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Ex05

Task 1) $E[(\hat{R}-R)^2] = E[(\hat{R}-E(\hat{R})+E(\hat{R})-R)^2]$

$= E[(\hat{R}-E(\hat{R}))^2+(E(\hat{R})-R)^2+2(\hat{R}-E(\hat{R}))(E(\hat{R})-R)]$

$= E[(\hat{R}-E(\hat{R}))^2]+E[(E(\hat{R})-R)^2]+2E[(\hat{R}-E(\hat{R}))(E(\hat{R})-R)]$

$= Var[\hat{R}] + Bias[\hat{R}]^2 + 2[\cancel{E(\hat{R})^2} - \cancel{E(\hat{R}R)} - \cancel{E(\hat{R})^2} + \cancel{E(\hat{R})E(R)}]$

$= Var[\hat{R}] + Bias[\hat{R}]^2$

Task 3)

$\hat{R}_T(f_{\theta^*}) = \frac{1}{N}\sum_{n=1}^{N} l_{eval}(\bar{y}_n, f_{\theta^*}(\bar{x}_n))$, $l_{eval} = \begin{cases} 0 & y = f_{\theta^*}(x) \\ 1 & y \neq f_{\theta^*}(x) \end{cases}$

$\Rightarrow \hat{R}_T(f_{\theta^*}) = \frac{1}{10}(0 \cdot 0 + 1 \cdot 4) = 0.4$

$S^2_{R_T} = \frac{\hat{R}_T(f_\theta)(1-\hat{R}_T(f_\theta))}{N} = \frac{0.4 \times 0.6}{10} = 0.024$

for a two-bounded confidence interval $1-2\delta = 0.95$

$\Rightarrow \delta = 0.05 \rightarrow \Phi^{-1} = 1.96$

$\varepsilon = S_{\hat{R}_T} \cdot \Phi^{-1}(1-2\delta) = \sqrt{0.024} \cdot (1.96) = 0.303$

$\Rightarrow CI = [\hat{R}_T(f_{\theta^*}) - \varepsilon, \hat{R}_T(f_{\theta^*}) + \varepsilon] =$

$[0.4 - 0.303, 0.4 + 0.303] = [0.097, 0.703]$

Task 2

In [ ]:

```python
import numpy as np
import matplotlib.pyplot as plt
from sklearn.metrics import mean_squared_error


# Input: number of samples N
# Output: one-dimensional data set of N points where y_n = sin(2 pi x_n) + epsilon_n as in lecture
def sine_data_set(N):
  np.random.seed(1234)
  x = np.random.uniform(0,1,(N))
  y = np.sin(x*2*np.pi)+np.random.normal(scale=0.2,size=(N))
  return x,y


# Input: one-dimensional inputs x as vector of length N, polynomial degree d
# Output: polynomial feature representation of the inputs as N x (d+1) matrix
def poly_features(x,d):
  X = np.zeros((x.shape[0],d+1))
  for i in range(0,d+1):
    X[:,i] = np.power(x,i)
  return X


# Input: instances X as N x M matrix, labels y as vector of length N, lambda for regularization
# Output: learned parameter vector
def fit_ridge_regression(X,y,lambda_param):
  N = X.shape[0]
  M = X.shape[1]
  return np.linalg.solve(X.T @ X + N*lambda_param*np.eye(M), X.T @ y)


# Input: instances X as N x M matrix, model theta
# Output: predictions of model theta on X
def predict_regression(X,theta):
  return X @ theta
```

