Deadline: Sun Jan. 21, 2024, 8:00 am Submit single unzipped PDF file on learn-web course "SoSe 2023: 3104 Modern Optimization Techniques

Instructions

Please following these instructions for solving and submitting the exercise sheet.

1. Student should clearly write his/her name, matriculation number and tutorial group number (i.e. "Group 1: Tuesday Tutorial", "Group 2: Wednesday Tutorial").
2. The submission should be made before the deadline, only through learnweb to your group submission link.
3. Should be submitted as a single unzipped PDF file on learn-web course "SoSe 2023: 3104 Modern Optimization Techniques".
4. Each student must submit an individual solution in-order to be eligible for bonus points.
5. Group submission are acceptable but will not contribute towards bonus points.

1 Constrained Minimization (12 points)

For the two following constrained problems, plot the level sets of f_0 and the given constraints to then graphically find $x^\star$.
a)

$$\begin{aligned} \text{minimize} \quad & f_0(x_1, x_2) = x_1^2 + x_2^2 \\ \text{subject to} \quad & h(x_1, x_2) = x_1 + 2x_2 = 3 \end{aligned}$$

Write down the KKT conditions for this optimization problem and analytically compute $x^\star$!

b)

$$\begin{aligned} \text{minimize} \quad & f_0(x_1, x_2) = x_1 + x_2 \\ \text{subject to} \quad & h(x_1, x_2) = x_1 - x_2 = 2 \\ & f_1(x_1, x_2) = x_1 \geq 0 \\ & f_2(x_1, x_2) = x_2 \geq 0 \end{aligned}$$

Reason why you cannot compute the dual problem for a linear program as this one!

2 Computing the Dual Problem (8 points)

Let us consider the following optimization problem:

$$\begin{aligned} \text{minimize} \quad & f_0(x_1, x_2) = x_1^2 + x_2^2 \\ \text{subject to} \quad & f_1(x_1, x_2) = x_1 + x_2 \leq 1 \\ & h(x_1, x_2) = x_2 - 2x_1 = 1/2 \end{aligned}$$

Compute the dual optimization problem as a function of λ and ν.