

Deadline: Sun Nov 14, 2021, 8:00 am Submit single unzipped PDF file on learn-web course "SoSe 2021: 3104 Modern Optimization Techniques"

Instructions

Please following these instructions for solving and submitting the exercise sheet.

1. Student should clearly write his/her name, matriculation number and tutorial group number (i.e. "Group 1: Tuesday Tutorial", "Group 2: Wednesday Tutorial").
2. The submission should be made before the deadline, only through learnweb to your group submission link.
3. Should be submitted as a single unzipped PDF file on learn-web course "SoSe 2021: 3104 Modern Optimization Techniques".
4. Each student must submit an individual solution in-order to be eligible for bonus points.
5. Group submission are acceptable but will not contribute towards bonus points.

1 Convex Sets and Convex Functions theory (10 points)

- (a) What is convexity and why is important in optimization? (2pt)
- (b) What is a convex set and what is a convex function? (2pt)
- (c) Let

$$D = \{x \in \mathbb{R}^2 | 0 \leq x_1 \leq 2 \wedge 0 \leq x_2 \leq 3\}$$

be a subset of \mathbb{R}^2 . Draw a sketch of D . Show, by using the definition, that D is a convex set. (3pt)

- (d) In the lecture were presented three different ways to determine the convexity of a function. What represents each of those definitions? You can use a sketch. (3pt)
 - (i) Jensen's inequality: $f(\theta x + (1 - \theta)y) \leq \theta f(x) + (1 - \theta)f(y)$; $\forall x, y \in \text{Dom}(f), \theta \in [0, 1]$
 - (ii) First-order Condition $f(y) \geq f(x) + (\nabla_x f(x))^T (y - x)$; $\forall x, y \in \text{Dom}(f)$, f differentiable
 - (iii) Second-order Condition: $\nabla_x^2 f \geq 0$; $\forall x, y \in \text{Dom}(f)$, f twice differentiable

2 Second Order Convexity Condition (10 points)

- (a) Determine the values of the parameters a and b so that the following functions are convex in their domains.
 - (i) $f(x, y, z) = ax^2 + y^2 + 2z^2 - 4axy + 2yz$ (2pt)
 - (ii) $f(x, y) = 4ax^2 + 8xy + by^2$ (2pt)
- (b) Determine the domains of the plane where the following functions are convex or concave by using the second order convexity condition.
 - (i) $f_1(x, y) = \ln(\sqrt[3]{xy})$ (3pt)
 - (ii) $f_2(x, y) = (x - 1)^2 + xy^2$ (3pt)