Politicians Competition in Persuading Voters

Amirreza Ahmadzadeh* and Pedram Pooyafar† ‡

2025-09-16

Abstract

We study a model of political persuasion in which two politicians with fixed policy platforms compete to persuade a heterogeneous electorate by designing public signals about a two-dimensional state of the world. Each politician commits ex-ante to an experiment that can be informative about only one issue, such as the urgency of climate change or the economic impact of immigration. Voters are Bayesian, care about different issues, and vote based on how the realized state aligns with the candidate's known policy. In equilibrium, when *persuasive advantage*—determined by prior beliefs and the distribution of voter preferences—is asymmetric, the more advantaged politician targets the issue where she has less support to sway voters inclined toward her opponent, while the less advantaged politician mixes across both issues. When persuasive advantages are symmetric, each politician focuses on the issue where she is weaker, but provides minimal information. Furthermore, the model shows that competition in persuasion leads each politician to commit to a specific signal structure in which they tell the truth when the state favors them, and may lie when it favors their opponent. This underscores how strategic communication and heterogeneous preferences shape political dynamics and media reporting, offering insights into the origins of media slant.

Keywords: Information, competition, media, heterogeneous preferences

* Toulouse School of Economics, `amirreza.ahmadzadeh@tse-fr.eu`

† Toulouse School of Economics, `pedram.pooyafar@tse-fr.eu`

‡ We are indebted to our advisor Karine Van der Straeten for guidance and support, to Johannes Hörner, Jean Tirole and Charles Angelucci for all the discussions, and we thank Anna Sanktjohanser, François Salanié, Sara Shahanaghi and Alex Smolin as well as audiences at the TSE Theory workshop for their comments and feedback. All errors are our own.

1 Introduction

In modern democracies, politicians face the dual challenge of persuasion and prioritization. They must appeal to a diverse electorate with heterogeneous preferences, yet they are limited in how they can communicate policy-relevant information. Voters care about different issues—some are motivated by environmental concerns, others by immigration, taxation, or public safety. Against this backdrop, politicians must decide not only *what* to say, but *which* issue to focus on. This paper examines how electoral competition shapes the strategic design of political messaging in the presence of multidimensional uncertainty and heterogeneous voter interests. Specifically, we study how politicians allocate persuasive effort when they can communicate information about only one issue, and how this constraint interacts with voters' preferences to shape equilibrium outcomes.

While a growing literature has studied media markets, political persuasion, and strategic communication, less is known about how competition between partisan information providers shapes the equilibrium pattern of information disclosure.[1] We contribute to this gap by developing a model in which politicians—each committed to a known, fixed policy platform—compete to sway voters by designing public signals about a multidimensional state of the world. Each politician can credibly release information about only one issue, and must decide which aspect of the policy space to emphasize. The uncertainty lies not in what each politician would do if elected, but in the underlying condition of the world that determines which policy is better aligned with voters' interests. For example, consider two policy issues: climate change and immigration. One dimension of the state may concern whether climate change is urgent and requires immediate action. Voters who prioritize environmental policy prefer the candidate who supports strong climate regulation if the state indicates urgency, and the deregulatory candidate otherwise. Meanwhile, other voters care primarily about immigration and base their decisions on whether current immigration levels are economically harmful or not. Voters observe the signal structure and realizations of both politicians' signals before casting their votes. Each politician can credibly design a signal about only one issue. While they must always commit to some signal structure, they may choose to be uninformative.

The model yields two core insights. First, the politician with greater *persuasive advantage* over another—determined by the proportion of supporters for R's policy in one group relative to the proportion of opponents in the other group of voters— tends to take the support of the voters whose issue preferences align more strongly with her platform for granted and does not simply reinforce support among those voters. Instead, she targets the issue on which

[1] See Strömberg (2015).

she is weaker, aiming to shift the beliefs of voters who might otherwise favor her opponent. As a result, the group that most favors the politician with more persuasive advantage may receive no informative communication about the issue they care most about.

Second, how much information is revealed in the equilibrium, which is a one to one mapping to voters' welfare, depends on how balanced the persuasive advantages are. When the two politicians have similar persuasive strength, each tends to specialize in a different issue, and both are more likely to be uninformative—resulting in minimal information disclosure overall. In contrast, when persuasive advantage is skewed—either because one politician dominates in support or because voters are more homogeneous in their issue priorities—both politicians may focus on the same issue and compete by offering more informative signals.

In the model, two politicians compete by designing public signal structures about a two-dimensional state of the world. Each politician commits to a fixed policy platform, and each dimension of the state corresponds to an issue—such as immigration or climate change—on which voters hold common priors. While both politicians observe the structure and realization of the full state only after committing to their strategies, each may design an experiment that conveys information about at most one issue. Importantly, these signal structures may be fully revealing, partially informative, or uninformative, depending on strategic incentives. Voters are Bayesian and observe both politicians' signal realizations before voting. Their preferences are heterogeneous: each voter cares about one issue, and chooses a candidate whose fixed policy position better aligns with the realized state on that issue.

The equilibrium demonstrates how politicians allocate their limited signaling capacity strategically across the two issues. A politician with greater persuasive advantage, tends to take her support for granted and focuses instead on persuading voters who are more likely to favor her opponent. This creates an asymmetry in information provision: the politician's natural base receive no valuable information about the issue they care about most as there is no competition on that issue between politicians. When persuasive advantages are similar across politicians, each focuses on the issue where they have relatively less support, rather than reinforcing their base. This mutual targeting leads to an equilibrium in which both politicians send signals that are persuasive but least informative, just to make the voters vote for them in the case of the signal realizations being in their favor. As a result, the information disclosure decreases. In the extreme case, when no politician has no persuasive advantage over another, voters are indifferent between the equilibrium outcome and receiving no information.

The equilibrium behavior has direct implications for media slant. Since each politician commits to a signal structure ex-ante and can only credibly signal on one issue, she chooses to be fully truthful only when the realized state supports her position. Although all voters are Bayesian and update rationally, the result is a skewed pattern of information provision: voters consulting a particular politician's signal are more likely to observe a realization favorable to that politician. Thus, the appearance of media slant in this model emerges not from misinformation or bias, but from the endogenous asymmetry in how information is disclosed and on which issue.

The rest of the paper is organized as follows. The next subsection discusses the related literature. Section 2 introduces the model, while section 3 gives more insights into voters' behavior by providing simple examples and section 4 simplifies the politicians' problem. Our main result is in section 5 in which we characterize the equilibrium, and section 6 provides some extensions of the model. Section 7 concludes the paper.

1.1 Literature review

The paper is related to the literature on Bayesian Persuasion with multi senders. Gentzkow and Kamenica (2016, 2017) define a Blackwell-connected environment if for any profile of others' strategies, each sender has a signal available that allows her to unilaterally deviate to any feasible outcome that is more informative. If the information environment is Blackwell-connected, any individual sender can generate as much information as all senders can do jointly. They show that every pure-strategy equilibrium outcome is no less informative than the collusive outcome (regardless of preferences) if and only if the information environment is Blackwell-connected. Our environment is not Blackwell-connected since each sender can be informative only about one issue.

Au and Kawai (2020) analyze a model of competition in Bayesian persuasion in which multi-senders persuade a receiver. Each sender privately observes his own type and can disclose information about his type (so the information environment is not Blackwell-connected). In our setting, both senders observe the two-dimensional state, and can disclose only one of them (in particular they can disclose information about the same dimension). Another difference is that in our setting there are two receivers. If we had one receiver, then both senders would focus on the same dimension. In this special case, the information environment is Blackwell-connected, and truth-telling (full disclosure) becomes the unique equilibrium. Au and Kawai (2021) consider a similar setting with only two senders. However, types are correlated.[2]

Another relevant literature pertains to papers that explore sender-receiver games where

[2]Finally, the common differences between this literature and our paper are multiple receivers with heterogeneous preferences.

the receiver's attention is limited. A closely related paper is Knoepfle (2024) in which there are many partially informed senders who dynamically compete for a decision maker's attention. The senders are assumed to choose their experiments (commitment for each period) before observing their signals. The receiver wants to match his action to the state, and senders only care about how many times the receiver paid attention to them. There are two main differences. First in our paper senders care about the final action of the receivers. Senders do not care if a receiver listens (experiments) to another sender, as long as the receiver chooses the favorable action. Second, in our paper, there are multiple receivers with heterogeneous preferences for different issues. While we focus on the case that receivers observe both signal structures, we show that our equilibrium is robust to assuming inattentive voters.

Another paper that studies media competition with limited attention is Innocenti (2022). In Innocenti (2022) receivers have heterogeneous prior beliefs about the state and can devote attention only to one media (sender). There are two main differences. First receivers have heterogeneous prior beliefs, as opposed to ours where receivers have common prior but heterogeneous preferences. Second, in Innocenti (2022) senders choose their experiments and at the same time (as opposed to ours) as receivers choose to devote attention to which sender. The paper argues that if the allocation of attention is chosen after persuasion takes place truth-telling is the equilibrium policy. Papers such as Che and Mierendorff (2019), and Leung (2020) consider an exogenous information environment (senders do not design an information structure), and study the receiver's problem with limited attention.

Our framework contributes to the literature on the supply-side driven bias. Gehlbach and Sonin (2014) highlights the interest of the government in "mobilizing" the individuals to take actions that are in favor of a political party but not the citizens' best interest. This force determines the degree of media bias in their model. Closest to their framework is Besley and Prat (2006). The theoretical framework is also close to Gentzkow and Shapiro (2006) as they both use Bayesian individuals, but the question that they answer is different in a sense that they focus on the trade-off between advertising revenue and bias. Chan and Suen (2009) considers policy preferences for media outlets. Their model is different from the latter two as they endogenize the choices available for the voters by the political parties' decisions. The third-party player, the media, has an exogenous policy preference. Another example is Anderson and McLaren (2012) where they show how politically motivated publisher can persuade Bayesian consumers.

