

# Intelligent Analysis of Biomedical Images

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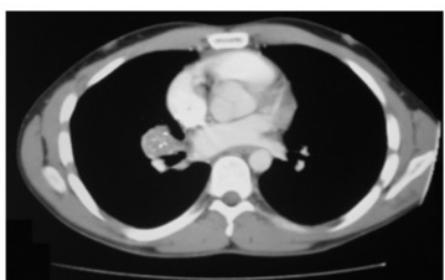
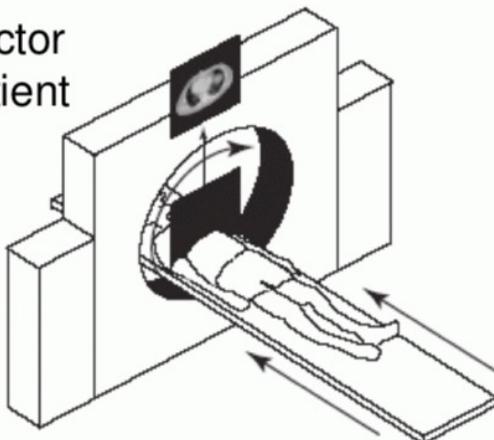
Fall 2023

Courtesy: Some slides are adopted from CSE 377 Stony Brook University  
and CS 473 U. Waterloo

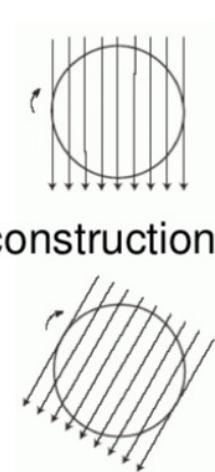
# Computed Tomography

# Overview

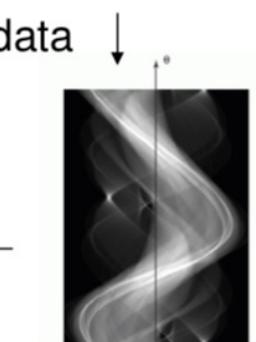
Scanning:  
rotate source-detector  
pair around the patient



reconstructed cross-  
sectional slice



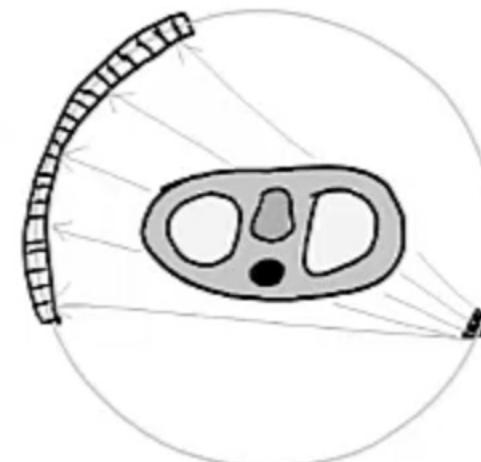
reconstruction routine



sinogram: a line for  
every angle

Also known as **CAT** (Computer assisted/aided tomography)

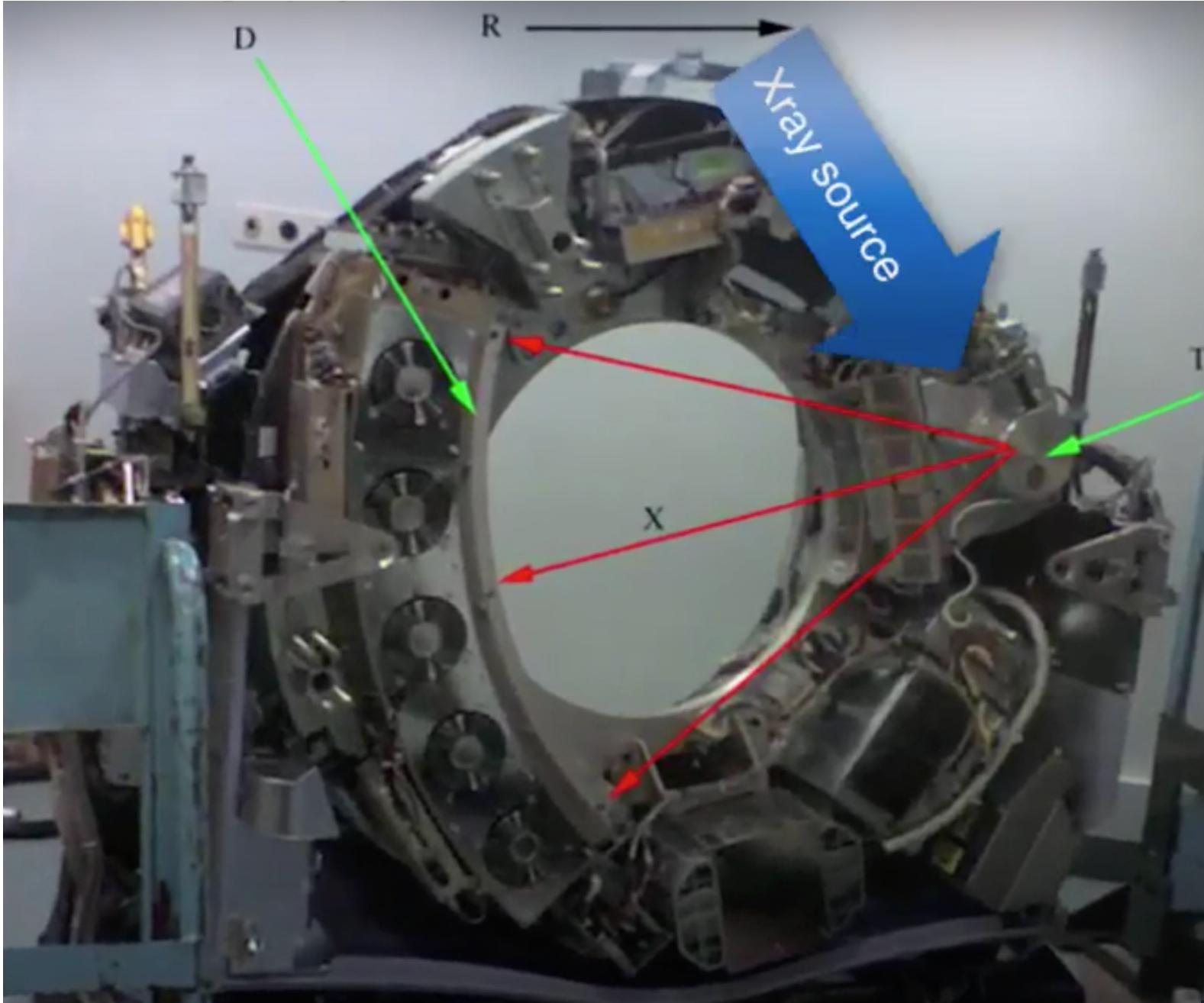
The idea is to resolve a single slice of an object using many x-ray projections.

