Incentive Allocation in Vertical Federated Learning Based on Bankruptcy Problem

Afsana Khan, Marijn ten Thij, Frank Thuijsman and Anna Wilbik

Department of Advanced Computing Sciences

Maastricht University

Maastricht, The Netherlands

Abstract. Vertical federated learning (VFL) is a promising approach for collaboratively training machine learning models using private data partitioned vertically across different parties. Ideally in a VFL setting, the active party (party possessing features of samples with labels) benefits by improving its machine learning model through collaboration with some passive parties (parties possessing additional features of the same samples without labels) in a privacy preserving manner. However, motivating passive parties to participate in VFL can be challenging. In this paper, we focus on the problem of allocating incentives to the passive parties by the active party based on their contributions to the VFL process. We formulate this problem as a variant of the Nucleolus game theory concept, known as the Bankruptcy Problem, and solve it using the Talmud’s division rule. We evaluate our proposed method on synthetic and real-world datasets and show that it ensures fairness and stability in incentive allocation among passive parties who contribute their data to the federated model. Additionally, we compare our method to the existing solution of calculating Shapley values and show that our approach provides a more efficient solution with fewer computations.

# Introduction

Organizations in recent times want to collaborate using data because data has become an increasingly valuable asset for businesses. By collaborating and sharing data, organizations can gain new insights, make better decisions, and identify new opportunities. Collaboration enables organizations to combine data from different sources and systems, and to analyze it in new and innovative ways. Hence, they often opt for aggregating all their data to a central server as a way to facilitate collaboration and gain the benefits of centralized data management. However, traditional centralized machine learning approaches can be problematic due to privacy concerns, scalability issues, and single point of failure risks. To address these issues, distributed machine learning approaches [17] have been developed, which involve training models on decentralized data sources and aggregating the results. This approach allows for better privacy, scalability, and fault tolerance, while still achieving similar or better performance compared to centralized methods. Federated learning (FL) [3], a form of distributed learning, solves the problem of centralized learning in organizations by addressing the concern of privacy. One of the main advantages of federated learning is that it allows organizations to train machine learning models on decentralized data without requiring data to be centralized or shared. This protects user privacy and data confidentiality by keeping data on user devices, rather than sending it to a central server for processing. In addition, federated learning can improve model performance by training models on more diverse data, which is particularly useful in organizations where data may be distributed across different locations or business units. Federated learning can be classified into three main types based on how the data is partitioned: horizontal federated learning (HFL), vertical federated learning (VFL), and hybrid federated learning. The choice among the types of FL depends on the nature of the data and the specific requirements of the application.

[3](http://arxiv.org/abs/2307.03515v1)

[arXiv:2307.03515v1 [cs.LG] 7 Jul 202](http://arxiv.org/abs/2307.03515v1)

* Horizontal FL: The data is partitioned based on samples, where each participating party holds a subset of the samples. The objective is to train a global model that can perform well on all samples. A practical example of horizontal federated learning can be in healthcare where multiple hospitals keep their patient data locally and collaboratively train a machine learning model to predict medical conditions. They share only aggregated statistics or model updates with a central server, ensuring privacy preservation while leveraging diverse patient data. The trained model can provide valuable insights for patient diagnosis and treatment across multiple institutions.
* Vertical FL: The data is partitioned based on features, where different parties hold different sets of features for the same samples. The objective is to train a model that can perform well on a joint set of features from all participating devices. For instance, a tax office has information on fraudulent tax returns filed by individuals, but they lack information on financial transactions that could help them identify potential fraudulent activity. To improve their fraud detection predictions, the tax office decides to collaborate with several banks to access transaction data that could help them better identify potential fraudulent activity.
* Hybrid FL: Hybrid FL is a combination of horizontal and vertical federated learning, where both sample and feature partitioning are used. This approach is used when the data is complex and both types of partitioning are required to train an accurate model. An good example of hybrid FL could be a car rental company willing to better predict the demand for their cars at airports collaborates with airline companies to access the number of passengers on flights to those airports. This would require combining data of multiple airlines (horizontal partition) and the car rental company (vertical partition).

Motivating data owners to participate in federated learning is crucial for its success. Incentives help encourage data owners to contribute their data, which is essential for improving model performance through diverse and representative data sources. Shi et al. [16] proposed a framework in an HFL setting based on the GaleShapley value to allocate proper incentives to participants in the federation based on their contributions. After measuring contributions, a reputation-driven reward allocation policy was used to distribute the incentive budget among participants. Zhang et al. [23] proposed a reward system for horizontal federated learning where clients can bid their prices, and a selector chooses clients to participate in FL rounds based on contribution quality. Reputation and payment depend on the quality of contributions, evaluated by a ranking of cosine similarity between client gradients and the final FL model. However, incentive mechanisms in HFL and VFL vary due to differences in the roles of the data owners and goals of the models. Unlike an HFL setting, there are two types of data owners or parties involved in the federation in VFL.

