Exercise 3

Class: Principle of Communication Systems

Semester: 4002

Deadline: **11:59 AM _ tuesday _ 27**/02/1401

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چند نکته

پاسخ تمارین را در همین فایل تکمیل کنید و در صورت نیاز نسبت به ارائه راه حل خود توضیح دهید -

دقت شود که کد های ارسالی دارای کامنت های توضیح خط به خط باشد و شکل ها نیز دارای لیبل و عنوان باشد

فایل نهایی بصورت فایل زیپ با اسم گفته شده در ابتدای ترم ارسال شود

به کد های مشابه نمره ای تعلق نمیگیرد

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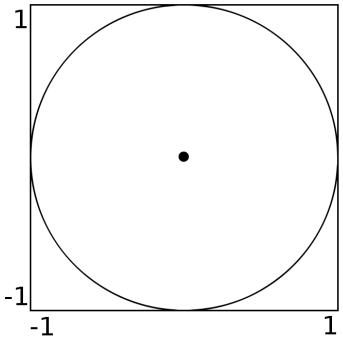
Question 1:

Monte Carlo estimation

The Monte Carlo method is a technique that can be used to approximate the distribution of random outcomes. This is done by recording the outcomes of running the same simulation multiple times.

The Monte Carlo method is very useful, especially to approximate solutions to problems that are analytically difficult. In this assignment, we will use the Monte Carlo method in a simple scenario, to numerically estimate the area of a shape.

- Consider a circle with radius 1, centered on (0, 0).



By generating *N* random points uniformly on the the circumscribing square and counting how many points *n* that fall within the circle, we can estimate the area of the circle by multiplying the area of the square by the fraction of points inside the circle

$$A_{\text{circle}} \approx A_{\text{square}} \cdot \frac{n}{N},$$

where in this case *A*_square = 4. To test if a point (*x, y*) is inside the circle, we can simply check if it magnitude of the vector from the center of the circle to (*x, y*) is less than one.

Problem definition

Write a function that estimates the area of a circle by Monte Carlo simulation. As input the function must receive two vectors containing the coordinates of the randomly drawn points. The function must return the estimated value of the area as output.

- Solution template

```
% function A = circleAreaMC(xvals, yvals)
%
% end
```

Input

xvals --> The x-coordinates of points (vector of decimal numbers)

yvals --> The y-coordinates of points (vector of decimal numbers)

Output

A --> Estimated value for the area of the circle (scalar decimal number)

Example

If we have randomly have drawn the following *N* = 5 points

(-0.1, 0.3) , (0.7, -0.1) , (0.8, 0.9) , (0.5, 0.6) , (-0.4, -0.3)

four of the points lies within the circle, and the area would be estimated as

$A \approx 3.2.(5.6)$

- **Test**

You can test your solution by running the following test script and checking that the output is as expected.

Test script Expected output

```
% circleAreaMC([-0.1, 0.7, 0.8, 0.5, -0.4], [0.3, -0.1, 0.9, 0.6, -0.3])
```

ans =

3.2000s

Discussion and futher analysis (optional)

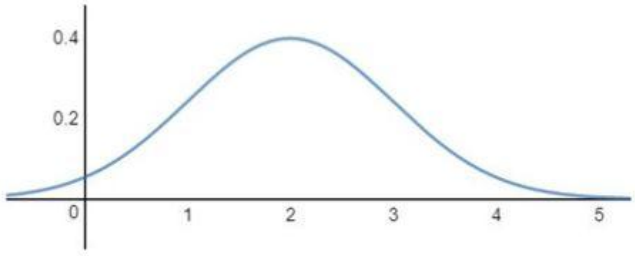
• Try calling your function for different number of points (like 10, 1000, and 1000000).

• Try calling your function multiple times with the same number of points.

• Try running the following code to plot an image of a circle along with your points.

```
% p = 0:0.01:2*pi;
% plot(sin(p), cos(p));
% axis equal; hold on;
% plot(xvals, yvals, 'o');
```

Question 2:



فرآیند تصادفی $x(t;\mu)$ به صورت زیر تعریف شده است:

$$x(t;\mu) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(t-\mu)^2} \quad , \quad \mu \sim N(0,1)$$

میانگین و تابع خود همبستگی این فرایند تصادفی را به دست آورید.

(رسم کنید و در زمان های 1 و 2 ثانیه بدست آورید.)

```
clc
clear all
```

```
% Describe mu and main random process
```

```
% Calculate Mean of random process and plot it
```

```
% Calculate auto correlation of random process and plot it
```

```
% Calculate Part 1 and 2  for time = 2, 3 s
```

Question 3:

3- 1000 عدد رندوم در بازه [0, 1] تولید کنید و هیستوگرام آن را رسم کنید. مجموعه اعداد تولیدی را به متغیر تصادفی با میانگین 5 و واریانس 9 تبدیل کنید سپس آن را گوسی کنید و نمودار هیستوگرام و توزیع احتمال داده ها را رسم نمایید.

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

```
clc
clear all
```

```
% write your code here
```

Question 4: (optional)

4- 1000 جفت اعداد (X, Y) رندوم تصادفی گوسی را تولید کنید و تابع توزیع احتمال متغیر تصادفی زیر را رسم کنید. x1 و x2 مستقل نیستند.

$$V = \sqrt{X^2+Y^2} \qquad \theta = \tan^{-1}(\frac{Y}{X})$$

- ***Binormal Distribution***

$$f_{X_1,X_2}(x_1,x_2)=\frac{1}{2\pi\sigma_{X_1}\sigma_{X_2}\sqrt{1-\rho_{X_1X_2}{}^2}}exp\left(-\frac{1}{2\left(1-\rho_{X_1X_2}{}^2\right)}\left[\frac{\left(x_1-m_{x_1}\right)^2}{\sigma_{X_1}{}^2}+\frac{\left(x_2-m_{x_2}\right)^2}{\sigma_{X_2}{}^2}-\frac{2\rho_{X_1X_2}\left(x_1-m_{x_1}\right)\left(x_2-m_{x2}\right)}{\sigma_{X_1}\sigma_{X2}}\right]\right)$$

```
clc
clear all
```

```
% write your code here
```