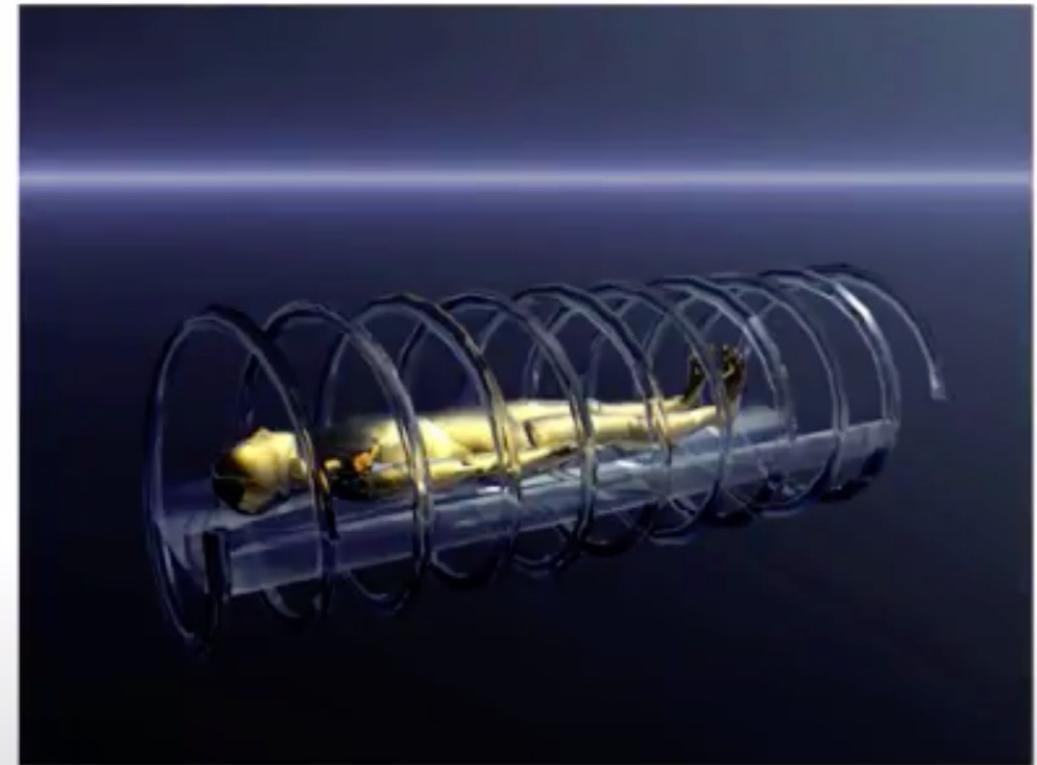
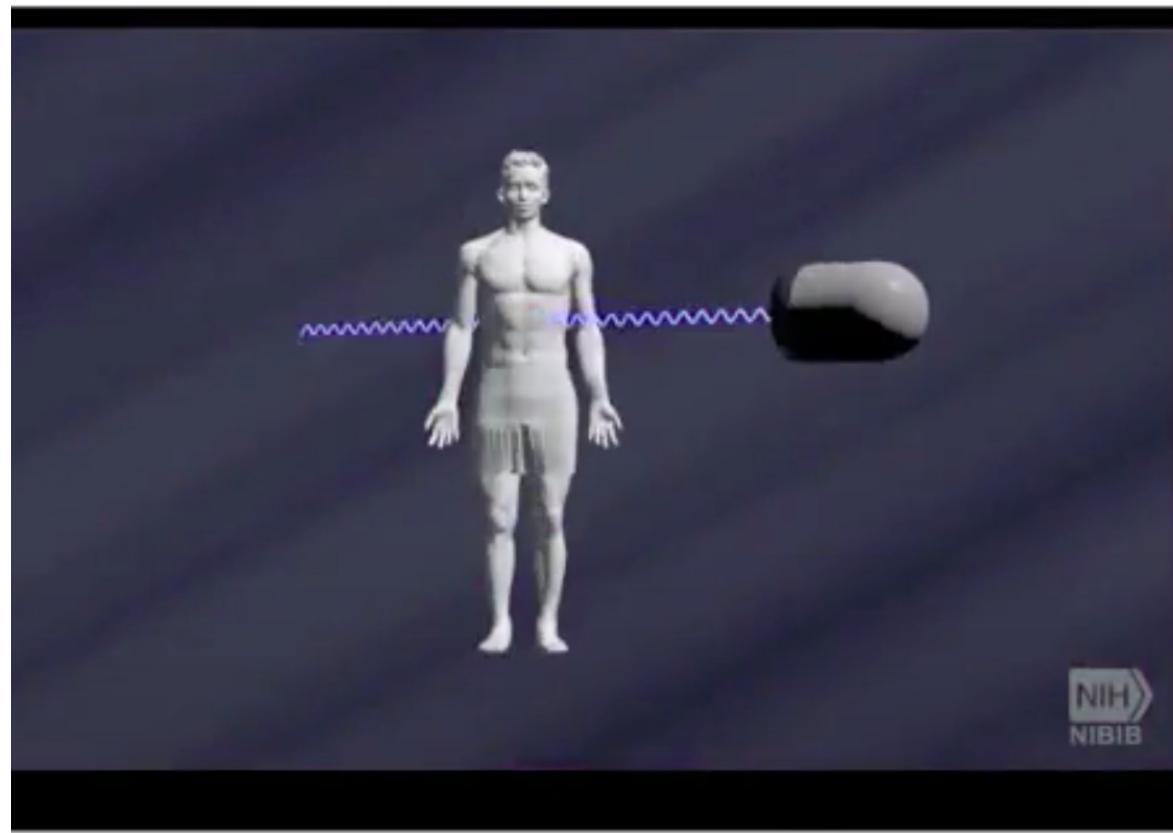




[https://www.google.com/url?  
sa=i&rct=j&q=&esrc=s&source=images&cd=&cad=rja&uact=8&ved=oahUKEwiX-6Cr9O7XAhWNw4MKHbYFBDwQjRIBw&url=https%3A%2F%2Fcommons.wikimedia.org](https://www.google.com/url?sa=i&rct=j&q=&esrc=s&source=images&cd=&cad=rja&uact=8&ved=oahUKEwiX-6Cr9O7XAhWNw4MKHbYFBDwQjRIBw&url=https%3A%2F%2Fcommons.wikimedia.org)

- Sir Godfrey Newbold Hounsfield
- Shared the 1979 Nobel Prize for Physiology or Medicine with Allan McLeod Cormack
- Electrical engineer, worked at EMI, Ltd ~ 1950s
- Helped develop the first business computer in Great Britain
- Given sabbatical to investigate new applications for this product



