**HW2**

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# Part 1

# 1) Convex sets and functions

### Q1

We define . Now we can define .

So, S is a convex set by definition!

### Q2

We define , meaning, . We want to show that L is convex by definition, meaning :

So, L is convex by definition!

### Q3

In order to show that g is convex by definition we would like to show: :

So, g is convex by definition!

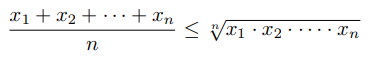
No. we would like to check the Hessian:

From HW1, Q1, we have seen that . So, given a vector , we will get:

And since f is convex, so is f(Ax) (since we can use the transformation y=Ax), it means that is PSD !

### Q4

We would like to show that:



We will use the mean inequality:

Now, we will take exponent on both sides of the equation, and that gives us exactly the desired equality!

### Q5

We define , and we know that are convex functions over , and that by definition of max, to every . So, we will get:

So, if we define , we will get:

Meaning, is convex by definition!

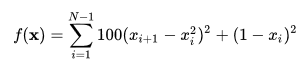
# Part 2

# 1) Gradient descent

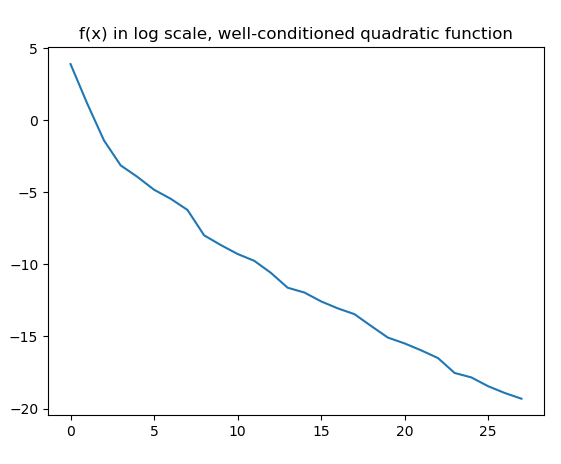
For the quardric function:

,

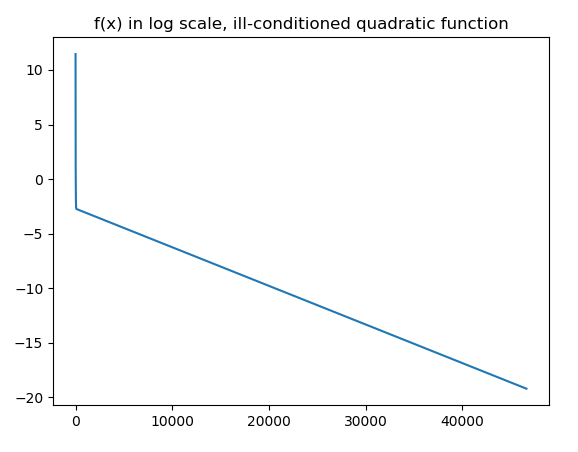
Rosenbrok function:



Grad(x) and H(x) in the apropiate manner (details in the python code).

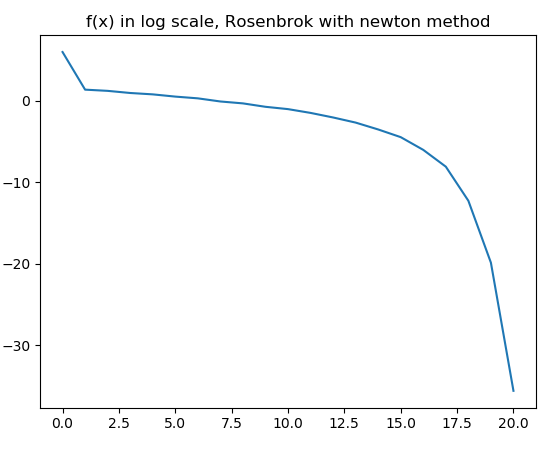
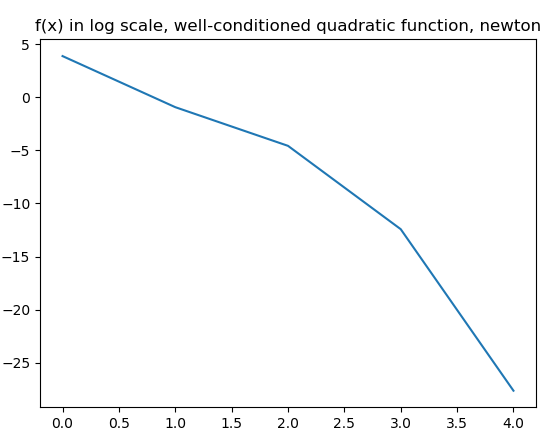
 

Note, for Rosenbrok function we used (-1,-1) as initial point.



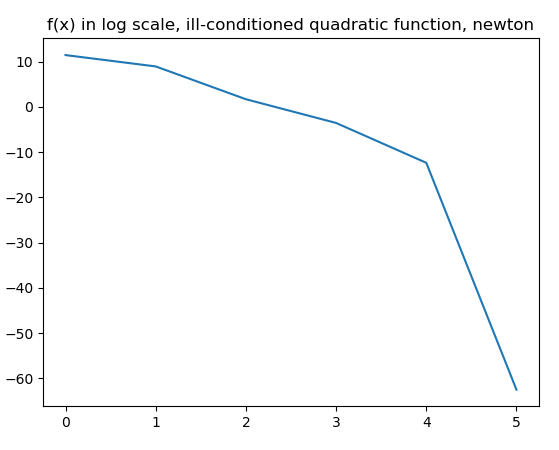
As we can see, hence the function is ill-conditioned, and the gradient directions is just an approximation the convergence is very slow. We can see that hence the gradients are close to zero (eigen value is close to zero) the function value decreases slowly.

# 2) Newton method

As we can see, the Rosenbrok and the well-conditioned quadric function are converging much faster using the newton method then using just first order Taylor expansion.

Note, for Rosenbrok function we used (-1,-1) as initial point.



As we can see, the ill-conditioned quadric function is converging very fast despite being ill-conditioned. This is a result of using the Newton method on a quadric function, that insures that the Taylor expansion is accurate and gets to a global minimum fast.