Graph Features 1

Machine Learning with Graphs

Department of Computer Science
University of Massachusetts, Lowell
Spring 2021
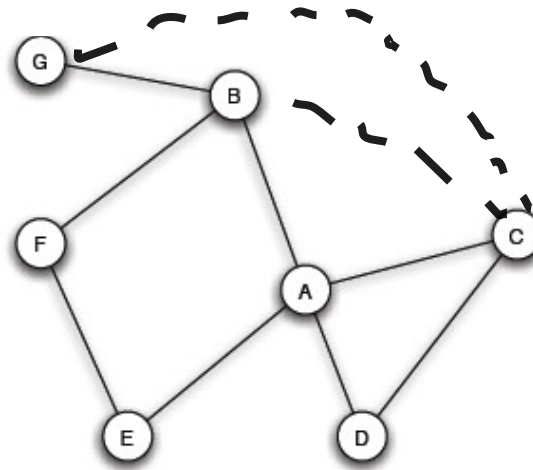
Hadi Amiri
hadi@cs.uml.edu

Lecture Topics

- Triadic Closure
  - Clustering Coefficient
  - Motifs
  - Bridges and Local Bridges
  - Tie Strength
  - Strong Triadic Closure
  - Local Bridges and Weak Ties
  - The Strength of Weak Ties
- Tie Strength in Real-World Nets
  - Neighborhood Overlap
  - Analysis on Facebook and Twitter
- Structural Holes

Question

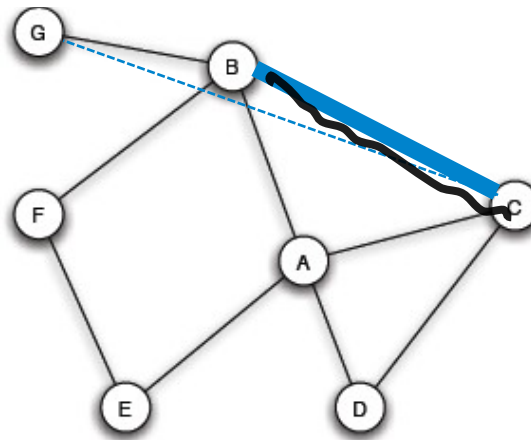In this friendship net, C-B is more likely to form or C-G?

Triadic Closure

- If two people in a network have a friend in common, then there is an increased likelihood they will become friends themselves.
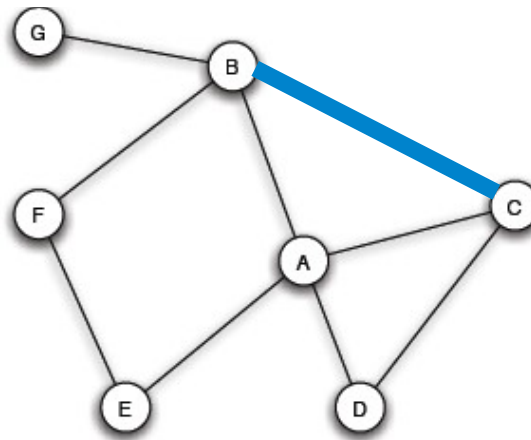
connected

Georg Simmel, 1900s

In this friendship net, C-B is more likely to form or C-G?
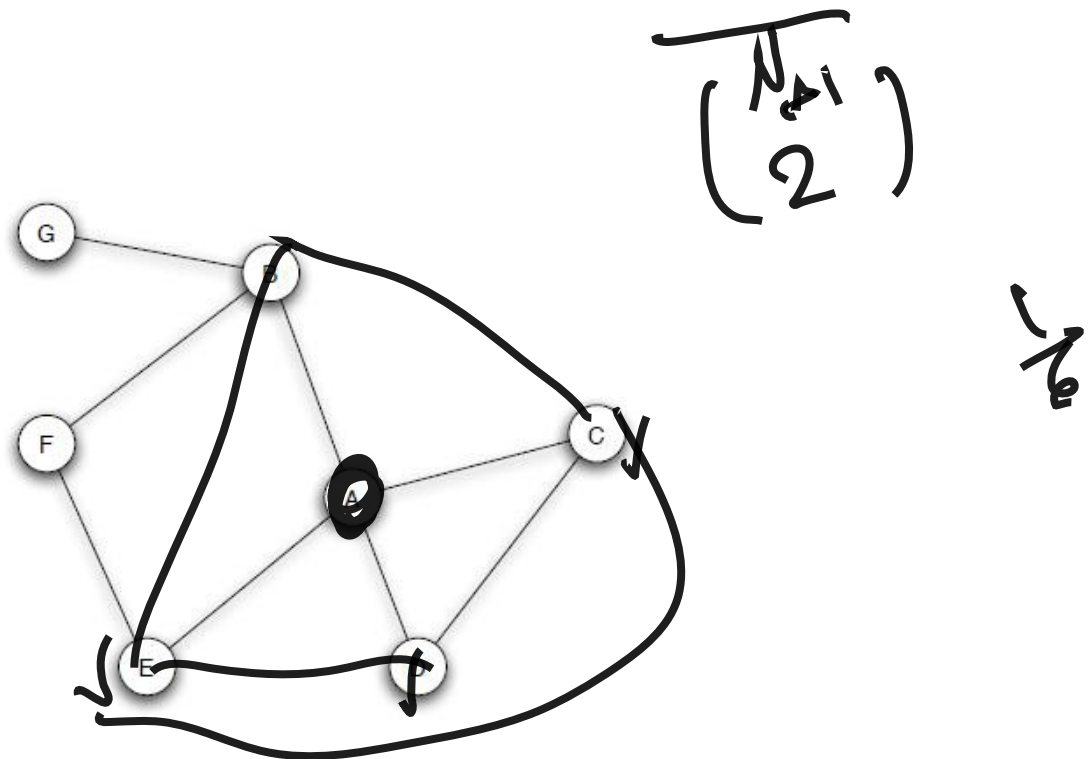
Triadic Closure- Cnt.

- The term "triadic closure" comes from the fact that the B-C edge has the effect of "closing" the third side of this triangle.

In this friendship net, C-B is more likely to form or C-G?

Prevalence of Triadic Closure

- Clustering Coefficient (CF)
  - CF of a node *A* is defined as the probability that two randomly selected friends of A are friends with each other.
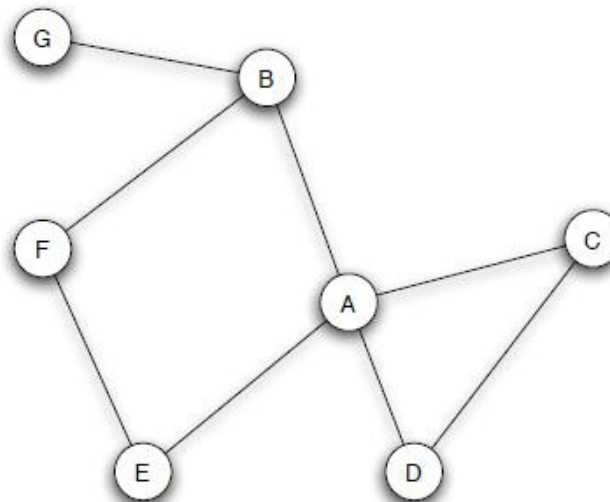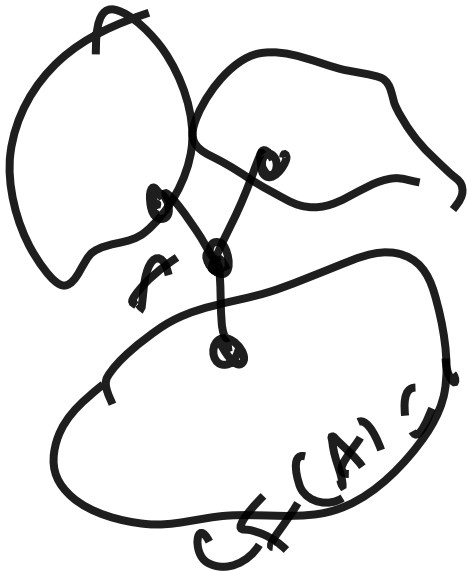
$$\binom{N_{\ge i}}{2}$$

Prevalence of Triadic Closure

- Clustering Coefficient (CF)
  - CF of a node *A* is defined as the probability that two randomly selected friends of A are friends with each other.

$$CF(A) = \frac{Number\ of\ connections\ btw\ A's\ friends}{Possible\ Number\ of\ connections\ btw\ A's\ friends}$$

Prevalence of Triadic Closure

- Clustering Coefficient (CF)
  - CF of a node *A* is defined as the probability that two randomly selected friends of A are friends with each other.
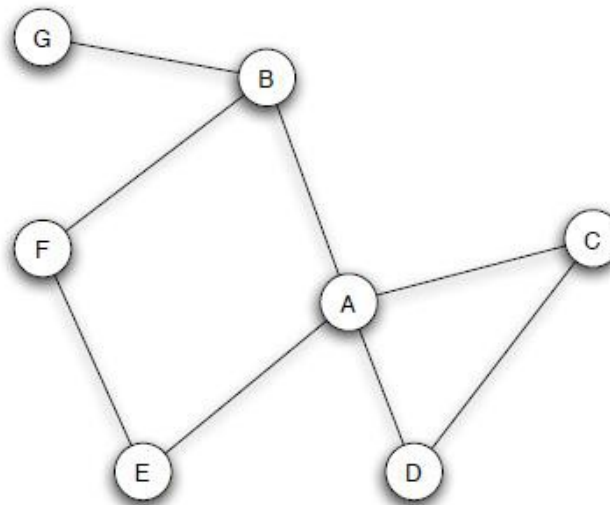
$$CF(A) = \frac{\textit{Number of connections btw A's friends}}{\textit{Possible Number of connections btw A's friends}}$$


- Structural notion
- Range: [0, 1]
- Measures how tightly clustered node neighbors are.