* Passive/Host Party: Parties that own the data (features) but not the labels or ground truth that is used to train a machine learning model
* Active/Guest Party: Parties that possess data with labels or ground truth and wish to train a machine learning model using the data held by the host parties

Hence, it is clear that the active party in a VFL is the one who is going to benefit from additional data from the passive parties. Therefore, it is essential for the passive parties to have the motivation to contribute to the learning process with their data. This can be achieved if there is a fair reward allocation mechanism to distribute the profit gained from the federation among the passive parties based on their contribution to the model. In this paper, we propose a method of profit distribution among participants of VFL process based on bankruptcy problem [1]. The idea behind the bankruptcy problem is to allocate the bankrupt’s estate in a fair and efficient way, taking into account the private claims of the creditors. The problem arises because the estate may not be sufficient to satisfy all the claims of the creditors, and thus a mechanism needs to be designed that fairly allocates the estate among the creditors based on their claims. In context of VFL, the host parties collaborate with the active party such that the active party is able to maximize the performance of the federated model. The profit from the maximization is then distributed among the passive parties depending on how much they have contributed to the final model.

The rest of the paper is organized as follows: In Section 2, related works on incentive mechanisms in federated learning settings have been presented. Section 3 introduces the Bankruptcy problem in general while Section 4 describes our proposed method. The experimental setup is then provided in Section 5 followed by the results in Section 6. Section 7 discusses some issues in VFL that could be solved using the proposed method. Finally, conclusions and future work are discussed in section 8.

# Related Work

This section provides an overview of the existing incentive mechanisms in federated learning. It highlights the different approaches that have been taken to incentivize the participating parties to contribute their data to the federated learning process. These approaches include concepts based on game theory, contract theory and auction theory.

## Game Theory

Some game theory concepts have been experimented with when it came to designing incentive mechanisms in federated learning environments. These approaches had two directions; contribution of participants and payment allocations. Shapley value [13] is the most commonly used technique for evaluating the contribution of the parties in the federation. On the other hand, the Stackelberg [20] game is based on data pricing and more focused on allocating monetary rewards to the parties from the profit obtained through the federation.

### Shapley Value

The Shapley value is a concept in cooperative game theory that measures the contribution of each player to the overall value created by a group of players. It is based on the idea that the value generated by a group is distributed fairly among its members according to their respective contributions. It takes into account all possible ways in which players can join a coalition and calculates their average marginal contributions to the coalition. In the context of FL, the Shapley value can be used to measure the marginal contribution of each party to the model performance by evaluating the change in model accuracy when a particular party’s data is included or excluded from the training process. From the computed Shapley value of each party, it is possible to incentivize participation by rewarding parties that contribute the most to the federated model performance.

Liu et al. [12] proposed a Guided Truncation Gradient Shapley (GTG-Shapley) approach for fairly evaluating participants’ contributions to the performance of federated learning models, without exposing their private data. The approach reconstructs FL models from gradient updates for Shapley value calculation and uses a guided Monte Carlo sampling approach combined with truncation to reduce computational costs. Another incentive mechanism has been proposed in [19] for federated learning based on the enhanced Shapley value method. The method measures income distribution by taking multiple influence factors into account as weights, using the analytic hierarchy process (AHP) [14] to find the corresponding weight value of each factor. Although most incentive mechanims are focused in HFL settings, there are some approaches proposed for VFL settings as well. Fan et al. [6] proposed a contribution valuation metric called *VerFedSV* based on Shapley value for vertical federated learning. The VerFedSV metric satisfies desirable properties for fairness and can be adapted to both synchronous and asynchronous vertical federated learning algorithms. Similarly, Wang et al. [18] used Shapley value in VFL to calculate the grouped feature importance of parties through approximation, and used it as a measure of importance to decide the contribution of each party.

### Stackelberg Game

Stackelberg game is a non-cooperative game concept in game theory where one player, called the leader, is allowed to make a move before the other player, called the follower, makes their move. The follower observes the leader’s move before making their own, and the leader takes into account the follower’s response when making their initial move. Since Stackelberg game is used to formulate the interactions between different players in the sell or procurement of common products, this makes it more appropriate for the incentive design in FL settings [15].

Zhan et al. [22] formulated the incentive mechanism for federated learning as a two-stage Stackelberg game where the parameter server announces a total reward, and each user determines its training strategy to maximize its own utility. The parameter server is the leader and the data parties are the followers in this game. In the context of VFL, the host parties can be modeled as followers, and the guest party as the leader. The guest party sets a price for each host party based on the value of its data and the importance of its contribution to the global model. Each host then decides whether to participate in the VFL process based on this price. If the price is too low, the host may choose not to participate, while if the price is too high, the organization may choose to keep its data and not share it [24].