The key assumption in our model and in this literature is that the decisions made by the owner or manager of the outlet cannot be swiftly changed without jeopardizing the credibility of the outlet. As a result, the players choose their "editorial policies" and then commit to them. Gehlbach and Sonin (2014) implicitly captures the delegation of what news to cover and how to do it to the reporters and editors. They assume the government considers the desired level of bias and structures the news operation based upon it by choosing a public editorial policy prior to the realization of the state of the world. Duggan and Martinelli (2011) has also a similar assumption in their model of electoral competition and media slant. They develop the notion of slant as filtering a multi-dimensional political reality into a one-dimensional policy spectrum and assigning relative weights to different policy issues in doing so. They see this bias as the outcome of editorial decisions made by the media outlet, however, they are agnostic about the mechanisms which are responsible for the bias. A related model is Chan and Suen (2008) in which they model the editorial policy of the media as only reporting whether a received signal is higher or lower than a specific threshold. Mullainathan and Shleifer (2005) study the role of competition and reader diversity in news accuracy. The newspapers in their model choose their slanting strategy first and then readers observe these choices and decide which one to consult.

In line with the literature, we assume Bayesian voters who want to match their votes with the state of the world, and we also consider the case that they cannot follow all the news there is in the extension. Gehlbach and Sonin (2014) and Duggan and Martinelli (2011) both assume that citizens can watch at most one news station due to the cost of information-processing. They both consider Bayesian voters who take into account the bias of the media when updating their beliefs. Chiang and Knight (2011) provides empirical evidence for this assumption. Moreover, they are assumed to vote sincerely. In the previous models, as well as Chan and Suen (2009, 2008) and Besley and Prat (2006) voters search for information to be able to make a better voting decision while the information is endogenously provided by the media.

The voters' rationality and the slant of the media in the equilibrium is reminiscent of Suen (2004). A voter who is inclined to vote for the Republicans has little interest in consulting a strongly Democratic newspaper even if they are aware the editors have insider information about the candidates. The voter assumes that the newspaper is likely to endorse the democratic party, and in the case where it does not, it would not change the vote of the voter. Such a rational demand for information determines why voters prefer the like-minded media. Closely related, Burke (2008) endogenizes the media market in a dynamic setting and shows that monopoly may provide unbiased news whereas in the presence of competition, highly biased news is provided whether the population has homogeneous priors or heterogeneous ones. Their focus is on viewership as it is assumed that the revenue of firms is strictly increasing in the number of viewers.

Another relevant literature focuses on models that examine voters with diverse preferences, considering the heterogeneity among individual voters. Maskin and Tirole (2019) develop a model of pork-barrel politics in which a government official tries to improve her reelection chances by spending on targeted interest groups. There are two main differences. First there is no explicit competition between politician and second there is no media competition. In Perego and Yuksel (2022) a finite identical number of firms compete to provide information to a finite number of Bayesian agents about a newly proposed policy with uncertain prospects. The probability of implementing the new policy depends on approval rate. Much like the present study, each politician (referred to as a "firm" in their paper) encounters limitations on how much information they can provide regarding various aspects of the state. The key distinction between the present paper and Perego and Yuksel (2022) is that the politicians are indistinguishable. They exhibit no bias and possess no advantage in implementing particular policies.

2 Model

There is a two-dimensional state of the world $\omega \in \Omega$. Each dimension corresponds to one issue of interest. Some examples of the issues can be foreign policy, environmental, and immigration. The state of the world with respect to any dimension can take two variables: $\{\ell, r\}$. For instance, people might wonder if climate change is real or not, and if it is real, the state is ℓ. The state of the world regarding each issue is the answer to the question: Which policy is better? For instance, the question can be whether it is better to build a wall to stop immigration or have better social security for immigrants. Denote two dimensions as $\omega = (\omega_1, \omega_2) \in \Omega = \Omega_1 \times \Omega_2 = \{\ell, r\}^2$.

There are two politicians L and R indexed by j who share a common prior $\mu = (\mu_1, \mu_2)$ with $\mu_i = Pr(\omega_i = r)$. Politicians play a simultaneous move game by designing and committing to an experiment that is informative about at most one dimension of ω.[3] Formally, consider the set of available signals on each issue $i \in \{1, 2\}$ by S_i. Then each politician j chooses an experiment $\pi_i^j \in \Pi_i$ consisting of a finite realization space S_i, which determines the focus of the experiment, and a family of likelihood functions over S_i, $\{\pi^j(\cdot|\omega_i)\}_{\omega_i \in \Omega_i}$, with $\pi^j(\cdot|\omega_i) \in \Delta(S_i)$. We allow for mixing, and denote each politician's action by $a_j \in \Delta(\Pi_1 \cup \Pi_2)$. We also put a restriction on the strategy space of players. We restrict each player's mixed strategy to be the union of finite disjoint full support intervals. Throughout the paper, without loss of generality, we restrict our attention to signal spaces that coincide with the state spaces, i.e. $S_i = \Omega_i$. Each politician after getting elected, implements fixed policies

[3]This captures the fact that newspapers or online news agencies can have only one main headline each day. Or news agencies can only cover one main topic each day.

specific to her. L would implement left-wing policies on both issues and R would implement right-wing policies.

There is a mass 1 of voters who share the same common prior μ as politicians. Each voter observes the outcomes of both experiments as well as the signal realizations and then decides to vote for one politician. Formally, the voter k observes π^L, π^R, subsequently signals $s^L, s^R \in \{s_\ell, s_r\}$, then votes $v_k \in \{L, R\}$.

Voters are heterogeneous in their preferences. Specifically, there are two groups of voters: G_1 with a share m_1 of the population and the group G_2 with share $m_2 = 1 - m_1$. Every voter in group G_i only cares about issue i and wants to match the politician with the state of the world in dimension ω_i.[4] Denote the utility of a voter in group G_i, when politician j is chosen by $u_j^{\omega_i}$. Assume $u_L^{\ell} = u_R^{r} = u > 0$[5], and otherwise 0. Throughout the paper we assume *sincere voting*, i.e. every voter votes as if she is the only voter and she is pivotal.

Politicians are vote-seekers, i.e. they want to maximize their expected share of votes. Ex-post payoff of politician R facing the vector of votes V is:

$$U_R(V) = \int \mathbb{1}_{v_k=R} \, dk, \text{ where } V = (v_k)_{k \in \{G_1 \cup G_2\}}$$

As all voters vote in this model, politicians are playing a constant-sum game and the ex-post payoff of politician L is simply $U_L(V) = 1 - U_R(V)$.

The timing of the game is depicted in figure 1. First politicians simultaneously choose a distribution over the set of all available experiments. Then, their choices are realized, and each voter observes the focus of each experiment as well as its signal structure. The voter then consult both experiments, and one signal for each experiment is going to be realized. Voters update their posteriors according to the Bayes rule and then vote for one politician. One politician would get elected according to the majority rule and would implement the fixed policies associated with her. Finally, the state of the world and payoffs are realized.

We assume a favorable tie-breaking rule wherever possible. Specifically, when a voter is indifferent between the two politicians after consulting both experiments, he chooses to vote for the designer whom his prior is closest to on the issue that he cares about.

We impose a notion of *sequentially rationality* on voters' behavior in any information set of the game by assuming sincere voting. Formally, we assume that voters always consult both experiments an update their beliefs according to Bayes' rule in all the infinitely many information sets available to them. Moreover, they always vote according to their posterior.

[4] For instance one politician wants to build a dam and another wants to build a wind turbine and voters who care about environmental issues want to know which policy is better.

[5] In other words, it is assumed that if the politicians' advantages in implementing policies are symmetric from the voters' point of view.

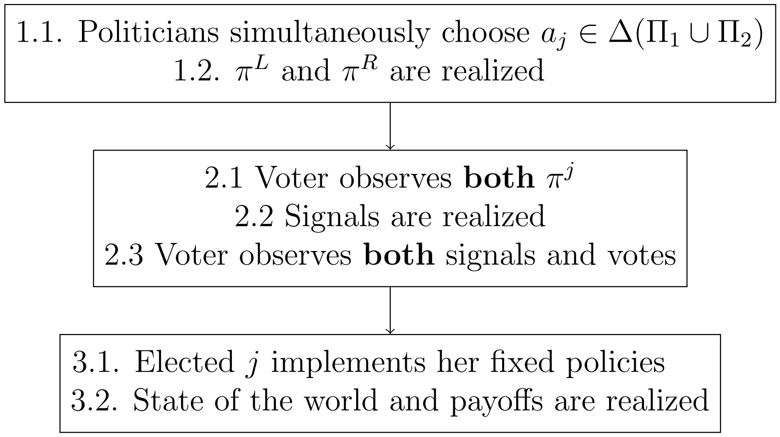

Figure 1: Timing

Fixing the voters' behavior as above, we focus on the Bayes-Nash equilibria of the politicians' game. Any equilibrium is then characterized by a tuple (a_L, a_R) in which a_j is politician j's strategy.

3 Voter's Behavior

Before analyzing the politicians' problem, let us study a special case to better understand the voter's behavior. Suppose that there is one issue, one voter, and two politicians. Moreover, assume that each politician reports according to the signal structure in table 1, i.e. they always tell truth if the state happens to be in their favor.[6]

(a) Experiment chosen by L: π^L

state/signal	s_ℓ	$s_{\mathscr{r}}$
ℓ	1	0
$\mathscr{r}$	$1-\lambda^L$	λ^L

(b) Experiment chosen by R: π^R

state/signal	s_ℓ	$s_{\mathscr{r}}$
ℓ	λ^R	$1-\lambda^R$
$\mathscr{r}$	0	1

Table 1: Signal structure of the experiments

Assume that the voter has a prior belief $\mu = Pr(\mathscr{r}) > 1/2$. In the absence of L, denote the expected utility of the voter when facing π^R and R chooses λ^R as $v^R(\lambda^R)$:

$$v^R(\lambda^R) = Pr(s_\ell)u + Pr(s_{\mathscr{r}})Pr(\mathscr{r}|s_{\mathscr{r}})u = (1-\mu)\lambda^R u + \mu u$$

The voter knows that when she observes a signal s_ℓ in favor of L from the experiment of R, then the state is ℓ with probability one. This is the conclusive signal in π^R. Hence she

[6]Later we show this restriction is without loss of generality.

votes for L and guarantees the utility u. If she receives the inconclusive signal s_r, which happens with $Pr(s_r)$, her posterior toward R becomes more extreme, and she votes for R and gets expected utility $Pr(r|s_r)u$. Therefore if $\lambda^R = 0$, R provides no information and the voter gets the expected utility of μu as she only votes according to her prior belief which is in favor of R. If R chooses $\lambda^R = 1$, then the experiment is fully revealing the state and provides expected utility of u for the voter.

