Interpolation resolved

In linear algebra terms, the FFT multiplies an arbitrary n-dimensional vector—which we have been calling the *coefficient representation*—by the $n \times n$ matrix

$$M_n(\omega) = \begin{array}{c} \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & \omega & \omega^2 & \cdots & \omega^{n-1} \\ 1 & \omega^2 & \omega^4 & \cdots & \omega^{2(n-1)} \\ & & \vdots & & \\ 1 & \omega^j & \omega^{2j} & \cdots & \omega^{(n-1)j} \\ & & \vdots & & \\ 1 & \omega^{(n-1)} & \omega^{2(n-1)} & \cdots & \omega^{(n-1)(n-1)} \end{bmatrix} \end{array} \begin{array}{l} \longleftarrow \text{ row for } \omega^0 = 1 \\ \longleftarrow \omega \\ \longleftarrow \omega^2 \\ \quad\ \vdots \\ \longleftarrow \omega^j \\ \quad\ \vdots \\ \longleftarrow \omega^{n-1} \end{array}$$

Master Theorem

If $T(n) = aT(n/b) + O(n^d)$ for constants $a > 0$, $b > 1$, $d \geq 0$, then

$$T(n) = \begin{array}{ll} O(n^d) & \text{if } d > \log_b a \\ O(n^d \log n) & \text{if } d = \log_b a \\ O(n^{\log_b a}) & \text{if } d < \log_b a \end{array}$$

Two nodes u and v of a directed graph are *connected* if there is a path from u to v *and* a path from v to u.

Property ***A directed graph has a cycle if and only if its depth-first search reveals a back edge.***

Proof. One direction is quite easy: if (u, v) is a back edge, then there is a cycle consisting of this edge together with the path from v to u in the search tree.

Conversely, if the graph has a cycle $v_0 \rightarrow v_1 \rightarrow \cdots \rightarrow v_k \rightarrow v_0$, look at the *first* node on this cycle to be discovered (the node with the lowest `pre` number). Suppose it is v_i. All the other

Property *Every directed graph is a dag of its strongly connected components.*

Property *Every dag has at least one source and at least one sink.*

The guaranteed existence of a source suggests an alternative approach to linearization:

Property *In a dag, every edge leads to a vertex with a lower* `post` *number.*

Property 1 *If the* `explore` *subroutine is started at node u, then it will terminate precisely when all nodes reachable from u have been visited.*

Property 2 *The node that receives the highest* `post` *number in a depth-first search must lie in a source strongly connected component.*

Property 3 *If C and C' are strongly connected components, and there is an edge from a node in C to a node in C', then the highest* `post` *number in C is bigger than the highest* `post` *number in C'.*

`pre/post` *ordering for* (u, v)	*Edge type*
$[_u \; [_v \;]_v \;]_u$	Tree/forward
$[_v \; [_u \;]_u \;]_v$	Back
$[_v \;]_v \; [_u \;]_u$	Cross

Tree edges are actually part of the DFS forest.

Forward edges lead from a node to a *nonchild* descendant in the DFS tree.

Back edges lead to an ancestor in the DFS tree.

Cross edges lead to neither descendant nor ancestor; they therefore lead to a node that has already been completely explored (that is, already postvisited).