## Auction Theory

Auction theory is a branch of economics that studies how auctions work and how bidders behave in different auction formats. It examines the optimal auction design and rules to maximize the seller’s revenue or social welfare, and analyzes the effects of auctions on efficiency, allocation, and distribution of resources [8]. This concept can be applied to federated learning by designing a mechanism for selecting the best models contributed by clients. The mechanism can determine the rewards paid to clients for their contributions and ensure that the contributions are of high quality.

A federated learning incentive mechanism has been proposed in [23] which the reputation and reverse auction theory has been applied. In this approach, the participants place bids for tasks, and reputation that acts as an indirect measure of their data quality and reliability. Furthermore, combining their reputation and bids under a limited budget, the participants are rewarded. Zheng et al. [21] proposed a lightweight and incentive compatible mechanism *FMore* for federated learning in the application of mobile edge computing (MEC) by extending the concept of multi-dimensional procurement auction [4]. This incentive mechanism encourages low-cost, highquality edge nodes to participate in the federated learning process. Moreover, Huong et al. [9] also proposed an auction game model in context of MEC for the federated learning service between the base station and mobile users, where the base station is the buyer and mobile users are sellers. The base station initiates and announces the FL task, and the mobile users decide the resources required for model training. Each mobile user submits a bid that includes the resources, local accuracy, and energy cost, and the base station acts as the auctioneer to select the winners and determine the payment.

## Contract Theory

Contract theory is a branch of economics that studies how contracts and incentives can be used to overcome problems of asymmetric information and achieve efficient outcomes. It focuses on the design of contracts that can align the interests of different parties and mitigate risks [2]. In federated learning, contract theory can be used to design incentive mechanisms that encourage participants to contribute highquality data and models to the shared learning process, while also ensuring fairness and privacy protection. This can involve the design of contracts that reward participants for their contributions. Liu et al. [11] proposed a contract-based aggregator for FL under a multi-dimensional contract model that considers two possible types of agents. The contract-based aggregator incentivizes agents to contribute high-quality data and computational resources by offering them contracts that are individually rational (IR) and incentive compatible (IC). In [5], data size and communication time of the participants have been considered as decision variables in the contract design for the federated learning process. For this approach, selected participants must satisfy the requirement of communication time and then the incentive design is simplified into a single-dimensional contract problem. Li et al. [10] considered the FL scenario in healthcare crowd sensing transformed the incentive design problem into an utility maximization problem and established an incentive mechanism based on the contract theory. Furthermore, Kang et al. [7] introduced an effective incentive mechanism combining reputation with contract theory to motivate high-reputation participants with high-quality data to participate in model learning.

However, most of the existing approaches for incentive allocation in federated learning deal with horizontally partitioned data, where the data is partitioned based on the samples. In contrast, vertically partitioned data is partitioned based on the features or attributes. To the best of our knowledge, there is no existing work that has specifically addressed the incentive allocation problem in vertically partitioned federated learning using game theoretic approaches apart from the Shapley value. This motivated us to propose a novel solution based on the bankruptcy problem with Talmud division where the characteristic function is the model performance gain. The experiments and results in this paper has demonstrated the efficiency of the solution.

# Preliminaries on Bankruptcy Problem

Bankruptcy problem involves dividing a set of available resources among a group of creditors with specific claims. This sort of problem has been studied in a game theoretic view considering it as a cooperative game where an Estate *E* is to be divided among *N* players having claims from the estate. A bankruptcy problem can be defined as an ordered pair

(*E*;*d*) ∈R×R*n* (1)

where 0 ≤ *d*1 ≤ *d*2*....* ≤ *dn* and 0 ≤ *E* ≤ *d*1 + *d*2 + *...* + *dn*

A division rule *f* in a bankruptcy problem (*E*;*d*) determines a solution or payout *f*(*E*;*d*) = (*f*1(*E*;*d*)*,f*2(*E,*;*d*)*....,fn*(*E*;*d*)) to every creditor such that

* *fi*(*E*;*di*) ≥ 0 for each *i* ∈ *N*
* P*ni*=1 *fi*(*E*;*d*) = *E*

Several division rules have been proposed to address this problem, each with its own principles and criteria for fairness. Some of the common division rules include:

* Proportional Rule: Under the proportional rule, each creditor receives a share of the estate that is directly proportional to their claim.
* Constrained Equal Awards Rule: The constrained equal awards (CEA) rule provides an equal distribution of an estate among creditors, while ensuring that no creditor receives more than what they have claimed.
* Constrained Equal Losses: The constrained equal losses (CEL) rule divides equally the difference between the aggregate claim and the estate, ensuring that no creditor ends up with a negative transfer.
* Talmud/Contest Garment Rule: According to this rule, if the estate is less than half the claim of the creditor with lowest claim, the CEA rule is applied on half of each of the creditor’s claims. Otherwise, each creditors receives half of their claims, and the CEL rule is implemented. Algorithm 1 explains how the Talmud division rule is applied in the bankruptcy problem.

