Cascading Behavior in Networks

Machine Learning with Graphs

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Lecture Topics

- **Modeling Diffusion**
- Cascades & Clusters
- Cascade Capacity

Diffusion

- In cascades, people **imitate** behaviors of others.

- Look at cascade from network structure perspective
  - How are individuals influenced by their immediate neighbors?
    - Compatibility with technology that friends use
    - Friends political views, etc.

- "Nodes" adopt a new behavior once a **sufficient proportion of their neighbors** have done so.

Diffusion- Cnt.

- A Networked Coordination Game
  - Nodes choose btw two possible behaviors: A and B.
  - If nodes v and w are linked, then they receive payoff if their behaviors match.
    - v and w both adopt A, each get a payoff of **a > o**;
    - v and w both adopt B, each get a payoff of **b > o**;
    - v and w adopt opposite behaviors, each get payoff of **o**.
- Nodes choice of behavior depends on choices made by all of its neighbors, taken together!

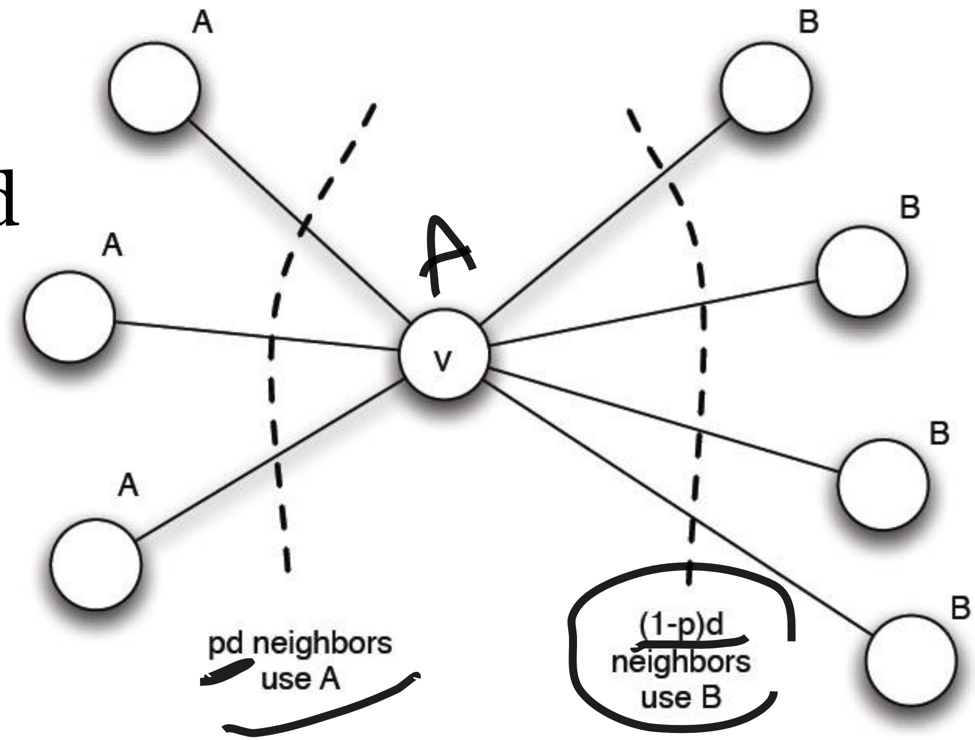
Diffusion- Cnt.

- p fraction of v's neighbors choose A
- $(1 - p)$ fraction choose B.
- v has d neighbors

- Which behavior should v adopt?

$(1-p) \times d \times b$

$p \times d \times a$

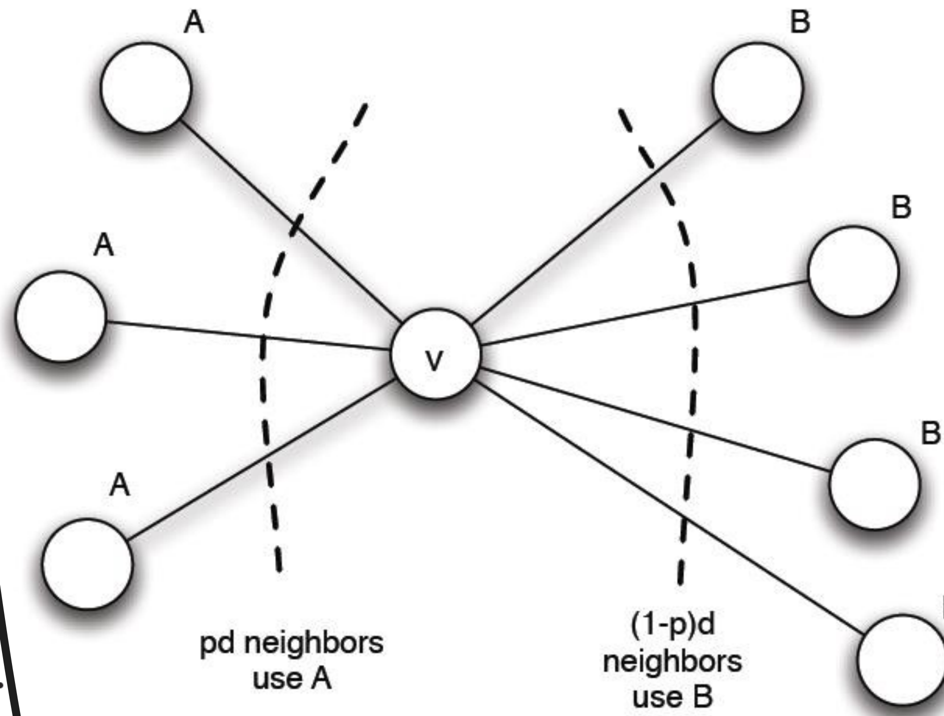


Diffusion- Cnt.

q = 90%!

- p fraction of v's neighbors choose A
- $(1 - p)$ fraction choose B.
- v has d neighbors
  - If v chooses A
    - payoff = p×d×a
  - If v chooses B
    - payoff = (1 – p)×d×b
  - A is the better if

$$pda \geq (1-p)db, \qquad p \geq \boxed{\frac{b}{a+b}}.$$



$$\boldsymbol{q = \frac{b}{a+b}}$$

q = 0.05 vs q = 0.95

Diffusion- Cnt.

