HW2- Gradient Descent method and Newton method

Convex sets and functions

Q1

Let C_1 and C_2 be convex sets, Show that $S = C_1 + C_2$ is a convex set.

Were $C_1 + C_2 = \{x_1 + x_2 : x_1 \in C_1, x_2 \in C_2\}$

Q2

Let $f(x)$ be a convex function defined over a convex domain C. Show that the level set $L = \{x \in C : f(x) \le \alpha\}$ is convex.

Q3

Let $f(x)$ be a smooth and twice differentiable convex function. Show that $g(x) = f(Ax)$ is convex, were A is a matrix of appropriate size.

Check positive semi-definiteness of Hessian.

Q4

Using Jensen inequality, prove arithmetic geometric mean inequality

$$\frac{x_1 + x_2 + \cdots + x_n}{n} \ge \sqrt[n]{x_1 \cdot x_2 \cdots x_n}$$

Q5

Show that if f_1 and f_2 are convex functions on a convex domain C. $g(x) = \max_{i=1,2} f_i(x)$ is a convex function.

Gradient Descent

1. Given Rosenbrock function:

   $$f(x) = f(x_1, x_2, ..., x_N) = \sum_{i=1}^{N-1} [(1-x_i)^2 + 100(x_{i+1} - x_i^2)^2]$$

   Implement in Matlab Gradient Descent method.
   In order to compute step size use Armijo inexact line search with the following parameters:

   $Starting\ point : x = (0, 0, 0..., 0)$

   $Initial\ step\ size : \alpha_0 = s = 1$

   $\sigma = 0.25$

   $\beta = 0.5$

   $\varepsilon = 10^{-5}$

   Use the following stopping criteria: $\|\nabla f(x)\| < \varepsilon$

   Plot the function's convergence curve i.e. plot $f(x_k) - p^*$ as a function of the iteration number k where p^* is the function optimal value.

   Use logarithmic scale for the *y*-axis, i.e. use semilogy for plotting this graph.
2. Repeat question 1 with your own well-conditioned quadratic problem.
3. Repeat question 1 with your own ill-conditioned quadratic problem.

Newton method

4. Implement in Matlab Newton method to find the minimum of Rosenbrock function (see previous question), Use Armijo inexact line search with the same parameters given in question 1 and the same stopping criteria $(\|\nabla f(x)\| < \varepsilon)$
   Use LDL decomposition in order to find newton direction, i.e. in order to solve the following set of linear equations $\nabla^2 f(x) d_{Newton} = -\nabla f(x)$
   Use given function 'mcholmz ' to compute LDL decomposition of the Hessian

   Plot function convergence curve i.e. plot $f(x_k) - p^*$ as a function of the iteration number k where p^* is the function optimal value.
   Plot data with logarithmic scale for the y-axis, i.e. use semilogy for plotting this graph.
   Your plot shows two phases in algorithm convergence rate, mark on the plot where approximately the convergence rate changes from linear to quadratic.
5. Repeat question 4 with your own well -conditioned quadratic problem.
6. Repeat question 4 with your own ill-conditioned quadratic problem.

Remark 1: for all questions Gradient and Hessian should be computed in analytic form. You should test it numerically before using in optimization.

Remark 2: Submit a word document with your analytical computations and all your graphs with a short description.

Remark 3: Submit your Matlab code.