Why not include eta?

η is included, just not interacted with x, i.e. excluded from the partial derivatives. The result in its most general form is

$$\frac{\partial \pi^u (x, \bar{\varepsilon})}{\partial x} = \bar{\Phi}_x \left(x, \bar{\varepsilon}, \boldsymbol{y}, \bar{S}^u (\boldsymbol{x}, \bar{\varepsilon})\right).$$

If we assume the partial derivative of $\bar{\Phi}$ with respect to x does not depend on η, then

$$\begin{aligned}\frac{\partial \pi^u (x, \bar{\varepsilon})}{\partial x} &= \bar{\Phi}_x \left(x, \bar{\varepsilon}, \boldsymbol{y}, \bar{S}^u (x, \bar{\varepsilon})\right) \\ &= \bar{\Phi}_x (x, \bar{\varepsilon}, \boldsymbol{y}),\end{aligned}$$

where the identified object on the right-hand-side is a structural function of the model and does not depend on the equilibrium outcome.

Without this assumption, i.e. allowing the derivative to depend on η, you suggested that we just define

$$\bar{\Phi}_x (x, \bar{\varepsilon}, \boldsymbol{y}) \equiv \bar{\Phi}_x \left(x, \bar{\varepsilon}, \boldsymbol{y}, \bar{S}^u (x, \bar{\varepsilon})\right), \tag{1}$$

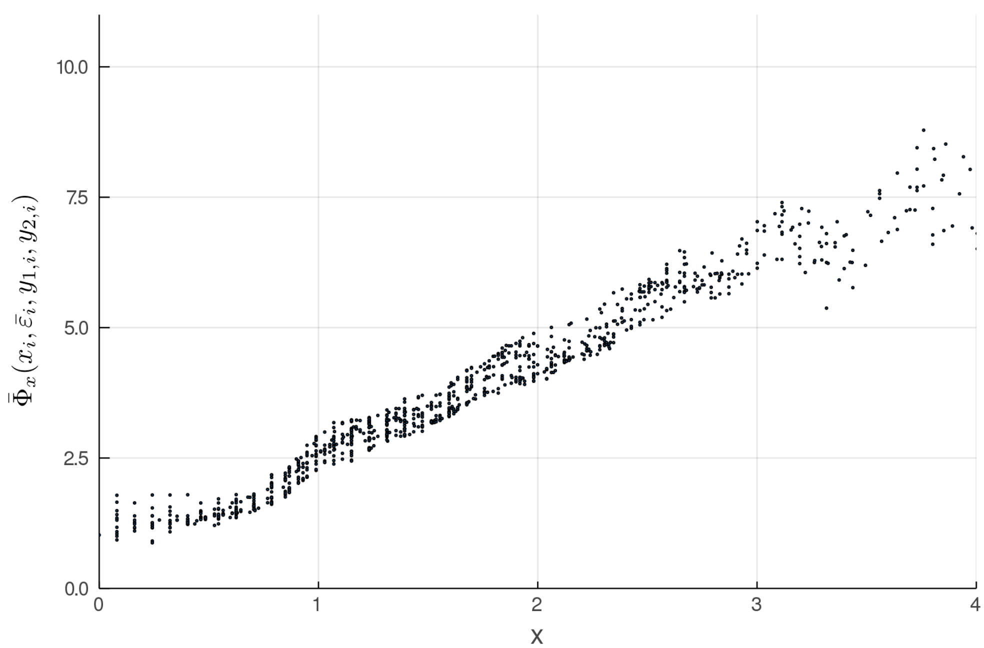
and we have

$$\frac{\partial \pi^u (x, \bar{\varepsilon})}{\partial x} = \bar{\Phi}_x (x, \bar{\varepsilon}, \boldsymbol{y}),$$