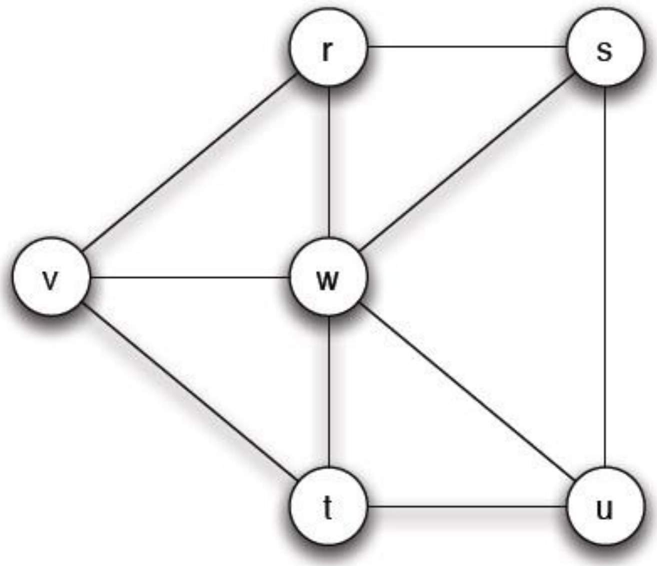
- Cascading behavior
  - Everyone adopts A,
  - Everyone adopts B,
  - **Intermediate state: some adopt A and some adopt B!**

Diffusion- Cnt.

- Suppose everyone initially use B as a default behavior.
- A small set of **initial adopters** decide to switch to A.
- Cascade may start:
  - some neighbors of initial adopters may switch to A, then their neighbors, and so forth
- Cascade stops if:
  - **Complete cascade**: every node switch over to A!
  - We reach a step where no node wants to switch! (coexistence btw A and B)
- That depends on:
  - the network structure,
  - the choice of initial adopters,
  - the value of the threshold q

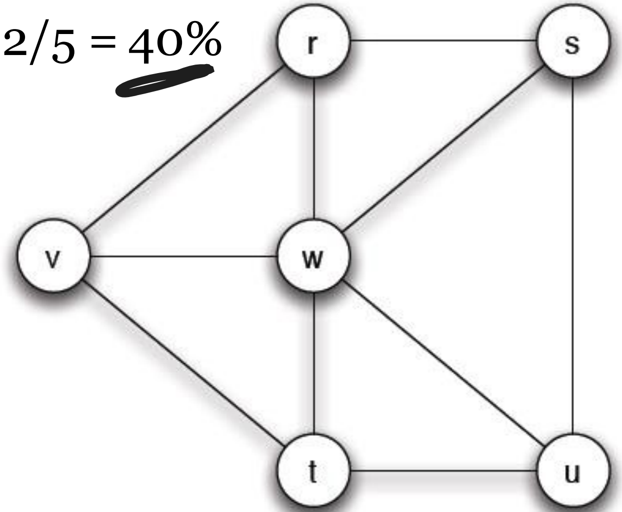
Diffusion- Cnt.

- Payoff a=3 and b=2.
- q = 2/5, nodes switch to A if at least 40% of their neighbors are using A!
- v and w are initial adopters of A!

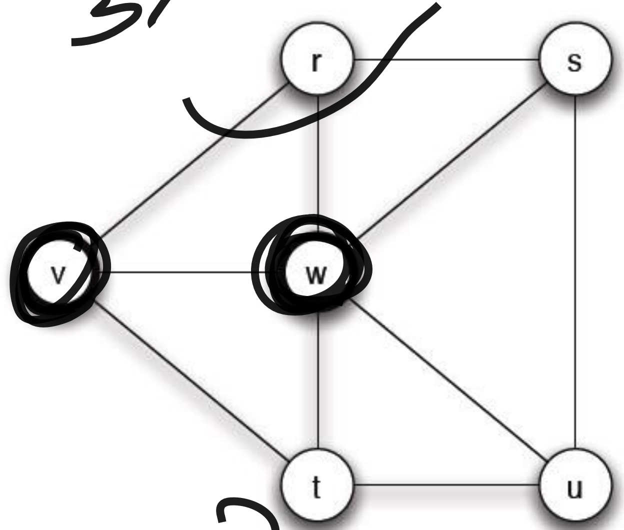


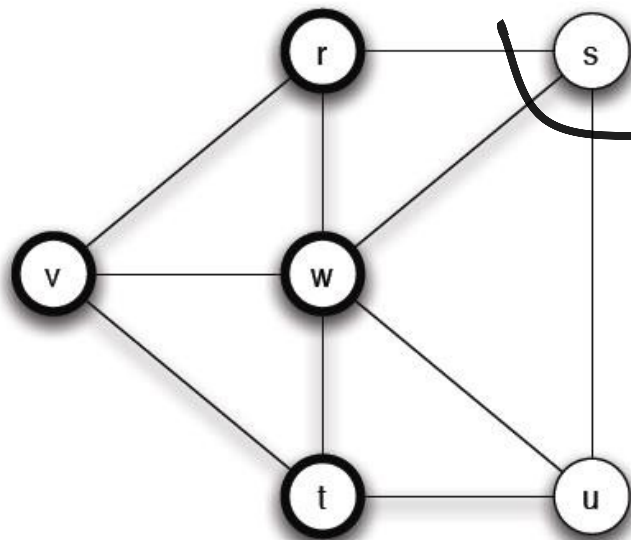
Diffusion- Cnt.

