1 The Model Specification

1.1 Valuation Functions

Upstream Valuation:

$$\Phi^u(\boldsymbol{x}, \boldsymbol{y}, \epsilon, \eta) = \beta_{11}^u x_1 y_1 + \beta_{12}^u x_1 y_2 + \beta_{21}^u x_2 y_1 + \beta_{22}^u x_2 y_2 + \beta_{13}^u x_1 \eta.$$

Downstream Valuation:

$$\Phi^d(\boldsymbol{x}, \boldsymbol{y}, \epsilon, \eta) = \beta_{11}^d x_1 y_1 + \beta_{12}^d x_1 y_2 + \beta_{21}^d x_2 y_1 + \beta_{33}^d \epsilon \eta.$$

1.2 Distribution of Characteristics:

The **observable characteristics** of the upstream and the downstream firms are distributed according to multivariate log-normal distributions.

The **unobservable characteristics** ϵ and η are distributed independently from (x_1, x_2) and (y_1, y_2). I consider two different parametric distributions for the unobservables: (i) Log-normal distribution with parameters $\mu = 0$ and different scale parameters; and (ii) Uniform distribution over $[0, 1]$.

Note that the variances of the unobservable distributions and the coefficients on the unobservable terms in the valuation functions **cannot be separately identified**. Thus, we fix the distribution parameters at a known value and only estimate the coefficients on the unobservables.

The estimator seems to be sensitive to the choice of the parametric distribution and performs better under the uniform distribution. The uniform distribution also has the advantage of being interpreted as quantiles of unobserved CEO's talent.

1.3 Restrictions

We impose the following restrictions for the model to be identified.

1. $\beta_{22}^d = 0$.
2. The parameters of the distributions of unobservable characteristics are known.
3. The sign of the coefficients on the unobservable terms in the valuations functions, i.e. β_{13}^u and β_{33}^d are known.

1.4 Unknown Parameters

The vector of parameters to be estimated is

$$\boldsymbol{\theta} = \left(\beta_{11}^u, \beta_{12}^u, \beta_{21}^u, \beta_{22}^u, \beta_{11}^d, \beta_{12}^d, \beta_{21}^d, \left|\beta_{13}^u\right|, \left|\beta_{33}^d\right|, \kappa\right).$$

Parameter κ is the equilibrium selection rule. Recall that the profits in the continuous matching game are unique up to an additive constant. The equilibrium selection rule κ is defined to be the median profit of the downstream firms, the equilibrium transfers are calculated after shifting the profits so that the median downstream profit is equal to κ. The equilibrium selection rule is one of the parameters in the model and is to be jointly estimated with the other parameters.

2 Correlation between β_{33}^d and κ

We restrict the parameter space to $\left|\beta_{33}^d\right|$ and the equilibrium selection rule κ by fixing all the other parameters at the truth. I estimate $\left(\left|\beta_{33}^d\right|, \kappa\right)$ from 100 replication for sample sizes of N=50, 100, 200, for the two cases of unobservable distributions: (i) Log-normal distribution; (ii) Uniform Distribution.

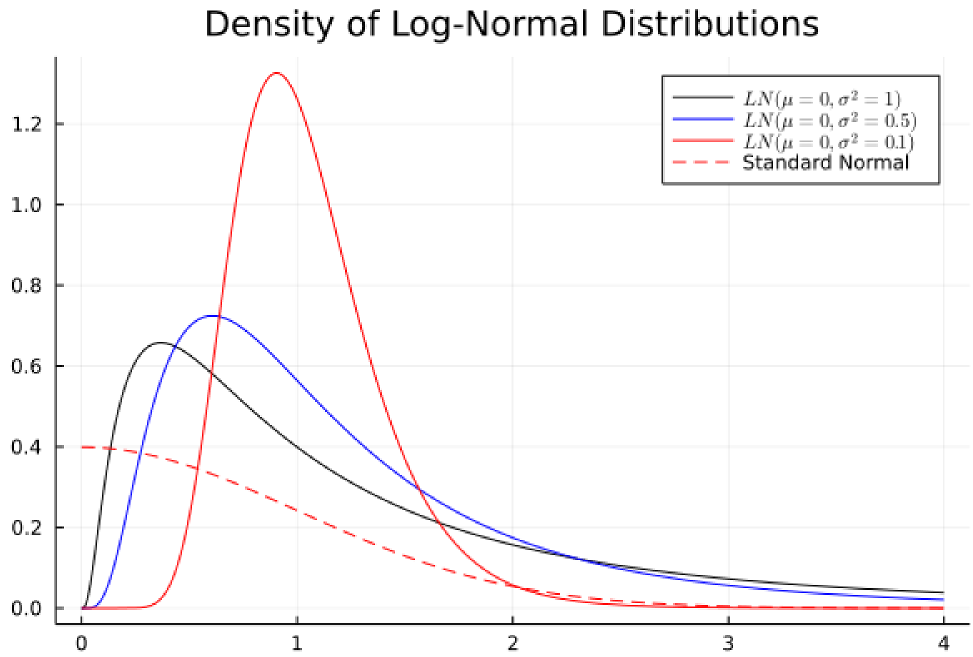

Figure 1: The densities of log-normal distributions with different scale parameter. The dashed line is the density of standard normal distribution.

Sample Size	Distribution of ϵ and η	$Cor\left(\hat{\beta}^d_{33}, \hat{\kappa}\right)$
50	$LN\left(\mu = 0, \sigma^2 = 0.1\right)$	0.994
50	$LN\left(\mu = 0, \sigma^2 = 0.5\right)$	0.951
50	$LN\left(\mu = 0, \sigma^2 = 1.0\right)$	0.912
50	Uniform $(0, 1)$	0.83
100	$LN\left(\mu = 0, \sigma^2 = 0.1\right)$	0.994
100	$LN\left(\mu = 0, \sigma^2 = 0.5\right)$	0.966
100	$LN\left(\mu = 0, \sigma^2 = 1.0\right)$	0.93
100	Uniform $(0, 1)$	0.831
200	$LN\left(\mu = 0, \sigma^2 = 0.1\right)$	0.996
200	$LN\left(\mu = 0, \sigma^2 = 0.5\right)$	0.981
200	$LN\left(\mu = 0, \sigma^2 = 1.0\right)$	0.961
200	Uniform $(0, 1)$	0.767

Table 1: Correlation between $\hat{\beta}^d_{33}$ and $\hat{\kappa}$ in a sample of estimates from 100 replications. All the other parameters are fixed at the truth. The estimator makes use of the same distribution of the unobservables as the true data generating process, i.e. there is no misspecification due to the unobservable distributions. The upstream and downstream unobservables are drawn from identical distributions.