Prevalence of Triadic Closure

- Empirical study by Bearman and Moody (2004):
  - Teenage girls who have a low clustering coefficient in their network of friends are significantly more likely to contemplate suicide than those whose clustering coefficient is high!
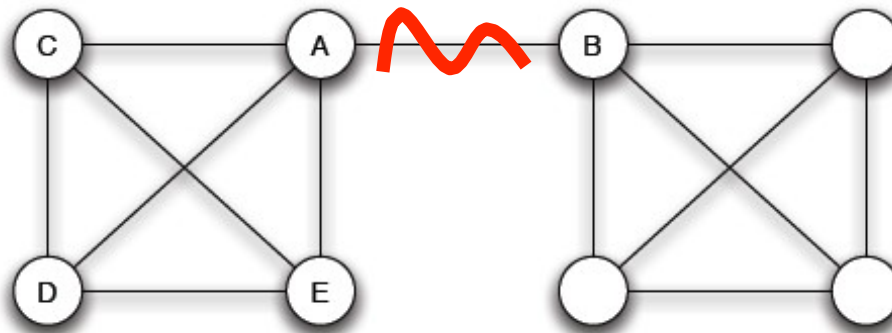
Motifs

- Motifs are sub-graphs (e.g. cycles, cliques, etc.,) within a nodes *ego graph.*
- Count more complex structures rather than triangles.
  - Transforms node feature extraction to a graph level task.

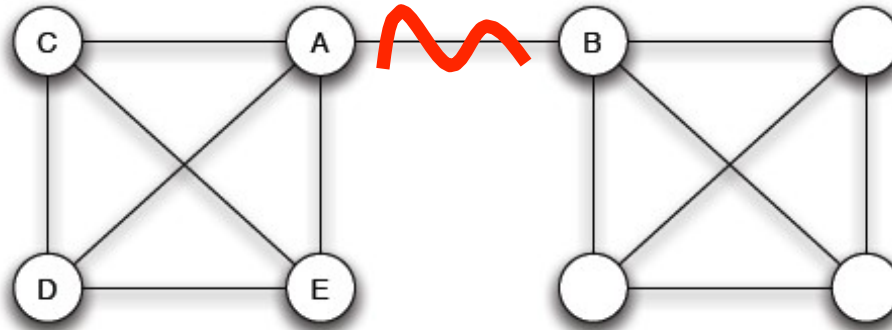https://mathinsight.org/
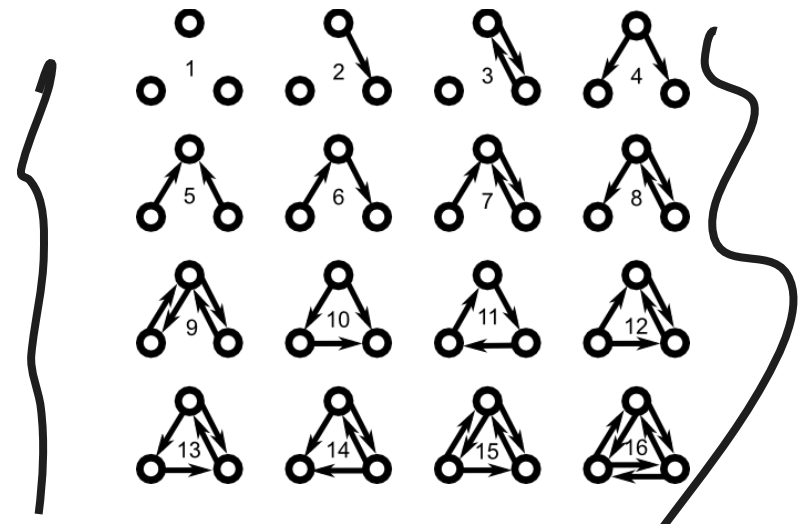
Bridges and Local Bridges

- The edge (A,B) is a bridge if **deleting** it put A and B into **two different connected components**.

Bridges and Local Bridges- Cnt.
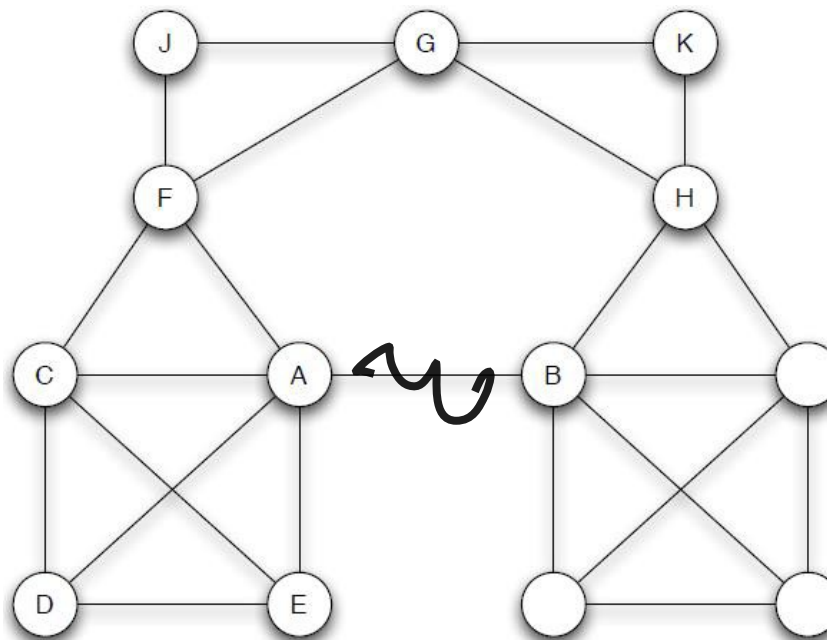
- Important points about Bridges:
  - A Bridge is the only route btw its endpoints!
  - Bridges provide access to parts of the network that are unreachable by other means!

- Aren't bridges rare in real-world networks?
  - Consider the availability of a giant component in real-world nets!
  - There could be others paths that connect two nodes!
    - **A-B**, **A**-F-G-H-**B**, etc.

Bridges and Local Bridges- Cnt.

- Local Bridges:
  - The edge (A,B) is a local bridge if A and B have **no friends in common**!
    - In other words, if deleting the edge would increase the distance btw A and B to a value strictly more than 2.
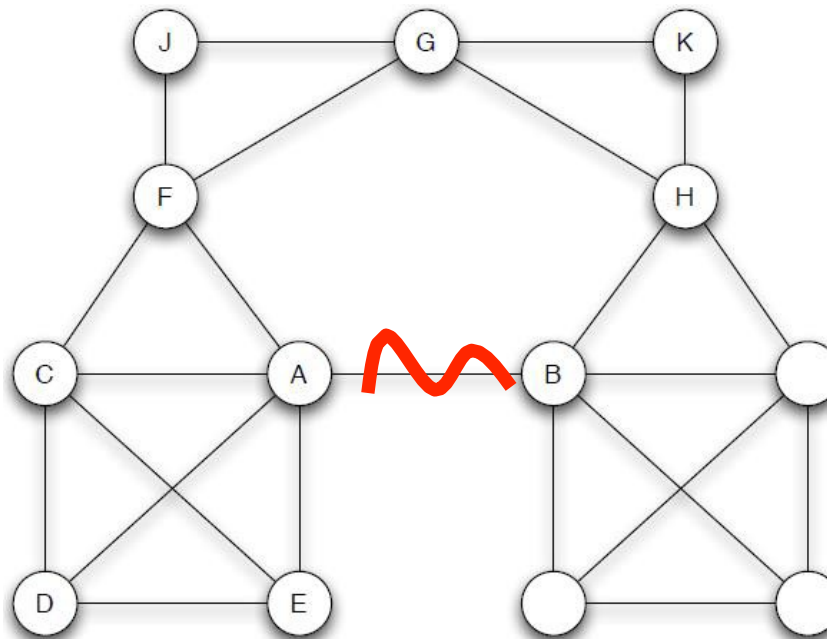
Bridges and Local Bridges- Cnt.

