In The Name of Allah

Pattern Recognition (Spring 2024)

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Homework #2: Density Estimation

Due Date: 1403.01.17

[Part1: Parametric Methods]

A) ML Parameter Estimate

a) Suppose a Normal Distribution $N(\mu_1, \Sigma_1)$ with the following mean and variance.
Note that in reality we do not access to this information.

$$\mu_1 = [0 \quad 5]^T \qquad \Sigma_1 = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

b) Generate N samples with N=10 from this density;
Now, we are going to estimate this density using an ML estimate. For this, we have to suppose a specific distribution for these samples. Based on the above simulation, we assume the density of the samples is Gaussian;
Estimate the parameter of the Gaussian using ML estimate; report the estimated parameters and compare them with the true values.

c) Plot the true density and the estimated density.

d) Repeat Section a and b for 20 times; Compute and report the bias and variance; Compare and discuss about the results.

e) Repeat Section a to d for N=100, N=1000. Discuss the effect of sample size in the performance of the estimation.

f) Repeat all the previous sections with the following mean and covariance; Compare the results with the previous ones.

$$\mu_2 = [5 \quad 0]^T \qquad \Sigma_2 = \begin{bmatrix} 1 & -1 \\ -1 & 4 \end{bmatrix}$$

B) [optional] MAP Parameter Estimate

Consider a Normal Distribution $N(\mu_1, \Sigma_1)$ as defined in Section A. Assume Σ_1 is known and μ_1 is unknown; Estimate the mean parameter of this density using MAP estimate. Suppose the prior of the mean is as follows:

$$P(\mu) = N(\mu_\mu, \Sigma_\mu) = (\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix})$$

note: repeat this section for N=10, 100, 1000

a) Complete the following table; Compare and discuss about the results.

#samples	True mean	Estimated mean		
		10	100	1000
μ_{ML}	$[0 \quad 5]^T$			
μ_{MAP}				

[Part2: Non-Parametric Methods]

C) <u>Non-Parameter</u>

a) Suppose a Mixture Normal Distribution $p(x) = 0.5\ N(\mu_1, \Sigma_1) + 0.5\ N(\mu_2, \Sigma_2)$ with the following means and variances.

$$\mu_1 = [0 \quad 5] \qquad \Sigma_1 = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \qquad \mu_2 = [5 \quad 0] \qquad \Sigma_2 = \begin{bmatrix} 1 & -1 \\ -1 & 4 \end{bmatrix}$$

<u>**Note that in reality we do not access to this information.**</u>

Generate N samples with N=200.

b) Using the generated samples in Section (a), estimate their density via the Parzen Window method for h=0.2, 0.4, 0.8, 1.6; plot the true density and the estimated densities for different h values.
c) Estimate their density via the k-nearest neighbors (KNN) method for k=1, 10, 30; plot the true density and the estimated densities for different k values.
d) Compare and discuss about the results.

Note:

- You are not allowed to employ any available codes from **others** or **on the internet**.
- Prepare a report in PDF format including the figures, answer to the questions and discussions mentioned in the homework.
- Make a folder including your report and your codes (Note that your code is needed to be self-comment)
- Submit all things in a zipped folder named as “YourNameYourFamily - Practical”+ “Exercise Number”+”Student Number”.rar

Good Luck