Robust Quadrupedal Locomotion through Asymptotic Stabilization of H-LIP on Dynamic Rigid Surfaces with General Vertical Motion

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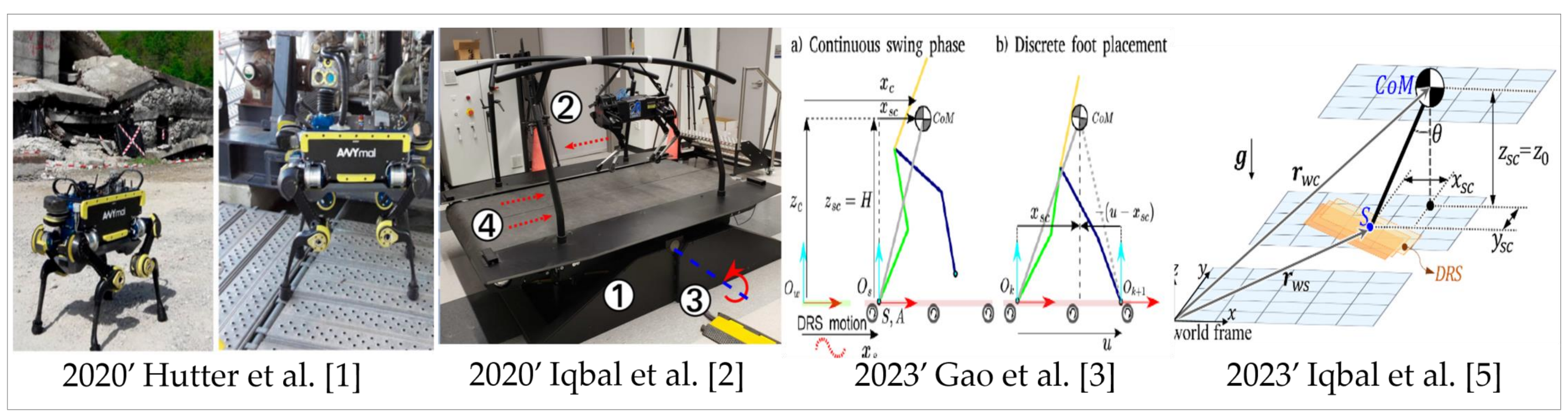
ABSTRACT

- We present our results on achieving robust quadrupedal locomotion by stabilizing a hybrid-linear inverted pendulum (H-LIP) stepping on surfaces with vertical motion.
- Our framework analyzes H-LIP stability, derives feasible and stable footsteps using quadratic programming (QP) based methods, and generates real-time trajectories for locomotion on surfaces with uncertain vertical motion.
- We design an optimization-based torque control law and validate our framework through hardware experiments.
- The validation results on hardware demonstrate robust locomotion on surfaces with uncertain and unknown motion, including external disturbances and uneven terrain.

BACKGROUND

- The quadrupedal robots show promise for real-world applications [1].
- However, achieving provably stable and robust locomotion on surfaces with dynamic motions poses challenges due to hybrid and explicitly time-varying dynamics [2].
- Our previous research has addressed legged locomotion on surfaces with periodic and accurately known motion [3]-[5].



- The proposed work addresses locomotion on a rigid surface with general (periodic or aperiodic), vertical, uncertain motions.

METHOD

- The overall framework is illustrated in Fig. 1.