In the absence of R, denote the expected utility of the voter voter when L chooses λ^L as $v^L(\lambda^L)$ which is:

$$v^L(\lambda^L) = \max\{Pr(s_r)u + Pr(s_\ell)Pr(\ell|s_\ell)u, Pr(r)u\} = \max\{\mu\lambda^L u + (1-\mu)u, \mu u\}$$

Again, the voter knows that if π^L sends a signal in favor of R, she learns state, votes for R, and secures utility u. However, if she receives signal s_ℓ, she votes for L only if her posterior becomes less than 1/2. In other words, if L does not provide her with a value greater than μu, which is the expected utility of voting without any information, the voter always votes for R. Intuitively, providing information can never make the voter worse off as she is Bayesian. If λ^L is big enough, she changes her vote to L upon receiving the signal s_ℓ and gets expected utility $Pr(\ell|s_\ell)u$. Note that here, if $\lambda^L = 1$, we are again in the fully revealing case, and the voter gets utility u, but if $\lambda^L = 0$, the voter would rather vote for R and get the same expected utility as voting in the absence of the information, which is μu.

The following lemma helps us pin down the voter's decision when she is facing both experiments by the politicians. As the voter is Bayesian, her decision only depends on whether her posterior is more in favor of R or L. The lemma states that only the experiment that provides her the higher value matters, as the other experiment will not provide her valuable information in a sense of changing her decision upon realizing any signal.

Lemma 1 ***(Voter's Behavior)***
For any pair of (λ^L, λ^R), the voter's voting behavior when facing both experiments is the same as if she is facing only one of the experiments that provides $\max\{v^L(\lambda^L), v^R(\lambda^R)\}$.

The intuition is as follows: Fix the prior $\mu > 1/2$. If the voter observes the conclusive signal, i.e. a signal that fully reveals the state, in any experiment, then the result follows from the fact that the voter learns the state perfectly. Moreover, if $v^L(\lambda^L) = \mu u$, then any experiment π^R provides weakly higher expected utility ($v^R(\lambda^R)$) than π^L, and since consulting π^L never ends up in voting for L, π^R dictates the behavior of the voter alone. The more interesting case is when $v^L(\lambda^L) > \mu u$, and the voter observes signals s_r from π^R and s_ℓ from π^L. Intuitively, receiving s_ℓ from π^L cannot be strong enough to change the posterior of the voter in favor of L after receiving s_r from π^R which is in favor of R. Otherwise, π^L would have brought her more expected value in the first place. In appendix A we provide full proof for lemma 1 when we do not restrict the experiments to the form of table 1.

We point out two remarks with respect to the previous lemma. First, as it is shown in the proof, it holds in a more general case in which politicians use any forms of experiments and the voter has any non-degenerate belief. Note that how the idea in the second part of the proof is independent of the signal structures, but just what value they bring for the voter. Second, restricting voters to choose only one experiment before the signal realization would not distort politicians' incentives as voter's decision remains the same whether she has access to the signals of both experiments, or she could choose to consult only one of the experiments.[7]

4 Politicians Problem

4.1 Experiment design

In this subsection, we plan to simplify the politician's problem. Note that every experiment $\pi_i \in (\Pi_1 \cup \Pi_2)$ can be characterized by a tuple (α_i, β_i) shown in table 2 where $\alpha_i, \beta_i \in [0, 1]$.

state/signal	s_ℓ	s_r
ℓ	α_i	$1-\alpha_i$
r	$1-\beta_i$	β_i

Table 2: an arbitrary experiment $\pi_i \in (\Pi_1 \cup \Pi_2)$

The following lemma helps us to characterize each experiment by only one variable.

Lemma 2 ***(Outcome equivalence)***
For any equilibrium of the game (a_L^*, a_R^*), there exists an equilibrium (b_L^*, b_R^*) such that the outcome of the game, i.e. politicians expected utilities and voters choices, is the same and every experiment $\pi_i^j \in supp(b_j^*)$, with $i \in \{1, 2\}$ and $j \in \{L, R\}$ has the signal structure in the form of table 3.

[7]There is ample evidence that voters cannot collect all the information there is. In our model, such inattentiveness can be added by adding an extra step of voters observing the choices of both experiments as before, but have to choose one to see a signal. Assuming sincere voting again, they will choose the one that provides them the higher expected utility, so the politicians' problem remains exactly the same as before.

(a) L-biased experiment source: π_i^L

state/signal	s_ℓ	s_r
ℓ	1	0
r	$1-\lambda_i^L$	λ_i^L

(b) R-biased experiment: π_i^R

state/signal	s_ℓ	s_r
ℓ	λ_i^R	$1-\lambda_i^R$
r	0	1

Table 3: Signal structure of the experiments on issue i

See appendix B for the proof.

lemma 2 allows us to restrict the choice set of politicians to experiments in the form of table 3. Politician L chooses only from the L-biased[8] experiments π^L and R chooses from the R-biased experiments π^R. With an abuse in notation, we denote each experiment π_i^j by $\lambda_i^j \in [0,1]$ that uniquely characterizes that experiment.

From now on, to make the problem more interesting assume that the prior on each issue is closer to a distinct politician. More specifically consider the case that $\mu_1 < 0.5 < \mu_2$. We denote issue 2 a *favored* issue for politician R and the issue 1 an *unfavored* issue for her.

To be able to compare politicians' choices, for any λ_i that L chooses for her π_i^L, consider the dagger version of the variable, $\lambda_i^\dagger$ as the choice for R that characterizes her π_i^R such that voters from group G_i are indifferent between choosing π^L and π^R, i.e. the value that both experiments bring for a voter from that group is the same: $v^L(\lambda_i) = v^R(\lambda_i^\dagger)$. The relation between these two variables are:

$$\lambda_i^\dagger = 1 - \frac{\mu_i}{1-\mu_i} + \frac{\mu_i}{1-\mu_i}\lambda_i, \qquad \lambda_i = 2 - \frac{1}{\mu_i} + \frac{1-\mu_i}{\mu_i}\lambda_i^\dagger, \qquad i \in \{1,2\}$$

This notation is going to help us write the politicians' expected utility more intuitively. To give more insight about the relation between these two variables, note that this relation is always linear, and for $\mu_i > 1/2$ we have $\lambda_i^\dagger < \lambda_i$ which shows if the prior is more towards R, then L has to choose a greater value of λ_i to make the voter indifferent between her experiment and the one designed by R with $\lambda_i^\dagger$. When the prior is less than 1/2, then R needs to provide greater value $\lambda_i^\dagger$ to make the voter indifferent and we have $\lambda_i^\dagger > \lambda_i$.

As a special case, let us assume there are only have two voters, one in G_1 and another in G_2. So half of the population, $m_1 = 1/2$ is in favor of L and another half in favor of R. An example of the expected payoff for L when the strategy profile is $(\lambda_2^L, \lambda_2^R)$ is as follows:

$$\mathbb{E}U_L(\lambda_2^L, \lambda_2^R) = 1/2 + 1/2 \times \left((1-\mu_2)\lambda_2^R + \left[1 - \mu_2\lambda_2^L - (1-\mu_2)\lambda_2^R\right]\mathbb{1}_{\lambda_2^L \geq \lambda_2^{\min}, \lambda_2^{\dagger,L} > \lambda_2^R}\right)$$

[8]Note that we assume that the voter is Bayesian, so the experiment cannot systematically bias her belief. The bias in the platforms mainly refers to the frequency of a signal favoring one state.

where $\lambda_2^{\min}$ is such that the value of π^L for the voter in G_2 is $v^L(\lambda_2^{\min}) = \mu_2 u$. The first part of the RHS, 1/2, is denoting what the voter in G_1 does when she is confronting the two experiments on issue 2. Since she only cares about issue 1, she votes for L in the absence of information on issue 1. So half of the population for sure is going to vote for him.

The second part depends on the competition on issue 2. If L does not provide enough information to persuade voters, i.e. $v^L(\lambda_2^L) < \mu_2 u$ which is equivalent to $\lambda_2^L < \lambda_2^{\min}$, or if he does not provide more information than his opponent, $v^R(\lambda_2^{\dagger,L}) < v^R(\lambda_2^R)$ or equivalently$\lambda_2^{\dagger,L} < \lambda_2^R$, then the voter votes as if she is only consulting the media of the right politician and hence, the expected share of $(1/2)(1-\mu_2)\lambda_2^R$ would vote for L. Otherwise, the voter behaves as if she is only listening to L's media and the expected share of votes for him would become $(1/2)(1-\mu_2\lambda_2^L)$. Because of the constant-sum nature of the game, the expected payoff for R is $\mathbb{E}U_R = 1 - \mathbb{E}U_L$.

To get more insights into the game, consider the following special cases:

Fully Informative Experiments: $(\boldsymbol{\lambda_1^L = \lambda_2^R = 1})$ In this case, the signal of each experiment will fully reveal the state on that issue. So the voters get to know the true state of the world with probability one before voting. This is the voters' most preferred outcome, and hence the social planner's, as their expected utilities are maximized. The expected payoff for R is $0.5\mu_1 + 0.5\mu_2$ since the state on issue 1 would be in favor of R with probability μ_1 and half of the population who care about this issue would vote for him and similarly the other half would see the state in favor of him with probability μ_2

Fully Uninformative Experiments: $(\boldsymbol{\lambda_1^L = \lambda_2^R = 0})$ This is the least desired outcome for the voters as no experiment provides any information for them. Voter 1 votes for L as on issue 1 she is in favor of L and voter 2 votes for R. So the expected payoff for each politician is 0.5.

Least Persuasive Experiments: $(\boldsymbol{\lambda_2^L = 2 - 1/\mu_2, \lambda_1^R = 1 - \mu_1/(1-\mu_1)})$ Consider the case in which L designs an experiment on issue 2 such that $v^L(\lambda_2^L) = \mu_2 u$ and R designs an experiment on issue 1 such that $v^R(\lambda_1^R) = (1-\mu_1)u$. In other words, L provides enough information for voter in G_2 such that she votes for L upon receiving s_ℓ. Similarly for voter in G_1 with respect to R. Note that the voter in each group is indifferent between these experiments and the experiments in the fully uninformative case as on expectation they are not better off.