A bankruptcy problem is not itself a coalitional game as coalitions are not observed here explicitly during the formulation. However, Aumann and Michael in [1] designed the bankruptcy problem (*E*;*d*) into a bankruptcy game and later proved theoretically that the, solution of a bankruptcy problem using Talmud’s division rule is the nucleolus of its corresponding bankruptcy game. Therefore, it can be



Algorithm 1 Talmud division algorithm for bankruptcy problem



Order creditors from lowest to highest claim

Divide estate equally among all creditors until the lowest creditor receives half of their claim

while Estate not empty and not all creditors have half their claim do

Remove lowest creditor

Divide estate equally among remaining creditors until the new lowest creditor receives half of their claim end while

Order creditors from highest to lowest claim while Estate not empty do

Allocate to highest creditor(s) until their loss equals the loss of the next highest creditor end while



derived that, by directly solving the bankruptcy problem using Talmud’s division rule, it is possible to determine the specific allocation of estate among the creditors without explicitly constructing the entire bankruptcy game. This simplifies the analysis and allows for a more direct and focused approach to the estate allocation aspect of the bankruptcy scenario.

# Proposed Method

In a VFL setting, the active party has access to the labels while the passive parties have access to additional features of the same samples. During the federation process, the passive parties provide valuable data to the active party which benefits it by improving its machine learning model, but the passive parties are not getting access to the final model or any of the benefits that come with it. Hence, the guest party needs to persuade the passive parties to contribute to the federated model based on incentives. In our proposed method we have designed this scenario as a bankruptcy problem to address the issue of incentive allocation among the participants. In this formulation, the passive parties in the vertical federated learning process are considered as creditors with specific claims on the profit obtained from the federation. The estate is defined as the difference in model performance between the federated model and the local model of the active party. The local model refers to the machine learning model that the active party would have built using only its own data, without collaborating with any other party. The federated model, on the other hand, is the machine learning model that is built collaboratively by all the parties involved in the VFL setup. To calculate the claim of a passive party, we can compare the performance of the federated model when the guest party collaborates only with that particular passive party to the local model of the active party. More specifically, the claim of a passive party can be defined as the improvement in model performance achieved by adding that only that particular party’s features to the federated model.

Let’s assume that there are *N* parties involved in a vertical federated learning setup. The federated learning algorithm aims to train a machine learning model *Mf* that minimizes a certain objective function *F*(*Mf*) over all parties’ datasets, subject to certain constraints.

Now, let’s define the local model of the active party (say, party i) as *Mi*, which is trained only on its own dataset *Di*, without any collaboration with other parties. We measure the performance of *Mi* using a certain evaluation metric, say *V* . Similarly, we can measure the performance of the federated model *Mf* using the same evaluation metric. The estate, denoted by *E*, is defined as the difference in performance between the federated model *Mf* and the local model *Mi*, i.e.,

*E* = *V* (*Mf*) − *V* (*Mi*) (2) Now, let’s consider a passive party (say, party *j*) that wants to claim credit for contributing to the federated model *Mf*. To calculate the claim of party *j*, we can compare the performance of *Mf* when party *j* collaborates alone with party *i* (the active party). More specifically, let’s denote the model when party *j* collaborates alone with party *i* by *Mij*. We can measure the performance of *Mij* using the evaluation metric *V* . The claim of party *j*, denoted by *Cj*, is then defined as the improvement in performance achieved by adding party *j*’s features to the local model of *i*, i.e.,

*Cj* = *V* (*Mij*) − *V* (*Mi*) (3)

Here, *Mij* is trained on both party *i*’s and party *j*’s data, while *Mi* is trained only on party *i*’s data. Therefore, the improvement in performance achieved by adding party *j*’s features to *i*-th local model represents the additional information that the passive party party *j* has contributed to the final model, beyond what the active party *i* could have achieved by training a local model only on its own data. Once the claims from the passive parties and estate of the VFL are obtained, the proposed method involves determining the Talmud bankruptcy problem solution and getting a payout vector represented as

*P* = [*p*1*,p*2*,...,pN*]

*N* (4) whereX*pi* = *E*

*i*=1

# Experimental Setup

This section outlines the experimental setup used to evaluate the proposed method. The classification problem was selected for simplicity and was used in the evaluation.

