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Ex10
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Task 2)

@ $f(x) = x_1^2 + x_2^2$

$$x_1 + x_2 = 1$$
$$-2x_1 + x_2 = \tfrac{1}{2}$$
$$\Bigg\} \rightarrow -x_1 + 2x_2 = \tfrac{3}{2}$$

$$\Rightarrow P = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \quad q = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad A = \begin{pmatrix} -1 & 2 \end{pmatrix} \quad a = \tfrac{3}{2}$$

$$\Rightarrow \begin{pmatrix} x_1^* \\ x_2^* \\ v^* \end{pmatrix} = \begin{pmatrix} 2 & 0 & -1 \\ 0 & 2 & 2 \\ -1 & 2 & 0 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 0 \\ \tfrac{3}{2} \end{pmatrix} = \begin{pmatrix} \tfrac{2}{5} & \tfrac{1}{5} & -\tfrac{1}{5} \\ \tfrac{1}{5} & \tfrac{1}{10} & \tfrac{2}{5} \\ -\tfrac{1}{5} & \tfrac{2}{5} & -\tfrac{2}{5} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ \tfrac{3}{2} \end{pmatrix} = \begin{pmatrix} -\tfrac{3}{10} \\ \tfrac{6}{10} \\ -\tfrac{6}{10} \end{pmatrix}$$

ⓑ $f(x) = x_1^2 + x_2^2 \qquad x_0 = (1,1)^T \xrightarrow{\quad Ax \neq a \Rightarrow \text{ infeasible start}\quad}$

$$-x_1 + 2x_2 = \tfrac{3}{2}$$

$$A = \begin{pmatrix} -1 & 2 \end{pmatrix} \qquad a = \tfrac{3}{2}$$

$$\nabla f(x) = \begin{pmatrix} 2x_1 \\ 2x_2 \end{pmatrix}, \quad \nabla^2 f(x) = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

First iteration

$$\begin{pmatrix} \Delta x_1^0 \\ \Delta x_2^0 \\ \Delta v_0 \end{pmatrix} = -\begin{pmatrix} \nabla^2 f(x_0) & A^T \\ A & 0 \end{pmatrix}^{-1} \begin{pmatrix} \nabla f(x_0) + A^T v_0 \\ A x_0 - a \end{pmatrix}$$

$$= -\begin{pmatrix} 2 & 0 & -1 \\ 0 & 2 & 2 \\ -1 & 2 & 0 \end{pmatrix}^{-1} \begin{pmatrix} 2 \\ 4 \\ \tfrac{5}{2} \end{pmatrix} = \begin{pmatrix} -\tfrac{2}{5} & \tfrac{1}{5} & \tfrac{1}{5} \\ -\tfrac{1}{5} & \tfrac{1}{10} & -\tfrac{2}{5} \\ \tfrac{1}{5} & -\tfrac{2}{5} & \tfrac{2}{5} \end{pmatrix} \begin{pmatrix} 2 \\ 4 \\ -\tfrac{5}{2} \end{pmatrix} = \begin{pmatrix} -1.7 \\ -1.6 \\ -2.4 \end{pmatrix}$$

$$\begin{pmatrix} x_1^1 \\ x_2^1 \\ v_1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} -1.7 \\ -1.6 \\ -2.4 \end{pmatrix} = \begin{pmatrix} 0.7 \\ 0.6 \\ -1.4 \end{pmatrix}$$

$$r(d,v) = \left\| \begin{pmatrix} \nabla f(x) + A^T v \\ Ax - a \end{pmatrix} \right\| = \left\| \begin{pmatrix} 2.8 \\ -1.6 \\ 0.4 \end{pmatrix} \right\| \approx 3.2 \not< 0.1$$

Second iteration

$$\begin{pmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta r_2 \end{pmatrix} = - \begin{pmatrix} \frac{-2}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{-1}{5} & \frac{-1}{10} & \frac{-2}{5} \\ \frac{1}{5} & \frac{-2}{5} & \frac{2}{5} \end{pmatrix} \begin{pmatrix} 2.8 \\ -1.6 \\ 0.4 \end{pmatrix} = \begin{pmatrix} -0.72 \\ -0.56 \\ 1.36 \end{pmatrix}$$

$$\begin{pmatrix} u_1^2 \\ u_2^2 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0.7 \\ 0.6 \\ -1.4 \end{pmatrix} + \begin{pmatrix} -0.72 \\ +0.56 \\ +1.36 \end{pmatrix} = \begin{pmatrix} -0.02 \\ 0.04 \\ -0.04 \end{pmatrix}$$

$$r(x_q v) = \left\| \begin{pmatrix} 0 \\ 0 \\ -1.4 \end{pmatrix} \right\| = 1.4 \quad \not< 0.1$$

iteration 3:

$$\begin{pmatrix} \Delta u_1 \\ \Delta x_2 \\ \Delta v_3 \end{pmatrix} = \begin{pmatrix} \frac{-2}{5} & \frac{-1}{5} & \frac{1}{5} \\ \frac{-1}{5} & \frac{-1}{10} & \frac{-2}{5} \\ \frac{1}{5} & \frac{-2}{5} & \frac{2}{5} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ -1.4 \end{pmatrix} = \begin{pmatrix} +0.28 \\ 0.56 \\ -0.56 \end{pmatrix}$$

$$\begin{pmatrix} u_1^3 \\ u_2^3 \\ v_3 \end{pmatrix} = \begin{pmatrix} -0.02 \\ 0.04 \\ -0.04 \end{pmatrix} + \begin{pmatrix} -0.28 \\ 0.56 \\ -0.56 \end{pmatrix} = \begin{pmatrix} -0.3 \\ 0.5 \\ -0.6 \end{pmatrix}$$

final $\begin{pmatrix} u_1 \\ x_2 \\ v \end{pmatrix} = \begin{pmatrix} -0.3 \\ 0.6 \\ -0.6 \end{pmatrix}$

$$r(x_q v) = \left\| \begin{pmatrix} -0.04 \\ 0.08 \\ 0 \end{pmatrix} \right\| = 0.08 < 0.1$$