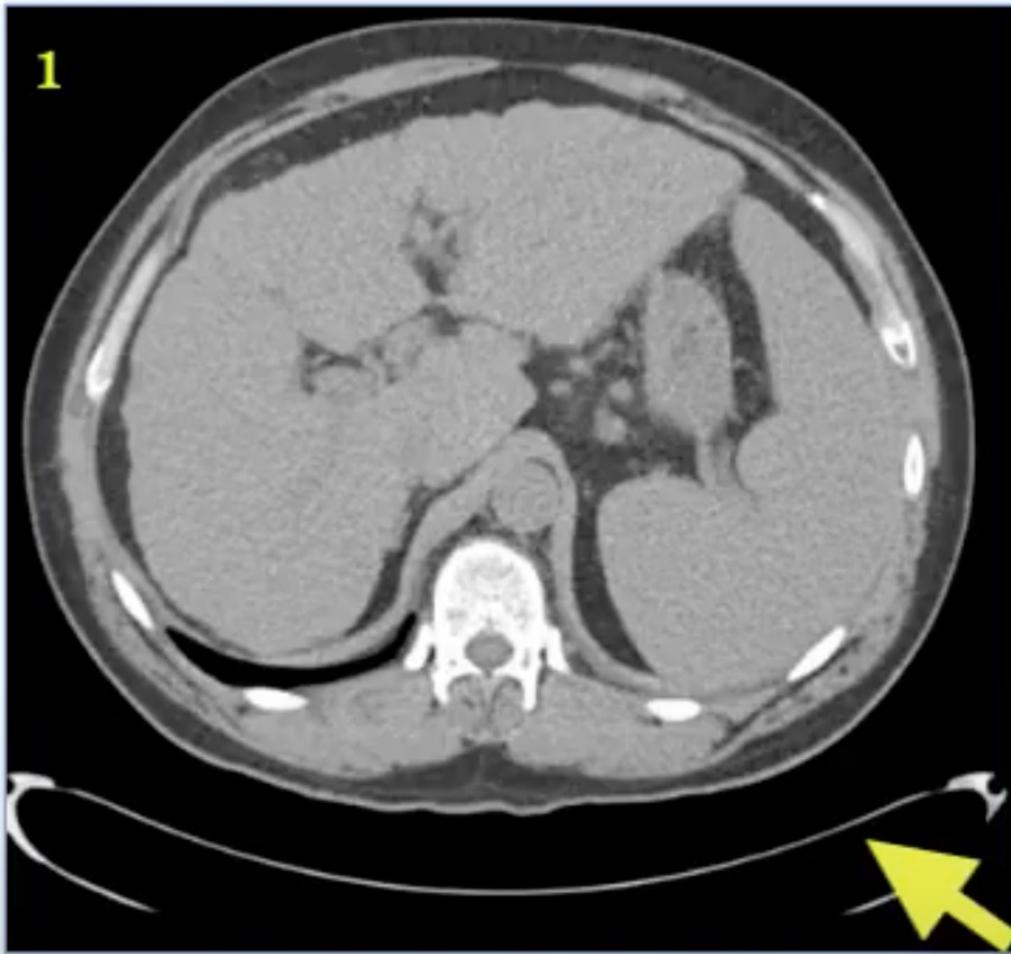


<https://youtu.be/l9swbAtRRbg>

[https://www.google.com/url?  
sa=i&rct=j&q=&esrc=s&source=images&cd=&cad=rja&uact=8&ved=oahUK  
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**Right**

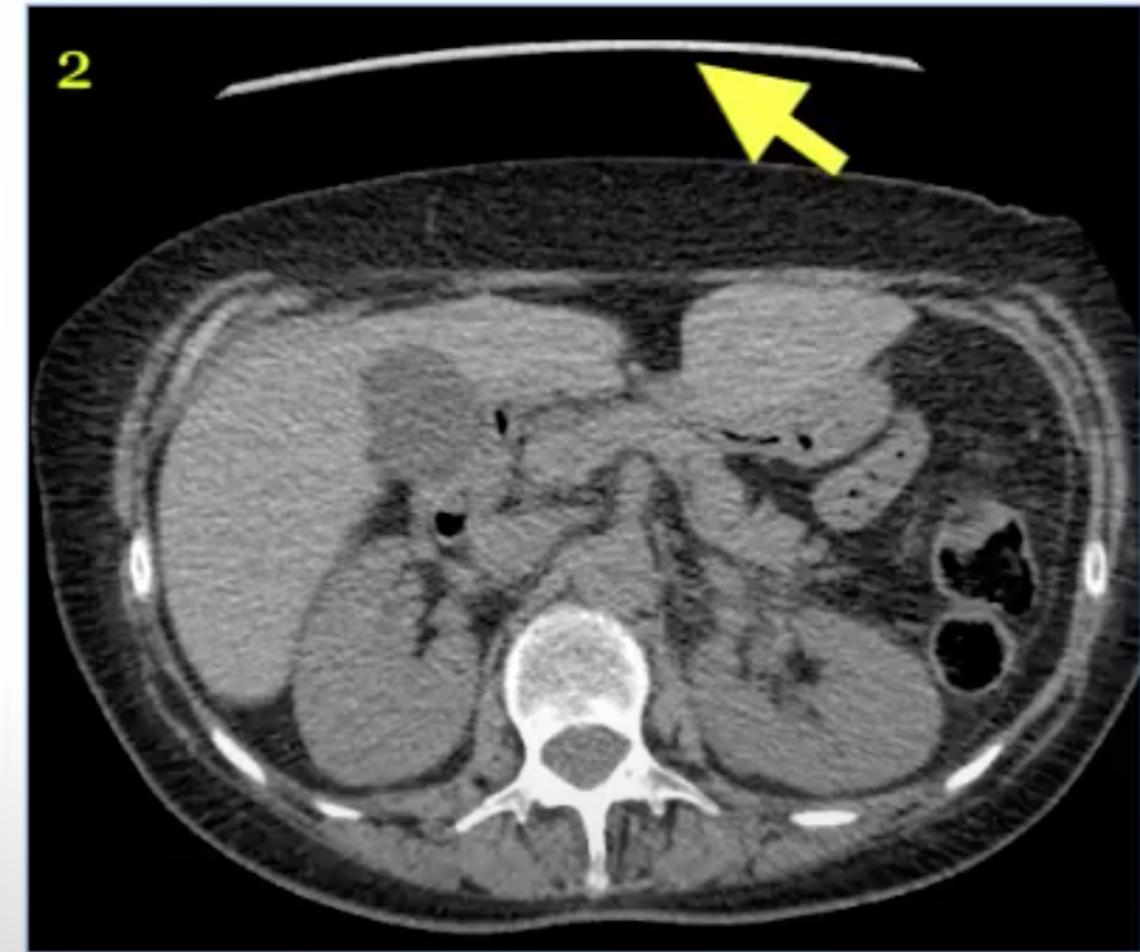
**Left**



**Supine**

**Right**

**Left**

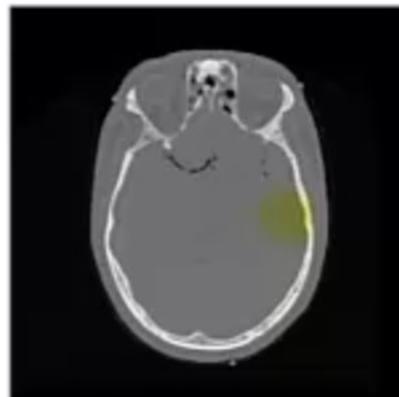


**Prone**

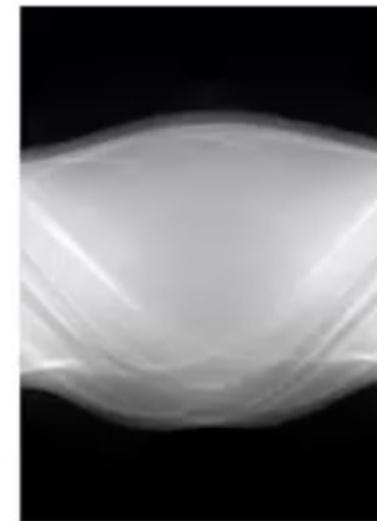
Note the position of the CT gantry (yellow arrow)

As the gantry rotates, the scanner collects a  
1D x-ray at each angle.