- Beside (A,B), is there any other local bridge in this net?
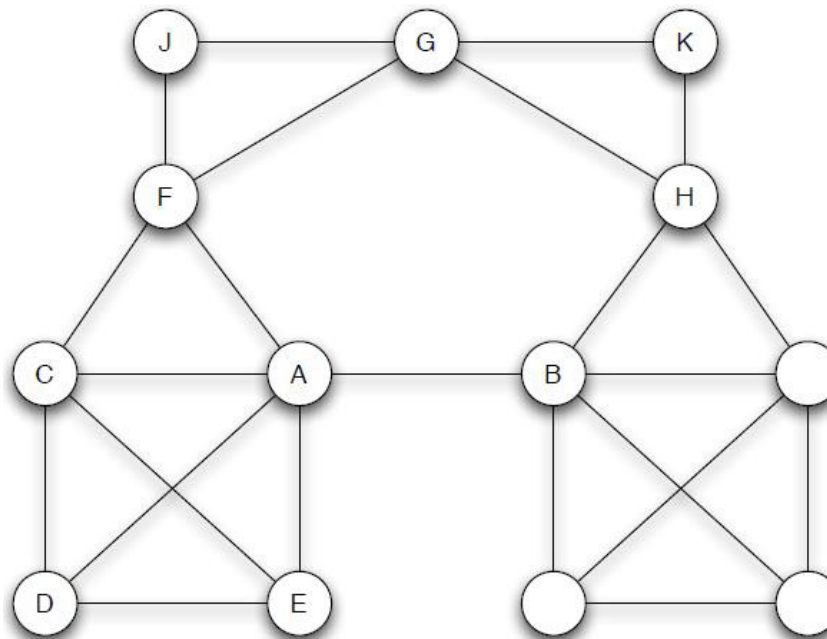
Bridges and Local Bridges- Cnt.

- Beside (A,B), is there any other local bridge in this net?
  - Local bridges never form the side of any triangle in the net!
  - Local Bridge→ **edge not in a triangle**!

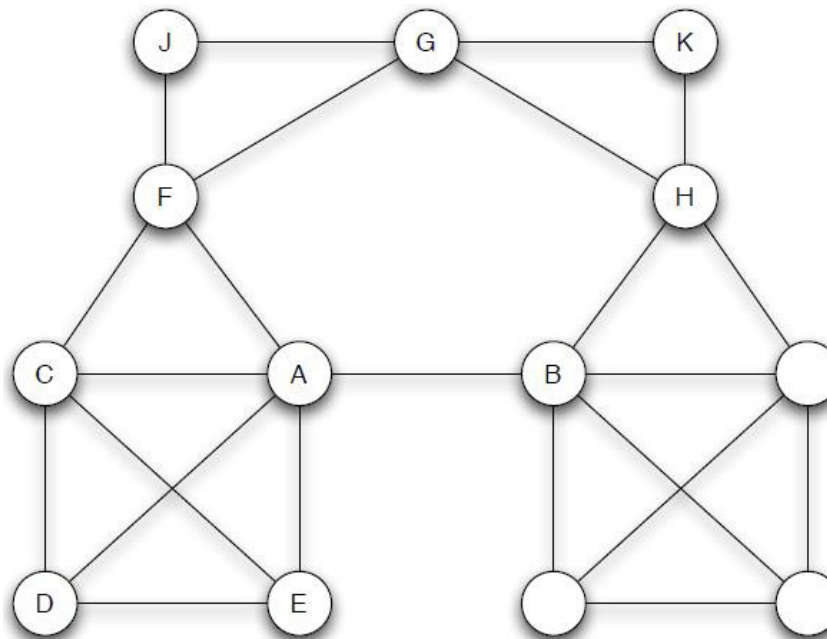# Bridges and Local Bridges- Cnt.

Length of the shortest path btw two nodes
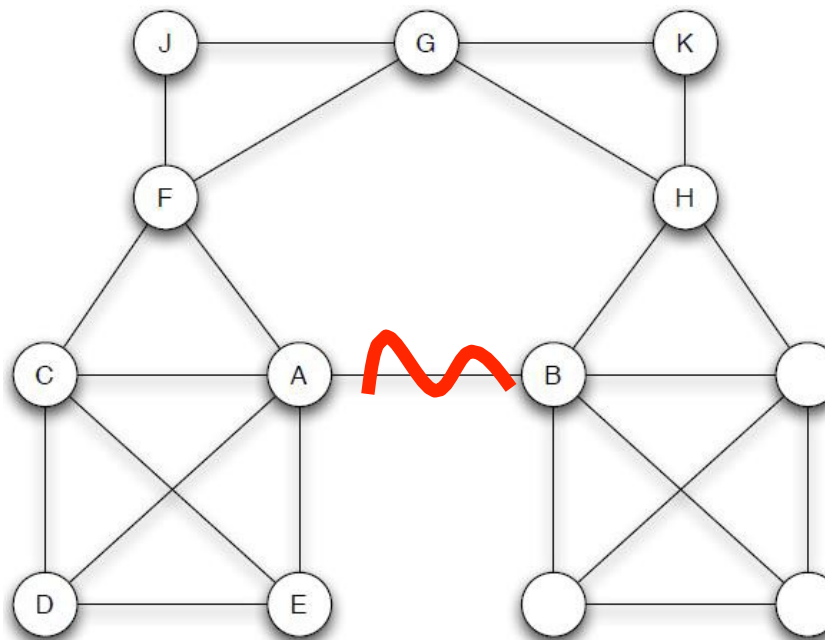
- ## Span of a Local Bridge:
  - Span of a local bridge is the distance btw its endpoints if the edge were deleted.
  - Span(A-B)=4

Local bridges with large span play roughly the same role as bridges:

Provide their endpoints with access to parts of the net that they would otherwise be far away from.

# Bridges and Local Bridges- Cnt.

- Granovetter's Experiment
  - 1960s
  - Q: "How do people find info about new jobs?"
    - People find the info through personal contacts
    - But: contacts were often *acquaintances* (weak ties) rather than *close friends* (strong ties)!
  - This is surprising as one would expect close friends to help you more than acquaintances!
  - Why are acquaintances most helpful?

Bridges and Local Bridges- Cnt.

- Why are acquaintances more important (in Granovetter's Experiment)?
  - A, C, D, and E will all tend to be exposed to similar sources of info, while A's link to B offers access to info A otherwise wouldn't necessarily hear about.
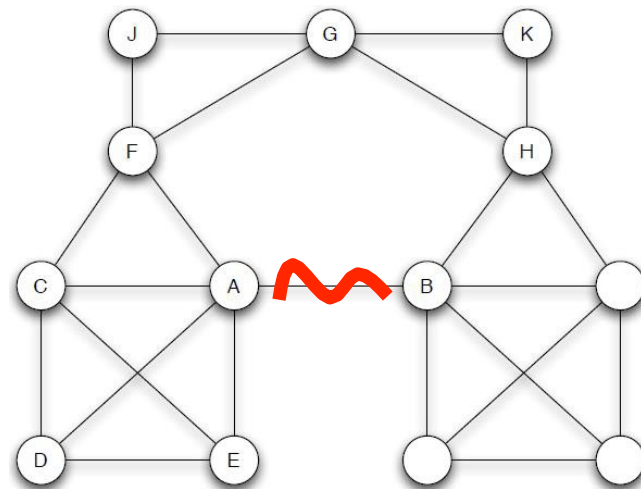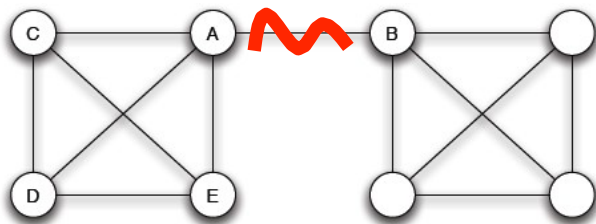
Tie Strength

- Links in networks have strength:
  - Friendship nets (close friends vs. acquaintances)
  - Telco nets (amount of time talking on the phone)
- We characterize edges / links as either:
  - Strong (corresponding to friends), or
  - Weak (corresponding to acquaintances)

Tie Strength- FB

Figure 3.8: Four different views of a Facebook user's network neighborhood

- **All Friends:** the largest representation of a person's network is the set of all people they have verified as friends.

