Xoring Networks

In this challenge, we propose to adopt the representation used in Sorting Networks to introduce **Xoring Networks**.

A *Xoring Network* of dimension `n` is a composed of

- `n` wires, each holding one bit (`0` or `1`),
- `m` XOR gates within the `n` wires,
- a final permutation `P` of the `n` wires.

Goal. The goal of the Xoring Network of dimension `n` denoted `XN[n, A, B]` is to transform the sequence `A` of `n`-bit input values `a_i` into the sequence `B` of `n`-bit output values `b_i`. That is, for all `i`, `XN[n, A, B](a_i) = b_i`.

Problem. On inputs `A` and `B`, the **Xoring Network Problem** (XNP) asks to find a Xoring Network that transforms sequence `A = (a_i)` into sequence `B = (b_i)` using **at most** `M` XOR gates. The associated decision problem **DXNP** asks whether a solution exists.

Note. Some XNP instances do not admit any solution, but some may have more than one.

Challenge. In this challenge, you will be asked to find solution for several XNP instances of increasing dimensions.

Format on an example

We now give the textual description for an XNP instance and the expected format for a corresponding solution.

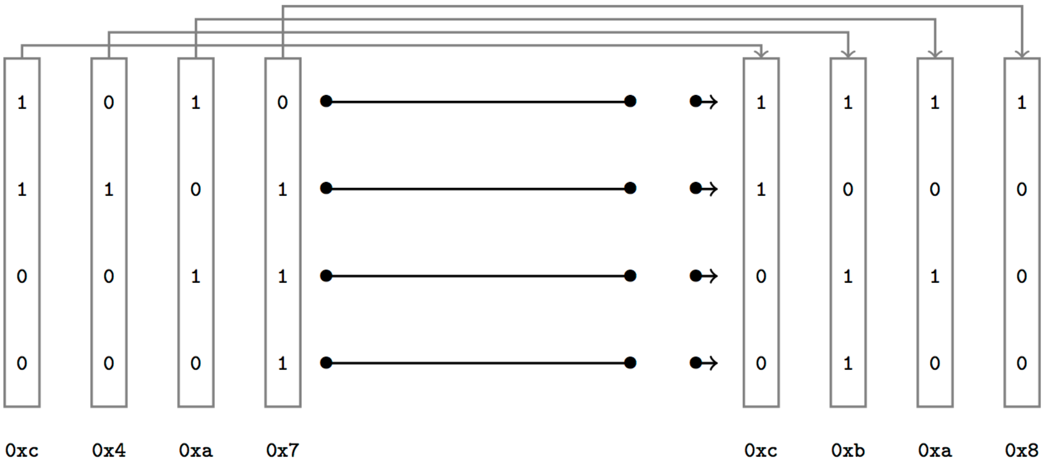
Consider the XNP instance given by

```
--------- BEGIN XNP ---------
4
6
0xc 0xc
0x4 0xb
0xa 0xa
0x7 0x8
---------- END XNP ----------
```

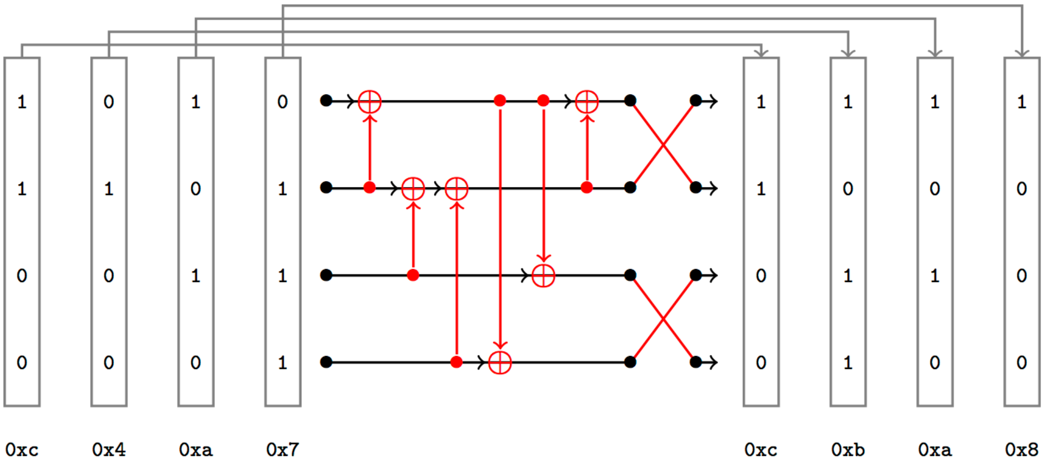
where :

- the first line contains `n` (here, `n = 4`),
- the second line contains `M` (here, `M = 6`),
- the remaining `n` lines contain each pair `(a_i, b_i)` of the sequences `A` and `B`. Here, `(a_0, b_0) = (0xc, 0xc)`, `(a_1, b_1) = (0x4, 0xb)`, etc.

This XNP instance example can be represented by this figure:



A **solution** of this XNP instance is represented in this figure:



This solution is described in textual form as:

```
----- BEGIN XNP SOLUTION -----
6
0 1
1 2
1 3
3 0
2 0
0 1
2
3
0
1
------ END XNP SOLUTION ------
```

where :

- the first line contains the number of XORs `m`, which must be at most `M`. Here, `m = 6`.
- the next `m` lines represent the sequence of XOR gates. Here, XOR Row 1 into Row 0, then XOR Row 2 into Row 1, etc.

- The remaining `n` lines describe the final permutation on the `n` wires. Here, `(3, 2, 1, 0)` is permuted to `(2, 3, 0, 1)`.

Challenge

Upon connection on the remote service for this challenge, you will receive several problems in the textual form described above, for instance,

```
--------- BEGIN XNP ---------
4
6
0x7 0x8
0x4 0xe
0x1 0x9
0xb 0x3
---------- END XNP ----------
```

and you will need to send back one solution for each problem in the same format as above to the server, for instance :

```
----- BEGIN XNP SOLUTION -----
6
0 2
2 1
3 0
1 0
0 3
0 2
3
2
1
0
------ END XNP SOLUTION ------
```

Good luck and have fun!