Advanced Computer Vision

Exercise Sheet 4

Winter Term 2023
Prof. Dr. Niels Landwehr
Dr. Ujjwal

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Hand in until: 05.12.2022 at 23:59
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Task 1 – Examples for Convolution Filters [15 points]

In this practical exercise we take a closer look at the 2D convolution operation. Based on the notebook *convolution.ipynb*, implement the 2D-convolution operation (for a single input and a single output channel) in the method *conv2d*. We would like to use 3×3 convolution kernels to implement the following image operations:

1. Blur an image;
2. Sharpen an image, by increasing the contrast between dark and bright regions to bring out features;
3. Detect vertical edges, that is, generate an output image that is bright where a vertical edge appears in the input image;
4. Detect horizontal edges, that is, generate an output image that is bright where a horizontal edge appears in the input image.

In the folder *images.zip* you find two example input images as well as example outputs for these operations. For each operation, define an appropriate kernel matrix K in the notebook and use it on the example images. Can you find kernel matrices such that your output matches the example outputs in *images.zip*?

Task 2 – Convolution as Linear Operation [15 points]

Assume you have an input tensor $\mathbf{X} \in \mathbb{R}^{4\times 4\times 2}$ given by

$$\mathbf{X}[:,:,0] = \begin{bmatrix} 11 & 12 & 13 & 14 \\ 15 & 16 & 17 & 18 \\ 19 & 20 & 21 & 22 \\ 23 & 24 & 25 & 26 \end{bmatrix} \qquad \mathbf{X}[:,:,1] = \begin{bmatrix} 27 & 28 & 29 & 30 \\ 31 & 32 & 33 & 34 \\ 35 & 36 & 37 & 38 \\ 39 & 40 & 41 & 42 \end{bmatrix}$$

On this input, we perform convolution with a kernel $K \in \mathbb{R}^{2\times 2\times 2}$ given by

$$\mathbf{K}[:,:,0] = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad \mathbf{K}[:,:,1] = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}.$$

We do not use padding, we use stride one, and there is a single output channel. Thus the output $\mathbf{Y}$ is of size $\mathbb{R}^{3\times 3}$.
Find a matrix $\mathbf{A} \in \mathbb{R}^{9\times 8}$ and a vector $\mathbf{b} \in \mathbb{R}^{8}$ such that $\mathbf{c} = \mathbf{A}\mathbf{b}$ is identical to a flattened version of the output $\mathbf{Y}$ (that is, $\mathbf{Y}$ is obtained by reshaping $\mathbf{c}$ to a 3×3 matrix). Matrix $\mathbf{A}$ and vector $\mathbf{b}$ must only contain elements that are also present in $\mathbf{X}$ or $\mathbf{K}$.

This idea also works for general inputs and kernel sizes. Assume an input of general dimension, $\mathbf{X} \in \mathbb{R}^{m\times l\times d}$, and a square kernel of general size, $\mathbf{K} \in \mathbb{R}^{k\times k\times d}$. What are the dimensions of $\mathbf{A}$ and $\mathbf{b}$?

Task 3 – Implement Convolution as Linear Operation [20 points]

Implement the transformation from an input $\mathbf{X}$ and convolution kernel $\mathbf{K}$ to a matrix product $\mathbf{Ab}$ derived in Task 2 for the general case as a Python notebook. Use it to compute the solution to the convolution given in Task 2.