

Task 1)

$$a) \nabla f = \begin{pmatrix} \frac{df}{dx} \\ \frac{df}{dy} \\ \frac{df}{dz} \end{pmatrix} = \begin{pmatrix} z^2 e^y \cos y \\ \cos y (xz^2 e^y) + xz^2 e^y (-\sin y) \\ 2xz e^y \cos y \end{pmatrix} = \begin{pmatrix} z^2 e^y \cos y \\ xz^2 e^y (\cos y - \sin y) \\ 2xz e^y \cos y \end{pmatrix}$$

$$b) \nabla g = \begin{pmatrix} \frac{dg}{dx} \\ \frac{dg}{dy} \\ \frac{dg}{dz} \end{pmatrix} = \begin{pmatrix} \frac{1}{x^2 y^2 + z^2} (2x) \\ \frac{1}{x^2 y^2 + z^2} (2y) \\ \frac{1}{x^2 y^2 + z^2} (2z) \end{pmatrix}$$

Task 2)

$$a) \frac{df}{dx} = -2(a-x) + 2b(y-x^2)(-2x) = -2a + 2x - 4bxy + 4bx^3$$

$$\frac{df}{dy} = 2b(y-x^2) = 2by - 2bx^2$$

$$\nabla f = \begin{pmatrix} \frac{df}{dx} \\ \frac{df}{dy} \end{pmatrix} = \begin{pmatrix} 4bx^3 + 2x(1-2by) - 2a \\ 2by - 2bx^2 \end{pmatrix}$$

$$b) \nabla f = 0 \Rightarrow \begin{cases} 4bx^3 + 2x(1-2by) - 2a = 0 \quad \textcircled{I} \\ 2by - 2bx^2 = 0 \Rightarrow y - x^2 = 0 \Rightarrow y = x^2 \quad \textcircled{II} \end{cases}$$

$$f^*(x,y) = (a-a)^2 + b(a^2 - a^2)^2 = 0 \quad \leftarrow (x,y)^* = (a, a^2)$$

$$\begin{aligned} \frac{df}{dx} &= 12bx^2 + 2(1-2by) & \frac{df}{dx} &= -4bx \\ \frac{df}{dy} &= -4bx & \frac{df}{dy} &= 2b \end{aligned} \Rightarrow H_f = \begin{pmatrix} 12bx^2 - 4by + 2 & -4bx \\ -4bx & 2b \end{pmatrix}$$

①