- **Reciprocal Communication:** as a measure of a sort of core network, we counted the number of people with whom a person had had reciprocal communications, or an active exchange of information between two parties.

- **One-way Communication:** the total set of people with whom a person has communicated.

- **Maintained Relationships:** to measure engagement, we took the set of people for whom a user had clicked on a News Feed story or visited their profile more than twice.
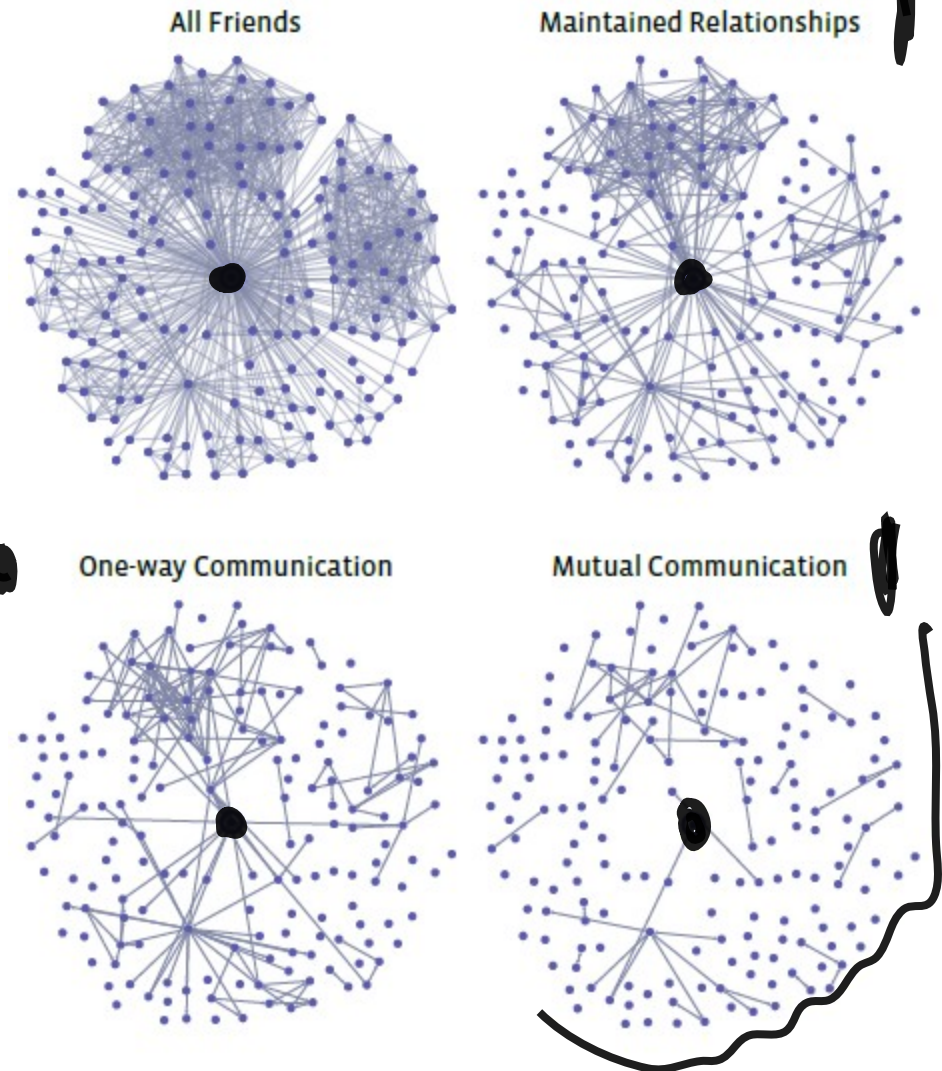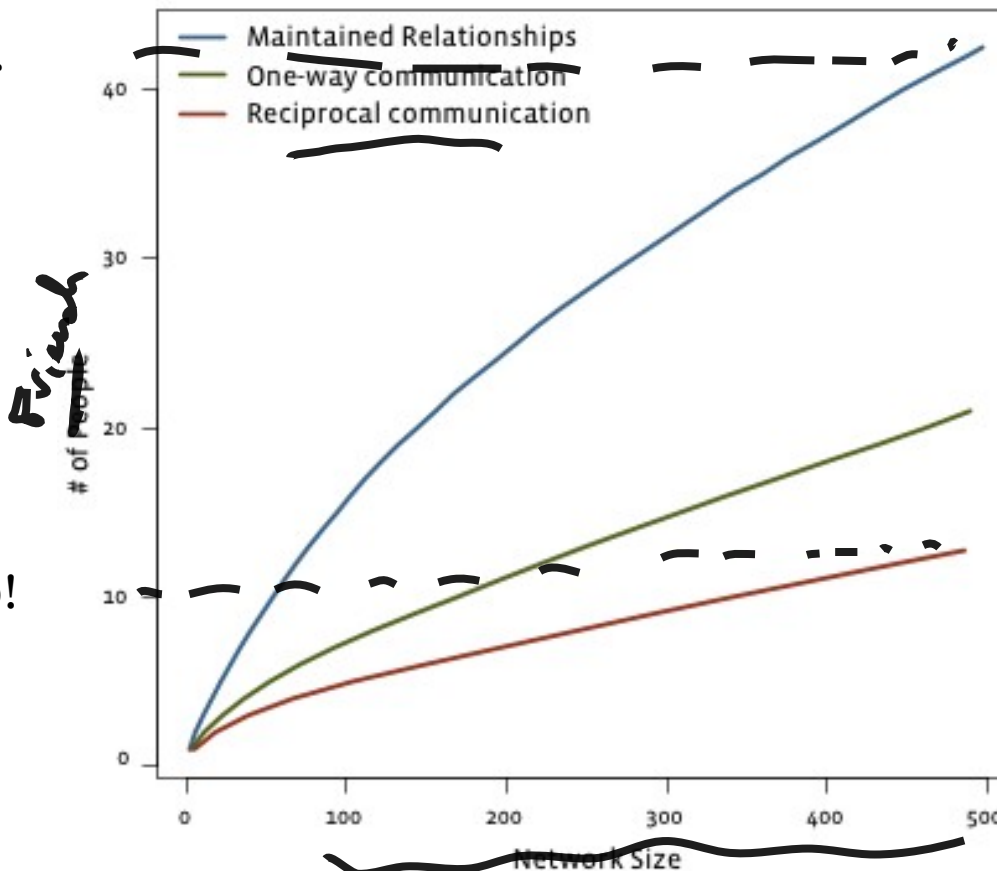
Tie Strength- FB- Cnt.

Number of users with whom a user communicate is generally under 20!

Number of users they follow even passively (e.g. by reading about them) is under 50!

Figure 3.9: The number of links corresponding to maintained relationships, one-way communication, and reciprocal communication as a function of the total neighborhood size for users on Facebook. (Image from [286].)

Number of friends on FB

Tie Strength- Twitter
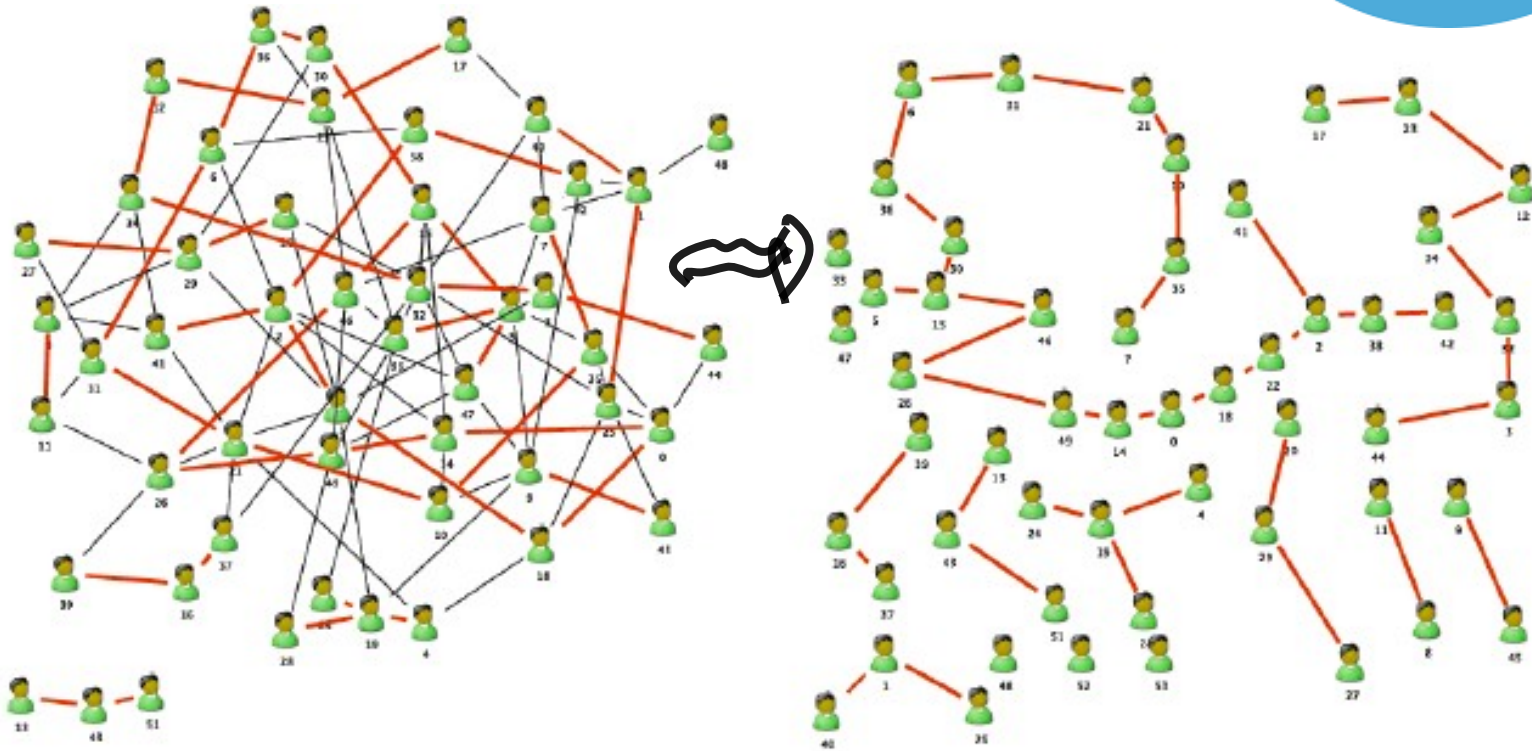
(a) All links are declared followees and the red links are actual friends.