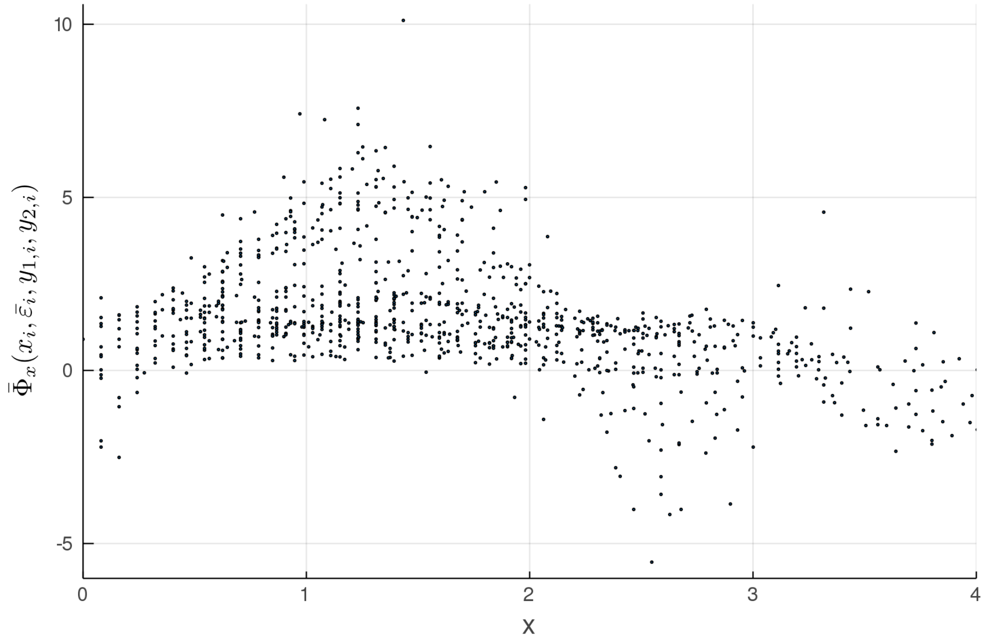
where $\bar{\Phi}$ is defined in (1). However, in this case $\bar{\Phi}_x (x, \bar{\varepsilon}, \boldsymbol{y})$ is no longer a structural function as it depends on the equilibrium outcome $\bar{S}^u (x, \bar{\varepsilon})$. Further, $\bar{S}^u (x, \bar{\varepsilon})$ is not identified in the data.

Could you plot the non parametric derivative rather than doing the parametric least squares exercise?

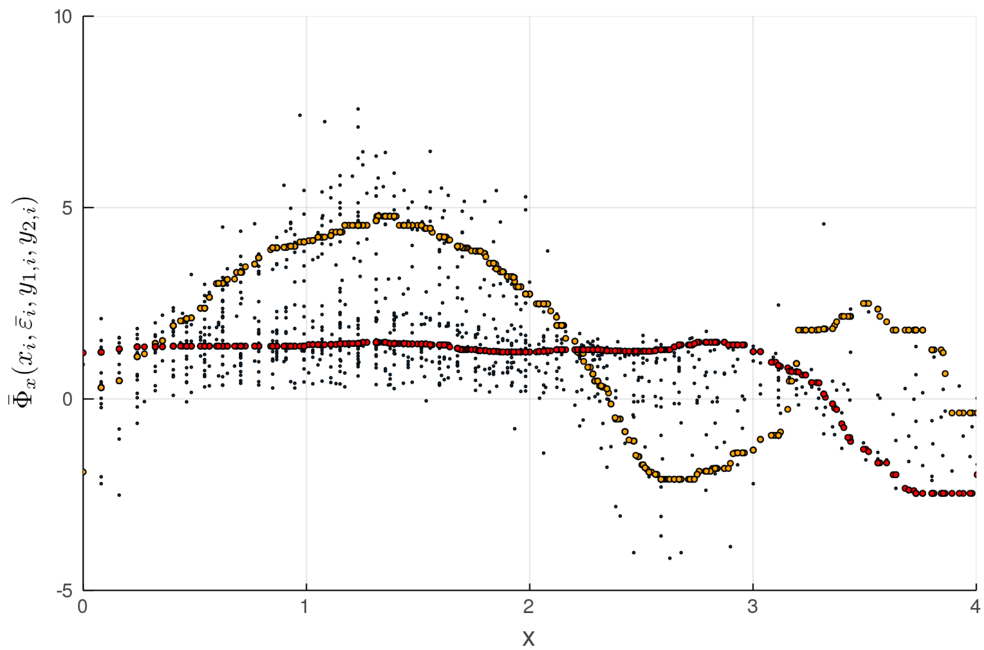
Yes. Here is a plot of partial derivatives from the same Monte-Carlo exercise. The horizontal axis is x_i, and the vertical axis is $\frac{\partial \pi^u (x_i, \bar{\varepsilon}_i)}{\partial x}$ or estimates of $\bar{\Phi}_x \left(x_i, \bar{\varepsilon}_i, \boldsymbol{y} = \bar{T}^u(x_i, \bar{\varepsilon}_i)\right)$.



Here is a plot of the partial derivatives for the data.



Instead of plotting the partial derivatives at $(x_i \bar{\varepsilon}_i)$, I can instead do it for a fixed $\bar{\varepsilon}$ for different x_i, i.e. $(x_i \bar{\varepsilon}_i)$. The orange points are the partial derivatives for $\bar{\varepsilon} = 0.8$, and the red ones are for $\bar{\varepsilon} = 0.4$.



Also, people typically report root integrated mean squared error for estimates of functions.

Correct! I didn't do this initially as I was focused on the MSE of parameter estimates. The IMSEs for the estimates of $\bar{\Phi}_x(x, \bar{\varepsilon}, \boldsymbol{y})$ decrease pretty well with the sample size in the Monte-Carlo exercise.

$IMSE_{250} = 0.97$, $IMSE_{500} = 0.66$, $IMSE_{1000} = 0.40$.