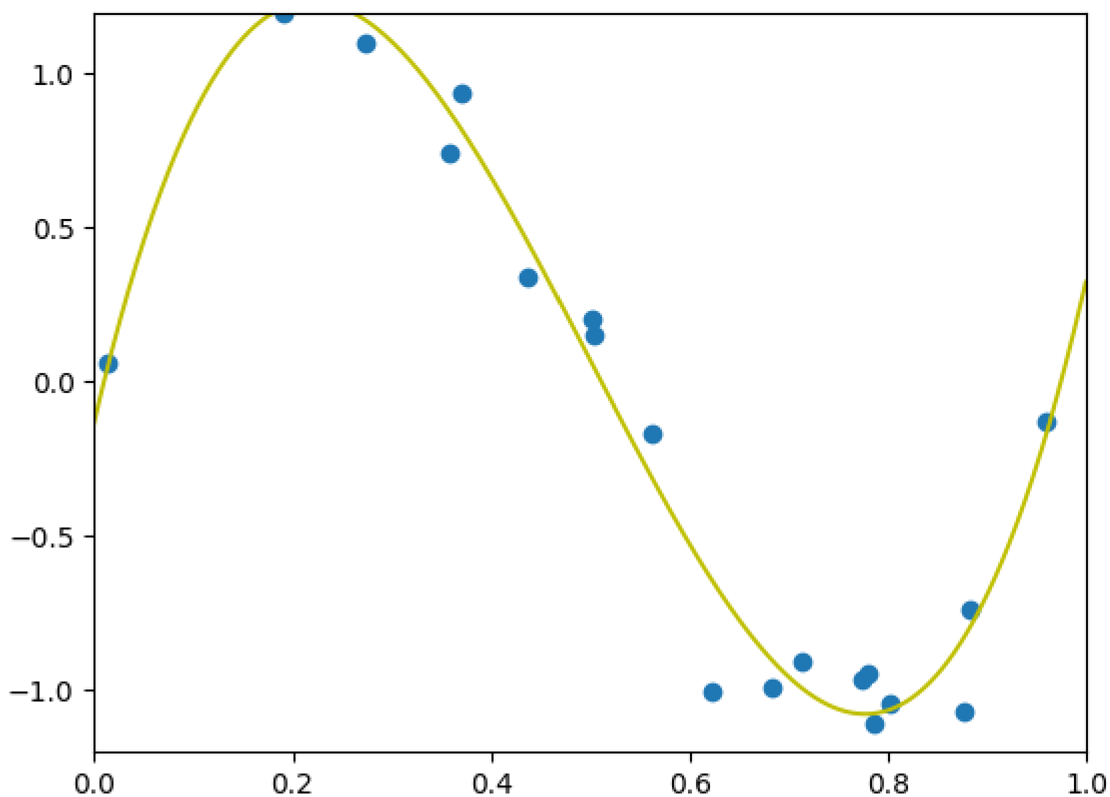
```python
# Input: instances X, labels y, number of cross-validation folds K, current fold k
# Output: train and test sets for fold k
def crossval_split(X,y,K,k):
  x_parts = np.array_split(X, K)
  y_parts = np.array_split(y, K)
  X_test = x_parts[k]
  y_test = y_parts[k]
  x_parts.pop(k)
  y_parts.pop(k)
  X_train = np.concatenate(x_parts)
  y_train = np.concatenate(y_parts)
  return X_train, y_train, X_test, y_test
```

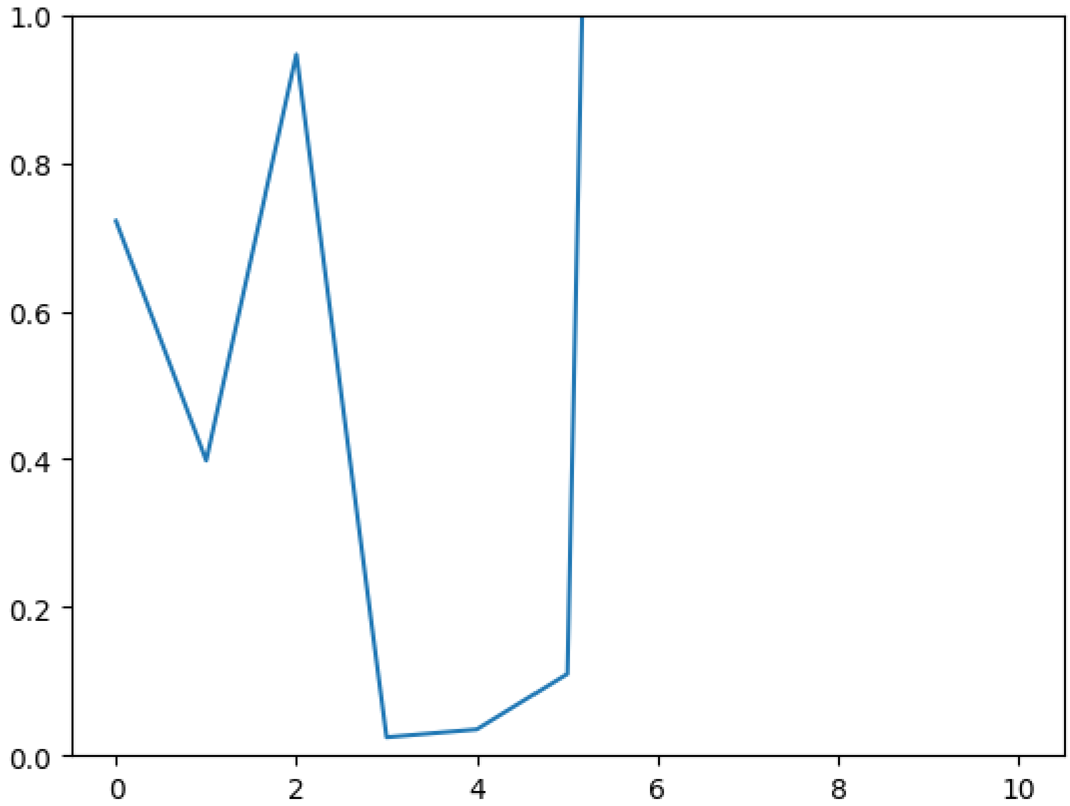
In [ ]:
```python
N = 20
x,y = sine_data_set(N)
plt.xlim([0,1])
plt.ylim([-1.2,1.2])
plt.scatter(x,y)
def GridSearch_d(x,y):
    lamb = 0
    best_loss = float('inf')
    best_model_theta = None
    best_d = None
    loss_history = []
    K = 4
    for d in range(0, 11):
        losses = 0
        for k in range(K):
            X_train, y_train, X_test, y_test = crossval_split(x,y,K ,k)
            X = poly_features(X_train,d)
            X_TEST = poly_features(X_test,d)
            theta = fit_ridge_regression(X,y_train,lamb)
            y_predict = predict_regression(X_TEST,theta)
            mse = mean_squared_error(y_test, y_predict)
            losses += mse
        losses /=  K
        loss_history.append(losses)
        if losses < best_loss:
            best_loss = losses
            best_model_theta = theta
            best_d = d
    print(f'best d is : {best_d}')
```

```
    X = poly_features(x,best_d)
    theta = fit_ridge_regression(X,y,lamb)
    grid = np.arange(0,1,0.001)
    plt.plot(grid,predict_regression(poly_features(grid,best_d),theta),'y')
    fig, ax = plt.subplots()
    ax.plot(range(0, 11), loss_history)
    ax.set_ylim(0, 1)
    plt.show()

GridSearch_d(x, y)
```

```
best d is : 3
```