(b) After removing the black links and reorganizing the network look simpler than before. This is the hidden network that matters the most.

Tie Strength- Twitter- Cnt.
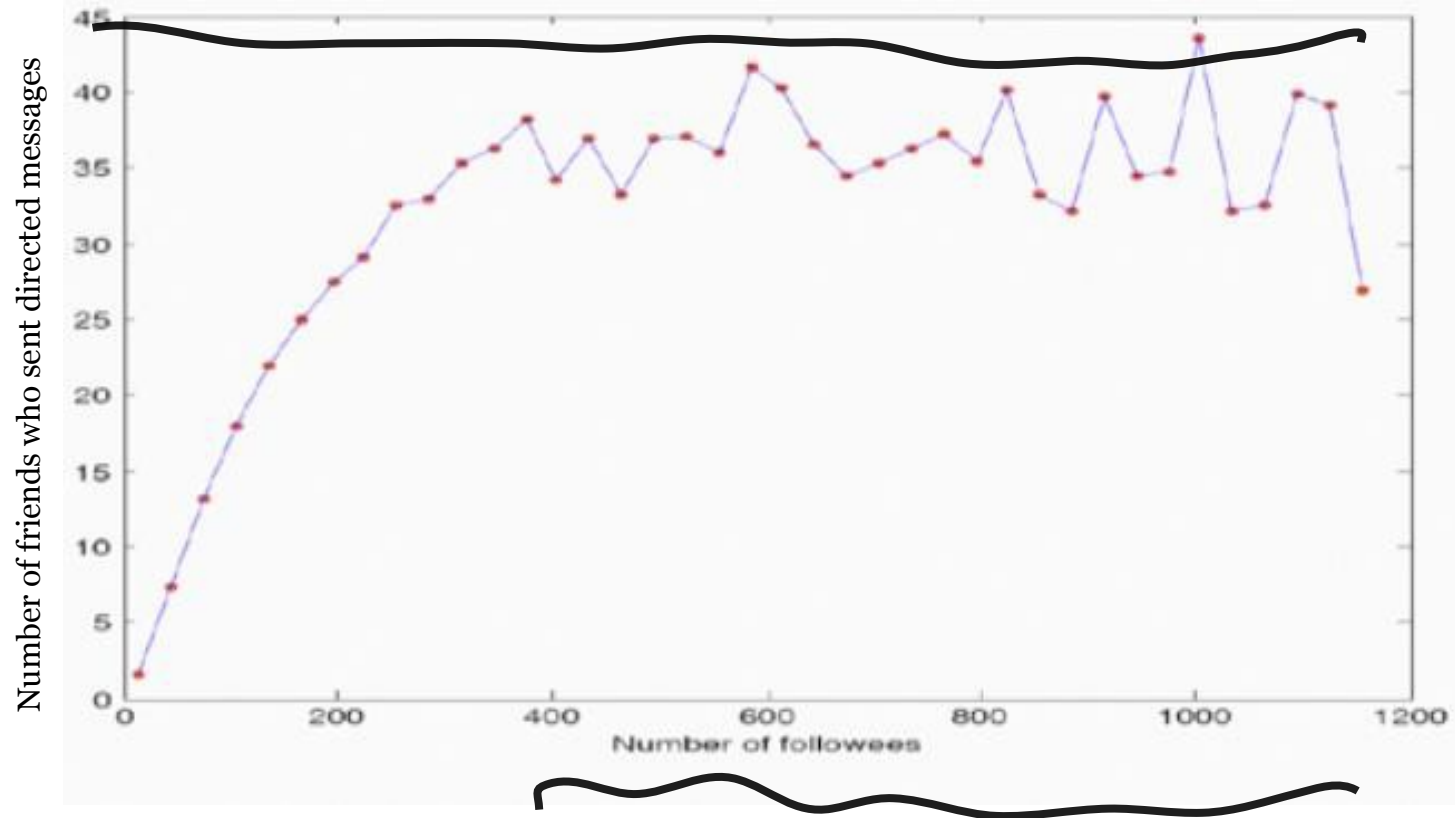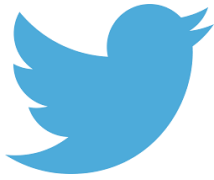
Even for users who maintain very large numbers of weak ties on-line, the number of strong ties remains relatively modest, in this case stabilizing at a value below 50 even for users with over 1000 followees.

Tie Strength- Cnt.

- The relative scarcity of strong ties in environments like Facebook and Twitter

Tie Strength- Cnt.

- The relative scarcity of strong ties in environments like Facebook and Twitter:
  - Each strong tie requires continuous investment of time and effort to be maintained
  - Even people who devote a lot of energy to building strong ties will eventually reach a limit, imposed simply by the hours available in a day, on the number of ties that they can maintain in this way.
  - This is while the formation of weak ties is governed by much milder constraints and such ties don't need to be maintained continuously!

The Strength of Weak Ties

- The dual role of weak ties - as weak connections but also valuable links - is the surprising strength of weak ties.

  □ Weak ties connect us to new sources of information, and their conceptual "span" in the social network (the local bridge property) is directly related to their weakness as social ties.

Summary

- Triadic Closure
  - friends who have common friends.
- Clustering Coefficient
  - fraction of triangles around an ego graph
- Motifs
  - complex structures around ego graphs
- Bridges
  - if removed their endpoints will be in different connected components.
- Local bridges
  - edges not in triangles!

Summary

- Two types of edges
  - Strong and weak ties
- Local bridges are weak ties
  - Local bridge adjacent to nodes involved in strong triadic closure (two strong ties imply a third strong/weak tie) must be a weak tie.

Tie Strength in Real-World Nets

- Granovetter's theory was untested on real-world large-scale networks!
- They are available now!

Tie Strength in Real-World Nets- Cnt.

- Onnela et al., (2007) studied who-talks-to-whom net:
  - A node is a user
  - An edge forms btw two users who made phone calls to each other in both directions
  - 20% of the national population (18-week observation period)
  - Mainly used for personal communication

- First Observation: a giant component covering 84% nodes!

Tie Strength in Real-World Nets- Cnt.

All nodes with distance less than six from the selected user (circled)

Real tie strengths: the aggregate call duration in minutes (see color bar).

