Advanced Computer Vision

Exercise Sheet 11

Winter Term 2023
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Available: 06.02.2024
Hand in until: 13.02.2024, until 23:59
Exercise session: 16.02.2024

Task 1 – A graph neural network forward pass [20 points]

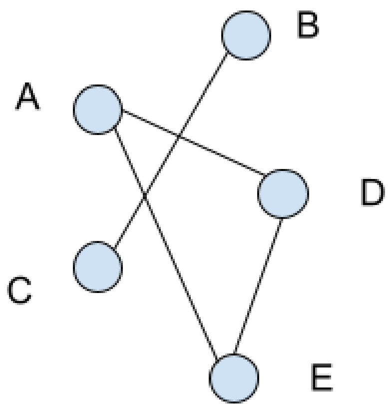

Figure 1: Graph neural network

Figure 1 shows a graph neural network topology with nodes indicated by alphabetical letters. The edges show the connectivity between the nodes. Given, the input features and weight matrices at input layer $l = 0$, the task is to compute the forward pass to obtain the node embeddings for $l = 1$.
The input featuress h_A^0 to h_E^0 (superscript 0 refers to the layer $l = 0$) respectively are : $h_A^0 = [2, 1]^T$, $h_B^0 = [0.03, 2]^T$, $h_C^0 = [4, 1.75]^T$, $h_D^0 = [-6.78, 1.02]^T$ and $h_E^0 = [-1, 1]^T$.
Following the notations in the lecture (see slides $19 - 26$), the weight matrices are :

- $W_0 = \begin{bmatrix} 1 & 2.01 \\ 2.3 & 3.4 \\ 1.7 & -0.91 \end{bmatrix}$
- $B_0 = \begin{bmatrix} -1.13 & 0.05 \\ 3.46 & 1.14 \\ -2.72 & 0.002 \end{bmatrix}$

Compute the node embeddings h_A^1 to h_E^1 using the equation specified on slide 20 of the lecture slides.. This task can be performed even without using a package like tensorflow or pytorch, using only numpy. Please note that there is no sample notebook for this task.

HINT: Try to be creative. Think of how you can achieve the computations using elementary matrix operations in a succinct way using numpy.

Task 2 – A graph neural network with different aggregation functions [30 points]

In this task, you will use the same network structure as in figure 1 with the same input features as specified in task-1. However, instead of computing the mean of node embeddings for neighborhood nodes, you will compute the sum. With these changes, compute the node embeddings for $l = 1$ to $l = 5$ such that $W_1 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix}$ and for $l > 1$, $W_{l+1} = W_l \; X \; W_l^T$ and $B_{l+1} = B_l \; X \; B_l^T$. Here X denotes matrix multiplication.