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١ الف

$X \sim \mathcal{N}(0,1)$

$Y \sim \mathcal{N}(0,1)$

$\ell = \max(X,Y) \rightarrow$ bigger randomvariable.

$m = \min(X,Y) \rightarrow$ smaller " "

$$\text{Correlation}(\ell,m) = \frac{E(\ell m) - E(\ell)E(m)}{\sqrt{Var(\ell)\,Var(m)}}$$

$$= \frac{E(XY) - E(\ell)E(m)}{1}$$

$\Rightarrow$ we need $E(\ell)$ and $E(m)$

$E(m+\ell) = E(m) + E(\ell) = E(x+y) = E(x) + E(y) = 0$

$E(m-\ell) = E(m) - E(\ell) = E(|x-y|)$,

$|x-y| \sim |w| \Rightarrow w \sim \mathcal{N}(0,\sqrt{2})$

$f_w(w) = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2\pi}} \times e^{-w^2/4}$

$E\{|w|\} = \int_{-\infty}^{+\infty} |w| f_w(w) = \frac{2}{2\sqrt{\pi}} \int_0^{\infty} w e^{-w^2/4} dw$ $\quad w^2 = u$, $2w\,dw = du$

$= \frac{1}{\sqrt{\pi}} \int_0^{\infty} e^{-u/4} \frac{du}{2} = \frac{1}{2\sqrt{\pi}} \left(-4e^{-u/4} \Big|_0^{\infty} \right) = \frac{-2}{\sqrt{\pi}}(0-1) = \frac{2}{\sqrt{\pi}}$

$\Rightarrow E(m+\ell) = 0$

$\left. E(m) - E(\ell) = \frac{2}{\sqrt{\pi}} \right\}$ $\begin{array}{l} E(m) = \pi^{-1/2} \\ E(\ell) = -\pi^{-1/2} \end{array}$

$\Rightarrow E(XY) - (-\pi^{-1}) = +\pi^{-1}$

$=$

1.b a random variable is independent of itself if it doesn't tell information about itself. so let c be a constant such that $P(X=c) = 1$. in this way occuring X doesn't give information tell us since we already know it is happening and it doesn't contain information in it.

1.c Yes, it can. w has extremly higher density value in other two percent that it's denst is bigge than z.

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2

a. $k < n \rightarrow$ Systematic Error did not occur.

Defining Events:

S: Systematic error occur | A: k judge convicts defendant

G: Defendent is guilty

$P(G \mid A, S')$ is our goal since $k < n$

$$\Rightarrow P(G \mid A, S') = \frac{P(A \mid G, S')\, P(G \mid S')}{P(A \mid S')} = \frac{c^k v^{n-k} (P)}{P(A \mid S'G)P(G \mid S') + P(A \mid S'G')P(G' \mid S')}$$

$$= \frac{p c^k (1-c)^{n-k}}{p c^k (1-c)^{n-k} + (1-p) v^k (1-v)^{n-k}} = P(G \mid A, S') \quad \square$$

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b. $P(G \mid A)$. $\Rightarrow$ This time A is $k = n$ judges.

$$\Rightarrow P(G \mid A) = \underbrace{P(G \mid A, S)}_{\text{I}}\ \underbrace{P(S \mid A)}_{\text{II}} + \underbrace{P(G \mid A, S')}_{\text{III}}\ \underbrace{P(S' \mid A)}_{\text{IV}}$$

I. $P(G \mid A, S) = \dfrac{\underbrace{P(A \mid G S)}_{1}\ \overbrace{P(G \mid S)}^{\text{independent} \to p}}{P(A \mid S)} = p$

II. $P(S \mid A) = \dfrac{\overbrace{P(A \mid S)}^{1}\ \overbrace{P(S)}^{s}}{P(A)}$, $\overbrace{P(A \mid S)}^{1}$

$P(A) \rightarrow \underbrace{P(A \mid S)}_{1}\ \underbrace{P(S)}_{s} + P(A \mid S')\, P(S')$

$$P(A \mid S') = \underbrace{P(A \mid S'G)}_{c^n}\ \underbrace{P(G \mid S')}_{p} + \underbrace{P(A \mid S'G')}_{v^n}\ \underbrace{P(G' \mid S')}_{(1-p)}$$

$$\Rightarrow \text{II}\quad P(S \mid A) = \frac{s}{s + (c^n p + v^n (1-p))(1-s)}$$