Anatomy

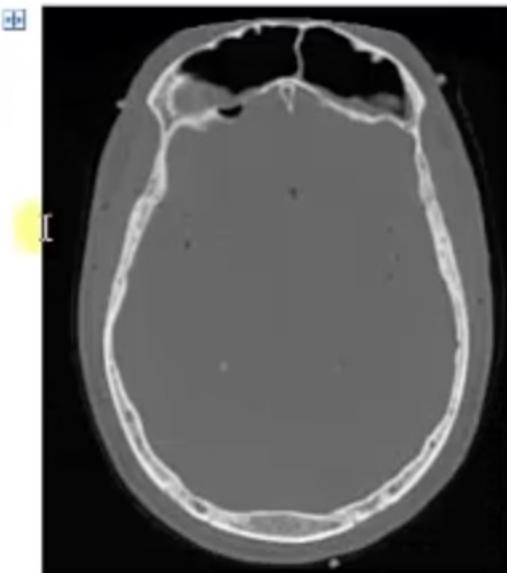


Radon Transform

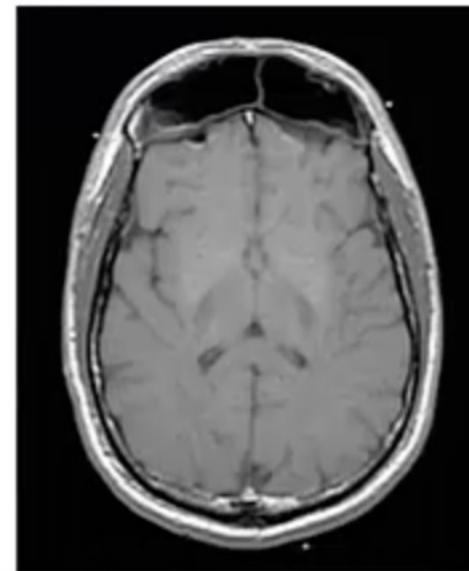


~~~~~  
CT is great for distinguishing bone from soft tissue, but not too good for distinguishing soft tissues apart.

CT



MRI



## Contrast Agents

Substances can be introduced to the body to add contrast. These are called contrast agents. They are especially useful for visualizing **vasculature** (arteries & veins)

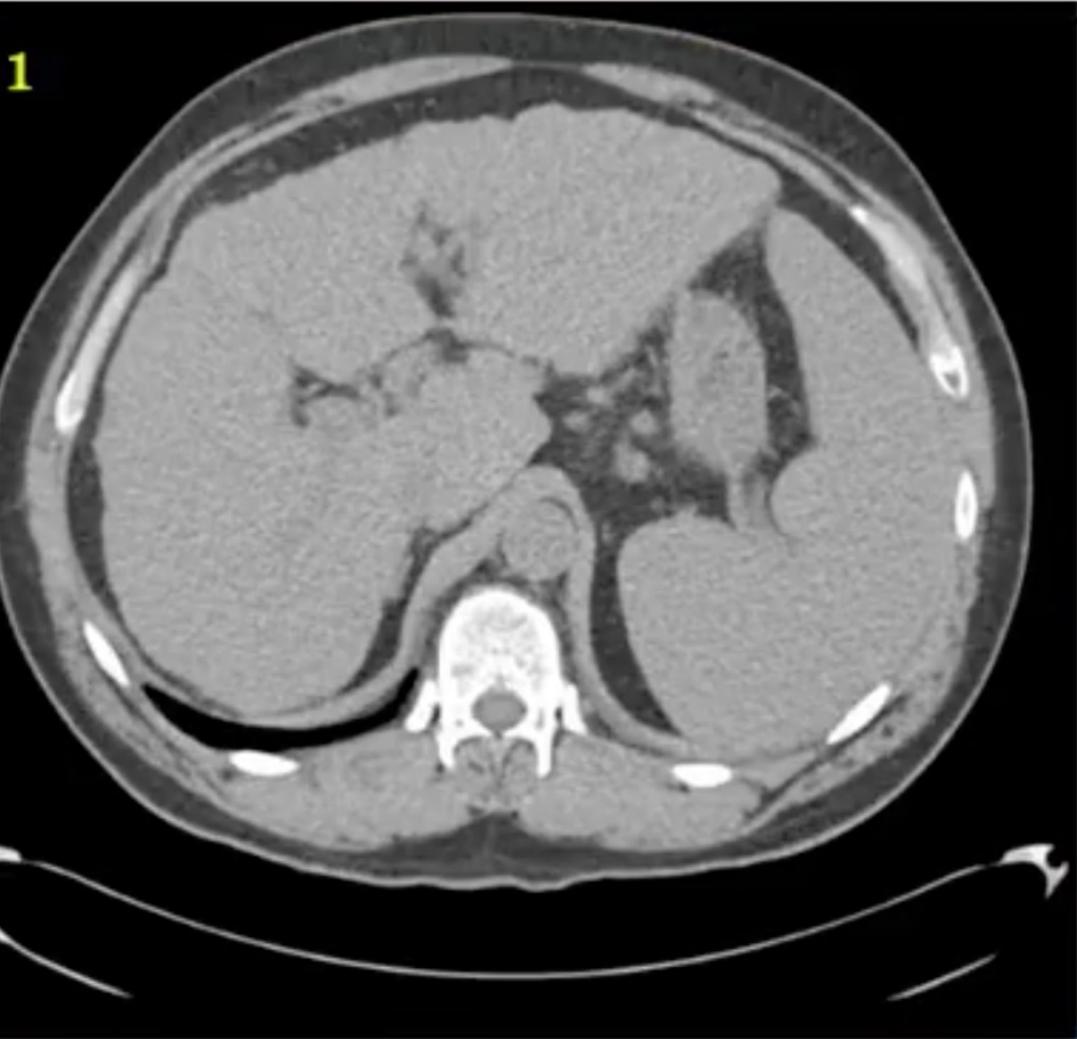
Without contrast agent



With contrast agent



1



2



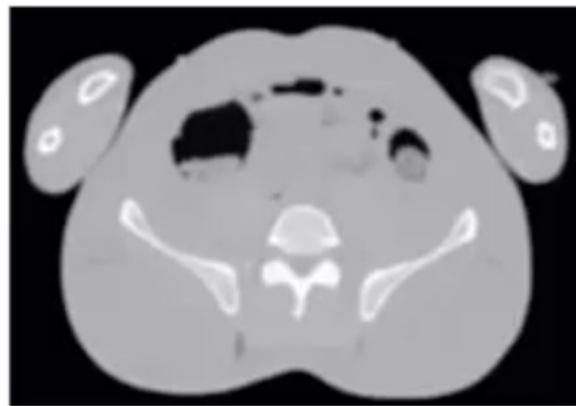
### **Examine the following 2 CT scans**

Which was performed with intravenous contrast?

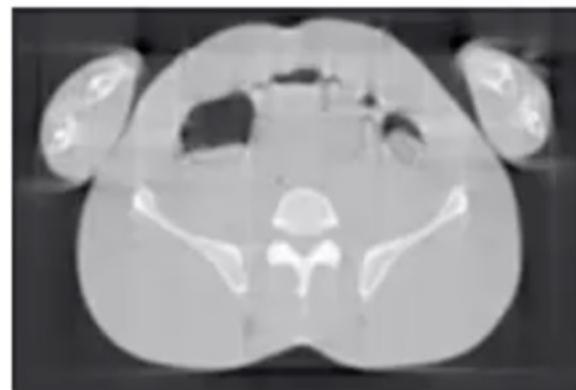
Which was performed without intravenous contrast?

## Motion Artifacts

It takes a few seconds to acquire one slice of CT data. If the patient moves during that acquisition, the resulting Radon transform will be inconsistent, and the reconstructed image will contain errors.



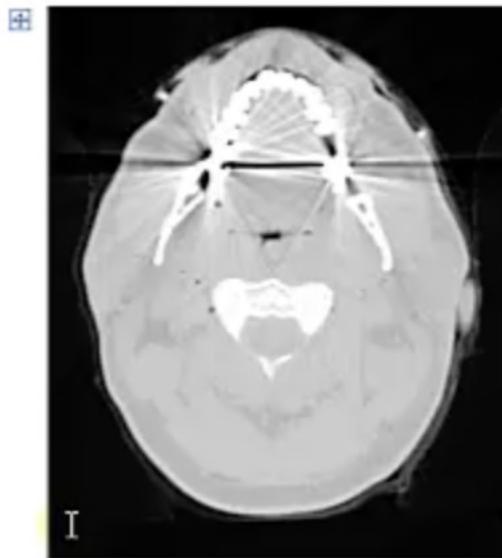
Motion-free



Patient moved  
3° half-way  
through the  
scan.

## Dense Object Shadowing

Highly radio-opaque objects in the field of view do not fit the model assumed by the imaging & reconstruction process. These include metal objects like dental fillings, bullets, artificial joints, etc.



## Ionizing Radiation

X-rays are a form of ionizing radiation. Ionizing radiation is radiation with high enough energy that electrons can be ejected out of their orbitals, creating ions. These ions, in large numbers, can cause tissue damage & damage DNA.

