

## 1. LINEAR PROGRAMMING

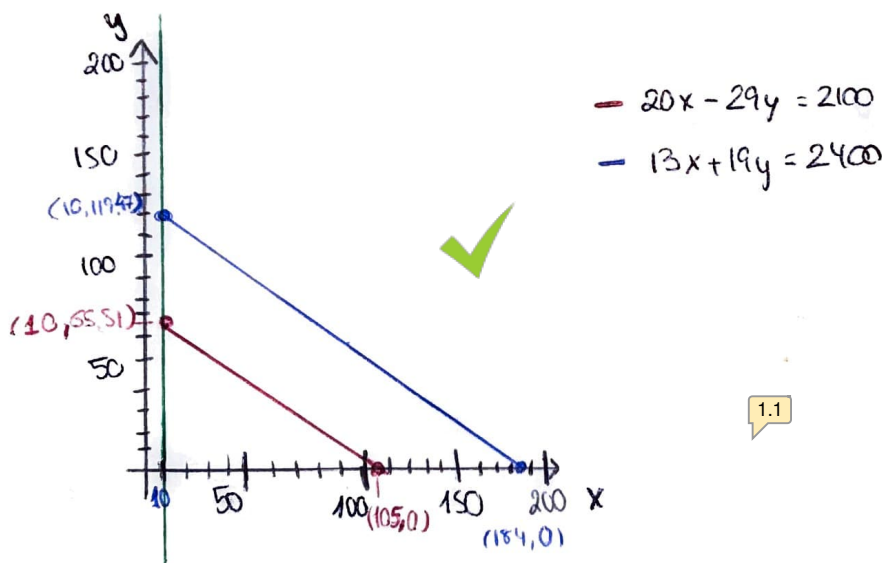
1. max.  $20x - 30y - 10t_m - 2t_c$ , where  $t_m$  = machine time  
 $t_c$  = craftsman time

s.t.  $13x + 19y \leq 2400$   
 $\hookrightarrow 40 \text{ hours} \times 60 \text{ min.}$

$20x + 29y \leq 2100$   
 $\hookrightarrow 35 \text{ hours} \times 60 \text{ min.}$

$x \geq 10$  : the company has to produce at least 10 items of X.

2.



When  $x = 10$ ,  $13 \cdot 10 + 19y = 2400$   
 $19y = 2400 - 130 \rightarrow y = \frac{2270}{19} = 119.47$  (10, 119.47)

$20 \cdot 10 + 29y = 2100$   
 $29y = 1900 \rightarrow y = \frac{1900}{29} = 65.51$  (10, 65.51)

When  $y = 0$ ,  $13x + 19 \cdot 0 = 2400$   
 $x = \frac{2400}{13} = 184.61$  (184.61, 0)

$20x + 29 \cdot 0 = 2100$   
 $x = \frac{2100}{20} = 105$  (105, 0)

## 2. OPTIMIZATION PROBLEMS

1.  $f_1(x) = x$

$$\frac{\partial f_1(x)}{\partial x} = 1 \neq 0$$

$$1 \neq 0$$

There is not minimum for in  $f_1(x)$ .

~~x can go~~  $f_1(x)$  can go from  $a$  to  $b$ , and  $f_1(x) = x$ , ~~Assuming~~ So  $a$  is the minimum value ~~because~~  $x^* = a$ ,  $p^* = a$ . ✓

2.  $f_2(x) = c \cos(x)$

$$f_2'(x) = -c \sin(x)$$

$$-c \sin(x) = 0$$

$$x = 0$$

2.1

$$f_2''(x) = -c \cos(x)$$

$$f_2''(0) = -c > 0$$

$p^* = -c$ , that happens at  $x^* = \pi$  or  $3\pi$

There is a minimum in  $-c$

$$x^* = -c$$

$x^*$  is not unique, depends on the  $c$

$$p^* = f_2(x^*) = c \cos(-c)$$

3.  $f_3(x) = \sin(x)$

$$f_3'(x) = \cos(x) = 0$$

$$x_1 = \arccos(0) = \frac{\pi}{2}$$

$$x_2 = 2\pi - \frac{\pi}{2} = \frac{3\pi}{2}$$

$$f_3''(x) = -\sin(x)$$

$$f_3''(\frac{\pi}{2}) = -1 < 0 \text{ not a minimum}$$

$$f_3''(\frac{3\pi}{2}) = 1 > 0 \quad x^* = \frac{3\pi}{2} \text{ and it is unique}$$

$$p^* = f_3(x^*) = \sin(\frac{3\pi}{2}) = -1$$

$$4. f_1(x) = \frac{\sin(x)}{x^2} = \sin(x) \cdot x^{-2}$$

$$f'_1(x) = \sin(x) \cdot (-2) \cdot x^{-3} + \cos(x) \cdot x^{-2} = \frac{-2\sin(x)}{x^3} + \frac{\cos(x)}{x^2} = 0$$

$$\frac{-2\sin(x) + x\cos(x)}{x^3} = 0 \rightarrow \underline{-2\sin(x) + x\cos(x) = 0}$$

$$x\cos(x) = 2\sin(x)$$

$$x = 2\tan(x)$$

$$x = 4.2748$$

$$\begin{aligned} f''_1(x) &= -2\sin(x) \cdot (-3)x^{-4} + (-2)\cos(x) \cdot x^{-3} + \cos(x) \cdot (-2)x^{-3} + x^2 \cdot (-\sin(x)) = \\ &= \frac{6\sin(x)}{x^4} + \frac{-2\cos(x)}{x^3} - \frac{2\cos(x)}{x^3} - \frac{\sin(x)}{x^2} = \frac{6\sin(x) - 2x\cos(x) - 2x\cos(x) - x^2\sin(x)}{x^4} = \\ &= \frac{6\sin(x) - 4x\cos(x) - x^2\sin(x)}{x^4} \end{aligned}$$

3.1

$$\cancel{f''_1(x)}$$

$$f''_1(4.2748) = -3.97 < 0$$

misscalculated

There is not minimum.

$$5. f_5(x) = x^3 - 3x$$

$$f'_5(x) = 3x^2 - 3 = 0$$

$$3x^2 = 3$$

$$x^2 = 1 \rightarrow x = \pm 1$$

$$f''_5(x) = 6x$$

$$f''_5(1) = 6 > 0 \leftarrow \begin{array}{l} \text{it's a minimum} \\ x^* = 1 \end{array}$$

$$f''_5(-1) = -6 < 0$$

$$x^* = 1 \quad \text{it is unique}$$

$$p^* = f(x^*) = 1^3 - 3 \cdot 1 = -2$$



# Index der Kommentare

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- 1.1 Here you should mention at which point the maximum profit occurs
- 2.1 it should be  $p^* = -c$  which happens at  $x^* = \pi$  or  $3\pi$  and so on ..
- 3.1 the formula for 2nd dervative is correct but the value of  $f''(4.27)$  is miscalculated