## Datasets

The proposed method was tested on a synthetic dataset as well as on two real world datasets from the UCI repository (Table 1)

|  |  |  |
| --- | --- | --- |
| Dataset | Instances | Features |
| Synthetic Dataset | 10000 | 20 |
| Heart Disease Dataset | 303 | 13 |
| Bank Marketing Dataset | 11162 | 15 |

Table 1. Overview of Datasets

* SyntheticDataset: Generated using *make\_classification* function provided by the scikit-learn library for binary classification and also added Gaussian noise to bring variation.
* Heart Disease Dataset: Contains medical features to predict the presence of heart disease.
* Bank Marketing Dataset: Contains data on direct marketing campaigns of a Portuguese banking institution to predict whether a client will subscribe to a term deposit or not.

The datasets were pre-processed to address issues such as missing values and duplicates. Additionally, categorical features in the datasets were encoded using One-Hot-Encoding. To simulate a vertically federated environment, the datasets were vertically divided into four partitions, with each partition representing local data of a party. Among them one of the parties was considered the active party with labels and the rest were passive parties.

|  |
| --- |
| Selected Features  Dataset  Synthetic Heart Disease Bank Marketing    *Pa* (active) target (label), features 1 through 3 output (label), age, sex deposit (label), poutcome  *Ph*1 (passive) features 4 through 7 chol, fbs, restecg, caa balance, housing, loan, contact, day  *Ph*2 (passive) features 8 through 14 oldpeak, exng, slp, cp month, campaign, pdays, previous  *Ph*3 (passive) features 15 through 20 thall, thalachh, trtbps age, job, marial, education, default  Table 2. Distribution of features across the involved parties for each experiment. |

## Model

The experimental setup consistes of an active party *Pa* and three passive parties *Ph*1*,Ph*2*,Ph*3. Each party holds some of the features required for the machine learning model. We used a vertical federated logistic regression model to train the model using the Algorithm 2 [25]. The datasets were randomly divided into training and test sets, with 70% of the data used for training and 30% for testing. To eval-



Algorithm 2 Vertical Federated Logistic Regression



1: *M* → Number of Parties

2: *T* → Number of Communication Rounds

3: *B* → Number of Batches

4: Training Data *X* = {*X*1*,X*2*,..,XM*}

5: Party 1 → Active Party

6: Party (2*..M*) → Passive Party

7: Initialize local model *θ*0*m,m ǫ* (1*..M*)

8: for *each communication round t* = 1*,*2*,...,T* do

9: for *each batch data Xbm ǫ* (*X*1*m,X*2*m,..,XBm*) do

10: for *each Party m* = 1*,*2*,..M* do

11: Compute *zbm* = *Xbmθtm*

12: if *m* 6= 1 then

13: send *zbm* to Party 1

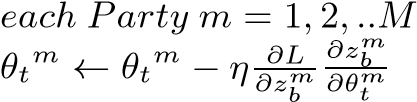
14: end if

15: end for

 and *L*(*yb,y*ˆ*b*) on Party 1

 on Party 1

18: Send *∂z∂Lbm* to Party *m,mǫ*(2*,k*)

19: fordo

20:

21: end for

22: end for

23: end for



uate the model’s performance, we used the F1-score as an evaluation metric. We calculated the F1-score for the local model of *Pa* when it uses only its local data, *M*(*Pa*) and as well as also computed F1scores for models trained with each combination of passive parties with the active party: *M*(*Pa,Ph*1)*,M*(*Pa,Ph*2)*,M*(*Pa,Ph*3) and *M*(*Pa,Ph*1*,Ph*2*,Ph*3)‘. Here, *M*(*Pa,Ph*1*,Ph*2*,Ph*3)) denotes the federated model.

## Estate and Claims for Bankruptcy Problem

The estate *E*, is considered to be the improvement in the federated model performance when using the combined data of all parties compared to using the data of *Pa* alone. The estate is calculated as the difference between the F1-score of the federated model and the F1score of the local model of *Pa*:

*E* = *F*1(*M*(*Pa,Ph*1*,Ph*2*,Ph*3)) − *F*1(*M*(*Pa*)) (5)

In addition, we calculated the claims for each passive party *Phi* where *i* ∈ 1*,*2*..n*, which measure their contribution to the overall model performance. The claims are computed as the difference between the F1-score of the federated model and the F1-score of the model trained with *Pa* and *Phi*:

*Claim*(*Phi*) = *F*1(*M*(*Pa,Phi*)) − *F*1(*M*(*Pa*)) (6)

# Results

This section presents the results of our proposed method for incentive allocation in VFL setting based on the bankruptcy problem where Talmud’s division rule has been applied. Additionally, we compared the solution which is the nucleolus of the bankruptcy prob-

lem with the corresponding Shapley values of the passive parties. The following subsections provide a detailed analysis of the results.

## Heart Disease Dataset

In Table 3, the *Estate* column shows the estate value, which measures the improvement in the model’s performance when using the combined data of all parties compared to using the data of *Pa* alone. In this case, the estate value is 39.33, indicating a significant improvement in the model’s performance when using the combined data of all parties. Here, *Ph*2 has the highest claim value of 35.27, followed by *Ph*1 with a claim value of 33.98, and *Ph*3 with a claim value of 28.43.