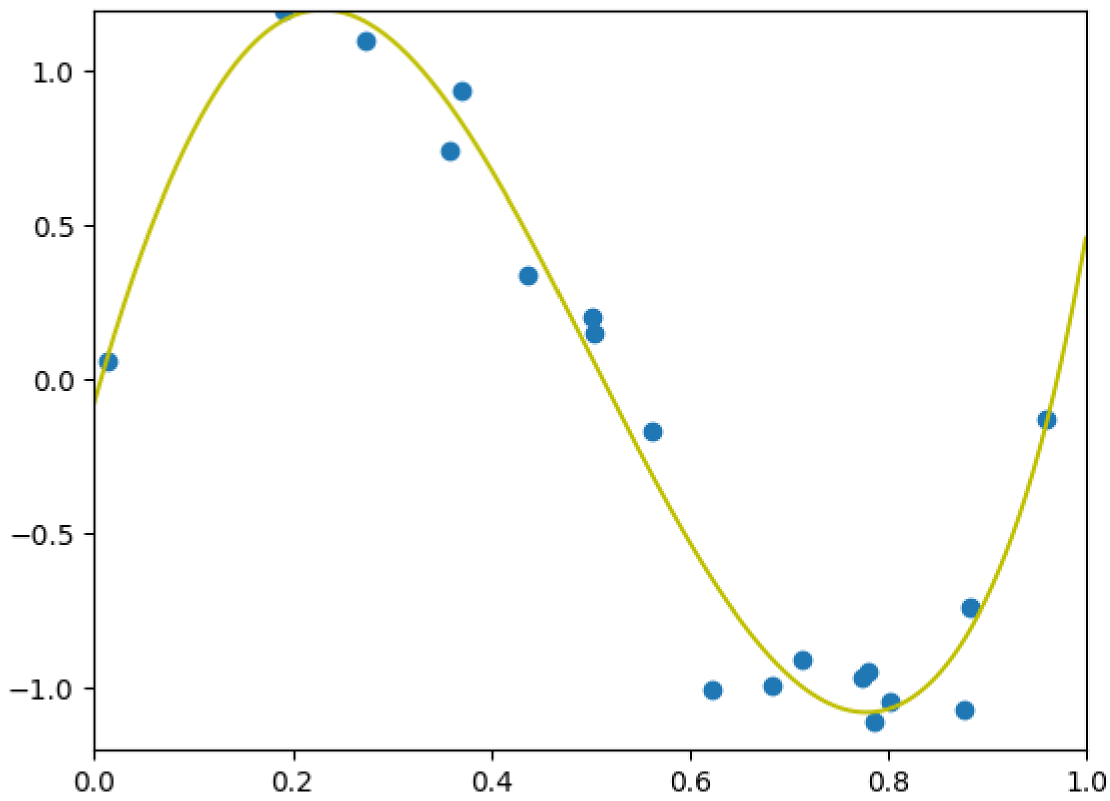
In [ ]:

```
N = 20
x,y = sine_data_set(N)
plt.xlim([0,1])
plt.ylim([-1.2,1.2])
plt.scatter(x,y)
def GridSearch_lamb(x,y):
    d = 10
    lambda_values = np.logspace(0, -9, num=10, base=10)
    best_loss = float('inf')
    best_model_theta = None
    best_lamb = None
    loss_history = []
    K = 4
    for lamb in lambda_values:
        losses = 0
        for k in range(K):
```