We are going to refer back to these cases and the two definitions of $\lambda_i^\dagger$ and $\lambda_i^{\min}$ in the later sections in the more general cases in which we have heterogeneous voters. We will do so since these concepts are independent of the share of the voters, as long as it is non-zero, and nothing about them is particular to the case of only two voters.

5 Equilibrium Analysis

In this section we first consider the case with two voters to provide the main insights of the equilibrium. We give intuition why in general there does not exist an equilibrium in pure strategies, and then characterize the equilibrium. We proceed by considering the more general case with different shares of voters in each group. Recall that $\mu_1 < 1/2 < \mu_2$ throughout the analysis.

Before we study the case with two voters, assume that there is only one voter in group G_2 and only L can choose an action. This is the case of seminal paper by Kamenica and Gentzkow (2011) in which there is one designer and one receiver. The best experiment that L can design is the experiment in the form of lemma 2 and provides $v^L(\lambda_2^L) = \mu_2 u$. The voter would vote for L upon receiving s_ℓ which happens with probability $2(1-\mu_2)$. Note that when signals are fully revealing, this probability is $(1-\mu_2)$. As long as s_ℓ is such that receiving it changes the posterior of the voter in favor of L, she would have incentives to choose smaller λ_2^L since higher value of it is going to increase the probability of sending signals in favor of her opponent.

Now assume that R can take an action as well. With two designers and one receiver, we are in the environment of Gentzkow and Kamenica (2016). Here, as the information environment is Blackwell-connected, for any non-truthful experiment of one politician, the other one can provide an experiment that is slightly more informative, i.e. provides higher expected utility for the voter, so that the voter votes according to her experiment. This will make politicians to compete à la Bertrand which results in the fully informative outcome $\lambda_2^L = \lambda_2^R = 1$. The main tradeoff for a politician is that whenever her experiment provides less value for the voter than her opponent's, then she prefers the truth-telling experiment than hers, but when her experiment provides more value for the voter than her opponent's, then she prefers to provide a smaller value as long as it is still higher than her opponent's. The intuition again is to make it less likely to send a signal in favor of her opponent.

5.1 Two Heterogeneous Voters

Now assume that there are two voters, one in G_1 who cares about issue 1 and another in G_2 who cares about issue 2. We saw at the start of section 5 that if there are voters only in one group, then there is a unique pure strategy equilibrium in which both politicians choose fully revealing experiments on the issue that the group of voters care about. But with heterogeneous voters that is not the case anymore. In general, with arbitrary beliefs and heterogeneous voters, no pure strategy equilibrium exists in which politicians play pure strategies with respect to both the focus of their experiments and the design of them. To see the intuition behind it, consider the following candidates.

1. Assume both politicians choose the same issue, e.g. 2. Then in equilibrium, they both should choose $\lambda_2^L = \lambda_2^R = 1$. But R then has an incentive to deviate and design her experiment to be informative on issue 1.

2. Assume each chooses the issue with the like minded group, that is politician L chooses 1 and politician R chooses 2. Then they should provide zero information as they already have the votes of the people. This will provide an incentive for them to change their focus and compete on the other issue.

3. Assume that R chooses 1 now and L chooses 2. R provides just enough information to change the posterior of the voters in his favor if they receive the inconclusive signal. L does the same and the candidate equilibrium becomes:

$$a_R^* = \lambda_1^{\min} = 1 - \frac{\mu_1}{1-\mu_1}, \quad a_L^* = \lambda_2^{\min} = 2 - \frac{1}{\mu_2}$$

   The expected share of voters for each politician is:

$$\mathbb{E}U_L(a_L^*, a_R^*) = (1-\mu_2) + 0.5(1-2\mu_1), \quad \mathbb{E}U_R(a_L^*, a_R^*,) = 0.5(2\mu_2 - 1) + \mu_1$$

Note that if any politician deviates, let's say L, for any $\lambda_1^L > 0$, lemma 1 states that the voters in G_1 would behave only according to her experiment, so she can get arbitrarily close to having 0.5 of expected share of votes. R can deviate to get close to the share of 0.5. So unless $\mu_1 = 1 - \mu_2$, one of the politicians is going to receive an expected share less than 0.5 and has an incentive to deviate. In fact the pure equilibrium exists only in the degenerate case of $\mu_1 = 1 - \mu_2$. This equality is pointing out that the belief of the voter in G_1 for R is exactly the same as the belief of the voter in G_2 for L. The relationship between these two beliefs is going to determine the equilibrium. Consider the following definition:

Definition 1 (Persuasive Advantage)

Denote $\frac{\mu_1}{1-\mu_2} =: Q$ *as the measure of persuasive advantage for politician R.*

The above equality in the degenerate case coincides with $Q = 1$ and we interpret it as the case where R has no persuasive advantage over L. For $Q > 1$, R prefers the imaginary case of truth-telling on both issues than the case with the absence of any information since: $Q > 1$ if and only if $0.5\mu_2 + 0.5\mu_1 > 0.5$. In some sense, this means that providing valuable information is more beneficial for R than for L. With μ_1 being greater than $1-\mu_2$, it depicts that voter 1 is more inclined toward R than voter 2 for L, so intuitively it would be easier for R to voter 1 than for the L to persuade her unfavored voter. For $Q < 1$, L would have more persuasive advantage and with $Q = 1$, they both are indifferent between the truth-telling case and the one with no information.

Going back to the candidate equilibrium in which each politician focus on her unfavored issue, with $Q > 1$, L would have incentive to focus on 1 and compete for her favored voter with R as loosely speaking, L is not gaining much by persuading the more *extreme* voter in 2 but losing more from the voter who is already in her favor, but more *easily* persuaded by R. In other words, $1 - \mu_2 < \mu_1$ would make voter 2 receiving a signal in favor of L less likely than voter 1 receiving a signal in favor of R. Finally, if they both focus on one issue, according to the first candidate equilibrium that we discussed above, they have to be truth-telling, which cannot be the case. Turns out that in equilibrium, R always focuses on issue 1 whereas L has to mix between the issues. Formally:

Proposition 1 (Equilibrium Characterization with Two Voters)

Assume $0 < \mu_1 < 1/2 < \mu_2 < 1$ *and restrict the politicians' actions to the experiments in the form of table 3. The Equilibrium is unique with respect to measure zero sets and:*

1. The only case in which the equilibrium exists in pure strategies is when $Q = 1$, *i.e. when the politicians are the same in terms of persuasive advantage. More specifically, each politician focuses on her unfavored issue and chooses the least persuasion:*

$$a_L^* = \lambda_2^{\min} := 2 - \frac{1}{\mu_2}, \qquad a_R^* = \lambda_1^{\min} := 1 - \frac{\mu_1}{1 - \mu_1}.$$

This coincides with the least persuasive outcome which is the least desired outcome for the voters.

2. When R *has persuasive advantage, i.e.* $Q > 1$, *then again she only focuses on her unfavored issue. However,* L *mixes between the two issues. She chooses* 1 *w.p.* $(1 - \dfrac{1}{Q})$ *and chooses her unfavored issue* 2 *w.p.* $\dfrac{1}{Q}$.

- ***Issue 2:*** *Whenever* L *chooses to focus on unfavored issue, i.e. issue 2, she chooses the least persuasion by playing* $\lambda_2^{\min}$.

- ***Issue 1:*** *Both politicians play distributions:* R *plays CDF* $G(\cdot)$ *with atom at* $\lambda_1^{\min}$ *w.p.*

$\dfrac{1}{Q}$ *and L plays an atomless CDF $F(\cdot)$ such that:*

$$supp(G) = [\lambda_1^{\min}, \frac{\mu_1}{1-\mu_1}\frac{1}{Q^2}], \qquad G(\lambda_1) = \frac{Q\sqrt{\mu_1/(1-\mu_1)}}{\sqrt{1-\lambda_1}},$$
$$supp(F) = [0, 1-\frac{1}{Q^2}], \qquad F(\lambda_1) = \frac{1}{Q-1}\big(\frac{1}{\sqrt{1-\lambda_1}} - 1\big).$$

Proof. See appendix C. ■

We now discuss some equilibrium properties when $Q > 1$, that is when R has more persuasive power. In this case, the voter in G_2 has more extreme belief toward R than the voter in G_1's belief toward L. This motivates R to solely focus on issue 1 as she knows she would not lose much even if L is the only politician who is giving information on issue 2. However, L has to mix between the two issues. She is indifferent between focusing on the extreme voter who cares about issue 2 and is already in favor of R and competing with R over the voter who cares about issue 1 and is more inclined toward herself. She mixes with probability $1/Q$. As Q increases, e.g. voter 2 becomes more extreme toward R, she prefers to focus on issue 1 with more probability and compete for her favored voter as she knows she would not gain much by trying to persuade the extreme voter on issue 2. As Q goes to 1, i.e. politicians become similar in terms of their persuasive power, we see the pure focus on the issues that R focuses on issue 1 with probability 1 and similarly L focuses on her unfavored issue 2

Moreover, L is sure that in equilibrium, R is never going to give information on issue 2, so whenever she is going to be informative about this issue, she designs the least persuasive experiment, in a sense that it gives enough information to the voter to vote for L when she sees the signal s_ℓ. Recall that because of our tie-breaking rule, whenever the voter is indifferent who to vote for, she votes for the designer of the experiment on the issue she cares about.

So the voter that is already in favor of R, the politician with persuasive advantage, does not get better off than the case with the absence of information. In some sense, R takes her support for granted and focuses solely on the issue in which she is not favored. Moreover, the expected share of votes that politician R gets in equilibrium is greater than both the fully truthful case and the case with no information. By the constant-sum nature of the game, L is getting less than the share she could get in the two extreme cases.

With R focusing only on issue 1 and L mixing between the issues, the competition can only happen on issue 1. R in general cannot play a pure experiment as L would choose an experiment that would provide the same expected utility for voter one, and hence make her to vote according to that experiment. This would incentivize R to design an even more informative experiment. These forces make both politicians mix between the experiments too while they are going to be informative on issue 1. An interesting property of the distribution that R plays is that providing high level of information is not in the support of her play. The intuition behind it is that if she designs *very high* informative experiments, then when L is competing on issue 1 has to provide informative experiments with higher probability, but then this makes voter 1 less interesting for R as the probability of this voter getting more accurate information increases and there is not much gain in persuading her. So R might as well focus back on her favored voter. In the extreme case that μ_2 goes to 1, that is the probability of voter 2 voting for R goes to 1, we come back to the simple example of two designers and only voter 1 in which the competition is fierce between two designers and they both provide fully informative experiments.