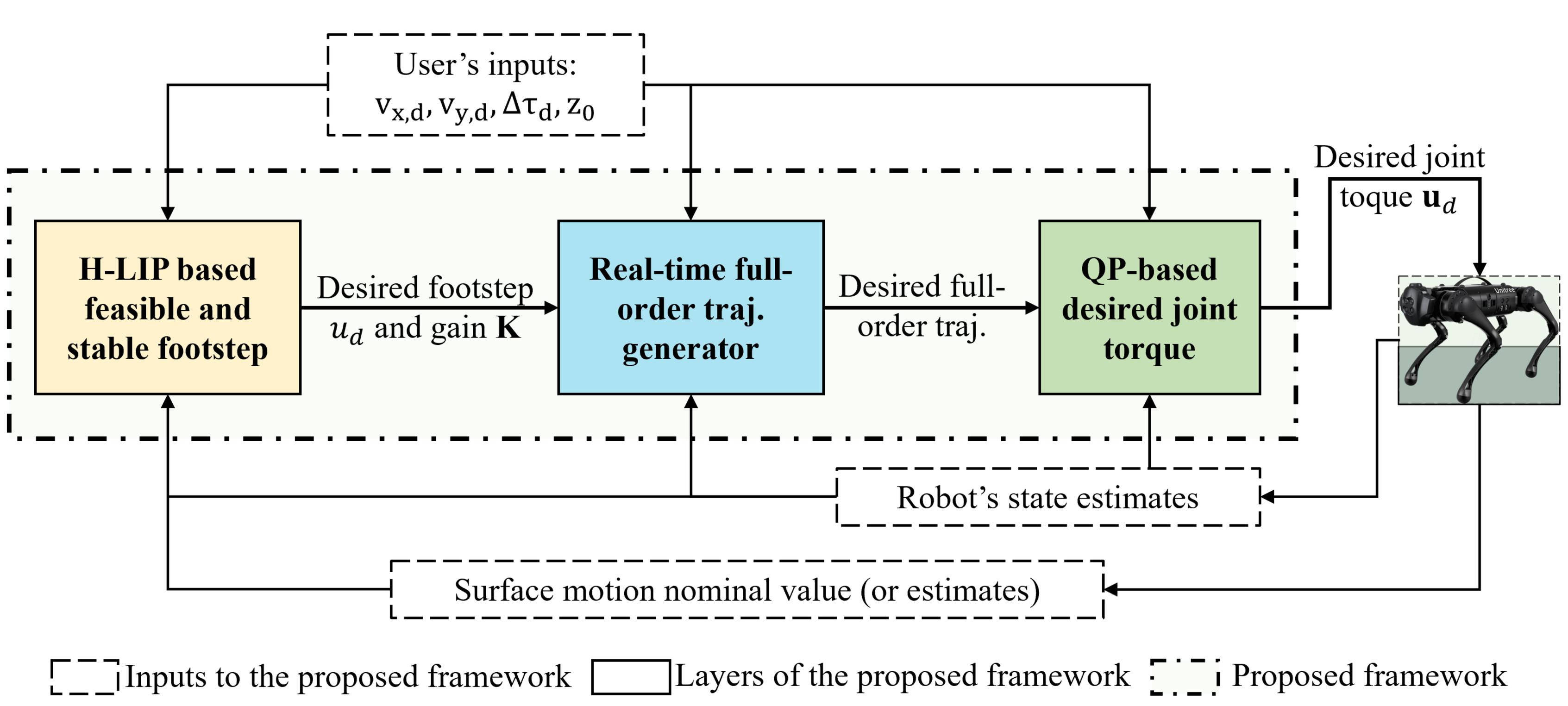


Fig. 1: Illustration of the overall proposed framework.

- **H-LIP Model and Stability Analysis.** We introduce a time-varying H-LIP model (illustrated in Fig. 2) to describe the essential robot dynamics associated with legged locomotion on a rigid surface with a vertical motion. We also derive the sufficient stability condition for the H-LIP model under a discrete-time foot-placement control law. Details of these analytical results are provided in [6].