A

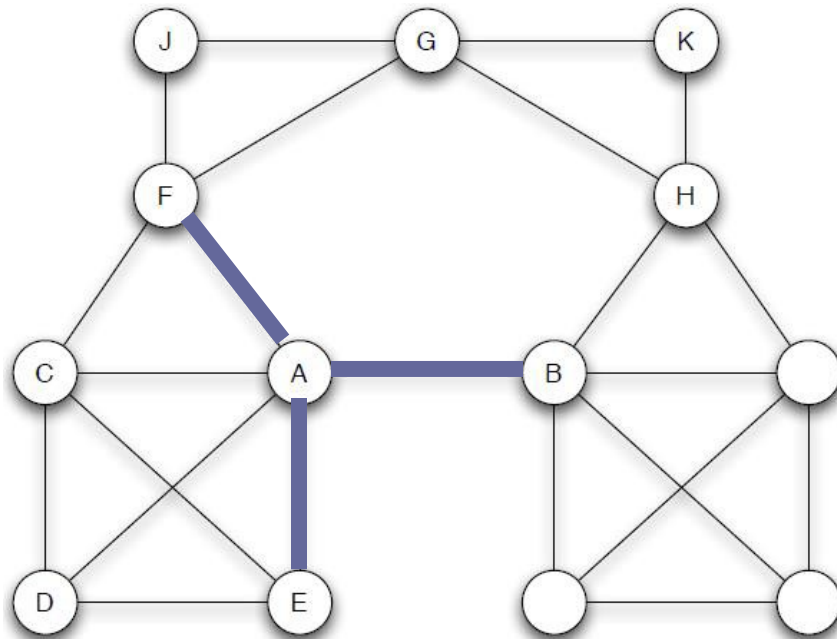100
10
1
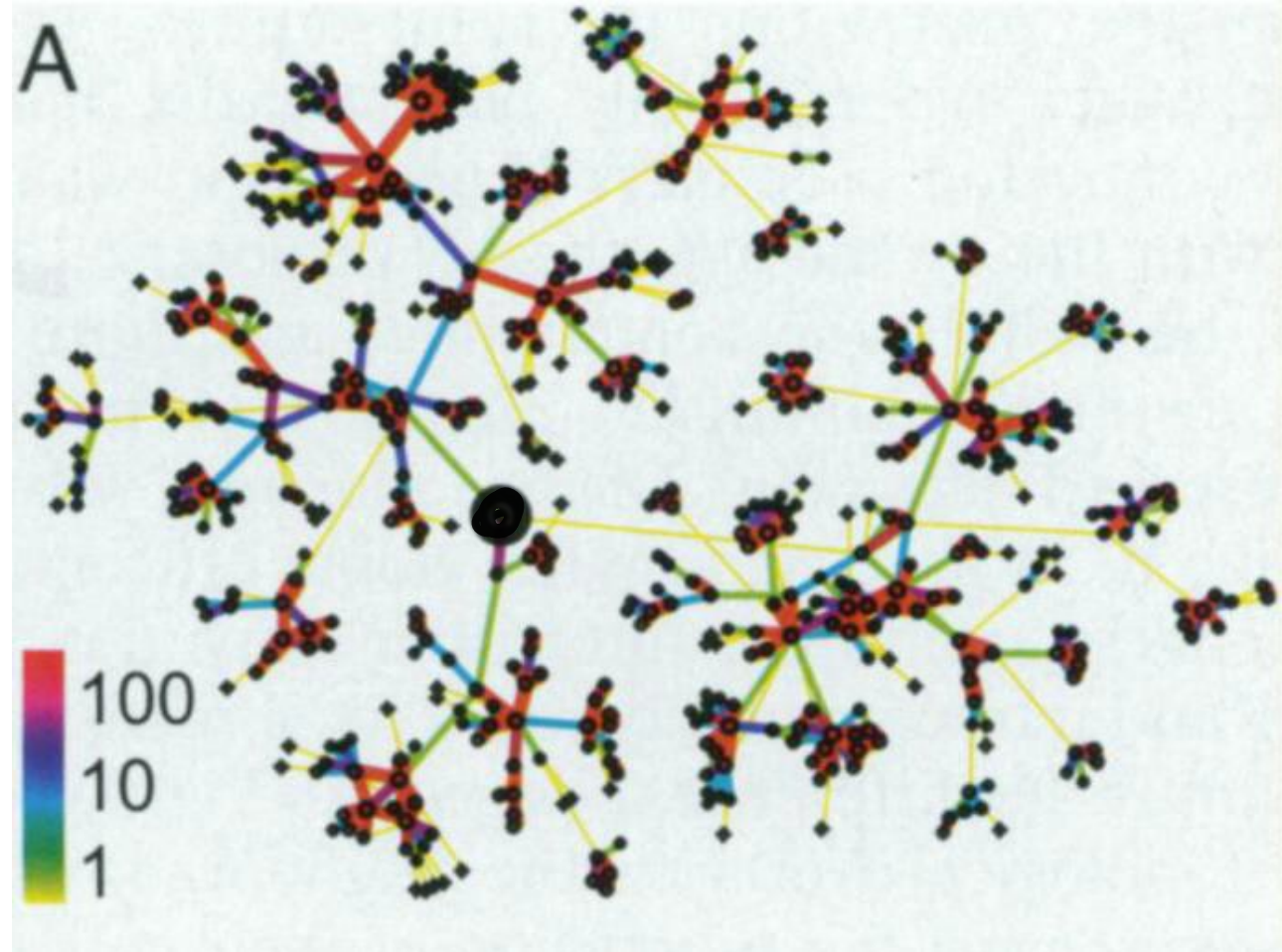
Tie Strength in Real-World Nets- Cnt.

- Relaxing the definitions (get numerical quantity):
  - Strength
    - Determined by the total number of minutes spent on phone calls between two nodes.
  - Local Bridges
    - Define *neighborhood overlap* for each edge!

- Neighborhood overlap of an edge connecting nodes A and B:

$$\frac{\text{number of nodes who are neighbors of } both \ A \text{ and } B}{\text{number of nodes who are neighbors of } at \ least \ one \text{ of } A \text{ or } B}$$

Don't count A and B here!

```
Nodes
-------  2
A-E     ---
A-G      4
A-B
```

- Neighborhood overlap of an edge connecting nodes A and B:

$$\frac{\text{number of nodes who are neighbors of } both \ A \text{ and } B}{\text{number of nodes who are neighbors of } at \ least \ one \ of \ A \text{ or } B},$$
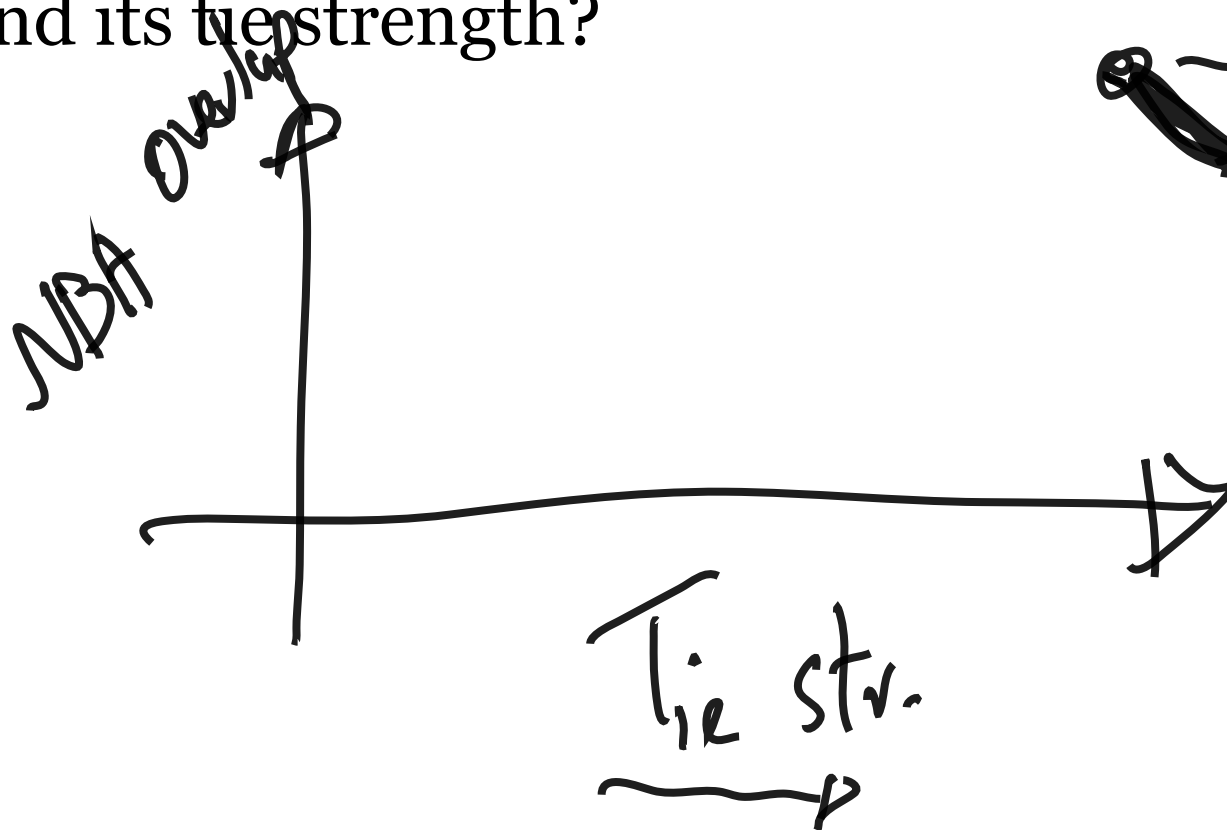

