Relay Coordination Using Linear Programming: A Comparative Study of Standard Inverse and Very Inverse Characteristics

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***Abstract*—This paper presents a linear programming (LP) approach to determine the optimal Time Multiplier Settings (TMS) for overcurrent relay coordination. Both Standard Inverse and Very Inverse relay characteristics are examined. The objective is to ensure selective operation of main and backup relays while minimizing operating times. By comparing results under each characteristic, I analyze the performance in terms of main and backup relay operation times, offering insights for engineers selecting appropriate relay curves and settings.**

***Index Terms*—Overcurrent relays, Linear programming, Relay coordination, Standard Inverse, Very Inverse, Time Multiplier Settings (TMS).**

I. Introduction

Overcurrent relay coordination ensures selective tripping and reliable protection in power systems. The main objective is to find Time Multiplier Settings (TMS) that allow main relays to operate quickly while ensuring backup relays operate correctly if the main relay fails. A Critical Time Interval (CTI) must be maintained between main and backup operations to achieve selectivity.

The problem setup, input data tables, and illustrative practices for overcurrent relay coordination are discussed in [1], where two-level fault currents and the operation of remote side relays are analyzed. This reference helps me understand how pickup currents (I_p) and fault currents (I_f) influence the coordination problem.

Two commonly used inverse-time characteristics are:

- *Standard Inverse*: Lower a and b leading to relatively faster operation for given I_f/I_p.
- *Very Inverse*: Higher a and b producing steeper time-current curves, often resulting in longer operating times for larger I_f/I_p ratios.

Each characteristic is defined by:

$$t = \text{TMS} \times \frac{a}{\left(\frac{I_f}{I_p}\right)^b - 1}. \quad (1)$$

II. Problem Setup and Equations

I consider 14 relays. For each relay i, the pickup current $I_p(i)$ and fault currents $I_{f,\text{main}}(i)$, $I_{f,\text{backup}}(i)$ are given. Backup fault currents are adjusted to ensure:

$$I_{f,\text{backup}}(i) \geq 1.1 \times I_p(i). \quad (2)$$

This assumption ensures the backup relay always sees a current greater than its pickup, preventing degenerate cases.

The coordination constraint:

$$t_{\text{backup}} - t_{\text{main}} \geq \text{CTI}. \quad (3)$$

This guarantees that the backup relay operates later, maintaining selectivity.

I define:

$$M(i) = \frac{a}{\left(\frac{I_{f,\text{main}}(i)}{I_p(i)}\right)^b - 1}. \quad (4)$$

All constraints and the objective function are linearized:

$$A \times \text{TMS} - \text{slack} \leq B. \quad (5)$$

I minimize:

$$\min \sum(\text{TMS}) + 10^6 \sum(\text{slack}), \quad (6)$$

where slack variables handle strict constraints. A large penalty (10^6) ensures slack is used minimally, maintaining realistic solutions.

Two scenarios:

- Standard Inverse: $a = 0.14, b = 0.02$
- Very Inverse: $a = 13.5, b = 1.0$

III. Network and Data

Figure 1 shows the 8-bus network . Relay pickup and fault currents are similarly arranged as in [1]. The following figures show the data tables for pickup currents and fault currents, providing the input parameters for the problem.

IV. Implementation

I solve the LP problem twice:

1) Compute $M(i)$ for chosen (a, b).
2) Construct matrices A, B from instructions mapping $M(i)$ values and conditions into linear constraints.
3) Solve using `optimproblem` in MATLAB. Slack variables ensure feasibility if constraints are tight.

This is done once for Standard Inverse and once for Very Inverse. I assume the linearization is valid and that CTI, TMS bounds, and slack penalties are chosen to achieve a realistic solution.

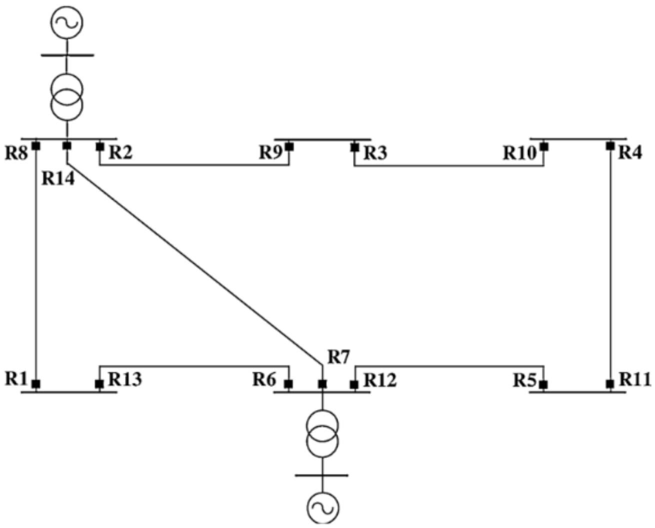

Fig. 1. Single-line diagram of the 8-bus network under study.

I_p (A)	CT ratio	Relays no.	I_p (A)	CT ratio	Relays no.
500	500/5	8	600	400/5	1
600	400/5	9	800	400/5	2
500	500/5	10	500	500/5	3
600	400/5	11	800	400/5	4
500	500/5	12	600	400/5	5
600	400/5	13	500	500/5	6
800	400/5	14	600	400/5	7

Fig. 2. Relay pickup currents and CT ratios .

