

Deadline: Sun Dec 17, 2023, 8:00 am Submit single unzipped PDF file on learn-web course "SoSe 2021: 3104 Modern Optimization Techniques"

Instructions

Please following these instructions for solving and submitting the exercise sheet.

1. Student should clearly write his/her name, matriculation number and tutorial group number (i.e. "Group 1: Tuesday Tutorial", "Group 2: Wednesday Tutorial").
2. The submission should be made before the deadline, only through learnweb to your group submission link.
3. Should be submitted as a single unzipped PDF file on learn-web course "SoSe 2023: 3104 Modern Optimization Techniques".
4. Each student must submit an individual solution in-order to be eligible for bonus points.
5. Group submission are acceptable but will not contribute towards bonus points.

1 Subgradients and Subdifferentials (10 points)

For the following functions, create a plot of them and find the points x_0 where they are not differentiable. Compute all the subdifferentials $\partial f(x_0)$ in the respective points!

a)

$$f(x) = \max \left\{ 0, \frac{1}{2}(x^2 - 1) \right\}$$

b)

$$f(x) = x^2 + |x - 1| + |x - 2|$$

c)

$$f(x) = \begin{cases} -3x - 2 & x \in (-\infty, -2] \\ x^2 & x \in (-2, 3) \\ 5x - 6 & x \in [3, \infty) \end{cases}$$

2 Subgradients and Subdifferentials of L1-Norm (10 points)

Let us consider the ℓ_1 Norm, which is defined by:

$$\|x\|_1 := \sum_{i=1}^n |x_i|$$

Compute the subdifferential of $\|\cdot\|_1$! Use the fact, that $\|\cdot\|_1$ can be written as maximum of 2^n linear functions

$$\|x\|_1 = \max \left\{ s^\top x \mid s_i \in \{-1, 1\} \right\}.$$