For these reasons, we actively limit the amount of ionizing radiation a person gets (the government keeps track of all your x-rays, CT scans, etc, and sets yearly & lifetime limits).

These dose issues are factored in when a doctor

## Speed

The speed of a CT scan is really only limited by how fast the x-ray source can be moved, and by x-ray flux. After all, the x-rays themselves move at the speed of light.

So, although patient motion can be an issue, CT is among the best for speed.

## Quantitative

The intensity values are in Hounsfield units, which is an absolute intensity scale.

$$\text{Recall: } H_{\text{air}} = -1000 \text{ HU} \quad H_{\text{water}} = 0 \text{ HU}$$

This helps when processing the images. For example, we know the range for bones, so we can do a pretty good job of deciphering which voxels are bone without even looking at the image.

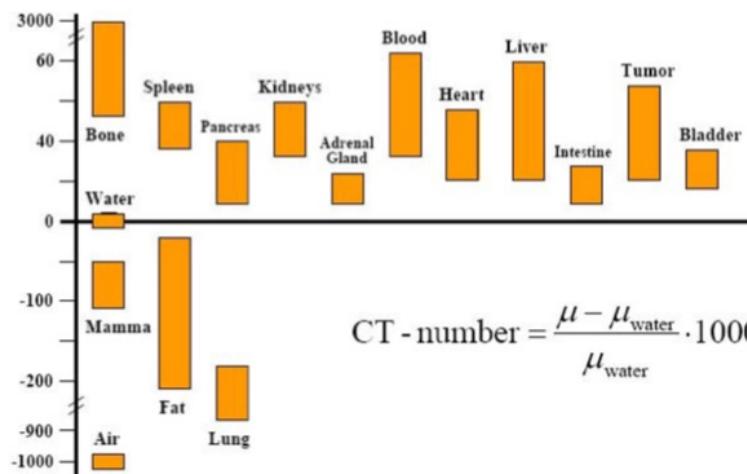
# Current technology

Principles derived by Godfrey Hounsfield for EMI

- based on mathematics by A. Cormack
- both received the Nobel Price in medizine/physiology in 1979
- technology is advanced to this day

Images:

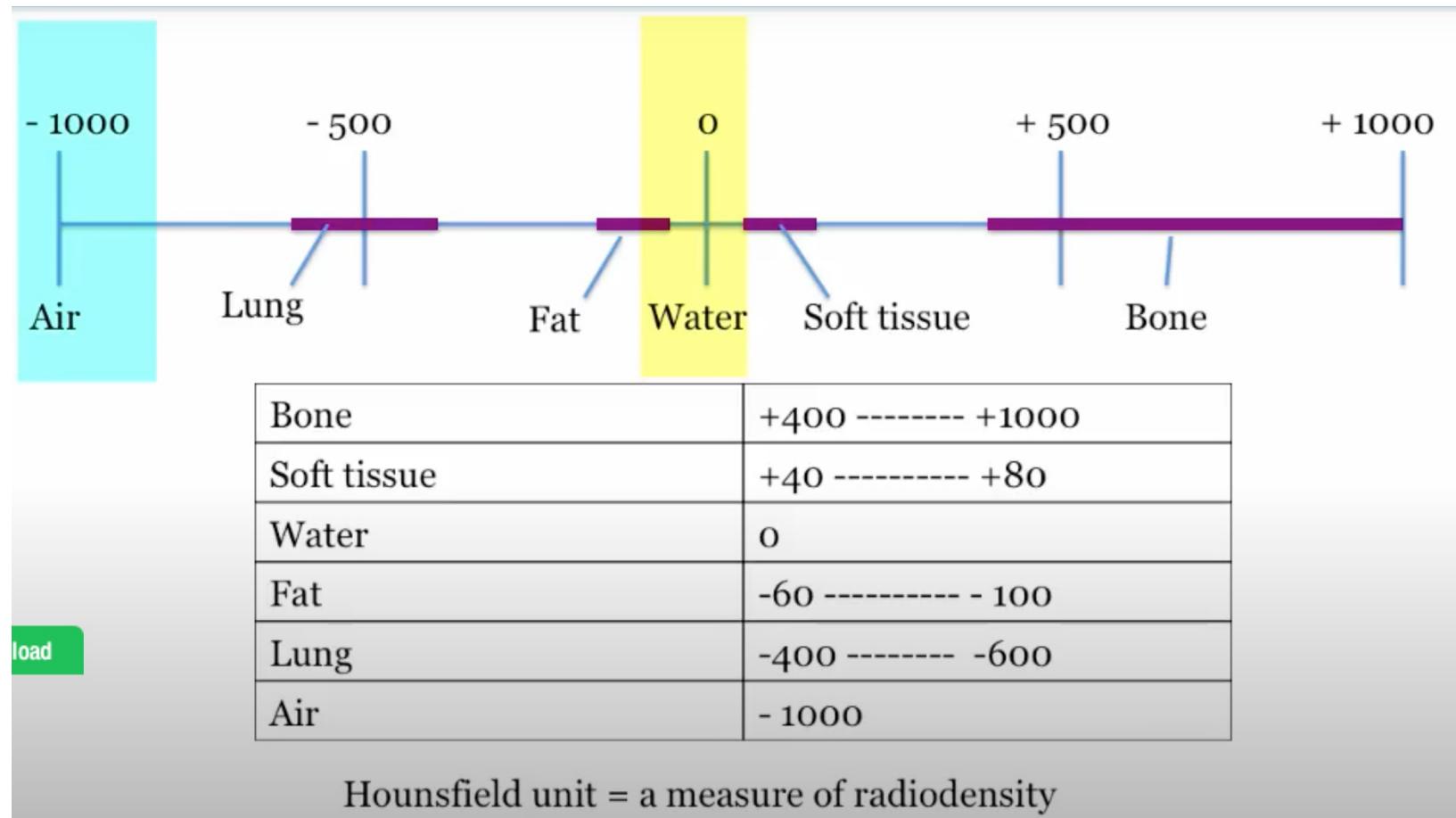
- size generally 512 x 512 pixels
- values in *Hounsfield units* (HU) in the range of -1000 to 1000



$\mu$ : linear attenuation coefficient

- due to large dynamic range, windowing must be used to view an image

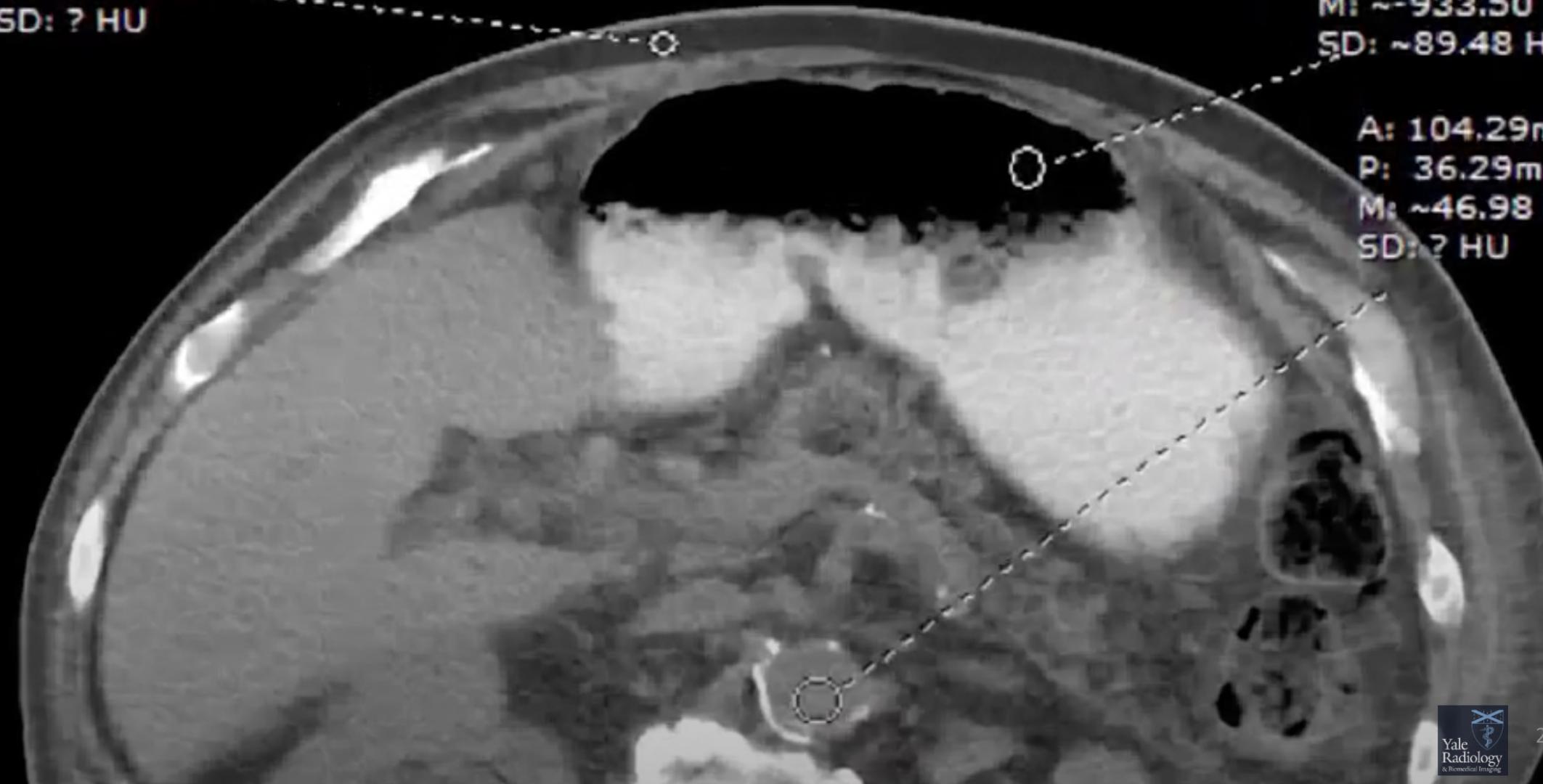
# Hounsfield units

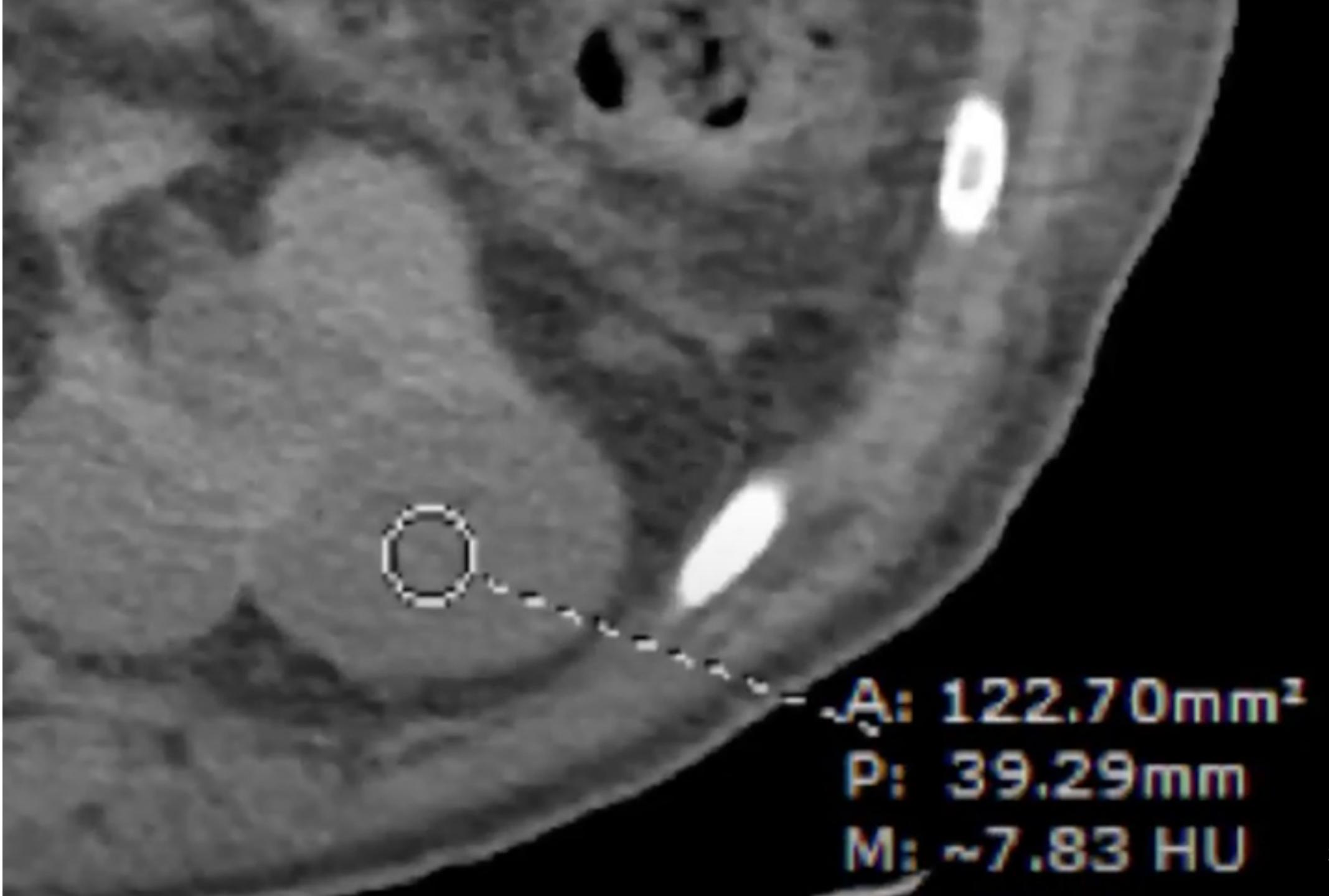


A: 31.97 mm<sup>2</sup>  
P: 20.19 mm  
M: ~82.77 HU  