Another property of the distribution that R plays is that the only pure experiment that she chooses to play with positive probability is the least persuasive one. She plays $\lambda_1^{\min}$ with probability $1/Q$ which is the same probability that L focuses on issue 2. The intuition behind it is that R wants to exploit the cases that L might not be giving information on issue 1 and the best thing that she can do when she is the only one focusing on issue 1 is to give enough information to voter 1 just to make her indifferent when voting upon receiving s_r. Note that as Q increases, that is as she gains more persuasive advantage, L focuses less on issue 2 as it becomes harder to persuade voter 2 and tries to compete for her favored voter. Since she would focus on issue 1 with more probability, R has to play the least persuasive experiment with lower probability as the risk of playing it is the voter would always behave according to L's experiment on issue 1 whenever it is available to consult. In the extreme case that Q goes to infinity, we are in the case of fully revealing experiments.

In equilibrium, the minimum levels of information (least persuasion: $\{\lambda_1^{\min}, \lambda_2^{\min}\}$) are played with positive probability. This probability ($= 1/Q^2$) increases as the politicians become more similar to each other in terms of their persuasive advantage.

The key assumption that eliminates truthful play from the support of any politician's equilibrium strategy is the restriction we impose on their focus. Intuitively, since they can no longer be truthful about both issues, they choose to *lie* more and be the least informative as possible.

The equilibrium in proposition 1 is robust to any tie-breaking rules for the voters when confronting both experiments on the same issue. More specifically, for any other tie-breaking rule that we had assumed, L plays the same distribution but with the open support at the minimum level.

Until now, we assumed that the shares of voters in two groups are equal. Now we examine the more general case in which these shares can be different. Turns out that with modifying the measure of persuasive advantage for R so that it incorporates the shares of voters in it, we end up in the exact equilibrium behavior as discussed.

5.2 Heterogeneous Voters with Heterogeneous Shares

When groups of voters have different shares, then intuitively one expects the group with a higher share to be more appealing to the politicians. For instance, even if the voters in G_2 are more extreme toward R on issue 2 than the voters in G_1 toward L on issue 1, L would still have interest in providing information for them if their share is high enough. Similarly, if G_1 are moderate toward L on issue 1, they would not still be worth R's focus if their share is insignificant. To capture this effect, we redefine our measure of persuasive advantage for R as follows:

Definition 2 (Persuasive Advantage)

Assume non-zero shares of m_1 for group G_1 and $m_2 = 1 - m_1$ for group G_2. Moreover, assume beliefs $0 < \mu_1 < 1/2 < \mu_2 < 1$. Redefine $\dfrac{m_1\mu_1}{m_2(1-\mu_2)} =: Q$ as a measure of persuasive advantage for politician R.

Similar to before, for $Q > 1$, R prefers the imaginary case of truth-telling on both issues than the case with the absence of any information since: $Q > 1$ if and only if $m_2\mu_2 + m_1\mu_1 > m_2$. With this new measure, the next proposition characterizes the equilibrium:

Proposition 2 (Equilibrium Characterization)

All the results in proposition 1 hold with the measure of persuasive advantage for politician R being defined according to definition 2.

Proof. See appendix C. ■

Proposition 2 points out that what matters for the politicians is not just the belief of the voters, but solely the intensity of their support. For instance, the persuasive advantage for politician R can increase not only by voters having more extreme belief toward her on issue 2, but also by having fewer people who support her on her favored issue. The intuition for the latter is by having fewer supporters, R would have less to lose by not focusing on her favored group and can focus more on the voters who will vote for L in the absence of information.

The uniqueness of the proposition 2 gives us more insights into the design of the experiments by the politicians. In combination with lemma 2, it shows that assuming biased experiments by the politicians in a way that they always commit to tell the truth when the state is in their favor is without loss. In lemma 2 we showed for any equilibrium, there exists an outcome equivalent equilibrium with the form of biased experiment. In proposition 2 we restrict attention to biased experiments and we constructed the unique equilibrium. The following proposition shows the equilibrium in proposition 2 is the unique equilibrium of the game without restricting attention to biased experiment. In other words, the biased experiments in the form of table 3 is the result of the competition.

Proposition 3 (Uniqueness of the Equilibrium)

The equilibrium mentioned in Proposition 2, is the unique equilibrium in the space of all experiments, i.e. restricting politicians' experiments to the experiments in table 3 is without loss.

Proof. In lemma 2 we proved that any equilibrium is outcome equivalent to the form of experiments of table 3. The politicians j is indifferent between such experiment and an arbitrary experiment π_i^j that brings the same value to the voters, only if the voters behave according to π_i^j with zero probability. In the unique equilibrium of the proposition 2, for every experiment choice that is realized by politician j, there is a non-zero probability that a group of voters behave according to the signal realization of that experiment. Hence, no equilibrium can exist in which an experiment not in the form of the table 3 is chosen by at least one politician. ■

Up until now we assumed that politicians only care about the expected share of votes. Depending on the election systems and the parties' preferences, it is worth examining the results when they are office-seekers, that is when they only care about the probability of getting elected for the office. In the extension of this paper, we study this case and compare voters' welfare in the two election regimes.

6 Extensions

6.1 Office-seeking Politicians

In this subsection, we briefly discuss what would happen if the politicians were office-seekers. As these preferences do not usually result in the same equilibrium outcome as the case where they only care about the share of votes, it is good practice to study both. Since politicians only care about the probability of winning in this case and since all voters receive the same signal if they consult the same media, only the group with the highest share of voters matters.

Normalizing the utility of winning to 1:

$$U_L^{\text{office}}(a_L, a_R) = \begin{cases} 0, & \text{if } U_L(a_L, a_R) < 1/2 \\ 1, & \text{otherwise} \end{cases}$$

Proposition 4 *If the politicians only care about the probability of winning, in the unique equilibrium, they choose the issue i with the highest share of the voters and they will provide the maximum amount of information: $\lambda_i^L = \lambda_i^R = 1$ where $m_i > 1/2$.*

Proof. Fix $m_2 > 1/2$. If politician j deviates to focus on issue 1, no matter of the votes of the voters on issue 1, voters in G_2 are going to determine the outcome of the election, as they all vote according to the signal they receive which fully reveals the state on issue 2.

To see the uniqueness, we proceed by showing that no equilibrium exists in which at most one politician focuses on issue 2. Assume the contrary that both focus on issue 1. Then the politician who does not get the votes of voters in G_2 in the absence of information is already losing the election as $m_2 > 1/2$, so she has an incentive to change her focus. If only one politician focuses on issue 2, then in the equilibrium candidate, she provides the least persuasive experiment. As a result, the other politician has always an incentive to change her focus on issue 2 and provide an experiment that brings more value to the voters so that the probability of them voting for her increases. One such deviation is the fully informative experiment. Finally, if both focus on issue 2, the only equilibrium candidate will be fully informative experiments which we have shown is the case. ■

An interesting exercise is to compare voters' welfare under two election regimes, one in which the share of votes determines the number of seats in some parliaments' or non Anglo-American presidential elections, and second in which politicians are office-motivated and only care about winning a specific election, and do not care about their popularity.

If in the equilibrium both politicians are present in the same issue with positive probability under the two regimes, then the office-motivated one provides more welfare for the voters as simply more information is released in this case. However, if more than half of the population has extreme priors toward one politician and if this politician happens to have the persuasive advantage, then the effect is not clear. Under each regime, information is going to be revealed on a different issue. In these circumstances, it is possible that the vote-seeking motives bring more welfare for the population as a whole.

6.2 Inattentive Voters

Now assume that voters can only consult one media. This assumption might make more sense as there is ample evidence that voters cannot follow all the news there is. Consider the timing in figure 2 for this extension.

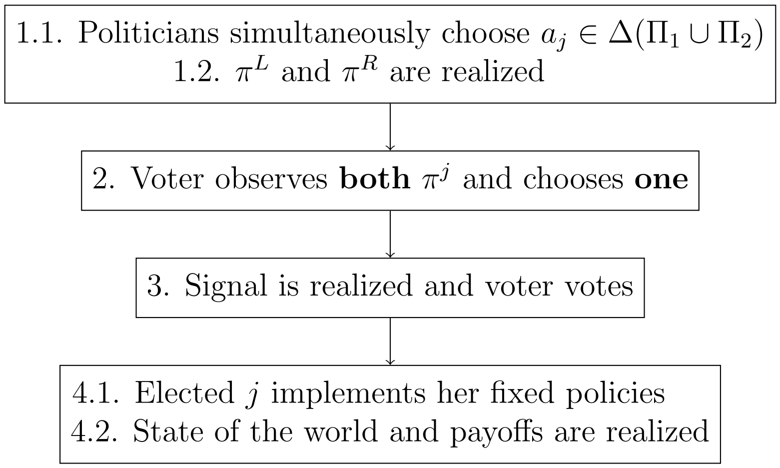

Figure 2: Timing of the game with Inattentive Voters

From the politicians' viewpoint, nothing changes since the voters' decisions remain the same according to lemma 1. When voters can choose only one experiment, they select the one that provides the highest expected utility. However, when they can choose both, the experiment that initially offered lower expected utility does not alter their decision. Either they receive a conclusive signal that confirms their updated posterior, or they receive a signal that modifies their posterior, but not enough to change their decisions. In short, politicians would adopt the same strategies as before.

Proposition 5 (Inattentive Voters)

If the voters are inattentive, in a sense that they have to choose only one experiment to consult, then all the results in proposition 2 and proposition 3 holds.

Proof. The result is a direct implication of lemma 1. ■

6.3 Voters with Convex Preferences

It aligns better with reality if we assume that voters care about both issues but assign different weights to them. When the weights voters assign to each issue are not close enough to each other, the results do not change. The intuition is that even knowing with certainty about the issue they care less about is not enough to change their decision or to vote against the politician on the issue they care more about.

Denote the utility of a voter from group G_i, when politician j is chosen by $(1-\alpha_i)u_j^{\omega_1} + \alpha_i u_j^{\omega_2}$ where α_i is the weight that they put on the second issue. Assume $\alpha_1 < 1/2 < \alpha_2$. As before consider $u_L^{\ell} = u_R^{r} = u > 0$.