The *Payout* column shows the payout allocated from the estate for each passive party, which is proportional to their claims. In this case, all parties receive the same payout of 13.11, which is lower than their claim values. This is because the total surplus value of the federated model is not enough to fully compensate all parties according to their claims. When the estate is relatively small compared to the sum of claims, it is divided equally and all parties receive the same payout. We think this makes sense, as it compensates the effort made for the federation, disregarding the individual contribution.

## Bank Marketing Dataset

In Table 3, for the bank marketing dataset, the estate was divided equally among *Ph*1, *Ph*2, and *Ph*3 until *Ph*3 received half of their claim value. Once *Ph*3 received half of its claims it was discarded temporarily and *Ph*2 became the next lowest creditor (according to Algorithm 1). The remaining estate was then again divided equally among *Ph*2 and *Ph*1 until *Ph*2 received half of their claim value. At this point, the estate was exhausted, and the remaining surplus value was not enough to fully compensate *Ph*1 according to its claims. Therefore, *Ph*1 received a payout that was the same as to *Ph*2.

## Synthetic Dataset

For the synthetic dataset, the estate was initially divided equally among the parties (*Ph*1, *Ph*2, and *Ph*3) until they received half of their claim value. After each party received half of their claim value, the remaining surplus value was allocated to the highest creditor, which was *Ph*2 with a claim value of 45*.*45. The surplus value was then allocated to *Ph*2 until their loss (7*.*46) equaled the loss of the next highest creditor, which was *Ph*3 with a claim value of 35*.*89 and a payout of 28*.*43.

|  |
| --- |
| Host Party Claim Host Party Payout Host Party Shapley Value  Description Estate  *Ph*1 *Ph Ph Ph Ph Ph Ph Ph Ph*    Table 3. Outcomes for the experiments performed in this study. Both property experiments were performed using the Heart Disease Dataset. For the Dummy  Player Property, Host Party *Ph*1 was selected as the dummy player. For the Symmetry property, Host Party *Ph*1 was defined to be the duplicate of *Ph*2. |

## Interpreting Payouts in Practical Setting

In our proposed approach, the payouts obtained by each party can be compared with the estate to find out what portion of the estate it received. This percentage can be used to divide the incentive in a practical setting where there is a budget for the federation. For example, if we consider the experimental results for the bank marketing dataset, the percentages of the payouts for *Ph*1*,Ph*2 and, *Ph*3 would be (10*.*055*/*28*.*03) ∗ 100 ≈ 36%, (10*.*055*/*28*.*03) ∗ 100 ≈ 36% & (7*.*92*/*28*.*03) ∗ 100 ≈ 28% respectively. Hence, if the total budget of the active party for this federation is lets say, $10,000, each of the parties will receive a compensation from this budget equal to their payout percentages.

Hence, this method can ensure that the participating parties are incentivized to contribute high-quality and unique data, while also ensuring that the incentive budget is allocated fairly among the parties.

## Comparison with Shapley Value

The Shapley value represents the average marginal contribution of each passive party to the overall model performance. It is important to note that the Shapley value and the payout (or nucleolus) may differ due to the specific formulation of the bankruptcy problem in the VFL setting. However, it has been observed across all the datasets (Table 3) that, parties with higher claims have higher Shapley values, indicating their higher contribution to the overall model performance. While the Shapley value provides insights into the relative importance of each party’s contribution to the model, it is not sufficient to determine the exact amount of incentive that needs to be allocated to each party.

Talmud’s bankruptcy solution is computationally more efficient than Shapley values because it requires fewer computations as the number of passive parties increases. Specifically, Talmud’s bankruptcy solution requires *O*(*n*) computations, where *n* is the number of passive parties, for computing the claims of each passive party and allocating the surplus value to each party based on their claims. On the other hand, computing Shapley values requires evaluating the performance of the model for all possible combinations of parties, which involves *O*(2*n*) computations. This exponential increase in computational complexity can be a significant challenge, especially for large values of *n*.

While there are efficient algorithms for approximating Shapley values, they may not be appropriate for VFL settings due to the inherent privacy constraints. Approximating Shapley values in VFL requires approximating the performance of the model for all possible combinations of parties, which can be challenging due to the need to maintain data privacy and security.

Moreover, approximations are never exact and can introduce errors in the computation of the Shapley values. These errors can lead to biased incentive allocation and undermine the fairness and efficiency of the VFL collaboration. In contrast, Talmud’s bankruptcy solution is a more robust and reliable approach for incentive allocation, as it is based on a well-established and proven method that is computationally efficient and fair.

