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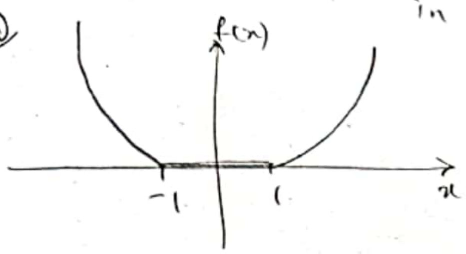
Ex7

Task I)

$$\delta f(x) = \begin{cases} x & x \le -1 \\ 0 & -1 < x < 1 \\ x & x \ge 1 \end{cases}$$

ⓐ



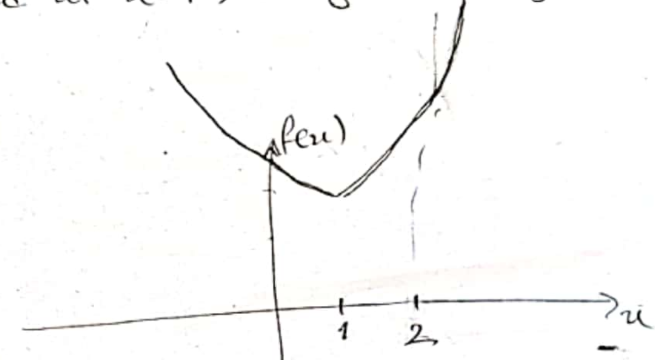
in Points : $\{-1, 1\}$ not differentiable and

at $x=-1$ the subgradients are $[-1, 0]$

and at $x=1$, they're $[0, 1]$.

ⓑ

$$f(x) = \begin{cases} x^2 - 2x + 3 & x \le 1 \\ x^2 + 1 & 1 < x < 2 \\ x^2 + 2x - 3 & x \ge 2 \end{cases}$$



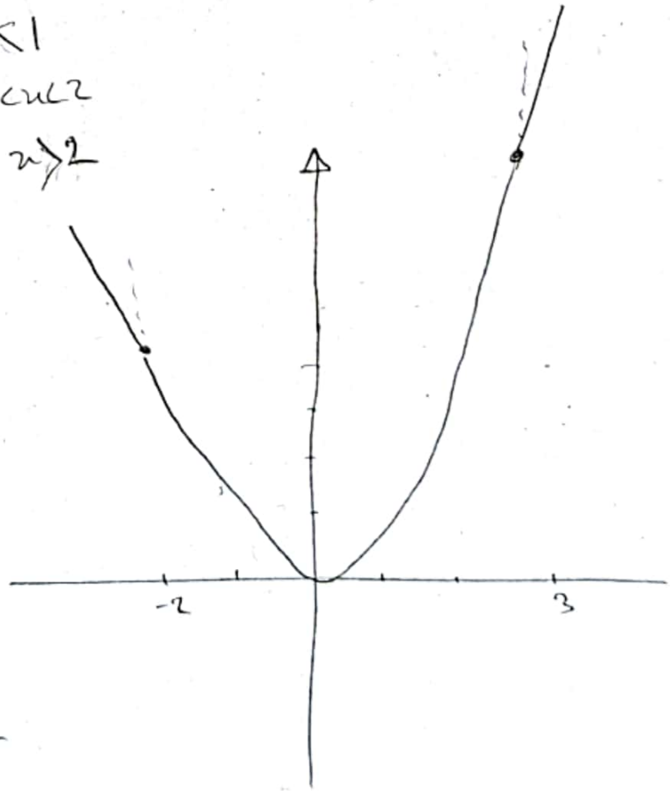
at $x = \{1, 2\}$, the function is not differentiable

at $x=1$ the subgradiants are $[0, 2]$ and at $x=2$ they're $[4, 6]$

$$\partial f(x) = \begin{cases} 2x - 2 & x < 1 \\ 2x & 1 < x < 2 \\ 2x + 2 & x > 2 \end{cases}$$

ⓒ

$$\partial f(x) = \begin{cases} -3 & x < -2 \\ 2x & -2 < x < 3 \\ 5 & x > 3 \end{cases}$$



at $x = \{-2, 3\}$ the function is not differentiable and the subgredients for $x=-2$ are $[-4, -3]$ and for $x=3$ are $[5, 6]$

Task II

$$\|x\|_1 = \sum_{i=1}^{n} |x_i| = \max \{ S^T \cdot x \mid S_i \in \{-1, 1\} \}$$

To find the correct S vector, we can choose $S_i = 1$ when $x_i > 0$ and $S_i = -1$ when $x_i < 0$ and $S_i = -1$ or 1 when $x_i = 0$. Therefore

$$g_i = \begin{cases} 1 & x_i > 0 \\ -1 & x_i < 0 \\ 1 \text{ or } -1 & x_i = 0 \end{cases}$$

and based on the max rule for subgradients (in the slide 10); we denote that the subdifferential for this max is the convex hull of the union of subdifferentials than can be generated this way: $\partial f(x) = \{ g \mid \|g\|_\infty \le 1, \; g^T x = \|x\|_1 \}$