For G_1 there is a maximum α_1 in which if voters in this group put a weight less than α_i on issue 2, then any information revealed to them on the second issue would not change their decision. Specifically, if they know truth-telling on the second issue and it's in the favor of the politician that they do not support on issue 1, they still prefer to vote for the politician that they support on issue one. This condition is satisfied for every $2\alpha_1 \leq 1-\mu_1/(1-\mu_1)$. Similar reasoning applies to the voters in G_2. For any weight $\alpha_2 \geq 1/2\mu_2$, no level of information on issue 1 has an impact on their decision.

Conjecture 1 *For more general weights, the results do not change qualitatively.*

7 Conclusion

In many important cases, voters have to rely on the information generated by a public experiment, such as media, as acquiring information is not feasible otherwise. We consider a case in which two politicians are competing in persuading voters with heterogeneous preferences. There are multiple issues to cover and politicians have to decide which topic they want to convey information about. This key restriction gives rise to novel behavior in politicians' competition. We observe in the equilibrium, a politician with a more persuasive advantage takes the support of a group of voters close to her for granted. Moreover, as politicians lose their persuasive advantage with respect to another, we see the competition is more relaxed in a sense that less information is going to be revealed to the voters. Finally, each politician commits to a specific signal structure that only lies when the state is not in their favor. We believe such observation provides more insights on why we see media slant in practice.

References

Anderson, Simon P. and John McLaren (2012), "Media mergers and media bias with rational consumers." *Journal of the European Economic Association*, 10, 831–859.

Au, Pak Hung and Keiichi Kawai (2020), "Competitive information disclosure by multiple senders." *Games and Economic Behavior*, 119, 56–78.

Au, Pak Hung and Keiichi Kawai (2021), "Competitive disclosure of correlated information." *Economic Theory*, 72, 767–799.

Besley, Timothy and Andrea Prat (2006), "Handcuffs for the Grabbing Hand? Media Capture and Government Accountability." *American Economic Review*, 96, 720–736.

Burke, Jeremy (2008), "Primetime Spin: Media Bias and Belief Confirming Information." *Journal of Economics & Management Strategy*, 17, 633–665.

Chan, Jimmy and Wing Suen (2008), "A spatial theory of news consumption and electoral competition." *The Review of Economic Studies*, 75, 699–728.

Chan, Jimmy and Wing Suen (2009), "Media as watchdogs: The role of news media in electoral competition." *European Economic Review*, 53, 799–814.

Che, Yeon-Koo and Konrad Mierendorff (2019), "Optimal dynamic allocation of attention." *American Economic Review*, 109, 2993–3029.

Chiang, Chun-Fang and Brian Knight (2011), "Media bias and influence: Evidence from newspaper endorsements." *The Review of Economic Studies*, 78, 795–820.

Duggan, J. and C. Martinelli (2011), "A Spatial Theory of Media Slant and Voter Choice." *The Review of Economic Studies*, 78, 640–666.

Gehlbach, Scott and Konstantin Sonin (2014), "Government control of the media." *Journal of Public Economics*, 118, 163–171.

Gentzkow, Matthew and Emir Kamenica (2016), "Competition in persuasion." *The Review of Economic Studies*, 84, 300–322.

Gentzkow, Matthew and Emir Kamenica (2017), "Bayesian persuasion with multiple senders and rich signal spaces." *Games and Economic Behavior*, 104, 411–429.

Gentzkow, Matthew and Jesse M. Shapiro (2006), "Media bias and reputation." *Journal of Political Economy*, 114, 280–316.

Innocenti, Federico (2022), "Can media pluralism be harmful to news quality?" *Available at SSRN 4257390*.

Kamenica, Emir and Matthew Gentzkow (2011), "Bayesian persuasion." *American Economic Review*, 101, 2590–2615.

Knoepfle, Jan (2024), "Dynamic competition for attention." URL https://arxiv.org/abs/2409.18595.

Leung, Benson Tsz Kin (2020), “Limited cognitive ability and selective information processing.” *Games and Economic Behavior*, 120, 345–369.

Maskin, Eric and Jean Tirole (2019), “Pandering and pork-barrel politics.” *Journal of Public Economics*, 176, 79–93.

Mullainathan, Sendhil and Andrei Shleifer (2005), “The market for news.” *American Economic Review*, 95, 1031–1053.

Perego, Jacopo and Sevgi Yuksel (2022), “Media competition and social disagreement.” *Econometrica*, 90, 223–265.

Strömberg, David (2015), “Media and politics.” *Annual Review of Economics*, 7, 173–205.

Suen, Wing (2004), “The self-perpetuation of biased beliefs.” *The Economic Journal*, 114, 377–396.

Appendix

A Proof of lemma 1

Proof. We prove this lemma in a more general case. Assume that politicians design π^j in the form of table 2. Denote L's choice by (α^L, β^L) and R's by (α^R, β^R). Fix prior $0 < \mu = Pr(\mathscr{r}) < 1$. Moreover, assume that each experiment is designed in a way that the signals can change the decision of the voter with prior μ, otherwise the result of the lemma is trivial. WLOG, assume that for each experiment, the posterior becomes greater than 1/2 upon receiving $s_{\mathscr{r}}$ and less than 1/2 after receiving s_ℓ. Formally, for each $j \in \{L, R\}$

$$Pr(\mathscr{r}|s^j_{\mathscr{r}}) > Pr(\ell|s^j_{\mathscr{r}}), \quad Pr(\ell|s^j_\ell) > Pr(\mathscr{r}|s^j_\ell)$$
$$\Rightarrow Pr(s^j_\ell|\ell) > Pr(s^j_\ell|\mathscr{r}) \Leftrightarrow \alpha^j > 1 - \beta^j$$

where $s^j_\ell, s^j_{\mathscr{r}}$ are realized according to the experiment π^j. Denote the expected value that π^j brings for the voter with prior μ by $v(\pi^j)$ where

$$v(\pi^j) = (Pr(\ell|s^j_\ell)Pr(s^j_\ell) + Pr(\mathscr{r}|s^j_{\mathscr{r}})Pr(s^j_{\mathscr{r}}))u$$

since the voter would vote for L if receiving s_ℓ and for R upon receiving $s_{\mathscr{r}}$. We abuse the notation by denoting $v(\pi^j) = v(\alpha^j, \beta^j) = v^j = (\alpha^j(1-\mu) + \beta^j\mu)u$.

The goal is to prove the following:

$$v^L < v^R \Rightarrow Pr(r|s^L_r, s^R_\ell) < 1/2 < Pr(r|s^L_\ell, s^R_r)$$

In other words, the voter would always follow the signals of π^R.

$$\begin{aligned}
Pr(r|s^L_\ell, s^R_r) = \frac{Pr(s^L_\ell, s^R_r|r)Pr(r)}{Pr(s^L_\ell, s^R_r|\ell)Pr(\ell) + Pr(s^L_\ell, s^R_r|r)Pr(r)} &> \frac{1}{2} \\
\Leftrightarrow Pr(s^L_\ell, s^R_r|r)Pr(r) &> Pr(s^L_\ell, s^R_r|\ell)Pr(\ell) \\
\Leftrightarrow (1-\beta^L)\beta^R\mu &> \alpha^L(1-\alpha^R)(1-\mu) \\
\Leftrightarrow \alpha^L\alpha^R(1-\mu) - \beta^L\beta^R\mu &> \alpha^L(1-\mu) - \beta^R\mu
\end{aligned}$$

To show the above we only need to show

$$\alpha^L\alpha^R(1-\mu) - \beta^L\beta^R\mu > \alpha^R(1-\mu) - \beta^L\mu$$

since

$$\begin{aligned}
v^R > v^L &\Leftrightarrow \alpha^R(1-\mu) + \beta^R\mu > \alpha^L(1-\mu) + \beta^L\mu \\
&\Leftrightarrow \alpha^R(1-\mu) - \beta^L\mu > \alpha^L(1-\mu) - \beta^R\mu
\end{aligned}$$

Now recall that $Pr(\ell|s^R_\ell) > Pr(r|s^R_\ell) \Leftrightarrow \alpha^R(1-\mu) > (1-\beta^R)\mu$ and $\alpha^L > 1-\beta^L$. So

$$\begin{aligned}
&\alpha^L\alpha^R(1-\mu) - \beta^L\beta^R\mu > \alpha^R(1-\mu) - \beta^L\mu \\
\Leftrightarrow\ & \beta^L\mu(1-\beta^R) > \alpha^R(1-\mu)(1-\alpha^L) > (1-\beta^R)\mu(1-\alpha^L) \Leftrightarrow \beta^L > 1-\alpha^L
\end{aligned}$$

Similar algebra will result in $Pr(r|s^L_r, s^R_\ell) < 1/2$ which concludes the proof. ■

B Proof of lemma 2

Proof. Fix an equilibrium (a^*_L, a^*_R). In the equilibrium, politician $j \in \{L, R\}$ is indifferent between choosing any experiment $\pi^j_i \in supp(a^*_j)$ with $i \in \{1, 2\}$ fixing $a^*_{k\neq j}$. Consider the expected utility of politician j in equilibrium which is the same when the strategy profile is $(\pi^j_i, a^*_{k\neq j})$. If the experiment π^j_i is chosen with positive probability by group G_i of voters, it provides an expected utility of v for them. There are two cases:

1. $v > \max\{\mu_i, 1-\mu_i\} \times u$: That is consulting the experiment is beneficial for the voters than voting without consultation. Then the experiment can change the action of voters

relative to the case that they were on their own. For any fixed v that satisfies this condition, there exists an experiment in the form of table 3 that provides the same v for the voters, but makes politician j strictly better off (same as Kamenica and Gentzkow (2011)).

2. $v = \max\{\mu_i, 1-\mu_i\} \times u$: That is the case where the voters do not benefit from the experiments. Here, the politician is indifferent between experiments that do not change the action of the voters, or in other words, any experiment that has zero value of information for the voters. One can assume that the politician chooses an uninformative experiment with the signal structure according to table 3.

Now consider the case in which π_i^j is chosen with zero probability by group G_i, that is the voters expected utilities are $v' > v$ in which v is their expected utility had they chosen to consult π_i^j. Similar to the second case above, the politician could choose an experiment in the form of table 3 that provides v for the voters.