Don't count A and B here!

```
Nodes      Neighborhood overlap
-------    --------------------
A-E        2/4
A-G        1/8
A-B        0/8 (Overlap = 0 for
                local bridges)
```

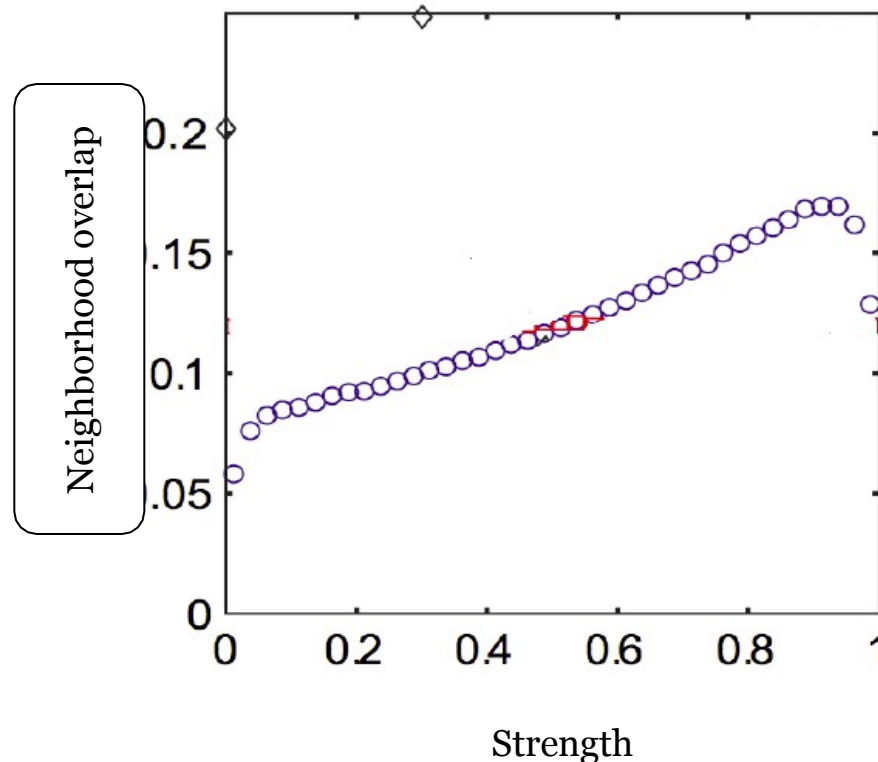Edges with very small neighborhood overlap can be considered as "almost" local bridges

Question 1.

- Relation between neighborhood overlap of an edge and its tie strength?

- Relation between neighborhood overlap of an edge and its tie strength?
  - Neighborhood overlap should grow as tie strength grows.

Question 2.

- How do weak ties serve to link different communities?

Question 2.- Cnt.

- How do weak ties serve to link different communities?

- Indirect Analysis:
  - **Delete edges** from the network one at a time, starting with the **strongest ties first**!
    - The giant component shrank steadily (its size decreases gradually).

Question 2.- Cnt.

- How do weak ties serve to link different communities?

- Indirect Analysis:
  - **Delete edges** from the network one at a time, starting with the **weakest ties first**!
    - The giant component shrank rapidly (its size decreases rapidly).

Question 2.- Cnt.

Size of the largest component
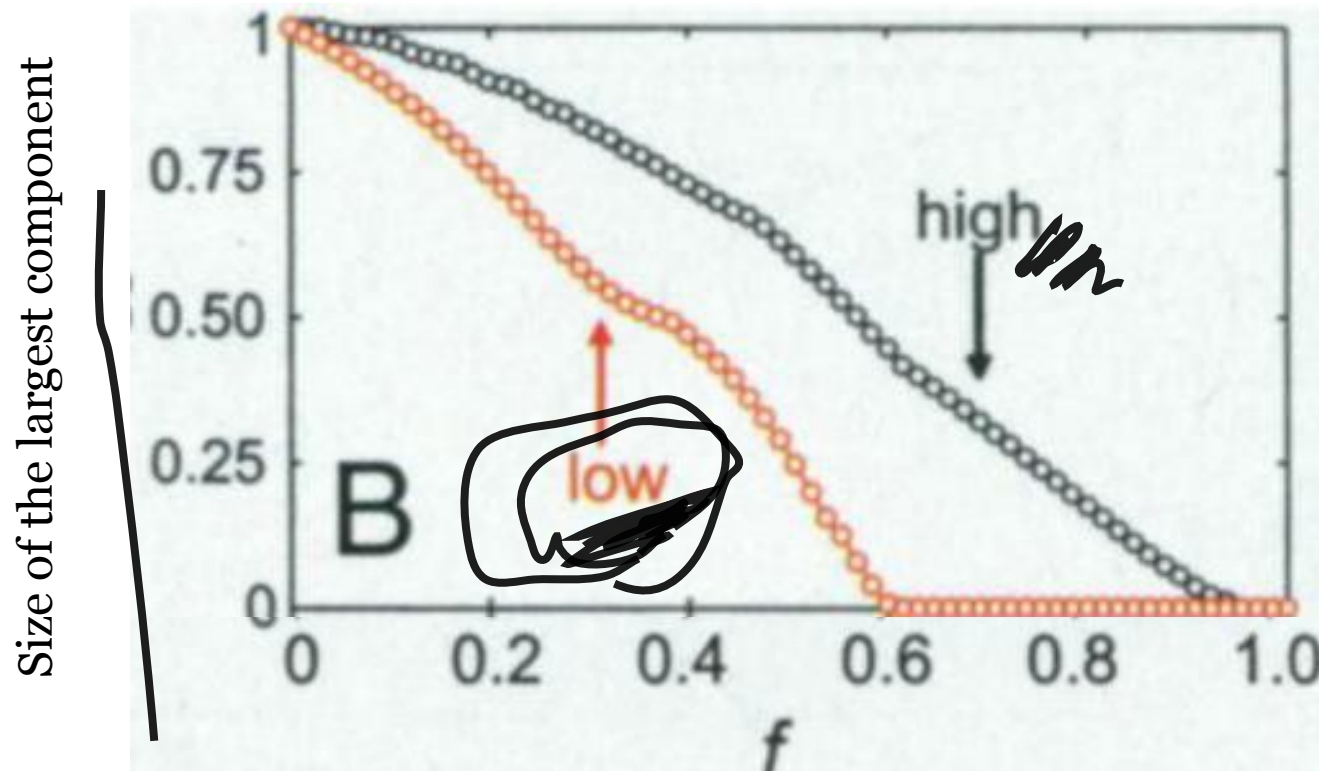
The fraction of removed links

The removal of high overlap links leads to the network's gradual shrinkage.

The removal of the low overlap links leads to a breakdown of the network.

Question 2.- Cnt.

- Results are consistent with the expectation that
  - weak ties provide the more crucial connective structure for holding together disparate communities!
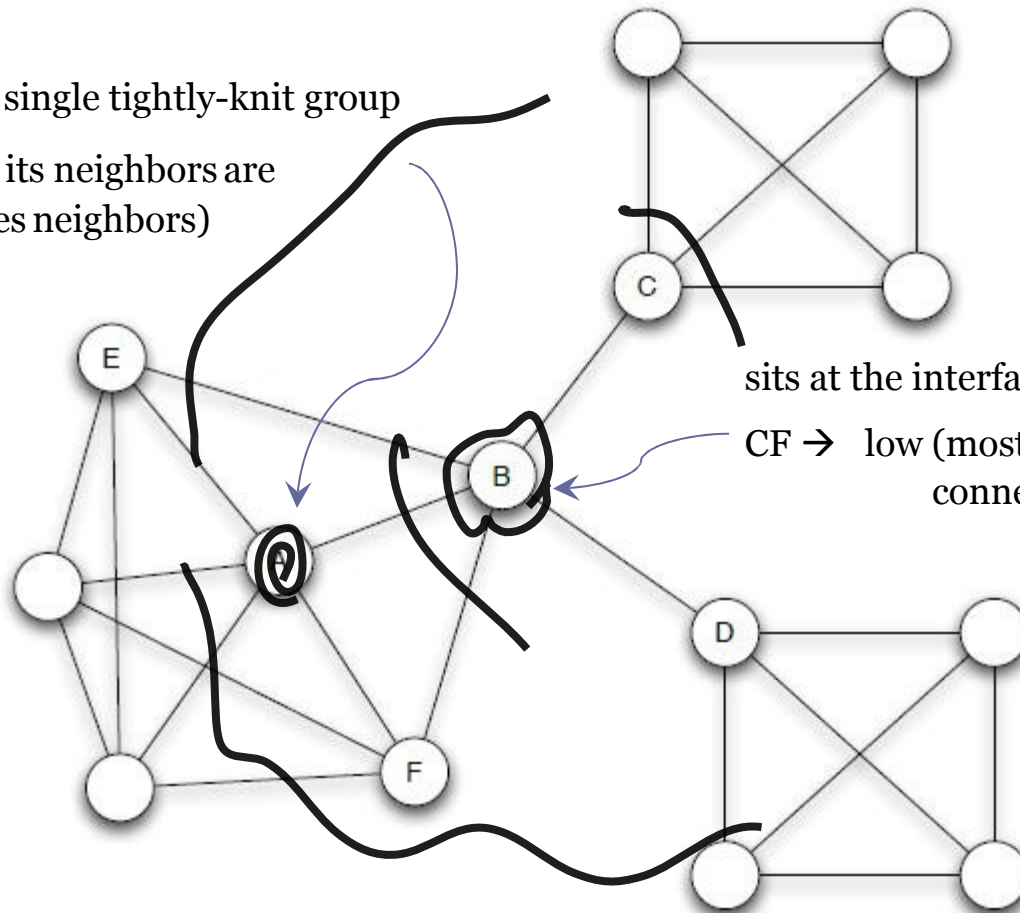
Structural Holes