# Discussion

In this section we discuss two common problems in VFL; identifying malicious data party and redundant data party and then, assess how our proposed approach and Shapley values have addresses them. The problem of malicious data parties can occur when a participating party intentionally provides data that is of poor quality or contains malicious content that can compromise the integrity of the model. On the other hand, redundant data party problem can occur when multiple participating parties provide similar or redundant data to the federated learning process. This can lead to an inefficient use of resources, as the same data is used multiple times, and can also lead to over-fitting of the model to the redundant data, reducing its ability to generalize to new data.

## Dummy Property

The malicious data problem in VFL can be has been tried to be solved by the dummy player property in co-operative game theory. In general, the dummy player refers to a player who does not contribute to the game’s value, and whose presence or absence in any coalition does not affect the total value generated by that coalition.

To simulate a dummy player or a party with malicious data in VFL, one of the passive parties (*Ph*1 in Table 3) has been assigned randomly generated data which represents its local data. Hence, this party should be the dummy player and not contribute to the federated model. We calculate the payouts using our proposed approach as well also the Shapley values for the Heart disease dataset as an example.

If a passive party does not contribute to any coalition in the FL process, they should not receive any share of the total profit from the federation. The Shapley value identifies dummy players by computing the average marginal contribution of a party to all possible coalitions. Since a dummy player or the malicious data party *Ph*1 contributes nothing to any coalition, their marginal contribution to every coalition is zero, and hence, the Shapley value assigns them a value of zero, which is consistent with the concept of dummy players in cooperative game theory.

Again, according to our proposed approach, if a party’s contribution to the federated model is negative or zero due to malicious data, their claim will also be zero or negative. Eventually applying Talmud’s division rule will ensure that, the party with claim less than or equal to zero is not given any incentive and only the estate is divided among parties with positive valued claims.

## Symmetry Property

The redundant party problem has been assessed with using the symmetry property in game theory. According to symmetry property, if two players have identical contributions to every coalition in the game, they should be treated equally and receive the same share of the total value of the game.

To simulate a redundant data party in VFL, one of the passive parties (here *Ph*1 has been assigned the same local data as *Ph*2. So basically, *Ph*1 is the duplicate of *Ph*2. As earlier, The payouts were calculated using our proposed approach as well also the Shapley values for the Heart disease dataset.

From Table 3 we observe that the Shapley values as well as payouts are the same for *Ph*1 and *Ph*2 since they both have redundant data. However, it is important to note that parties with the same Shapley values may not necessarily have the same data, and dropping one of them might not always be the best decision. In some cases, parties with the same Shapley values may be contributing equally to the federated model, even if their data is different.

Similarly, in our proposed approach, equal payouts may not necessarily indicate that the claimants have the same data. The equal payouts simply mean that the claimants are receiving a fair share of the model performance gains based on their respective contributions. Therefore, while the symmetry property is an important property in cooperative game theory, it should be used in conjunction with party selection before the VFL process to identify duplicate or similar contributing parties.

# Conclusion

One of the emerging challenges in VFL which is allocating incentives to passive parties in vertically partitioned federated learning has been addressed in our paper. The proposed approach, based on the Bankruptcy Problem using Talmud’s division rule, ensures fairness and stability in incentive distribution among the passive parties. Through experiments on both synthetic and real-world datasets, we have demonstrated that our method is comparable to the existing approach of using Shapley values while also surpasses it in terms of computational efficiency. This efficiency is particularly valuable when dealing with large-scale federated learning scenarios involving numerous parties and features. Overall, our approach presents a practical solution for motivating passive parties to contribute their private data, thus enhancing the accuracy and reliability of machine learning models while preserving privacy. Future research could explore incorporating party selection mechanism in VFL before the training process begins and then finally allocate incentives in the end which would make the whole incentive allocation mechanism more efficient. We will also test this approach on real world use cases in near future.