$q = 2/5 = 40\%$



$\frac{2}{3} > 40\%$





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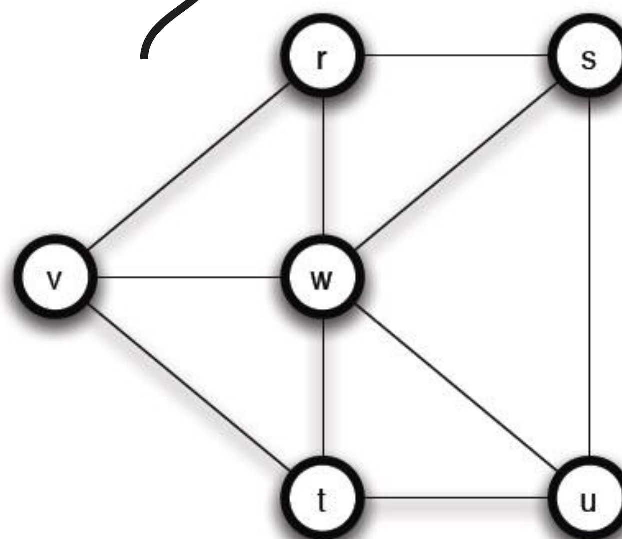
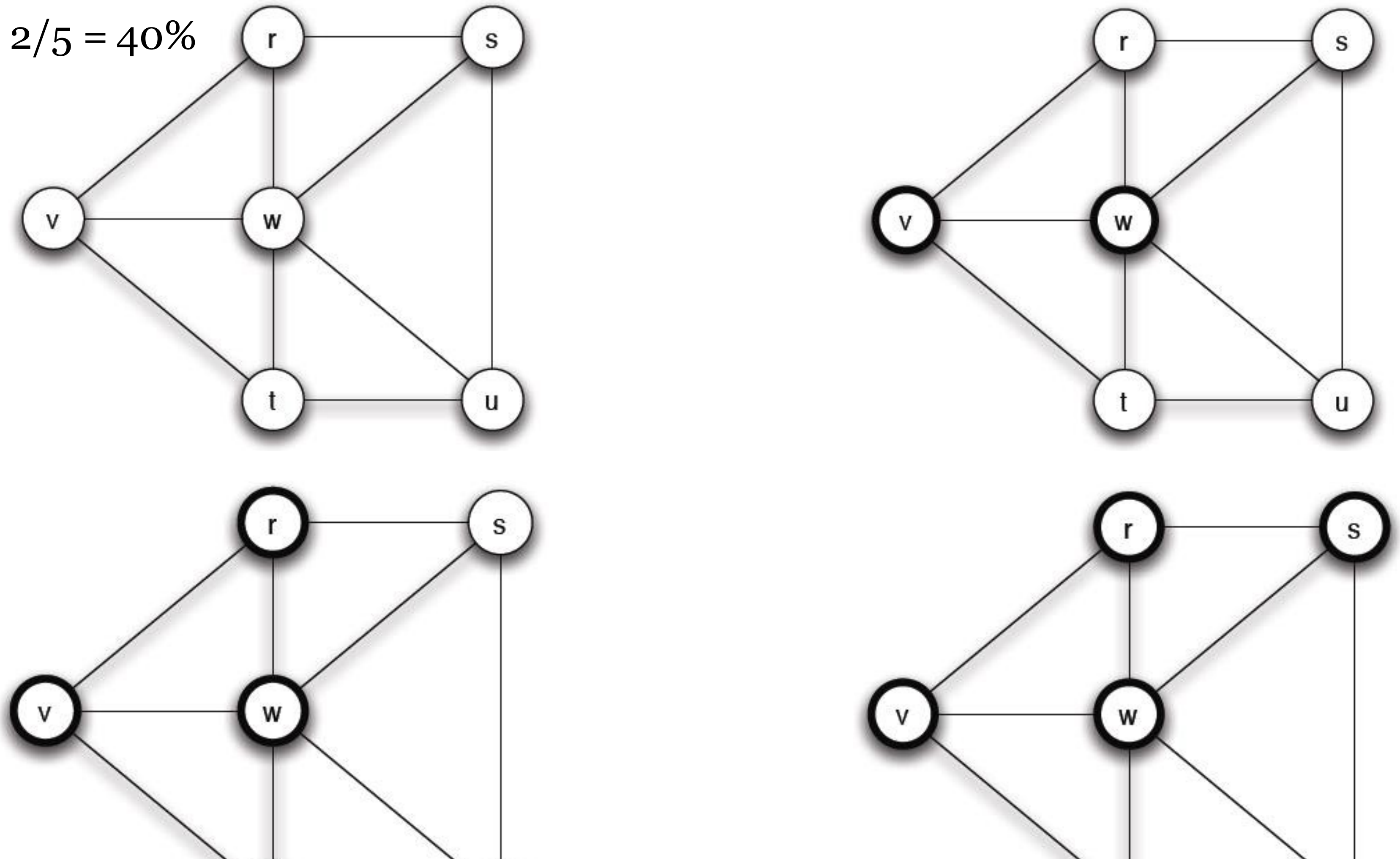


Diffusion- Cnt.

$q = 2/5 = 40\%$

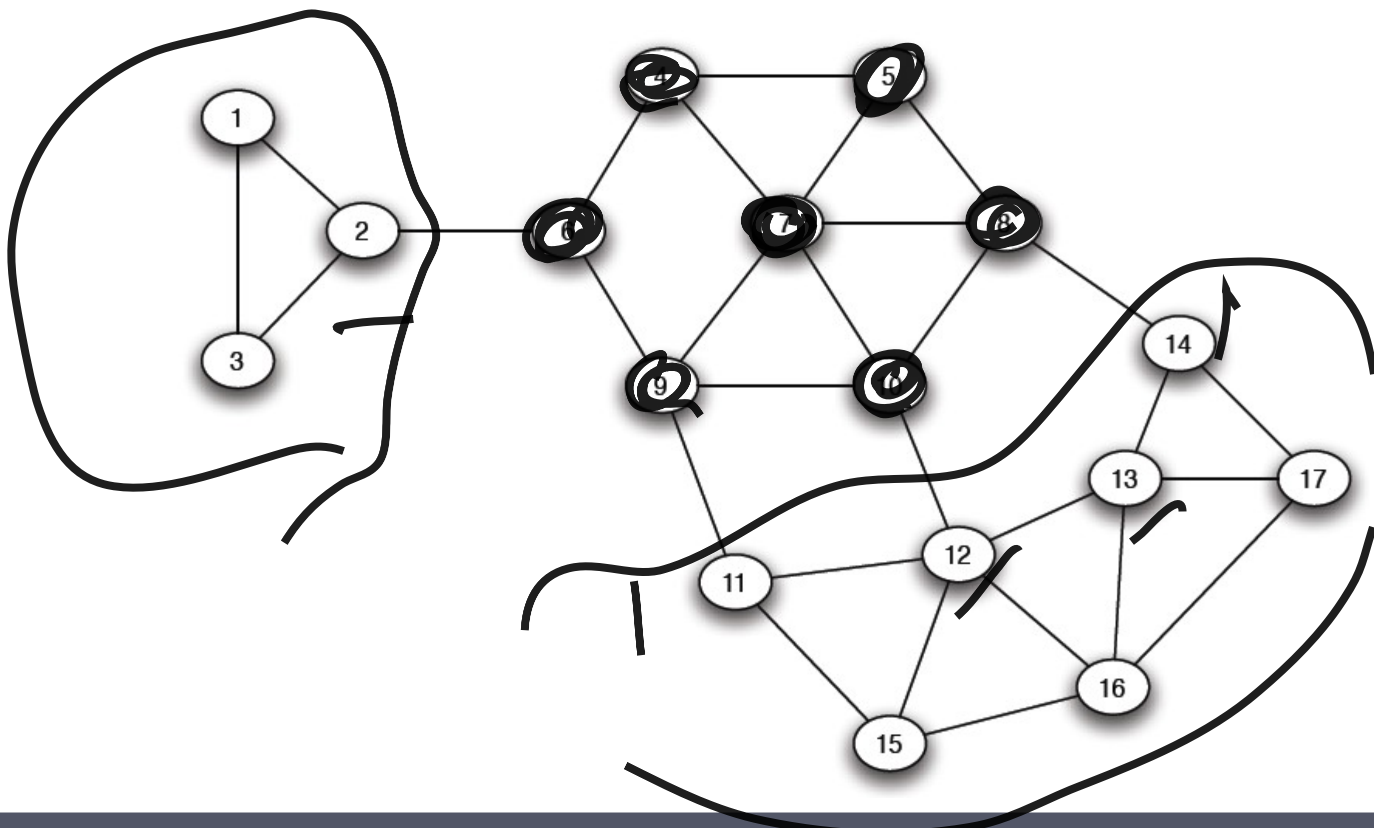


chain reaction: *v* and *w* aren't able to get *s* and *u* to switch by themselves, but once they've converted *r* and *t*, this provides enough leverage.

Diffusion- Cnt.