SD: ? HU

A: 72.33 mm<sup>2</sup>  
P: 30.25 mm  
M: ~933.50 HU  
SD: ~89.48 HU

A: 104.29 mm<sup>2</sup>  
P: 36.29 mm  
M: ~46.98 HU  
SD: ? HU



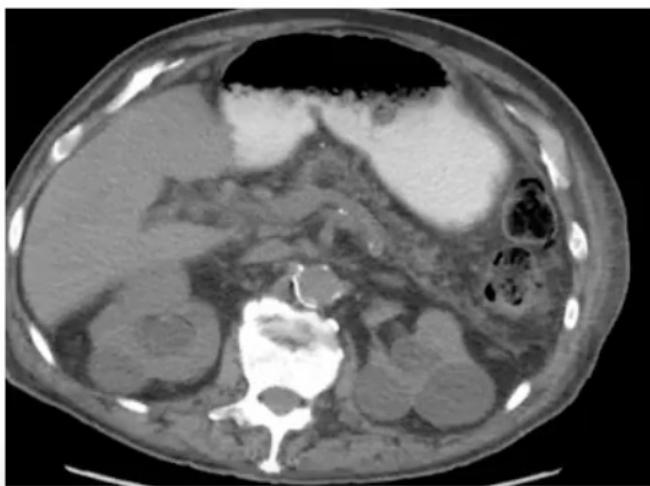


A: 122.70 mm<sup>2</sup>  
P: 39.29 mm  
M: ~7.83 HU

# Windowing

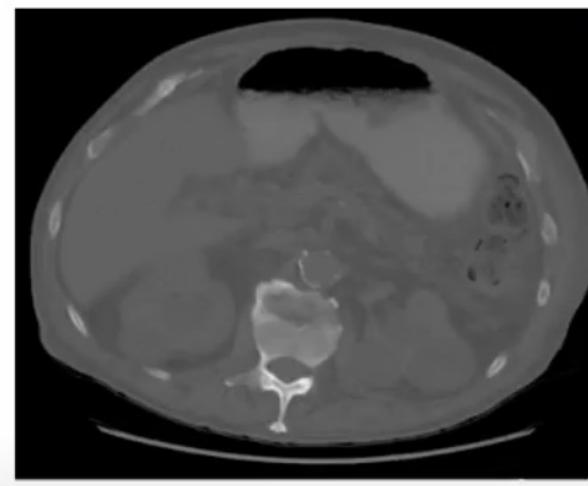
**Window width/level = allows to adjust contrast levels**

$2000/16 =$   
125  
=> Different  
shade of gray  
seen every  
125 HU



**Abdomen**

$500/16 = 31$   
=> Different  
shade of  
gray seen  
every 31  
HU



**Bone**

# Projection coordinate system

The parallel-beam geometry at angle  $\theta$  represents a new coordinate system  $(r, s)$  in which projection  $I_\theta(r)$  is acquired

- rotation matrix  $R$  transforms coordinate system  $(x, y)$  to  $(r, s)$ :

$$\begin{pmatrix} r \\ s \end{pmatrix} = R \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

that is, all  $(x, y)$  points that fulfill

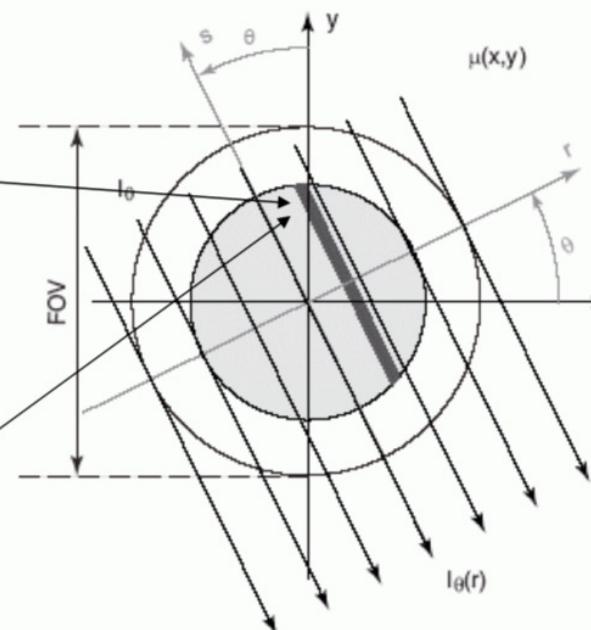
$$r = x \cos(\theta) + y \sin(\theta)$$

are on the (ray) line  $L_{(r, \theta)}$

- $R^T$  is the inverse, mapping  $(r, s)$  to  $(x, y)$

$$\begin{pmatrix} x \\ y \end{pmatrix} = R^T \begin{pmatrix} r \\ s \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} r \\ s \end{pmatrix}$$

$s$  is the parametric variable  
along the (ray) line  $L_{(r, \theta)}$



# Projection

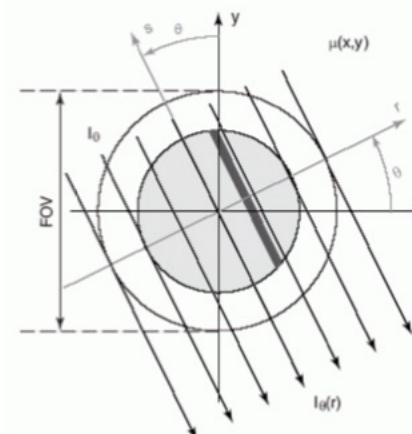
Assuming a fixed angle  $\theta$ , the measured intensity at detector position  $r$  is the integrated density along  $L_{(r,\theta)}$ :

