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Ex9

Task 1)

ⓐ KKT conditions:

1. Primal feasibility: $g_p(x_1, x_2) = x_1 + 2x_2 - 3 = 0$ ①

2. dual feasibility: $\lambda \geq 0$

3. Complementary slackness: $\lambda_q h_q = 0 \quad \forall q$

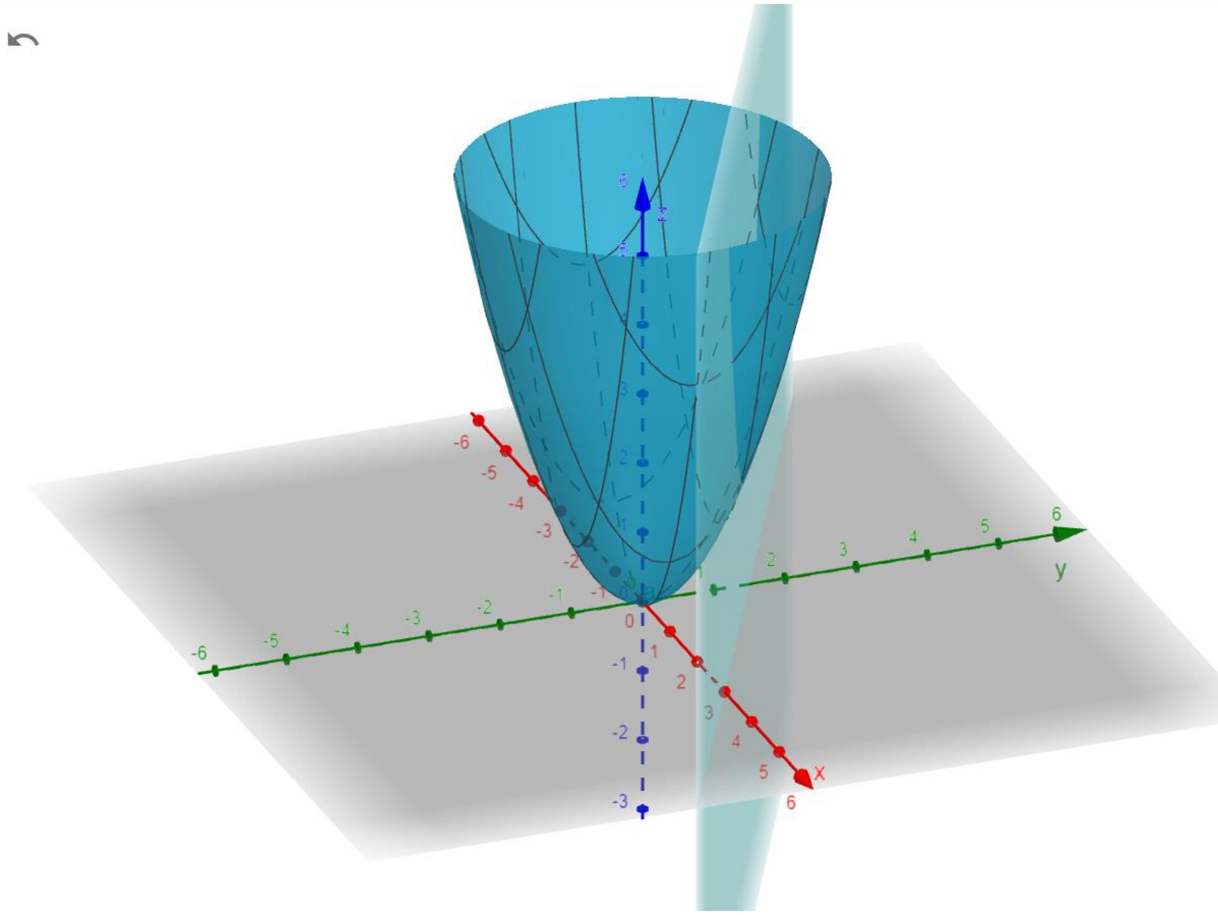
→ Since we don't have inequality constraints, these are satisfied

4. Stationarity: $\nabla f(x) + \sum_p^P v_p \nabla g_p(x) + \sum_q^Q \lambda_q \nabla h_q(x) = 0$

$$\Rightarrow \begin{pmatrix} 2x_1 \\ 2x_2 \end{pmatrix} + v \begin{pmatrix} 1 \\ 2 \end{pmatrix} = 0 \Rightarrow \begin{cases} 2x_1 + v = 0 & ② \\ 2x_2 + 2v = 0 & ③ \end{cases}$$

from ①, ②, ③ we get the optimum solution $x^* = \left(\frac{3}{5}, \frac{6}{5}\right)$, $v^* = \frac{-6}{5}$

The function is plotted as below. And we can find the solution based on the graphics.



ⓑ $L(x, v, \lambda) = f(x) + \sum_{p=1}^{P} v_p g_p(x) + \sum_{q=1}^{Q} \lambda_q h_q(x)$

$= x_1 + x_2 + (x_1 - x_2 - 2)v - \lambda_1 x_1 - \lambda_2 x_2$

$\nabla_x L = \begin{pmatrix} 1 + v - \lambda_1 \\ 1 - v - \lambda_2 \end{pmatrix} = 0$ and $f(x)$ is unbounded

and based on a theorm, if the primal is unbounded, the dual is infeasible.

Task2)

$$L(x,\lambda,\nu) = f_0(x_1,x_2) + \nu h_1(x_1,x_2) + \lambda f_1(x_1,x_2)$$

$$= x_1^2 + x_2^2 + \nu(x_2 - 2x_1 - 0.5) + \lambda(x_1 + x_2 - 1)$$

$$\nabla_x L = \begin{pmatrix} 2x_1 - 2\nu + \lambda \\ 2x_2 + \nu + \lambda \end{pmatrix} = 0 \Rightarrow x^* = \left(\frac{2\nu-\lambda}{2}, \frac{-\nu-\lambda}{2}\right)$$

$$\Rightarrow L(x^*,\lambda,\nu) = \left(\frac{2\nu-\lambda}{2}\right)^2 + \left(\frac{-\nu-\lambda}{2}\right)^2 + \nu\left(\frac{-\nu-\lambda}{2} - 2\cdot\left(\frac{2\nu-\lambda}{2}\right) - 0.5\right)$$

$$+ \lambda\left(\frac{2\nu-\lambda}{2} + \frac{-\nu-\lambda}{2} - 1\right)$$

$$= \frac{5\nu^2 - 2\nu\lambda + 2\lambda^2}{4} + \frac{-5\nu^2 + \lambda\nu - \nu + \lambda\nu - 2\lambda^2 - 2\lambda}{2}$$

$$= \frac{-5\nu^2 + 2\nu\lambda - 2\lambda^2 - 2\nu - 4\lambda}{4}$$

Hence the dual problem is:

$$g(\lambda,\nu) = -\frac{5\nu^2}{4} + \frac{\nu\lambda}{2} - \frac{\lambda^2}{2} - \frac{\nu}{2} - \frac{\lambda}{2}$$

$$\lambda \geq 0$$