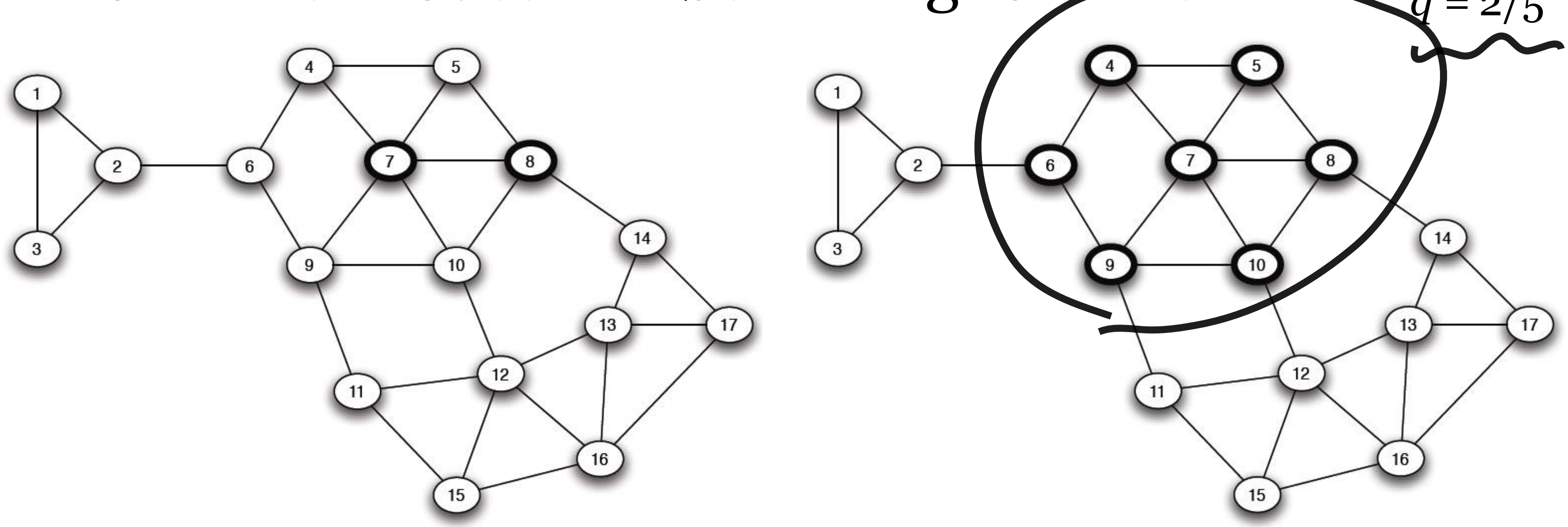
Sol.

- a=3 and b=2.
- $q = 2/5$
- 7 and 8 are initial adopters of A!



Diffusion- Cnt.

- Takes 3 steps for the cascade to stop!
  - 5 and 10 switch to A, then
  - nodes 4 and 9, then
  - node 6.
- No further nodes will be willing to switch!



Tightly-knit communities in the network can hinder the spread of a behavior.

Diffusion- Cnt.

- What are useful strategies to push adoption of A (assume A and B are competing technologies)?

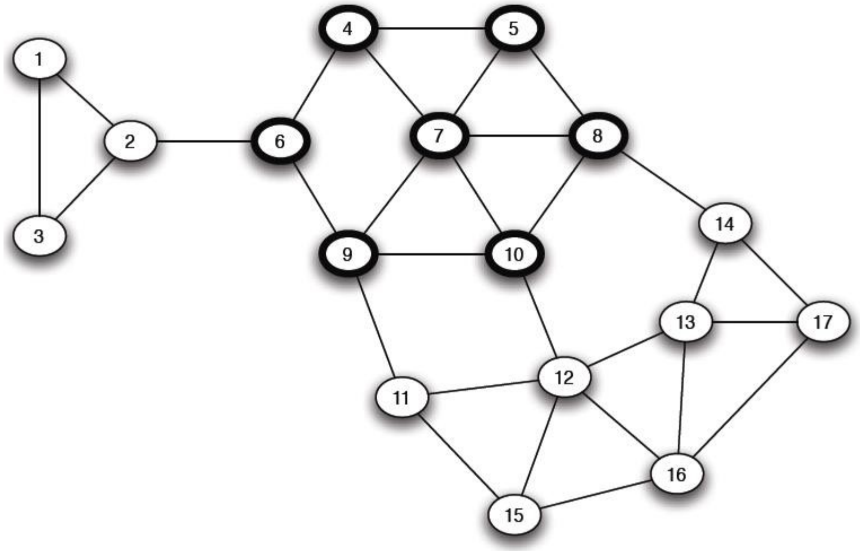
Diffusion- Cnt.

- Strategies that are useful to push adoption of A
  - Change the payoff
    - Say from a = 3 to a = 4!
    - q drops from 2/5 down to 1/3
      - then all nodes will switch to A in the above example.

$q = b/(a+b)$.

Diffusion- Cnt.

- Strategies that are useful to push adoption of A
  - Convince a small number of key nodes in the part of the network using B to switch to A
    - Choose carefully so as to get the cascade going again!
    - Convince 12?
    - Convince 14?



Lecture Topics

- Modeling Diffusion
- **Cascades & Clusters**
- Cascade Capacity

Cascades & Clusters

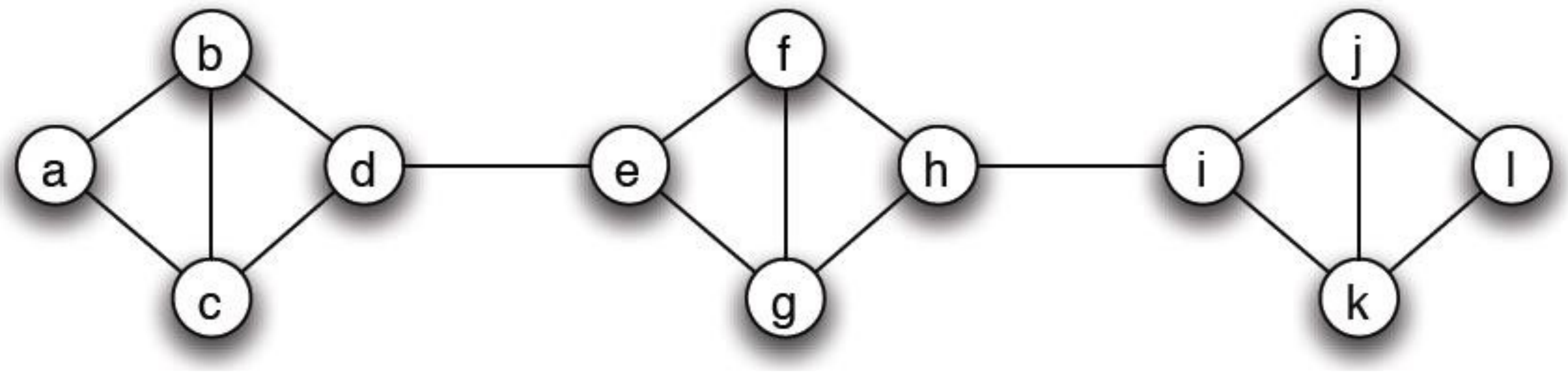
- Question: What makes a cascade stop? Or prevents it from breaking into all parts of a network?

