Deadline: Sun Jan. 14, 2023, 8:00 am Submit single unzipped PDF file on learn-web course "SoSe 2021: 3104 Modern Optimization Techniques

Instructions

Please following these instructions for solving and submitting the exercise sheet.

1. Student should clearly write his/her name, matriculation number and tutorial group number (i.e. "Group 1: Tuesday Tutorial", "Group 2: Wednesday Tutorial").
2. The submission should be made before the deadline, only through learnweb to your group submission link.
3. Should be submitted as a single unzipped PDF file on learn-web course "SoSe 2023: 3104 Modern Optimization Techniques".
4. Each student must submit an individual solution in-order to be eligible for bonus points.
5. Group submission are acceptable but will not contribute towards bonus points.

1 Linear Regression with Coordinate Descent (10 points)

Let us revisit our linear regression problem from last exercise sheet with same data:

Shouting Frequency	Cursing Frequency	Pleasure Level (Response)
1.5	2	10
3	2.5	15.5
4.5	3	21

We want to find the parameter vector $\beta = (\beta_0, \beta_1, \beta_2)$ that minimizes the loss over all instances x_i:

$$\mathcal{L}(X, \beta, y) = \sum_{i=1}^{3} (x_i \beta^\top - y_i)^2$$

a) Explain in your own words, what is the difference between Coordinate Descent and a normal Gradient Descent (3 pts).

b) Do two epochs using coordinate descent. Report the errors and the overall loss after each epoch, with an initial $\beta = (1, 1, 1)$ (7 pts).

2 Exercise 16: Coordinate Descent (10 points)

a) Show that coordinate descent fails for the following function: (5 pts)

$$g(x) = |x_1 x_2| + 0.1(x_1 + x_2)$$

Hint: Verify that the algorithm terminates after one step while $\inf_x g(x) = -\infty$

b) Let

$$L(x) = f(x) + ||x||_1$$

be L1-regularized minimization with $f(x)$ convex and differential and $\lambda \leq 0$. Assume the algorithm converges in a fixed point x^*. show that x^* is optimal, i.e. it minimizes L (5 pts).

Hint:Use the subdifferential you have seen in the previous lecture and exercise sheet. $||x||_1 = |x|$