# References

1. Robert J Aumann and Michael Maschler, ‘Game theoretic analysis of a bankruptcy problem from the talmud’, *Journal of economic theory*, 36(2), 195–213, (1985).
2. Patrick Bolton and Mathias Dewatripont, *Contract theory*, MIT press, 2004.
3. Keith Bonawitz, Hubert Eichner, Wolfgang Grieskamp, Dzmitry Huba, Alex Ingerman, Vladimir Ivanov, Chloe Kiddon, Jakub Konecnˇ y, Ste-` fano Mazzocchi, Brendan McMahan, et al., ‘Towards federated learning at scale: System design’, *Proceedings of machine learning and systems*, 1, 374–388, (2019).
4. Yeon-Koo Che, ‘Design competition through multidimensional auctions’, *The RAND Journal of Economics*, 668–680, (1993).
5. Ningning Ding, Zhixuan Fang, and Jianwei Huang, ‘Incentive mechanism design for federated learning with multi-dimensional private information’, in *2020 18th International Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks (WiOPT)*, pp. 1–8. IEEE, (2020).
6. Zhenan Fan, Huang Fang, Zirui Zhou, Jian Pei, Michael P Friedlander, and Yong Zhang, ‘Fair and efficient contribution valuation for vertical federated learning’, *arXiv preprint arXiv:2201.02658*, (2022).
7. Jiawen Kang, Zehui Xiong, Dusit Niyato, Shengli Xie, and Junshan Zhang, ‘Incentive mechanism for reliable federated learning: A joint optimization approach to combining reputation and contract theory’, *IEEE Internet of Things Journal*, 6(6), 10700–10714, (2019).
8. Paul Klemperer, ‘Auction theory: A guide to the literature’, *Journal of economic surveys*, 13(3), 227–286, (1999).
9. Tra Huong Thi Le, Nguyen H Tran, Yan Kyaw Tun, Minh NH Nguyen, Shashi Raj Pandey, Zhu Han, and Choong Seon Hong, ‘An incentive mechanism for federated learning in wireless cellular networks: An auction approach’, *IEEE Transactions on Wireless Communications*, 20(8), 4874–4887, (2021).
10. Li Li, Xi Yu, Xuliang Cai, Xin He, and Yanhong Liu, ‘Contract theory based incentive mechanism for federated learning in health crowdsensing’, *IEEE Internet of Things Journal*, (2022).
11. Yuan Liu, Mengmeng Tian, Yuxin Chen, Zehui Xiong, Cyril Leung, and Chunyan Miao, ‘A contract theory based incentive mechanism for federated learning’, in *Federated and Transfer Learning*, 117–137, Springer, (2022).
12. Zelei Liu, Yuanyuan Chen, Han Yu, Yang Liu, and Lizhen Cui, ‘Gtgshapley: Efficient and accurate participant contribution evaluation in federated learning’, *ACM Transactions on Intelligent Systems and Technology (TIST)*, 13(4), 1–21, (2022).
13. Alvin E Roth, ‘Introduction to the shapley value’, *The Shapley value*, 1–27, (1988).
14. Thomas L Saaty, *What is the analytic hierarchy process?*, Springer, 1988.
15. Yunus Sarikaya and Ozgur Ercetin, ‘Motivating workers in federated learning: A stackelberg game perspective’, *IEEE Networking Letters*, 2(1), 23–27, (2019).
16. Zhuan Shi, Lan Zhang, Zhenyu Yao, Lingjuan Lyu, Cen Chen, Li Wang, Junhao Wang, and Xiang-Yang Li, ‘Fedfaim: A model performancebased fair incentive mechanism for federated learning’, *IEEE Transactions on Big Data*, (2022).
17. Joost Verbraeken, Matthijs Wolting, Jonathan Katzy, Jeroen Kloppenburg, Tim Verbelen, and Jan S Rellermeyer, ‘A survey on distributed machine learning’, *Acm computing surveys (csur)*, 53(2), 1–33, (2020).
18. Guan Wang, Charlie Xiaoqian Dang, and Ziye Zhou, ‘Measure contribution of participants in federated learning’, in *2019 IEEE international conference on big data (Big Data)*, pp. 2597–2604. IEEE, (2019).
19. Xun Yang, Weijie Tan, Changgen Peng, Shuwen Xiang, and Kun Niu, ‘Federated learning incentive mechanism design via enhanced shapley value method’, *Wireless Communications and Mobile Computing*, 2022, (2022).
20. Yugang Yu, George Q Huang, and Liang Liang, ‘Stackelberg gametheoretic model for optimizing advertising, pricing and inventory policies in vendor managed inventory (vmi) production supply chains’, *Computers & Industrial Engineering*, 57(1), 368–382, (2009).
21. Rongfei Zeng, Shixun Zhang, Jiaqi Wang, and Xiaowen Chu, ‘Fmore: An incentive scheme of multi-dimensional auction for federated learning in mec’, in *2020 IEEE 40th international conference on distributed computing systems (ICDCS)*, pp. 278–288. IEEE, (2020).
22. Yufeng Zhan, Peng Li, Zhihao Qu, Deze Zeng, and Song Guo, ‘A learning-based incentive mechanism for federated learning’, *IEEE Internet of Things Journal*, 7(7), 6360–6368, (2020).
23. Jingwen Zhang, Yuezhou Wu, and Rong Pan, ‘Incentive mechanism for horizontal federated learning based on reputation and reverse auction’, in *Proceedings of the Web Conference 2021*, pp. 947–956, (2021).
24. Zhixian Zhang, Xinchao Li, and Shiyou Yang, ‘Data pricing in vertical federated learning’, in *2022 IEEE/CIC International Conference on Communications in China (ICCC)*, pp. 932–937, (2022).
25. Hangyu Zhu, Jinjin Xu, Shiqing Liu, and Yaochu Jin, ‘Federated learning on non-iid data: A survey’, *Neurocomputing*, 465, 371–390, (2021).