Machine Learning

Exercise Sheet 11

Winter Term 2023 | Available: 02.02.2024
Prof. Dr. Niels Landwehr | Hand in until: 08.02.2024 11:59am
Dr. Ujjwal | Exercise sessions: 12.02.2024/14.02.2024

Task 1 – Bayesian Network Design [10 points]

We consider the following domain that describes how to start a car engine. We are trying to start the engine of our car. The engine could either start or not start, which we represent by a binary variable e with $e = 1$ if the engine starts and $e = 0$ otherwise. There are various reasons for failing to start the engine: the tank could be empty (random variable t with $t = 0$ indicating an empty tank and $t = 1$ indicating a full tank), or the starter motor could not rotate (random variable r with $r = 0$ indicating a non-rotating starter motor and $r = 1$ indicating that the starter motor rotates). To rotate, the starter motor requires a full battery (random variable b with $b = 1$ for full battery and $b = 0$ for empty battery) and it must not be defective (random variable d with $d = 1$ indicating a defective starter motor and $d = 0$ indicating a non-defective one). We can observe the condition of the tank indirectly through the electric fuel gauge: if the tank is full and the battery provides enough power for the fuel gauge to work, the fuel gauge shows full ($f = 1$), otherwise it shows empty ($f = 0$).

Construct a Bayesian network over the binary random variables e, t, r, b, d, and f. Show the graph structure G and the respective (conditional) distributions associated with the nodes in tabular form. For the conditional distributions, try to set realistic numerical probabilities (note: these are almost never exactly 0 or 1).

Task 2 – D-separation [10 points]

In this task, we derive independence relations between variables in the graphical model that you designed in Task 1. Using the D-separation criterion, show for each of the following possible independence relations whether they hold or not:

a) $d \perp e \mid b$

b) $b \perp t \mid \emptyset$

c) $b \perp e \mid r$

d) $t \perp d \mid e$

Task 3 – Bayesian Network Inference [10 points]

Again using the Bayesian network that you designed in Task 1, using naive inference (see Slide 10 and Slide 50 in the lecture) compute the probability that the tank is really empty ($t = 0$) given that we have observed an empty fuel gauge ($f = 0$).

Task 4 – Separating Set [10 points]

Let $G = (V, E)$ be the graph structure of a graphical model (with V the set of nodes and E the set of edges), and let x be a node in G. We will study the question of which set M of nodes we have to observe such that the node x is independent of all other nodes in G given M. A minimal set M that has this property will be called *separating set*. That is, M is a minimal set with $x \notin M$ and

$$\forall x' \in V \setminus (\{x\} \cup M) : x' \perp x \mid M. \tag{1}$$

Characterize the set M concisely and argue why it is minimal and has the separating property. Hint: D-separation.