Cascades & Clusters

- Question: What makes a cascade stop? Or prevents it from breaking into all parts of a network?
  - A cascade comes to stop when it runs into a **dense cluster** (tightly-knit **communities** & **homophily**),
  - This is the **only** thing that causes cascades to stop!

Cascades & Clusters- Cnt.

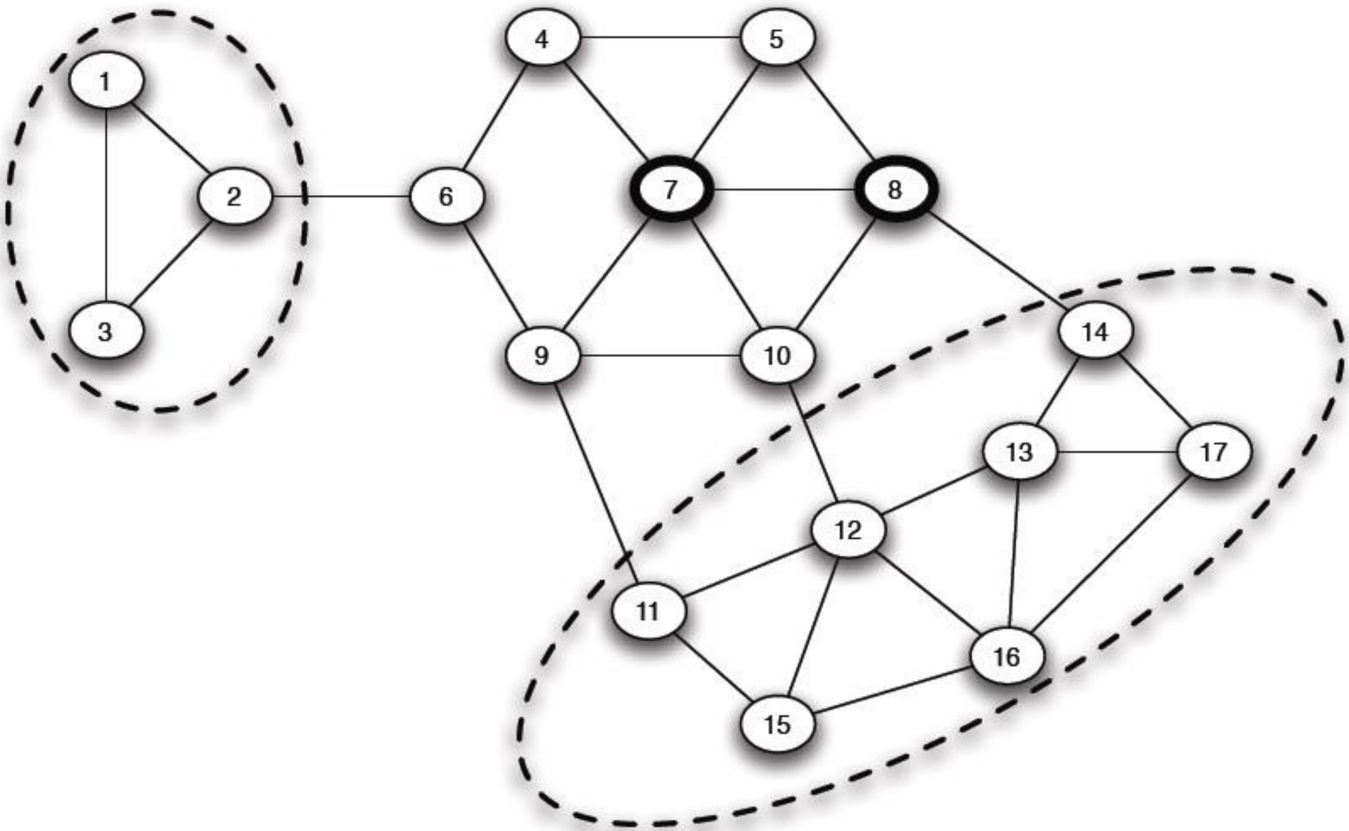
- **Cluster Density**
  - A cluster with density *p* is a set of nodes such that each node has **at least** *p* fraction of its neighbors in the set.



Cascades & Clusters- Cnt.

- **Claim**: Given initial adopters of A & threshold q:
  i. If remaining network contains a cluster of density greater than $1 - q$, then no complete cascade.
  ii. If there is no complete cascade, the remaining network contains a cluster of density $> 1 - q$.

Cascades & Clusters- Cnt.



q = 2/5 = 40%

Cluster density = 2/3 = 66%

Cluster density > (1-q) = 60%

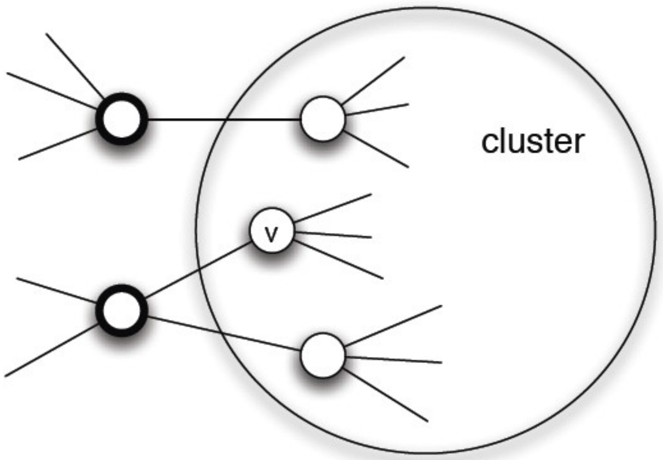
Cascades & Clusters- Cnt.

i. If remaining network contains a cluster of density greater than $1 - q$, then no complete cascade.

Cascades & Clusters- Cnt.

i. If remaining network contains a cluster of density greater than 1 – q, then no complete cascade.

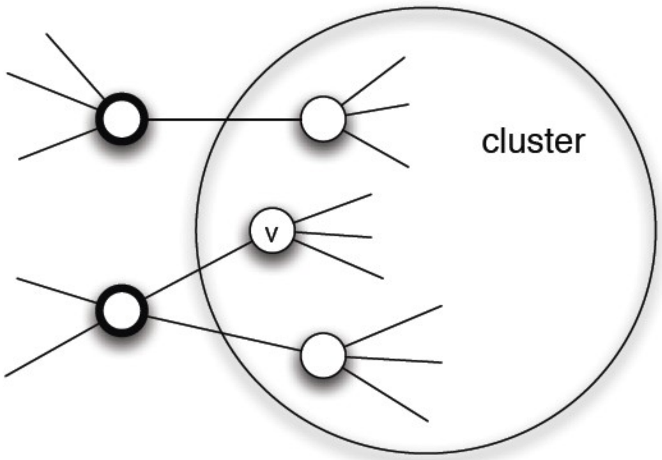
- **Solution**
  - Assume there is a node inside the cluster (density > 1-q) that adopts A
  - Let v be the **first** node that does so.