Backup relays fault current (A)	Main relays fault current (A)	Backup relay no.	Main relay no.	Backup relays fault current (A)	Main relays fault current (A)	Backup relay no.	Main relay no.
1890	6093	7	8	3232	3232	6	1
2484	2484	10	9	996	5924	1	2
2344	3883	11	10	1890	5924	7	2
3707	3707	12	11	3556	3556	2	3
987	5899	13	12	3783	3783	3	4
1874	5899	14	12	2401	2401	4	5
2991	2991	8	13	1874	6109	14	6
996	5199	1	14	987	5223	13	7

Fig. 3. Passing fault currents from main and backup relay pairs.

V. Results and Discussion

Tables I and II present the relay-wise TMS and computed main/backup times for both runs. The Average TMS is a key metric for comparing overall aggressiveness of settings.

For Standard Inverse, the average TMS is about 0.1678, relatively low. This suggests faster overall main operation times. Some backup times are high (e.g., Relay 2), indicating certain conditions or slack usage allow large backup margins.

For Very Inverse, the average TMS is about 0.2602, higher than the Standard Inverse case. This leads to generally slower main operations and, in some cases, extremely long backup times (e.g., Relay 2 at 14.8854 s). The Very Inverse characteristic thus trades off speed for more pronounced time-current behavior.

TABLE I
Standard Inverse Run Results

Relay	TMS	MainTime(s)	BackupTime(s)
1	0.0500	0.2044	0.2044
2	0.1793	0.6144	5.7148
3	0.2948	0.8144	1.5316
4	0.2194	1.0144	1.0144
5	0.3254	1.2144	1.2144
6	0.0500	0.2196	0.2196
7	0.1406	0.4144	0.8544
8	0.2250	0.6144	1.1689
9	0.2088	1.0144	1.0144
10	0.2434	0.8144	1.0860
11	0.1628	0.6144	0.6144
12	0.1498	0.4144	1.5311
13	0.0500	0.2144	0.2144
14	0.0500	0.1835	1.5937

Average TMS: 0.1678

TABLE II
Very Inverse Run Results

Relay	TMS	MainTime(s)	BackupTime(s)
1	0.0500	0.1539	0.1539
2	0.2701	0.5694	14.8854
3	0.6182	0.7694	3.0023
4	0.2474	0.9694	0.9694
5	0.4595	1.1694	1.1694
6	0.0500	0.1775	0.1775
7	0.2512	0.3694	1.5973
8	0.4718	0.5694	2.2911
9	0.2255	0.9694	0.9694
10	0.3856	0.7694	1.4115
11	0.2184	0.5694	0.5694
12	0.2955	0.3694	4.0951
13	0.0500	0.1694	0.1694
14	0.0500	0.1228	2.7551

Average TMS: 0.2602

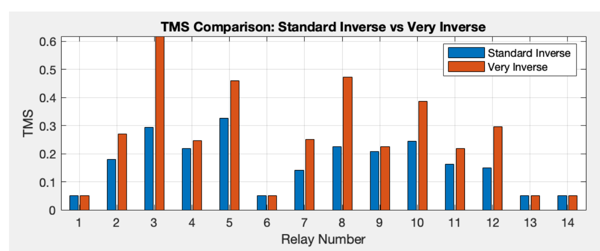

Fig. 4. TMS comparison: Standard Inverse vs Very Inverse. The Very Inverse scenario yields generally higher TMS values.

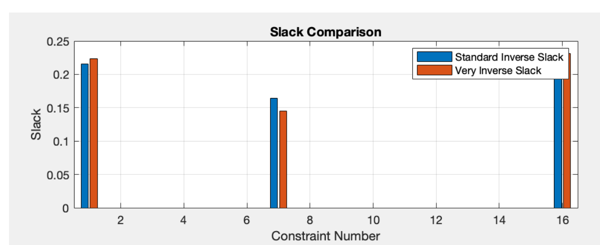

Fig. 5. Slack comparison for both scenarios. Slack indicates tight constraints that required relaxation.

Fig. 4 shows that the Very Inverse scenario generally leads to higher TMS. Fig. 5 reveals slack usage in both scenarios, meaning certain constraints (likely related to maintaining CTI under minimal margin) are challenging to satisfy without relaxation. This is expected in complex coordination scenarios.

VI. Conclusion

By referencing [1] for foundational data and issue clarification, I connect the theoretical relay coordination principles to practical LP-based solutions. The Standard Inverse scenario provides lower TMS and quicker main relay operations. The Very Inverse scenario, while feasible, demands higher TMS, resulting in slower operations and, in some cases, significantly longer backup times.

These insights help me select appropriate characteristics and TMS settings. If rapid clearance is paramount, a Standard Inverse approach may be preferable. If system conditions require a Very Inverse curve, I must accept higher TMS and operation times, but potentially greater selectivity in certain fault conditions. The presence of slack indicates that the constraints are inherently strict, and minor relaxation is necessary to achieve a feasible solution.

References

[1] N. Mohammadzadeh, R. M. Chabanloo, and M. G. Maleki, "Optimal coordination of directional overcurrent relays considering two-level fault current due to the operation of remote side relay," *Electric Power Systems Research*, vol. 175, p. 105921, 2019.

Author Biography

Amirreza Shafiei received the B.Sc. degree from Amirkabir University of Technology, Tehran, Iran, in 2022, in electrical engineering. He is currently a Junior graduate student of Electrical Engineering at Shahid Beheshti University, Tehran. His research interests mainly focus on integrating dynamic security assessment and transient stability assessment in smart grids utilizing artificial intelligence techniques.