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Ex 10

Task 1) let x_A be any point in the hyperplane, hence $x_A^T\theta + b = 0$ (I)

Imagine a point x_0 with a d distance with the plane.

$$\Rightarrow x_A + d = x_0 \Rightarrow x_A = x_0 - d \overset{(I)}{\Rightarrow} (x_0 - d)^T\theta + b = 0 \Rightarrow x_0^T\theta + b = d^T\theta$$

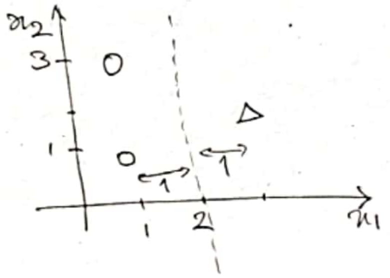
because d is paralel to the normal vector of the hyperplane, we have $d = \lambda\theta$

$$\Rightarrow x_0^T\theta + b = \lambda\theta^T\theta \Rightarrow \lambda = \frac{x_0^T\theta + b}{\theta^T\theta} \Rightarrow d = \frac{x_0^T\theta + b}{\theta^T\theta}\theta$$

$$\|d\| = \sqrt{d^Td} = \sqrt{\lambda^2\theta^T\theta} = \lambda\sqrt{\theta^T\theta} \Rightarrow d = \frac{x_0^T\theta + b}{\theta^T\theta}\cdot\sqrt{\theta^T\theta} = \frac{x_0^T\theta + b}{\sqrt{\theta^T\theta}} = \boxed{\frac{x_0^T\theta + b}{\|\theta\|}}$$

Task 2)

(a)



$$x_1 = 2 \Rightarrow (-1\ \ 0)\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + 2 = 0$$

$$\Rightarrow \theta = (-1\ \ 0),\ \underline{b = 2}$$

$\|\theta\| = 1$ already normalized here

$c = 1$

(b) hard margin primal problem is:

minimize $\frac{1}{2}\|\theta\|^2$

s.t. $y_n(x_n^T\theta + b) \geq 1$ $\xrightarrow{\text{for all}}$ data points

$$\begin{cases} 1(1\ \ 1)\begin{pmatrix} -1 \\ 0 \end{pmatrix} + 2 = 1 \geq 1 \ \checkmark \\ 1(1\ \ 3)\begin{pmatrix} -1 \\ 0 \end{pmatrix} + 2 = 1 \geq 1 \ \checkmark \\ -1(3\ \ 2)\begin{pmatrix} -1 \\ 0 \end{pmatrix} + 2 = 1 \geq 1 \ \checkmark \end{cases}$$

→ inequality constraints are satisfied

for the $x_1 = (1\ .\ 1)$ we have $\theta_1 + \theta_2 + 2 \geq 1 \Rightarrow \theta_1 + \theta_2 \geq -1$

$x_2 = (1\ \ 3) \rightarrow \theta_1 + 3\theta_2 \geq -1$

$x_3 = (3\ \ 2) \rightarrow -3\theta_1 - 2\theta_2 \geq -1$

→ these show that any other θ_1, θ_2 would result in a higher norm

(c) $\theta^* = \sum_{n=1}^{3} \alpha_n^* x_n y_n \Rightarrow \begin{pmatrix} -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}\alpha_1 + \begin{pmatrix} 1 \\ 3 \end{pmatrix}\alpha_2 - \begin{pmatrix} 3 \\ 2 \end{pmatrix}\alpha_3$

$$\Rightarrow \begin{cases} \alpha_1+\alpha_2-3\alpha_3=-1 \\ \alpha_1+3\alpha_2-2\alpha_3=0 \end{cases} \quad \text{I}$$

also we have $\sum y_n \alpha_n = 0 \Rightarrow \alpha_1+\alpha_2-\alpha_3=0$ and $\forall \alpha \geq 0$ II

$\xrightarrow[\text{Solving these system of linear equations we get}]{\text{I}, \text{II}}$ $\boxed{(\alpha_1^*, \alpha_2^*, \alpha_3^*) = (\frac{1}{4}, \frac{1}{4}, \frac{1}{2})}$

(3) k must be positive indefinite; but the kernel function provided is <u>not</u>. here is an example:

$$\begin{matrix} a=(1,2) \\ b=(0,1) \end{matrix} \rightarrow k(a,b) = \frac{-2}{\sqrt{5}} < 0$$