Cascades & Clusters- Cnt.

i. If remaining network contains a cluster of density greater than $1 - q$, then no complete cascade.

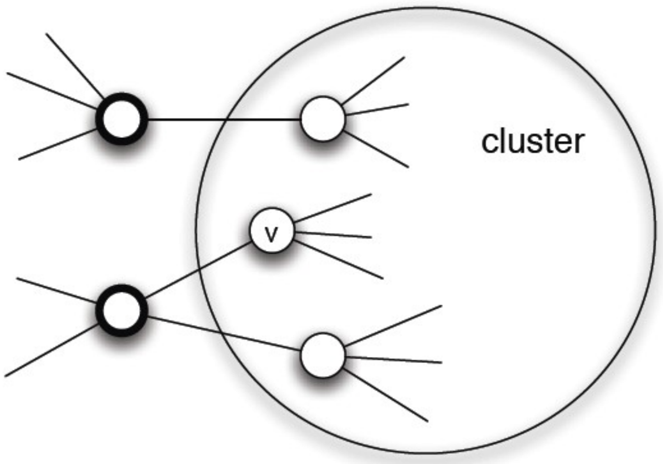
- **Solution**
  - The only neighbors of v that were using A at the time it decided to switch were **outside** the cluster.



Cascades & Clusters- Cnt.

i. If remaining network contains a cluster of density greater than $1 - q$, then no complete cascade.

- **Solution**
  - But, more than a 1-q fraction of v's neighbors are inside the cluster,
  - Thus less than a q fraction of v's neighbors are outside the cluster.
    - Thus v cannot adopt A



clusters block the spread of cascades

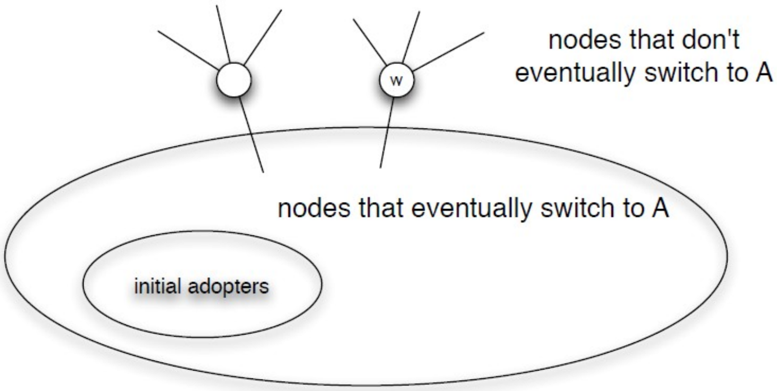
Cascades & Clusters- Cnt.

ii. If there is no complete cascade, the remaining network contains a cluster of density $> 1 - q$.

Cascades & Clusters- Cnt.

ii. If there is no complete cascade, the remaining network contains a cluster of density > 1 – q.

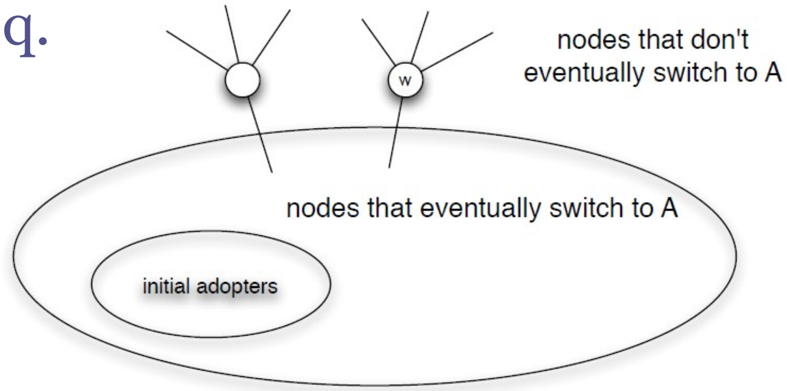
- **Solution**
  - Run the process until it stops!
    - there are nodes using B that don't want to switch.
    - let S denote such nodes.



Cascades & Clusters- Cnt.

ii. If there is no complete cascade, the remaining network contains a cluster of density > 1 – q.

- **Solution**
  - Run the process until it stops!
    - consider any node $w \in S$
    - fraction of w's neighbors using A is < q.
    - fraction of w's neighbors using B is > 1 – q.
    - This holds for any node $w \in S$
      - S is a cluster of density > 1 – q.



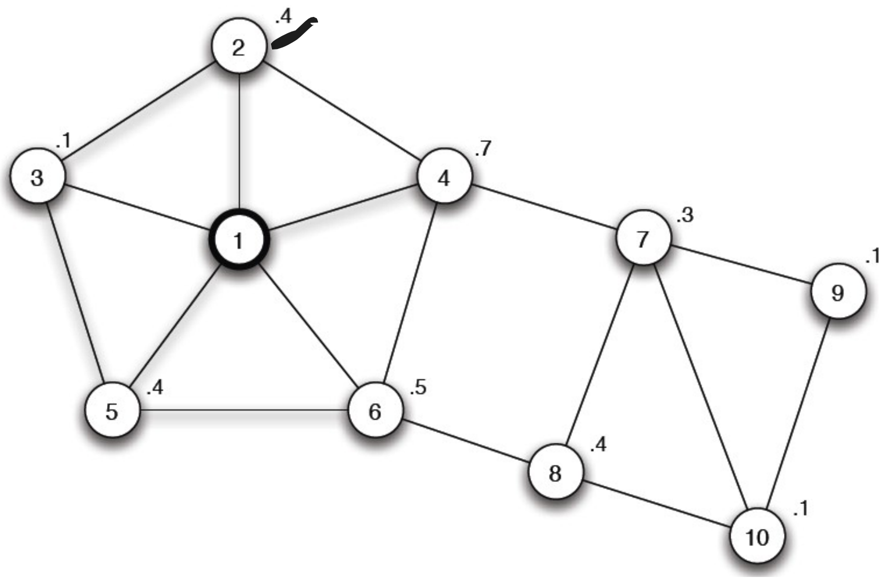
Whenever a cascade comes to a stop, there's a cluster that can be used to explain why.

