CSE/AMS 547 Discrete Mathematics October 17, 2024

Homework Three

Instructor: David Gu *TA: Aditya Patwardhan*

Due by Thursday, October 17, 2024, 3:25 pm.
Please complete **ALL** the following eight exercises and provide solutions.

From the textbook's exercises:
1). 4.32
2). 4.33
3). 4.47
4). 5.14
5). 5.16
6). 5.37

7). For any non-negative integer n ($n \geq 0$), (1). please prove the following expression is an integer; (2). please analyze whether the conclusion in (1) is still true when $n < 0$.

$$\frac{1}{5}n^5 + \frac{1}{3}n^3 + \frac{7}{15}n$$

8). For any integers: x,y,u, and v, please prove,

$$\gcd(a, b) \leqslant \gcd(xa + yb, ua + vb)$$

Bonus Points (5 points)
To earn bonus points, your solutions should satisfy the following requirements:

- Please first provide solutions for the above eight exercises.
- To earn full points, please provide **two** methods to solve the following question.
- Sufficient intermediate steps are needed to earn full points.
- The 25% rule for leaving blank does NOT apply here.

B1). Please list all the non-negative integer pairs of (a, b) that satisfy both of the following conditions,

$$\gcd(a, b) = 10, \text{and } \operatorname{lcm}(a, b) = 100.$$