(III) $P(G|A, S') =$ Same as Previous Part $\rightarrow k = n$

$$P(G|A, S') = \frac{p c^n}{p c^n + (1-p) \nu^n}$$

(IV) $P(S'|A) = 1 - P(S|A) = \boxed{1 - \text{(II)}}$

$$\Rightarrow P(G|A) = p \left(\frac{s}{s + \left(c^n p + \nu^n (1-p)\right)(1-s)} \right)$$

$$+ \frac{p c^n}{p c^n + (1-p)\nu^n} \left(1 - \frac{s}{s + \left(c^n p + \nu^n (1-p)\right)(1-s)} \right)$$

(c) By ~~increasing $n \rightarrow \infty$ we can see that $P(G|A) \rightarrow P$ which means in in Jewish law if $k=n$ then the defendent would be set free although we are seeing that here if $n \rightarrow \infty$ the $P(G|A)$ is p which is the probability that defendant is guilty~~

as the n increases, chance of systematic error increases, and when sytematic error rises judges votes are independent of guiltiness of victim

$n \rightarrow \infty \quad P(G|A) \rightarrow P \rightarrow$ So No!

3

Since The Poisson Distribution is often used in situations where we are counting the number of succses in a particular region or interval of time, and there are a large number of trials, each with a small probability of success. So number of emails that are arriving at Bob's email has poisson distribution

3 Y r.v indicate time it is spend to wait

X " " # email at time t

Z " " amount the spend on each email

I " " work related email or personal

$$E(Y) = E(XZ) \overset{ind}{=} E(X)E(Z) = \lambda E(Z)$$

$$E(E(Z|I)) = E(Z) =$$

$$E(E(Z_w I + (1-I)Z_p | I)) = E(\mu_w I + (1-I)\mu_p)$$

$$E(Y) = (\mu_w p + \mu_p q)\lambda$$

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أرام 3

$$Var(Y) = Var(XZ) \overset{\text{independent}}{=} \underbrace{Var(X)}_{\lambda}\ Var(Z)$$

$$Var(Z) : E(Var(Z|I)) + Var(E(Z|I))$$

$$Var(Z|I) = Var(I Z_w + Z_p(1-I)|I)$$

$$E(Var(Z|I)) = E(I^2 \sigma_w^2 + (1-I)^2 \sigma_p^2)$$

$$= p\,\sigma_w^2 + q\,\sigma_p^2$$

$$E(Z|I) = E(Z_w I + Z_p(1-I)|I)$$

$$\Rightarrow Var(E(Z|I)) = Var(I\mu_w + (1-I)\mu_p)$$

$$= \mu_w^2\, p(1-p) + p(1-p)\mu_p^2$$

$$= \lambda\left(p\sigma_w^2 + q\sigma_p^2 + \mu_w^2 pq + pq\,\mu_p^2\right)$$

5 for the last person there are only two choices to sit the first chair and the last one; because for e.y ith sit would be occupied either by ith person or some one whose place is occupied. so the last person should either sit on first chair or the last one.

4

we define an indicator random variable that shows whether the i th number is a min or not

$$E(I_1 + \dots + I_n)$$

$$= \sum_{i=2}^{n-1} E(I_i) + E(I_1) + E(I_n)$$

$p(I_1) = E(I_1) = \frac{1}{3} \Rightarrow$ it has three diffient permutation and one of them is desired

$$\Rightarrow \frac{n-2}{3} + \frac{1}{2} + \frac{1}{2} = \frac{n+1}{3}$$

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6 we try to count possibilities

A: at least one of them is boy and born in tuesday:

$$P(B|A) = \frac{P(A, B)}{P(A)}$$

$P(A,B) \rightarrow$ probability of A happens and other child be also boy so both childs should be boys so we do not have and choice in children therefore we have 13 possibility of being born at least in tuesday.

$$BB\ \underset{\uparrow}{\underline{7}}\ \underline{T_{uesday}} + BB\ \underline{T}\ \underset{d}{\underline{6}} = 13$$

we have counted one time TT

$$P(A,B) = \frac{13}{\binom{4}{2}\binom{7}{1}\binom{7}{1}}$$

$P(A) \Rightarrow$ at least one of them is boy and born in Tuesday

we try to approach the complement way:

$$4 \text{ ways} \left\{ \begin{array}{l} GG \rightarrow \underline{7}\ \underline{7} \\ BG \rightarrow \underline{6}\ \underline{7} \\ GB \rightarrow \underline{7}\ \underline{6} \\ BB \rightarrow \underline{6}\ \underline{6} \end{array} \right\} \Rightarrow \frac{49 + (42 \times 2) + 36}{\binom{7}{1}\binom{7}{1}\binom{4}{2}}$$

$$1 - (\quad) = \frac{27}{\binom{4}{2}\binom{7}{1}^2}$$

$$= \frac{13}{27}$$

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