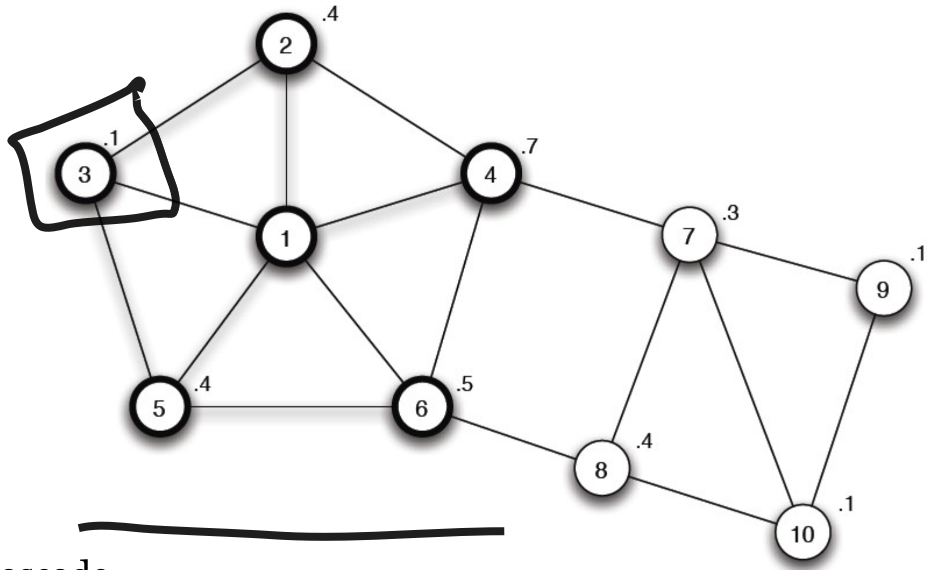
Extensions of Cascade Model

- **Heterogeneous thresholds**
  - each nodes v has a **node-specific** threshold (q_v) for adopting a behavior!
- v has d neighbors of whom a p fraction have behavior A, and a (1 – p) fraction have behavior B:
  - Payoff from choosing A is pda_v
  - Payoff from choosing B is $(1 - p)db_v$.
- A is better for v if
  - $pda_v > (1 - p)db_v$.

$$p \geq \frac{b_v}{a_v + b_v}.$$

Extensions of Cascade Model- Cnt.





- Without node-specific thresholds, there would no cascade.
- The extremely low threshold of node 3 lead to diffusion.

> The power of **influential nodes** is correlated to the extent to which such nodes have access to easily **influenceable nodes**.

- Clusters are still obstacle to cascades
- A ***blocking cluster*** is a set of nodes for which each node v has > 1-q_v fraction of its neighbors in the set.
  - Heterogeneous cluster density: node-specific threshold for the fraction of friends to have in cluster.

Lecture Topics

- Modeling Diffusion
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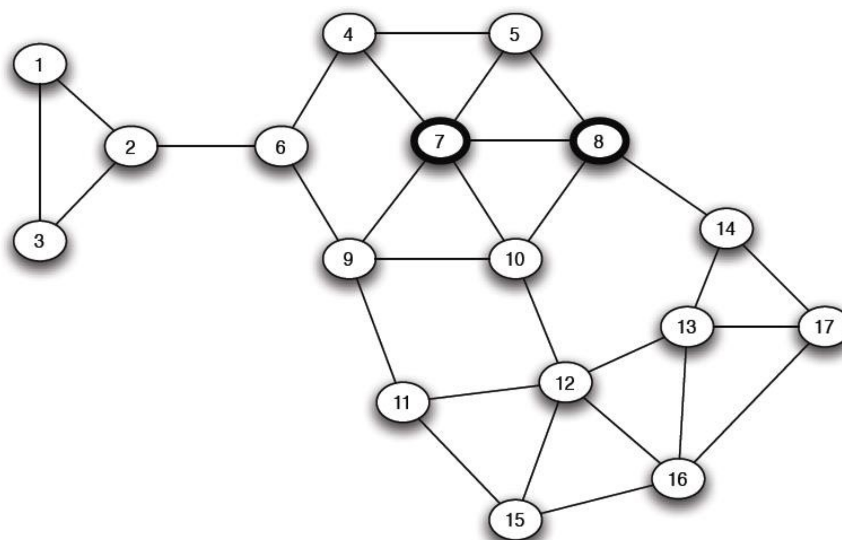
- **Cascade Capacity**

Cascade Capacity

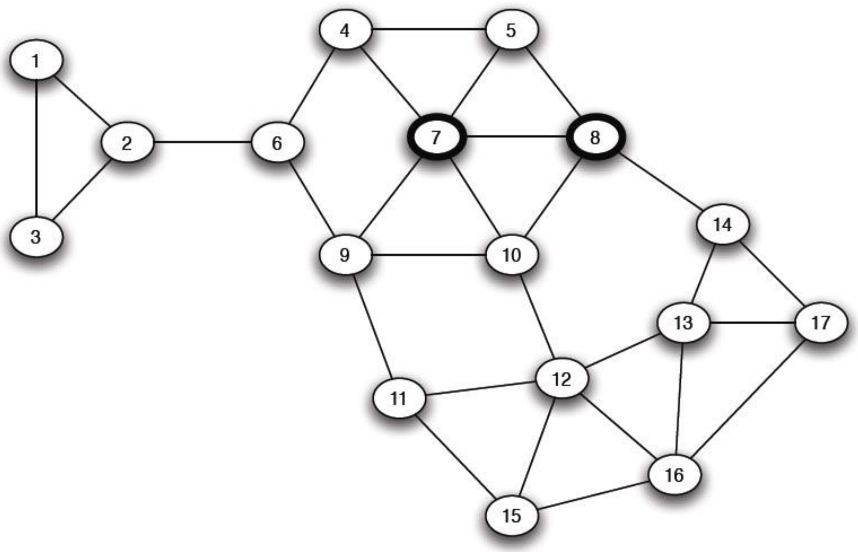
- **Cascade capacity** of a network: The maximum *q* for which some **small** set (***finite set***) of initial adopters can cause a **complete cascade**!

q = \ol ,



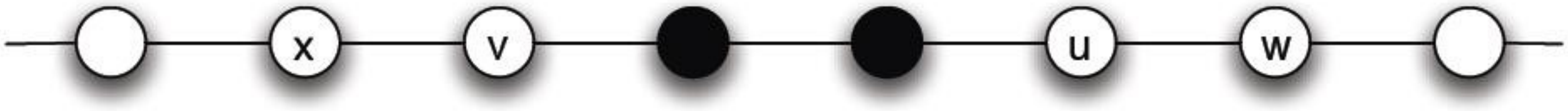
Cascade Capacity

- **Cascade capacity** of a network: The maximum q for which some **small** set (***finite set***) of initial adopters can cause a **complete cascade**!
  - Indicates how different network structures are **hospitable** to cascades!



Cascade Capacity- Cnt.

- Let S be the small set of early adopters of A.
- What is cascade capacity?
  - the maximum *q* for complete cascade?



Cascade Capacity- Cnt.

