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Ex6

Task1

a) $\nabla f = \begin{pmatrix} \dfrac{1}{1+u+2y} \\[3mm] \dfrac{2}{1+u+2y} \end{pmatrix}$, $\nabla^2 f = \begin{pmatrix} \dfrac{-1}{(1+u+2y)^2} & \dfrac{-2}{(1+u+2y)^2} \\[3mm] \dfrac{-2}{(1+u+2y)^2} & \dfrac{-4}{(1+u+2y)^2} \end{pmatrix}$

b) $\det(\nabla^2 f) = \dfrac{1}{(1+u+2y)^4}\left[4 - 4\right] = 0. \Rightarrow$

Hessian is not invertible.

c) Since Hessian was not invertible, and $\nabla^2 f^{-1}$ does not exist, we can't compute the update step.

Task2 The algorithm used here is the same as slide 24.

A is the H^{-1} and $A_0 = I$

$\nabla f = \begin{pmatrix} 2u_1 \\ u_2 \end{pmatrix}$ $X_0 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $\mu = 0.001$ $k = \{1, 2\}$

iteration 1 :

$$\Delta x_0 = -A_0 \nabla f(x_0) = -\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 2(1) \\ (2) \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \end{pmatrix}$$

$$u_1 = x_0 + u \Delta x_0 = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + 0.001 \begin{pmatrix} -2 \\ -2 \end{pmatrix} = \begin{pmatrix} 0.998 \\ 1.998 \end{pmatrix}$$

$$S_1 = u_1 - x_0 = \begin{pmatrix} 0.998 \\ 1.998 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} -0.002 \\ -0.002 \end{pmatrix}$$

$$G_1 = \nabla f(u_1) - \nabla f(x_0) = \begin{pmatrix} 2(0.998) \\ 1.998 \end{pmatrix} - \begin{pmatrix} 2(1) \\ (2) \end{pmatrix} = \begin{pmatrix} -0.004 \\ -0.002 \end{pmatrix}$$

$$A_1 = A_0 + \frac{(S_1 - A_0 G_1)S_1^T + S_1(S_1 - A_0 G_1)^T}{S_1^T G_1} - \frac{(S_1 - A_0 G_1)^T G_1}{(S_1^T G_1)^2} S_1(S_1^T)$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{\left[\begin{pmatrix} -0.002 \\ -0.002 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} -0.004 \\ -0.002 \end{pmatrix}\right](-0.002 \; -0.002) + \begin{pmatrix} -0.002 \\ -0.002 \end{pmatrix}\left[\begin{pmatrix} -0.002 \\ -0.002 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} -0.004 \\ -0.002 \end{pmatrix}\right]}{(-0.002 \; -0.002)\begin{pmatrix} -0.004 \\ -0.002 \end{pmatrix}}$$

$$- \frac{\left[\begin{pmatrix} -0.002 \\ -0.002 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} -0.004 \\ -0.002 \end{pmatrix}\right]^T \begin{pmatrix} -0.004 \\ -0.002 \end{pmatrix}}{\left[(-0.002 \; -0.002)\begin{pmatrix} -0.004 \\ -0.002 \end{pmatrix}\right]^2} \cdot \begin{pmatrix} -0.002 \\ -0.002 \end{pmatrix}(-0.002 \; -0.002)$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} -0.667 & -0.333 \\ -0.333 & 0 \end{pmatrix} - \begin{pmatrix} 0.222 & -0.222 \\ 0.222 & -0.222 \end{pmatrix} = \begin{pmatrix} 0.555 & -0.111 \\ -0.111 & 1.222 \end{pmatrix}$$

iteration 2)

$$\Delta x_1 = -\begin{pmatrix} 0.555 & -0.111 \\ -0.111 & 1.222 \end{pmatrix}\begin{pmatrix} 2(0.998) \\ 1.998 \end{pmatrix} = \begin{pmatrix} -0.886 \\ -2.22 \end{pmatrix}$$

$$u_2 = \begin{pmatrix} 0.998 \\ 1.998 \end{pmatrix} + 0.001\begin{pmatrix} -0.886 \\ -2.22 \end{pmatrix} = \begin{pmatrix} 0.997 \\ 1.996 \end{pmatrix}$$

$$S_2 = \begin{pmatrix} 0.997 \\ 1.996 \end{pmatrix} - \begin{pmatrix} 0.998 \\ 1.998 \end{pmatrix} = \begin{pmatrix} -0.001 \\ -0.002 \end{pmatrix}$$

$$G_2 = -\begin{pmatrix} 2(0.998) \\ 1.998 \end{pmatrix} + \begin{pmatrix} 2(0.997) \\ 1.996 \end{pmatrix} = \begin{pmatrix} -0.002 \\ -0.002 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} 0.555 & -0.111 \\ -0.111 & 1.222 \end{pmatrix} + \begin{pmatrix} 0.037 & 0.000333 \\ 0.000333 & -0.146 \end{pmatrix} - \begin{pmatrix} -0.00611 & -0.012222 \\ -0.01222 & -0.02444 \end{pmatrix}$$

$$= \begin{pmatrix} 0.598 & -0.098 \\ -0.098 & 1.098 \end{pmatrix}$$