$$\begin{aligned} I_\theta(r) &= I_0 \cdot e^{- \int \mu(x,y) ds} \\ &= I_0 \cdot e^{\int_{L_{r,\theta}} \mu(r \cdot \cos \theta - s \cdot \sin \theta, r \cdot \sin \theta + s \cdot \cos \theta) ds} \end{aligned}$$

For a continuous energy spectrum:

$$I_\theta(r) = \int_0^\infty I_0(E) \cdot e^{\int_{L_{r,\theta}} \mu(r \cdot \cos \theta - s \cdot \sin \theta, r \cdot \sin \theta + s \cdot \cos \theta) ds} dE$$

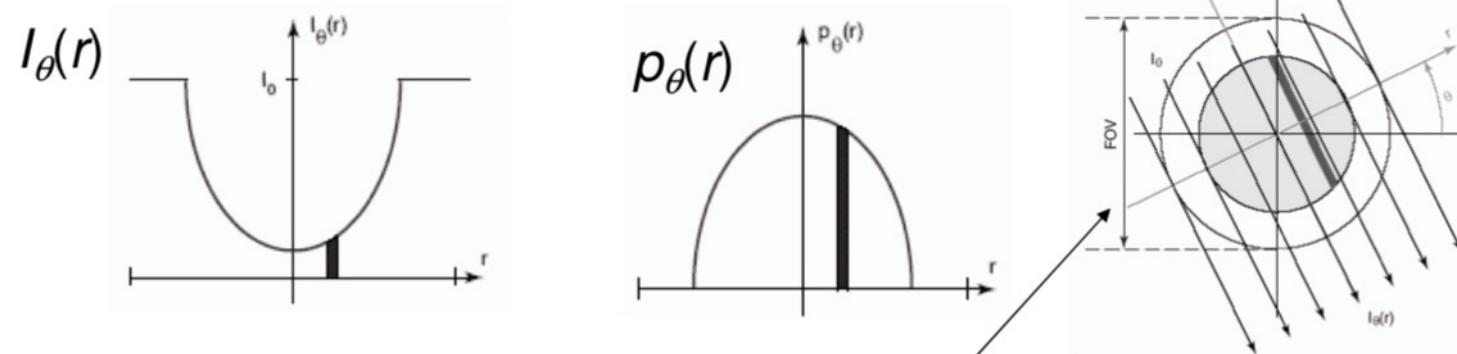
But in practice, it is assumed that the X-rays are monochromatic



# Projection profile

Each intensity profile  $I_\theta(r)$  is transformed into an attenuation profile  $p_\theta(r)$ :

$$p_\theta(r) = -\ln \frac{I_\theta(r)}{I_0} = \int_{L_{r,\theta}} \mu(r \cdot \cos \theta - s \cdot \sin \theta, r \cdot \sin \theta + s \cdot \cos \theta) ds$$



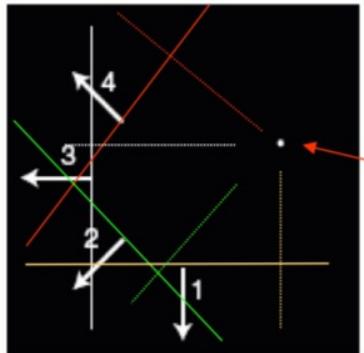
- $p_\theta(r)$  is zero for  $|r| > FOV/2$  ( $FOV$  = Field of View, detector width)
- $p_\theta(r)$  can be measured from  $(0, 2\pi)$
- however, for parallel beam views  $(\pi, 2\pi)$  are redundant, so just need to measure from  $(0, \pi)$

# Sinogram

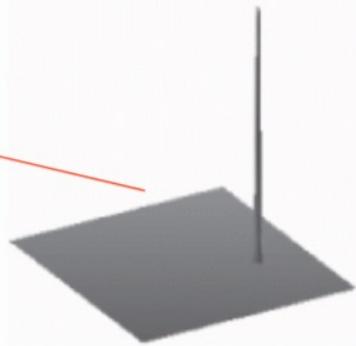
Stacking all projections (line integrals) yields the *sinogram*, a 2D dataset  $p(r, \theta)$

To illustrate, imagine an object that is a single point:

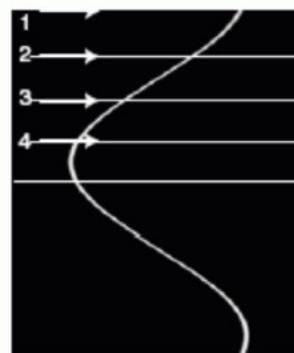
- it then describes a sinusoid in  $p(r, \theta)$ :



projections



point object



sinogram



# Radon Transform

The transformation of any function  $f(x,y)$  into  $p(r,\theta)$  is called the *Radon Transform*