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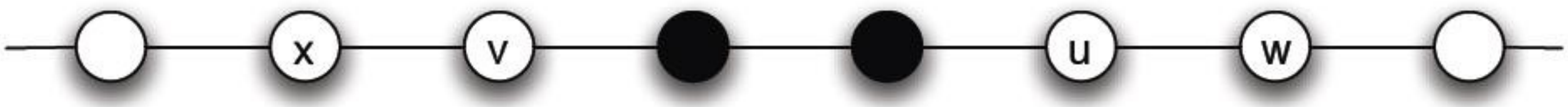
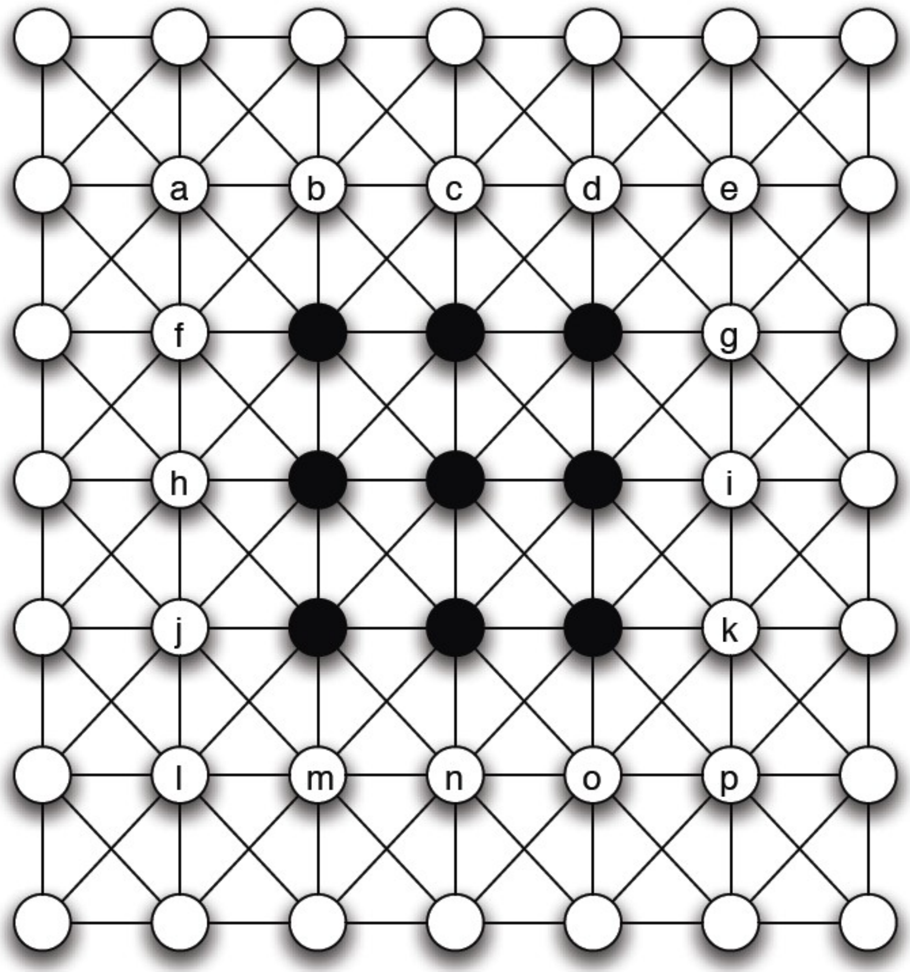


If $q <= ½$, complete cascade.
If $q > ½$, no finite set of initial adopters can get any node to switch to A.

Cascade capacity= ½

Cascade Capacity- Cnt.

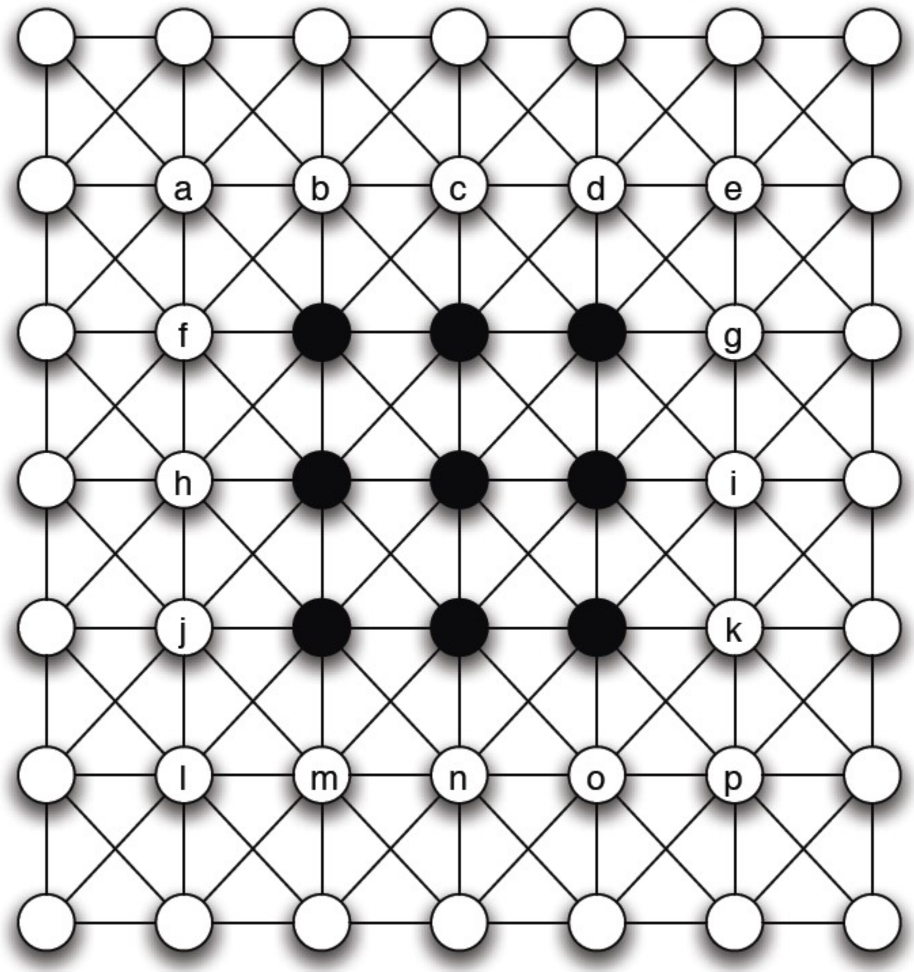


Cascade Capacity- Cnt.

If q <= 3/8, then there is a complete cascade: first to the nodes *c*, *h*, *i*, *n*; then to nodes *b*, *d*, *f*, *g*, *j*, *k*, *m*, *o*; and then to others

If q > 3/8, no node will choose to adopt A.

Cascade Capacity=3/8



Cascade Capacity- Cnt.

- How easy cascades propagate in a network with *large* cascade capacity?

Cascade Capacity- Cnt.

- How easy cascades propagate in a network with *large* cascade capacity?
  - Cascades happen more “easily”!
  - they happen even for behaviors A that don't offer much payoff advantage over the default behavior B.

Cascade Capacity- Cnt.

- What is the maximum possible value of cascade capacity?

Information diffusion on Twitter

- https://snikolov.wordpress.com/2012/11/12/information-diffusion-on-twitter/

Reading

- Ch.19 Cascading Behavior in Networks [NCM]