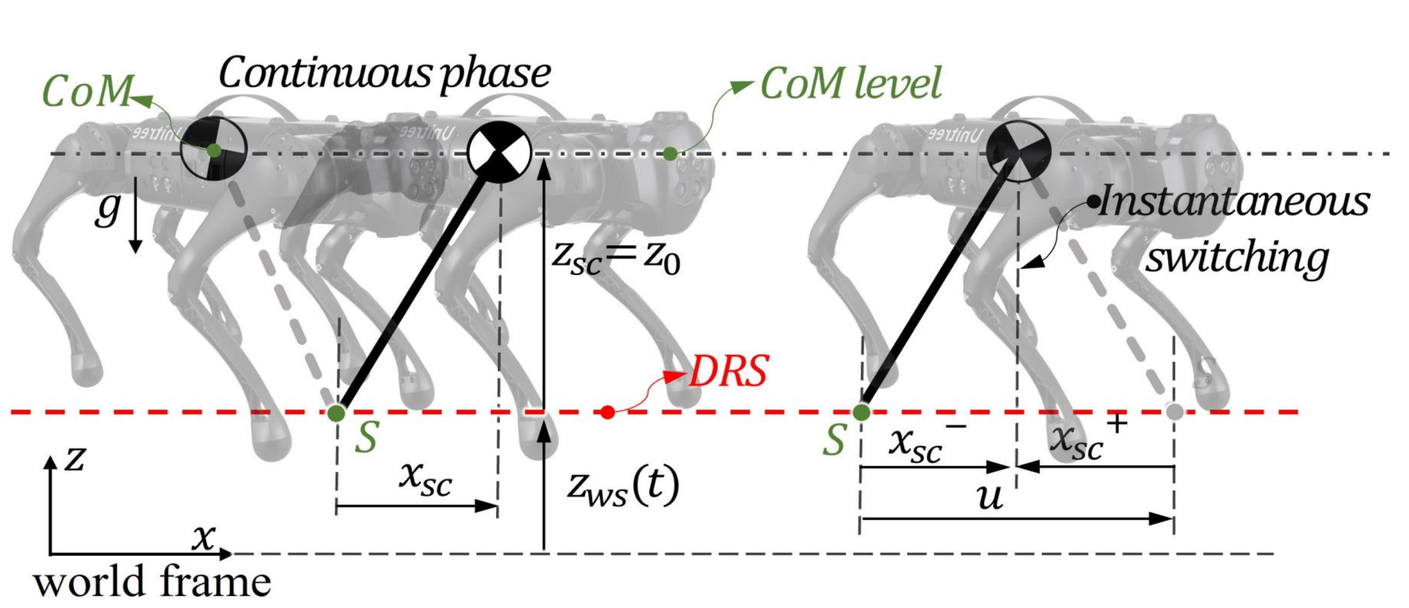


Fig. 2: Illustration of the H-LIP model on a dynamically moving surface (DRS).

- **QP-based Optimization of Foot-Stepping Control Gains.** To enforce the proposed stability condition and necessary kinematic constraints (e.g., maximum step length), we formulate the following QP to optimize in real-time the gains of the foot-placement controller:

$$\min_{\mathbf{K}} \frac{1}{2}\mathbf{K}\mathbf{S}\mathbf{K}^T + \mathbf{K}\mathbf{c} \quad \text{s.t.} \quad \mathbf{E}\mathbf{K}^T < \mathbf{b},$$

where $\mathbf{K} \in \mathbb{R}^{1\times 2}$ represents the control gain, $\mathbf{S} \in \mathbb{R}^{2\times 2}$ denotes the Hessian matrix of the cost function, $\mathbf{c} \in$ is the gradient vector of the cost function, $\mathbf{E} \in \mathbb{R}^{6\times 2}$ is a matrix used to form the linear constraints representing the kinematic feasibility and stability condition, and $\mathbf{b} \in \mathbb{R}^{6\times 1}$ is a vector of constraint bounds.

EXPERIMENTAL RESULTS

- Hardware experiments on a Unitree Go1 robot trotting on a rocking treadmill, which possesses a general, vertical, unknown motion, confirm the locomotion robustness under the proposed control approach, as demonstrated through the snapshots of the experiment videos in Fig. 3. The experiment video is available at https://youtu.be/S7ysMQp0Vyo.

