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Ex05

Task 1) $E[(\hat{R}-R)^2] = E[(\hat{R}-E(\hat{R})+E(\hat{R})-R)^2]$

$= E[(\hat{R}-E(\hat{R}))^2 + (E(\hat{R})-R)^2 + 2(\hat{R}-E(\hat{R}))(E(\hat{R})-R)]$

$= E[(\hat{R}-E(\hat{R}))^2] + E[(E(\hat{R})-R)^2] + 2E[(\hat{R}-E(\hat{R}))(E(\hat{R})-R)]$

$= Var[\hat{R}] + Bias[\hat{R}]^2 + 2[\cancel{E(\hat{R})^2} - \cancel{E(\hat{R}R)} - \cancel{E(\hat{R})^2} + \cancel{E(\hat{R})E(R)}]$

$= Var[\hat{R}] + Bias[\hat{R}]^2$

Task 3)

$\hat{R}_T(f_{\theta^*}) = \frac{1}{N}\sum_{n=1}^{N} l_{eval}(\bar{y}_n, f_{\theta^*}(\bar{x}_n))$, $l_{eval} = \begin{cases} 0 & y = f_{\theta^*}(x) \\ 1 & y \neq f_{\theta^*}(x) \end{cases}$

$\Rightarrow \hat{R}_T(f_{\theta^*}) = \frac{1}{10}(0 \cdot 0 + 1 \cdot 4) = 0.4$

$S^2_{R_T} = \frac{\hat{R}_T(f_\theta)(1-\hat{R}_T(f_\theta))}{N} = \frac{0.4 \times 0.6}{10} = 0.024$

for a two-bounded confidence interval $1-2\delta = 0.95$

$\Rightarrow \delta = 0.05 \rightarrow \Phi^{-1} = 1.96$

$\varepsilon = S_{\hat{R}_T} \cdot \Phi^{-1}(1-2\delta) = \sqrt{0.024} \cdot (1.96) = 0.303$

$\Rightarrow CI = [\hat{R}_T(f_{\theta^*}) - \varepsilon, \hat{R}_T(f_{\theta^*}) + \varepsilon] =$

$[0.4 - 0.303, 0.4 + 0.303] = [0.097, 0.703]$