

Objective

Our objective is to code a pricing routine for a derivative contract paying $(S_T - K)^+$ in USD at a pre-specified expiration date T , where S_T is the price of STOXX50E denominated in EUR and K a given strike price in USD. The contract “knocks-out” if on a specified date $T_1 < T$ the 3-month USD LIBOR is above a known barrier level L^* .

We will simulate:

$$\mathbb{E}^{\mathbb{Q}^d} \left[(S_f(T) - K)^+ \cdot (\mathbb{1}_{L_{T_1 - T_1 + \delta} < L^*}) \cdot \left(e^{-\int_t^T r du} \right) \middle| \mathcal{F}_t \right]$$

1 Inputs:

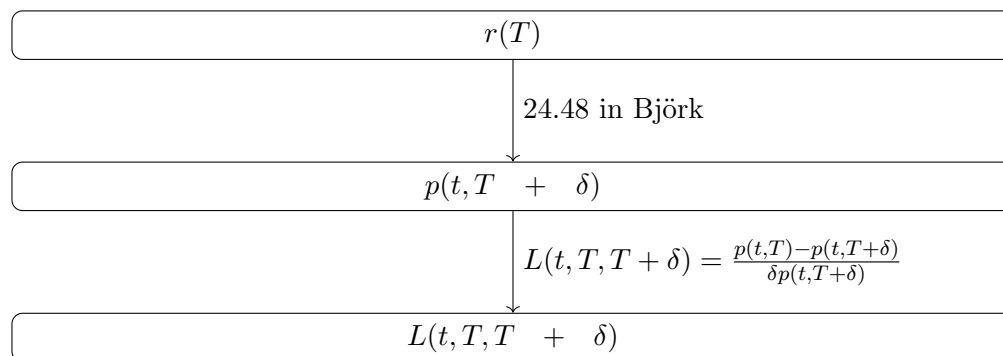
We will apply two-factor Monte Carlo to simulate:

$$\begin{cases} \frac{dS}{S} = (r_f - \rho_{SX}\sigma_X\sigma_S)dt + \sigma_S dW^{\mathbb{Q}^d} \\ dr_d = (\Theta(t) - br)dt + \sigma_r dZ^{\mathbb{Q}^d} \end{cases}$$

where:

- r_f : is the constant foreign exchange rate
- ρ_{SX} : is the historic correlation of the stock and the foreign exchange rate
- σ_X : is the foreign exchange volatility
- σ_S : is the stock volatility. In the foreign currency, we will use a European option price to back out σ_S from Black Scholes *i.e.* we will obtain the implied volatility using a solver because, by Girsanov’s theorem, changing measure does not change volatility.
- σ_r : Assuming known caplet prices (which we will obtain from Bloomberg) we will Use the equation for $C(t, T)$ found on page 386 of Björk to back out σ_r in the same manner as σ_S .ⁱ

2 Order of Operations:



ⁱWe assume volatility is a piecewise step function

3 Formulas:

1. Hull-White Term Structure (24.48 in Björk)

$$p(t, T) = \frac{p^*(0, T)}{p^*(0, t)} \exp \left\{ B(t, T)f^*(0, t) - \frac{\sigma^2}{4a} B^2(t, T)(1 - e^{-2at}) - B(t, T)r(t) \right\}$$

2. Bond Options (24.9 in Björk)ⁱⁱ

$$c(t, T, K, S) = p(t, S)N(d) - p(t, T) \cdot K \cdot N(d - \sigma_p)$$

where

$$d = \frac{1}{\sigma_p} \log \left\{ \frac{p(t, S)}{p(t, T)K} \right\} + \frac{1}{2} \sigma_p,$$
$$\sigma_p = \frac{1}{a} \left\{ 1 - e^{-a(S-T)} \right\} \cdot \sqrt{\frac{\sigma^2}{2a} \left\{ 1 - e^{-2a(T-t)} \right\}}$$

ⁱⁱIn Björk's formula we will assume a mean reversion rate, a , of about 3% or 4%.