Thus, for any equilibrium (a_L^*, a_R^*), we can build an equilibrium (b_L^*, b_R^*) with every $\pi_i^j \in supp(b_j^*)$ having the same signal structure as table 3, such that the expected utility of the politicians and the outcomes are the same in the two equilibria. ■

C Proof of proposition 1 and proposition 2

Proof. We proceed by proving proposition 2 as it is the general version of proposition 1. First, we prove that in any equilibrium when $Q \neq 1$, only one politician mixes between the issues. Then, we show that on the issue that is on the path of the play of both politicians, they play distribution with support that starts from the minimum level of persuasion specific to each politician. Finally, we construct the equilibrium.

Lemma 3
No equilibrium exists in pure strategies when $Q \neq 1$.

Proof. We go through the candidate pure equilibria and show that there exists an incentive for deviation in each one of them.

1. Assume both choose the same issue, e.g. 2. Then in equilibrium, they both should choose $\lambda_2^L = \lambda_2^R = 1$. This way, the politician that is not getting any votes from G_1 has an incentive to deviate and be active on issue 1.

2. Assume each chooses the issue with the like minded group, that is politician R chooses 2 and politician L chooses 1. Then they should provide zero information as they already

have the votes of the people. This will provide an incentive for them to change their focus and compete on the other issue.

3. Assume that R chooses 1 now and L chooses 2. R provides just enough information to change the posterior of the voters in his favor if they receive the inconclusive signal. L does the same and the candidate equilibrium becomes:

$$a_R^* = \lambda_1^{\min}, \quad a_L^* = \lambda_2^{\min}$$

The expected share of voters for each politician is:

$$\mathbb{E}U_L(a_L^*, a_R^*) = 2(1-\mu_2)m_2 + (1-2\mu_1)(1-m_2),$$
$$\mathbb{E}U_R(a_L^*, a_R^*,) = (2\mu_2 - 1)m_2 + 2\mu_1(1-m_2)$$

Note that if any politician deviates, let's say L, for any $\lambda_1^L > 0$, the voters in G_1 would choose her experiment, so he can get arbitrarily close to having $m_1 = 1 - m_2$ of expected share of votes. R can deviate to get close to the share of m_2. Writing the conditions, the degenerative equilibrium happens iff $Q = 1$. ■

Now we characterize how the politicians would mix between the issues in any equilibrium:

Lemma 4 (Mixing between issues)

In any equilibrium, only one politician mixes between the issues. Moreover, the support of the politicians' strategies on both issues, in which they focus on with positive probability contains $\lambda_1^{\min}$ and $\lambda_2^{\min}$ which denote the minimum persuasion they prefer doing if they were alone on one issue.

Proof. Note that no equilibrium exists in which politicians choose to focus on the same issue and play mixed on the informativeness of their experiments. This is a direct result of the previous lemma as the only equilibrium candidate in this case is for both to be truthful and as we have shown earlier, always one politician has a deviation.

For the proof of the first part of this lemma, we just need to prove no equilibrium exists in which both politicians play mixed between both issues. Assume by the contrary that it exists. Choose the minimum λ_1^L in the support of L's strategy on issue 1 by $\lambda_1^{L,\min}$ and the minimum λ_2^L in the support of L's strategy on issue 2 by $\lambda_2^{L,\min}$. In equilibrium, L should be indifferent playing each pure strategy in the support of her equilibrium strategy, i.e. $\mathbb{E}U_L(\lambda_1^{L,\min}, a_R^*) = \mathbb{E}U_L(\lambda_2^{L,\min}, a_R^*)$. When L plays $\lambda_1^{L,\min}$, she should win the attention of a positive share of voters G_1 when R is present on issue 1, otherwise, L has a deviation to $\lambda_2^{L,\min}$. This means that R has to lose the attention of the voters when she is playing the minimum λ_1^R in her support. Accordingly, L has to win the attention of a positive share of voters when playing $\lambda_2^{L,\min}$, otherwise, she has deviation to play $\lambda_1^{L,\min}$ and win the attention of voters on issue 1 with positive probability. Thus, R has to lose the attention of the voters when she is playing the minimum λ_2^R in her support as well. With similar reasoning for R, she should win the attention of some group of voters with positive probability when playing the minimums in her support which is a contradiction. Thus, Only one politician can mix between issues in any equilibrium.

The proof of the second part of the lemma is immediate based on the proof of the first part. Consider the case that only L mixes between issues. In equilibrium, R cannot focus on 2 as she is losing the attention of the voters when choosing $\lambda_2^{R,\min}$, so she has a trivial deviation to $\lambda_1^R = 1$. When focusing on 1, she is losing the attention of voters whenever she is choosing the minimum in her support, i.e. $\lambda_1^{R,\min}$ and L is present on issue 1. Thus, the best that R can do is to choose $\lambda_1^{R,\min} = \lambda_1^{\min}$ to get the highest utility when L is not present there. Moreover, as L faces no competition when she is present on 2, she always chooses $\lambda_2^{\min}$ there. The same reasoning applies when R mixes between the two issues and L focuses only on one. ■

Now recall that the politicians' strategies when playing mixed on the informativeness levels are restricted to finite disjoint intervals in which they play full support on. The following lemma shows that in any equilibrium, the number of such intervals is one.

Lemma 5 (No holes)

In any equilibrium, the politicians cannot have non measure zero holes on the support of their strategies regarding the same issue.

Proof. The proof is similar to the proof of the second part of lemma 4. Assume that R stays on issue 1 and there is a non measure zero interval $(\underline{\lambda}_1, \overline{\lambda}_1)$ inside the minimum and maximum support of her strategy in which she does not play. This would incentivized L to have a hole as well as now she prefers and λ_1^L that corresponds to $\underline{\lambda}_1$ to the one that corresponds to any $\lambda_1 \in (\underline{\lambda}_1, \overline{\lambda}_1)$. By corresponding we mean the voters are indifferent between choosing either one of the experiments and have to choose by the tie-breaking rule. Moreover, R cannot play atom at $\overline{\lambda}_1$ because whenever L plays an λ_1^L that corresponds to $\overline{\lambda}_1$, she should win with similar reasoning as the one in the previous lemma and as a result, R would strictly prefer $\underline{\lambda}_1$ to $\overline{\lambda}_1$ and cannot be indifferent between the two. The same holds if we assume L has a non measure zero interval between the min and max of her support. ■

With the help of previous lemmas, we can focus on the cases in which the politicians choose distributions with connected supports. More specifically, we need to search for all the equilibria in a special class of strategies such that when $Q < 1$:

- R's strategy is $a_R = G(\cdot)$ where $G(\cdot)$ is a CDF with support $[\lambda_1^{\min}, \lambda_1^{\dagger,\max}]$, atom at $\lambda_1^{\min}$, and differentiable on $(\lambda_1^{\min}, \lambda_1^{\dagger,\max}]$. Denote the pdf by $g(\cdot) > 0$.

- L's strategy a_L is to play $\lambda_2^{\min}$ with probability r and $F(\cdot)$ with probability $1-r$ where $F(\cdot)$ is a CDF with support $[0, \lambda_1^{\max}]$, and differentiable on its support. Denote the pdf by $f(\cdot) > 0$.

Note that above, we assumed that the politicians do not play atoms inside any open interval in the support of their strategies. The reason for that is when one plays an atom there, it induces a hole in the strategy of the other player as a deviation, but then lemma 5 applies. The following lemma pins down the equilibrium restricting to the above class of strategies:

Lemma 6

There exists a unique equilibrium in the aforementioned class of strategies.

Proof. We write the necessary and sufficient conditions for a strategy to be an equilibrium and show that only one strategy satisfies thess conditions. Consider an equilibrium (a_L, a_R). Throughout the proof, assume $\lambda_1^R \sim G(\cdot)$ and $\lambda_1^L \sim F(\cdot)$. To find the $G(\lambda_1^{\min})$, note that L should be indifferent between playing $\lambda_2^{\min}$ and 0_1 in equilibrium where 0_1 denotes $\lambda_1 = 0$:

$$\mathbb{E}U_L(0_1, a_R) = \mathbb{E}U_L(\lambda_2^{\min}, a_R)$$

$$\Leftrightarrow (1-m_2)\Big[G(\lambda_1^{\min}) + (1-\mu_1)(1-G(\lambda_1^{\min}))\mathbb{E}[\lambda_1^R|\lambda_1^R > \lambda_1^{\min}]\Big] = (1-m_2)(1-\mu_1)\mathbb{E}[\lambda_1^R] + 2m_2(1-\mu_2)$$

$$\Leftrightarrow (1-m_2)G(\lambda_1^{\min}) = (1-m_2)(1-\mu_1)\lambda_1^{\min}G(\lambda_1^{\min}) + 2m_2(1-\mu_2)$$

$$\Leftrightarrow 2(1-m_2)\mu_1 G(\lambda_1^{\min}) = 2m_2(1-\mu_2) \Leftrightarrow G(\lambda_1^{\min}) = \frac{m_2(1-\mu_2)}{(1-m_2)\mu_1} = \frac{1}{Q}.$$

Note that if $G(\lambda_1^{\min}) = 0$ then $\mathbb{E}U_L(0_1, a_R) < \mathbb{E}U_L(\lambda_2^{\min}, a_R)$. Now, to pin down the distribution that R plays, i.e. G, we check the indifference conditions for every $\lambda_1^L = \lambda_1$ that L plays on issue 1. Using $\lambda_1^R = \lambda_1^\dagger$ for R that make voters indifferent between choosing λ_1 and $\lambda_1^\dagger$:

$$\mathbb{E}U_L(0_1, a_R) = \mathbb{E}U_L(i, a_R) \Leftrightarrow$$

$$(1-m_2)\Big[G(\lambda_1^{\min}) + (1-\mu_1)\int_{\lambda_1^{+,\min}}^{\lambda_1^{\dagger,\max}} \lambda_1^R dG\Big] = (1-m_2)\Big[G(\lambda_1^\dagger)(1-\lambda_1\mu_1) + (1-\mu_1)\int_{\lambda_1^\dagger}^{\lambda_1^{\dagger,\max}} \lambda_1^R dG]\Big]$$

Where I use the following definition for $\lambda_1^{+,\min}$: $\int_{\lambda_1^{\min}}^{x^\dagger} \lambda_1^R dG = G(\lambda_1^{\min})\lambda_1^{\min} + \int_{\lambda_1^{+,\min}}^{x^\dagger} \lambda_1^R dG$.

