IEOR 4735 **Amir Oskoui**
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Objective

Our objective is to code a pricing routine for a derivative contract paying $(S_T - K)^+$ in USD at a pre-specified expiration date T, where S_T is the price of STOXX50E denominated in EUR and K a given strike price in USD. The contract "knocks-out" if on a specified date $T_1 < T$ the 3-month USD LIBOR is above a known barrier level L^*.

We will simulate:

$$\mathbb{E}^{\mathbb{Q}^d}\left[(S_f(T) - K)^+ \cdot \left(\mathbb{1}_{L_{T_1 - T_1+\delta} < L^*}\right) \cdot \left(e^{-\int_t^T r\,du}\right) \Big| \mathcal{F}_t\right]$$

1 Inputs:

We will apply two-factor Monte Carlo to simulate:

$$\begin{cases} \frac{dS}{S} = (r_f - \rho_{SX}\sigma_X\sigma_S)dt + \sigma_S dW^{\mathbb{Q}^d} \\ dr_d = (\Theta(t) - br)dt + \sigma_r dZ^{\mathbb{Q}^d} \end{cases}$$

where:

- r_f : is the constant foreign exchange rate
- ρ_{SX} : is the historic correlation of the stock and the foreign exchange rate
- σ_X : is the foreign exchange volatility
- σ_S : is the stock volatility. In the foreign currency, we will use a European option price to back out σ_S from Black Scholes *i.e.* we will obtain the implied volatility using a solver because, by Girsanov's theorem, changing measure does not change volatility.
- σ_r : Assuming known caplet prices (which we will obtain from Bloomberg) we will Use the equation for $C(t,T)$ found on page 386 of Björk to back out σ_r in the same manner as σ_S.[i]

2 Order of Operations:

$$\begin{array}{c} \boxed{\quad r(T) \quad} \\ \Big\downarrow \text{ 24.48 in Björk} \\ \boxed{\quad p(t, T + \delta) \quad} \\ \Big\downarrow\ L(t,T,T+\delta) = \frac{p(t,T) - p(t,T+\delta)}{\delta p(t,T+\delta)} \\ \boxed{\quad L(t, T, T + \delta) \quad} \end{array}$$

[i]We assume volatility is a piecewise step function

3 Formulas:

1. Hull-White Term Structure (24.48 in Björk)

$$p(t,T) = \frac{p^*(0,T)}{p^*(0,t)} \exp\left\{ B(t,T)f^*(0,t) - \frac{\sigma^2}{4a}B^2(t,T)(1-e^{-2at}) - B(t,T)r(t) \right\}$$

2. Bond Options (24.9 in Björk)[ii]

$$c(t,T,K,S) = p(t,S)N(d) - p(t,T)\cdot K \cdot N(d-\sigma_p)$$

where

$$d = \frac{1}{\sigma_p}\log\left\{\frac{p(t,S)}{p(t,T)K}\right\} + \frac{1}{2}\sigma_p,$$
$$\sigma_p = \frac{1}{a}\left\{1 - e^{-a(S-T)}\right\} \cdot \sqrt{\frac{\sigma^2}{2a}\left\{1-e^{-2a(T-t)}\right\}}$$

[ii] In Björk's formula we will assume a mean reversion rate, a, of about 3% or 4%.