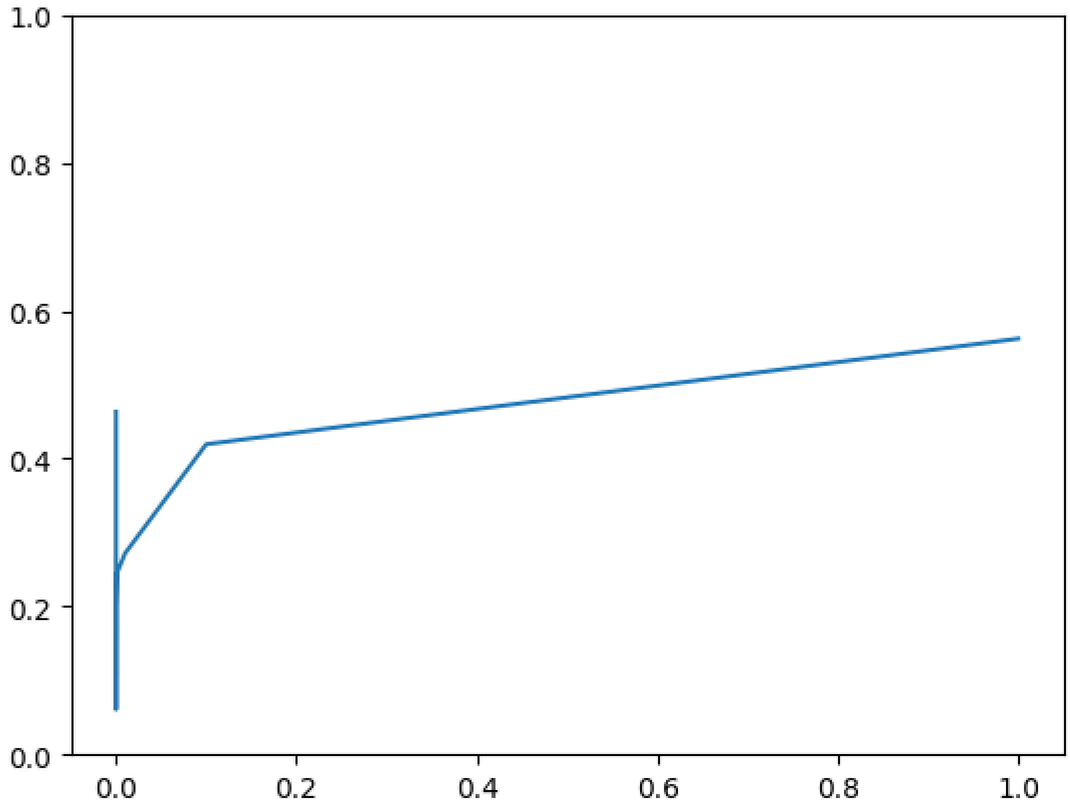
```
            X_train, y_train, X_test, y_test = crossval_split(x,y,K ,k)
            X = poly_features(X_train,d)
            X_TEST = poly_features(X_test,d)
            theta = fit_ridge_regression(X,y_train,lamb)
            y_predict = predict_regression(X_TEST,theta)
            mse = mean_squared_error(y_test, y_predict)
            losses += mse
        losses /=  K
        loss_history.append(losses)
        if losses < best_loss:
            best_loss = losses
            best_model_theta = theta
            best_lamb = lamb
    print(f'best lambda is : {best_lamb}')
    X = poly_features(x,d)
    theta = fit_ridge_regression(X,y,best_lamb)
    grid = np.arange(0,1,0.001)
    plt.plot(grid,predict_regression(poly_features(grid,10),theta),'y')
    fig, ax = plt.subplots()
    ax.plot(lambda_values, loss_history)
    ax.set_ylim(0, 1)
    plt.show()

GridSearch_lamb(x, y)
```

```
best lambda is : 1e-06
```



1.0
0.5
0.0
−0.5
−1.0
0.0
0.2
0.4
0.6
0.8
1.0




```
In [ ]:
```