$$\begin{aligned} p(r,\theta) &= R\{f(x,y)\} \\ &= \int_{-\infty}^{\infty} f(r \cdot \cos \theta - s \cdot \sin \theta, r \cdot \sin \theta + s \cdot \cos \theta) ds \end{aligned}$$

The Radon transform has the following properties:

- $p(r,\theta)$  is periodic in  $\theta$  with period  $2\pi$

$$p(r,\theta) = p(r,\theta+2\pi)$$

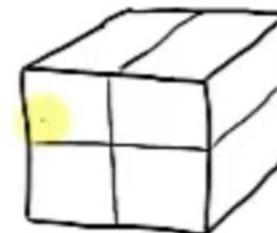
- $p(r,\theta)$  is symmetric in  $\theta$  with period  $\pi$

$$p(r,\theta) = p(-r,\theta \pm \pi)$$

## Resolution

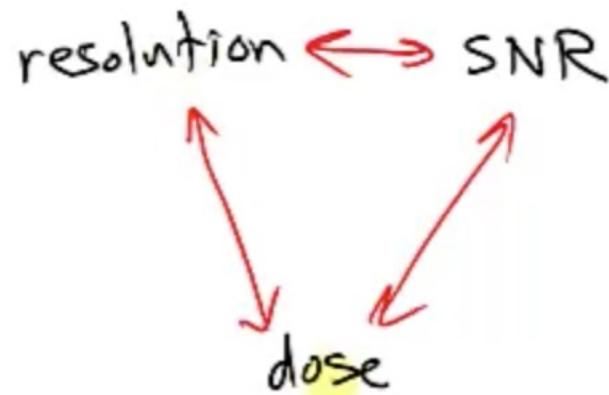
CT scanners designed for human diagnostic imaging are generally capable of producing images with voxels of size  $0.5\text{mm} \times 0.5\text{mm} \times 1\text{mm}$

Typical image size is



## Tradeoffs

We could achieve higher resolution & higher signal-to-noise ratio (SNR) if we could increase the x-ray dose. So CT scans are carefully designed with all 3 factors in mind to minimize the dose, while still acquiring images that serve their diagnostic purpose.

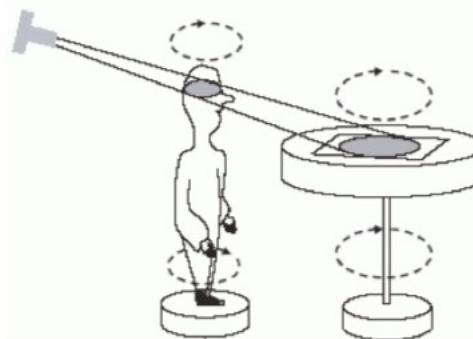


# Reconstruction concept

Given the sinogram  $p(r, \theta)$  we want to recover the object described in  $(x, y)$  coordinates

Recall the early axial tomography method

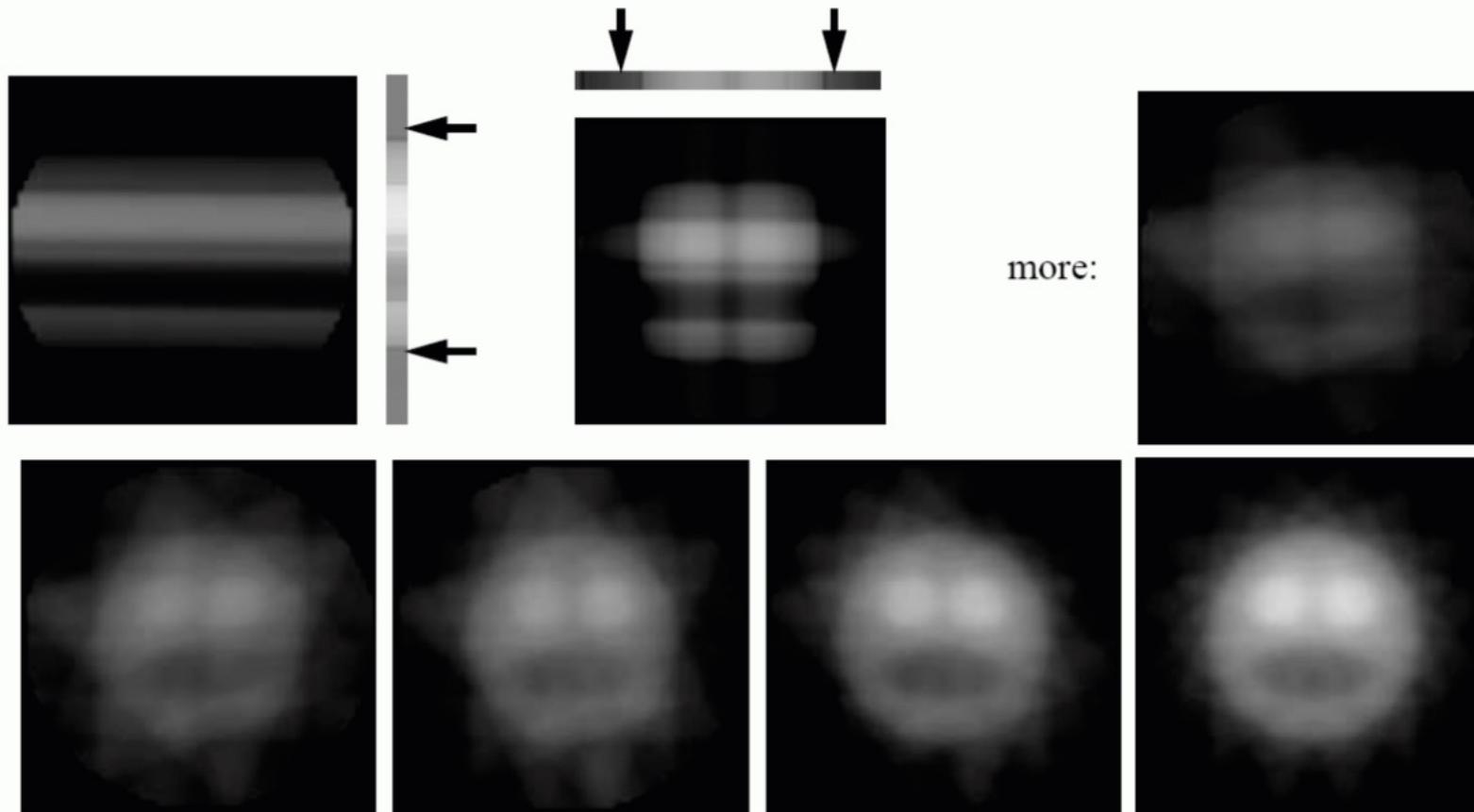
- basically it worked by subsequently “smearing” the acquired  $p(r, \theta)$  across a film plate
- for a simple point we would get:



This is called *backprojection*:

$$b(x, y) = B\{ p(r, \theta) \} = \int_0^\pi p(x \cdot \cos \theta + y \cdot \sin \theta, \theta) d\theta$$

# Backprojection (illustration)



# Backprojection (cont.)