Recall from before $\lambda_i = 2 - \dfrac{1}{\mu_i} + \dfrac{1-\mu_i}{\mu_i}\lambda_i^{\dagger}$, substituting λ_1 with $\lambda_1^{\dagger}$, we would have $1 - \lambda_1\mu_1 = (1-\mu_1)(2-\lambda_1^{\dagger})$:

$$\Leftrightarrow (1-\mu_1)\int_{\lambda_1^{\dagger,\min}}^{\lambda_1^{\dagger}} \lambda_1^R dG = G(\lambda_1^{\dagger})(2-\lambda_1^{\dagger})(1-\mu_1) - \frac{1}{Q} \xRightarrow{\frac{d}{d\lambda_1^{\dagger}}} G(\lambda_1^{\dagger}) = 2g(\lambda_1^{\dagger})(1-\lambda_1^{\dagger})$$

With initial condition $G(\lambda_1^{\min}) = \frac{1}{Q}$ and denote $c \coloneqq \dfrac{\mu_1}{1-\mu_1}$, solving for the ODE:

$$G(\lambda_1^{\dagger}) = \frac{\sqrt{c}}{Q\sqrt{1-\lambda_1^{\dagger}}}$$

Since G is a CDF with support $[\lambda_1^{\min}, \lambda_1^{\dagger,\max}]$, we have:

$$G(\lambda_1^{\dagger,\max}) = 1 \Leftrightarrow \lambda_1^{\dagger,\max} = 1 - \frac{c}{Q^2} \Leftrightarrow \lambda_1^{\max} = 1 - \frac{1}{Q^2}$$

So $\lambda_1^{\max}, \lambda_1^{\dagger,\max}$, and the distribution G have been uniquely determined by necessary conditions. To find a relation between the maximum in the support λ_1, i.e $\lambda_1^{\max}$ and the probability that L plays mixed between the issues, we use the zero-sum nature of the game. In equilibrium, R is indifferent between playing $\lambda_1^{\dagger,\max}$ and λ_1^R with λ_1^R in G's support, so his equilibrium share is:

$$\mathbb{E}U_R^* = \mathbb{E}U_R(a_L^*, \lambda_1^{\dagger,\max}) = (1-m_2)(1-\lambda_1^{\dagger,\max}(1-\mu_1)) + m_2[1-2r(1-\mu_2)]$$

Similarly, L's equilibrium share of votes $\mathbb{E}U_L^*$ is equal to $\mathbb{E}U_L(\lambda_1^{\max}, a_R^*) = (1-m_2)(1-\lambda_1^{\max}\mu_1)$. Note that the share of voters who vote for politicians should be equal to 1:

$$\begin{aligned}
&\mathbb{E}U_R^* + \mathbb{E}U_L^* = 1 \Leftrightarrow (1-m_2)(1-\lambda_1^{\dagger,\max}(1-\mu_1)) + m_2[1-2r(1-\mu_2)] = 1-(1-m_2)(1-\lambda_1^{\max}\mu_1) \\
&\qquad \Leftrightarrow (1-m_2)\mu_1(2-\lambda_1^{\max}) + m_2[1-2r(1-\mu_2)] = 1-(1-m_2)(1-\lambda_1^{\max}\mu_1) \Leftrightarrow \\
&2(1-m_2)\mu_1(1-\lambda_1^{\max}) = 2rm_2(1-\mu_2) \Leftrightarrow \lambda_1^{\max} = 1 - \frac{m_2(1-\mu_2)}{(1-m_2)\mu_1}r = 1 - \frac{r}{Q} = 1 - \frac{1}{Q^2} \Rightarrow r = \frac{1}{Q}
\end{aligned}$$

To pin down the distribution for L, we check the indifference conditions in for every value that R plays in the support of G. Assume again that for i that L plays, $\lambda_1^{\dagger}$ is the counterpart that R has to play to make voters indifferent:

$$\mathbb{E}U_R(a_L,\lambda_1^\dagger) = \mathbb{E}U_R(a_L,\lambda_1^{\dagger,\max}) \Leftrightarrow r\Big[(1-m_2)(1-(1-\mu_1)\lambda_1^\dagger)$$
$$+m_2(2\mu_2-1)\Big] + (1-r)\Big[(1-m_2)\big[F(\lambda_1)(1-(1-\mu_1)\lambda_1^\dagger + \mu_1\int_{\lambda_1}^{\lambda_1^{\max}} \lambda_1^L dF\big] + m_2\Big]$$
$$= r\Big[(1-m_2)(1-(1-\mu_1)\lambda_1^{\dagger,\max}) + m_2(2\mu_2-1)\Big] + (1-r)\Big[(1-m_2)(1-(1-\mu_1)\lambda_1^{\dagger,\max} + m_2\Big]$$

Recall $\lambda_i^\dagger = 1 - \frac{\mu_i}{1-\mu_i} + \frac{\mu_i}{1-\mu_i}\lambda_i$, substituting $\lambda_1^\dagger$ with λ_1, we would have $1-(1-\mu_1)\lambda_1^\dagger = \mu_1(2-\lambda_1)$:

$$\Leftrightarrow r(2-\lambda_1) + (1-r)\Big[F(\lambda_1)(2-\lambda_1) + \int_{\lambda_1}^{\lambda_1^{\max}} \lambda_1^L dF\Big] = 2-\lambda_1^{\max}$$
$$\Leftrightarrow \lambda_1^{\max} - \lambda_1 = (1-r)\Big[(1-F(\lambda_1))(2-\lambda_1) - \int_{\lambda_1}^{\lambda_1^{\max}} \lambda_1^L dF\Big]$$

Now we substitute for $\lambda_1^{\max}$ and r with the equalities that we derived before: $\lambda_1^{\max} = 1 - \frac{1}{Q^2}$, $r = \frac{1}{Q}$:

$$\Leftrightarrow \frac{\lambda_1^{\max}}{1-r} = 1 + \frac{1}{Q} = \Big[(1-F(\lambda_1))(2-\lambda_1) - \int_{\lambda_1}^{\lambda_1^{\max}} \lambda_1^L dF\Big] + \frac{Q\lambda_1}{Q-1}$$
$$\xrightarrow{\frac{d}{d\lambda_1}} 1 - F(\lambda_1) + 2F(\lambda_1)(1-\lambda_1) = \frac{Q}{Q-1}$$

Solving for the above ODE with initial condition $F(\lambda_1^{\max} = 1 - \frac{1}{Q^2}) = 1$ results in:

$$F(\lambda_1) = \frac{1}{Q-1}\Big[\frac{1}{\sqrt{1-\lambda_1}} - 1\Big]$$

Note that when searching for $F(\cdot)$ we did not assume that it is atomless at 0, but this is the only solution given the unique $G(\cdot)$ that we have determined earlier.

Now we check if the politicians have incentives to deviate or not. First, L does not want to deviate to λ_2 where $\lambda_2 \in (\lambda_2^{\min}, 1]$ since $\lambda_2^{\min}$ is the best he can do when choosing to talk about issue 2:

$$\mathbb{E}U_L(a_L^*, a_R^*) = \mathbb{E}U_L(\lambda_2^{\min}, a_R^*) > \mathbb{E}U_L(\lambda_2, a_R^*) \Leftrightarrow$$
$$(1-m_2)(1-\mu_1)\mathbb{E}[\lambda_1^R] + 2m_2(1-\mu_2) > (1-m_2)(1-\mu_1)\mathbb{E}[\lambda_1^R] + m_2(1-\lambda_2\mu_2) \Leftrightarrow 1 > \mu_2(2-\lambda_2)$$

Moreover, for $\lambda_2 < \lambda_2^{\min}$, voters of G_2 would always vote for R. Another possible deviation for L is to choose λ_1 where $\lambda_1 > \lambda_1^{\max} = \frac{1}{Q}$. Again, he does not have an incentive to deviate this way since conditional on winning the voters' attention, a politician wants to provide minimum information possible:

$$\begin{gathered}\mathbb{E}U_L(a_L^*, a_R^*) = \mathbb{E}U_L(\lambda_1^{\max}, a_R^*) > \mathbb{E}U_L(\lambda_1, a_R^*) \\ \Leftrightarrow (1-m_2)(1-\lambda_1^{\max}\mu_1) > (1-m_2)(1-\lambda_1\mu_1) \Leftrightarrow \lambda_1 > \lambda_1^{\max}\end{gathered}$$

With the same reasoning, R does not have an incentive to deviate to $\lambda_1^{\dagger}$ where $\lambda_1^{\dagger} > \lambda_1^{\dagger,\max}$. His best deviation is to play 0_2. The reason for that is L is only playing $\lambda_2^{\min}$ on issue 2, so if R plays 0_2, every voter in group G_2 would choose his media and all of them would eventually vote for him. Thus any deviation of $\lambda_2^R > 0$ is strictly dominated by giving no information when talking about issue 2. Now checking for this best deviation:

$$\begin{gathered}\mathbb{E}U_R(a_L^*, a_R^*) = \mathbb{E}U_L(a_L^*, \lambda_1^{\min}) > \mathbb{E}U_R(a_L^*, 0_2) \Leftrightarrow \\ r\Big[2(1-m_2)\mu_1 + m_2(2\mu_2 - 1)\Big] + (1-r)\Big[(1-m_2)\mu_1\mathbb{E}(\lambda_1^L) + m_2\Big] > m_2 + (1-r)(1-m_2)\mu_1\mathbb{E}(\lambda_1^L) \\ \Leftrightarrow 2(1-m_2)\mu_1 + m_2(2\mu_2-1) > m_2 \Leftrightarrow (1-m_2)\mu_1 > m_2(1-\mu_2) \Leftrightarrow Q > 1.\end{gathered}$$

With similar reasoning as above it is easy to see why $(\lambda_2^{\min}, \lambda_1^{\min})$ is an equilibrium in the degenerate case $Q = 1$. Since politicians are talking about separate issues, the best they can do is to be the least informative. Moreover, if they deviate, the best deviation is again to deviate to talking about the other issue and provide the minimum level of information. The condition $Q = 1$ makes each politician indifferent between equilibrium play and his best deviation. ■

Lemma 2 and lemma 3 to lemma 6 show the equilibrium is unique in the class of all strategies available to the politicians. ■