- Networks: tightly-knit groups connected by weak ties

  - some nodes are positioned at the interface between multiple groups,
  - others are positioned in the middle of a single group.

Weak ties

Strong ties
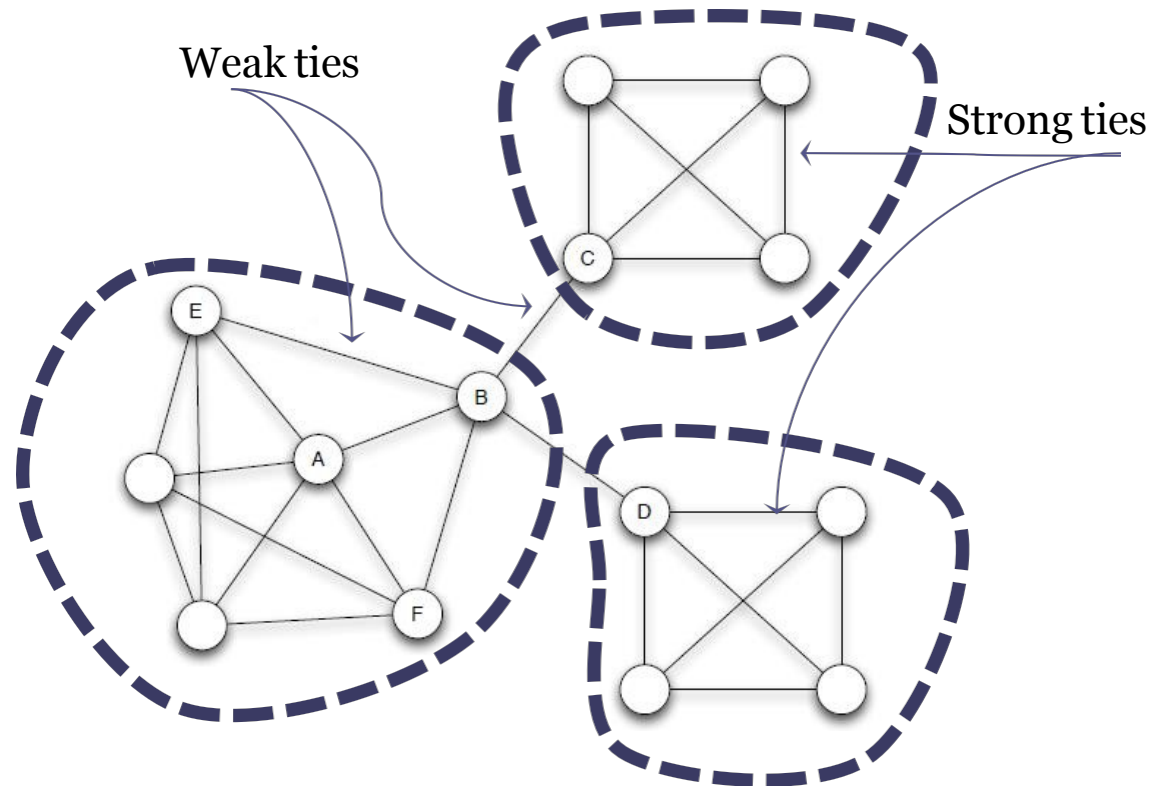
Structural Holes- Cnt.

sits at the center of a single tightly-knit group

CF → high (most of its neighbors are themselves neighbors)

sits at the interface between several groups

CF → low (most of its neighbors are not connected to each other)

Empirical studies shows that an individual's success within a company is highly correlated to his/her access to local bridges!

Structural Holes- Cnt.

Structural hole: the "empty space" in the net btw 2 sets of nodes that don't interact closely!

A node with multiple local bridges spans a structural hole in the net.
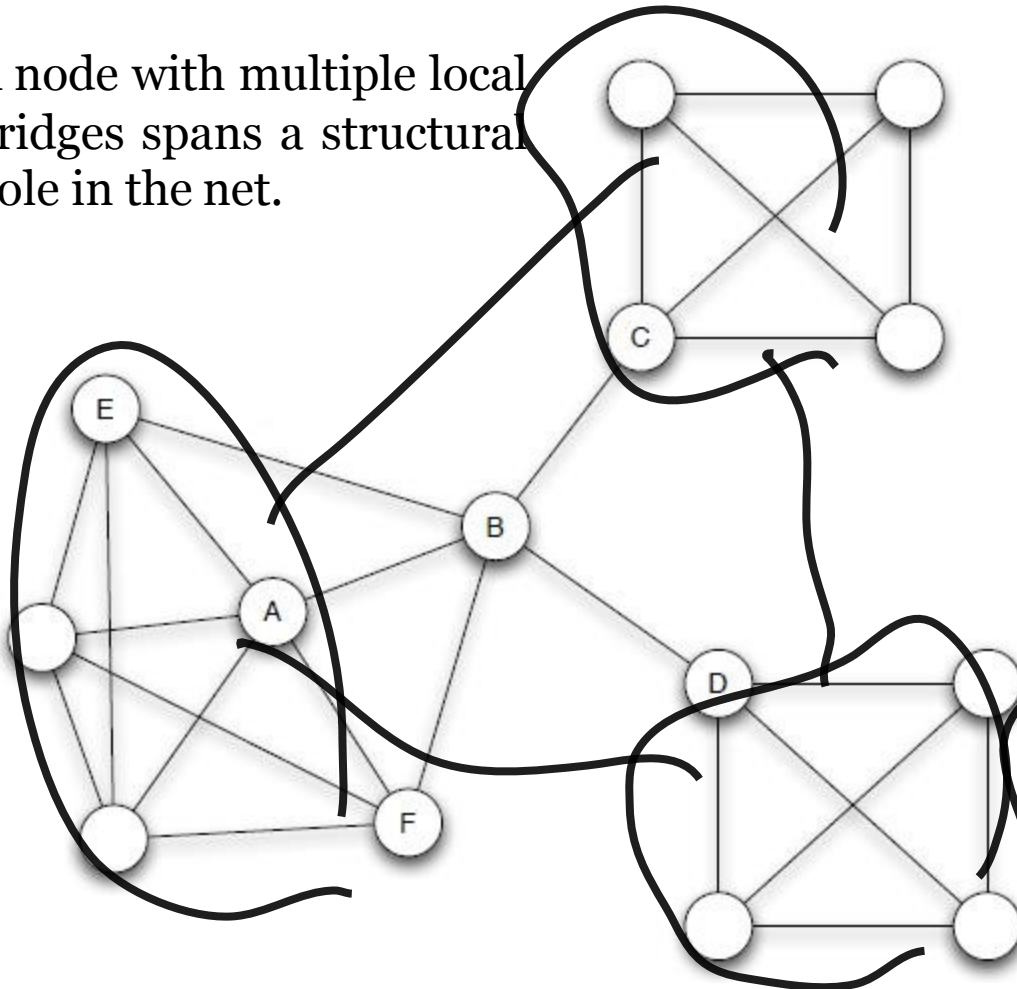
B has early access to info!

B is a gatekeeper and controls the ways in which groups learn about info. It has power!

B may try to prevent triangles from forming around the local bridges it is part of!

How long these local bridges last before triadic closure produces short-cuts around them?

Reading

- Ch.03 Strong and Weak Ties [NCM]
- Structure and tie strengths in mobile communication networks. Onnela, et al. National Academy of Sciences. 2007.