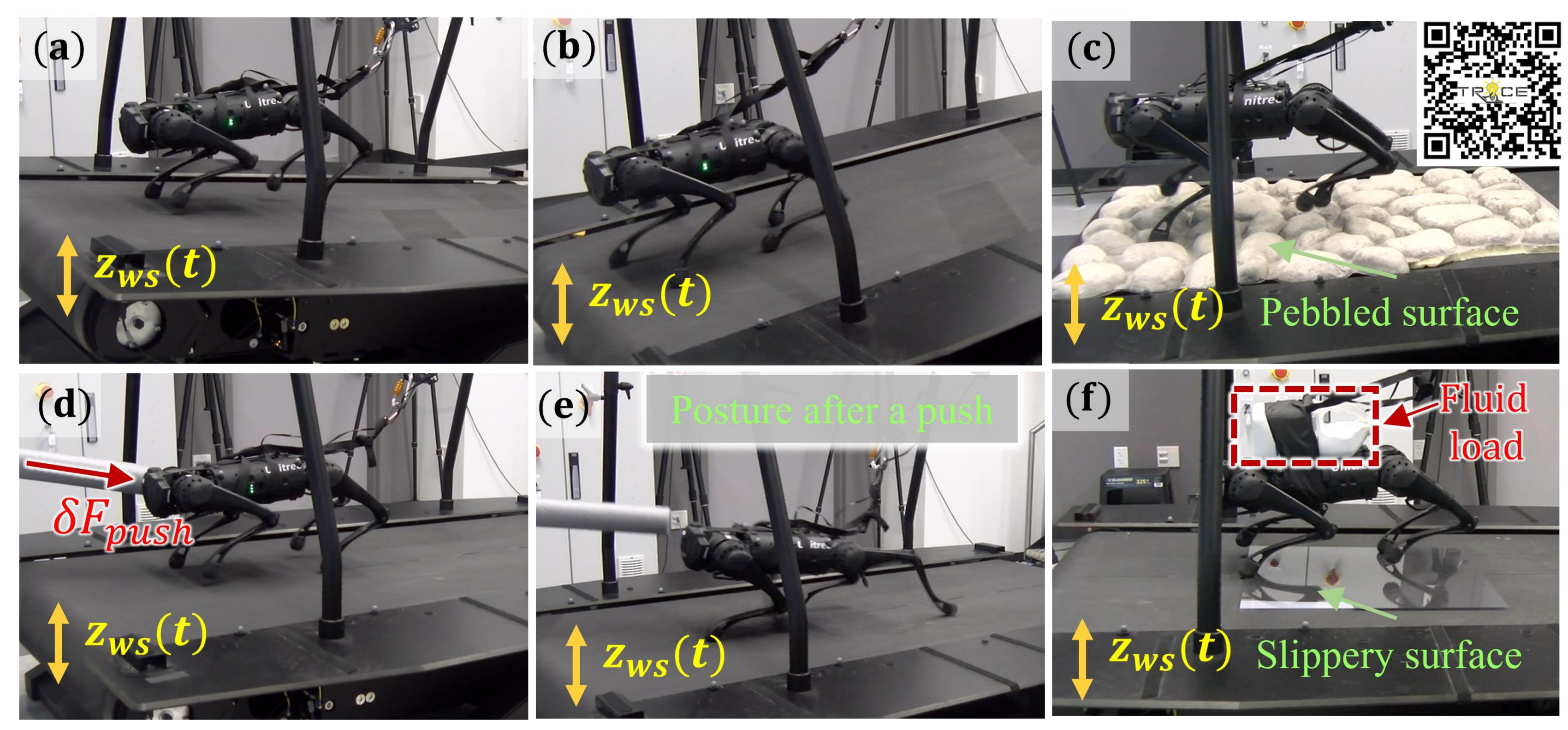


Fig 3: (a) and (b) show experiments under nominal surface motions. (c) to (f) show snapshots of the various robustness experiments on a rocking treadmill with general and uncertain dynamic motions.

CONCLUSION

- This poster summarizes a reduced-order model based quadruped control method that exploits the provable stabilization of a time-varying H-LIP model to achieve robust underactuated trotting on a vertically oscillating surface.
- Various hardware experiments confirm the robustness of the proposed method under different types of uncertainties, including uncertain surface motion, surface unevenness, intermittent pushes, and external loads.
- Our future work will focus on integrating the proposed model-based approach and reinforcement learning to enable rapid and provable adaptation to general classes of real-world uncertainties beyond those considered in this study.

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