x$
2. $f_2 : \mathbb{R} \longrightarrow \mathbb{R}$ with $f_2(x) = c \cdot cos(x) \qquad c > 0$
3. $f_3 : [0, 2\pi] \longrightarrow \mathbb{R}$ with $f_3(x) = sin(x)$
4. $f_4 : [0, 2\pi] \longrightarrow \mathbb{R}$ with $f_4(x) = \frac{sin(x)}{x^2}$
5. $f_5 : \mathbb{R} \longrightarrow \mathbb{R}$ with $f_5(x) = x^3 - 3x$