Ultimate Force and Drift Prediction of Stone-masonry Walls Using Multiple Linear Regression Analysis

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Abstract

The study aims at predicting the ultimate force and displacement capacity of stone-masonry walls using multiple linear regression. To do so, a database including 104 samples has been gathered from different testing campaigns where the parameters influencing the resistance of the walls have been measured. A second goal is to find the most effective predictor variables by using two different procedures, partial F-test and Akaike Information Criterion.

Contents

1 Introduction

Stone-masonry buildings are among the first man-made structures, and the existence of natural stone resources had a great influence on the foundation of cities [1]. Aesthetic beauty, low cost, durability and the availability of natural stone result in the use of stone blocks in old buildings [1–3]. Stone-masonry structures are composite structures consisting of units (stone), mortar and in-fill material [4, 5]. In former times, buildings were not constructed to resist induced seismic loads, so stone-masonry buildings are among the most vulnerable structures to earthquakes, heightening the need to assess the load-bearing capacity of their walls. In order to simulate earthquake actions on stone-masonry walls, quasi-static tests are conducted in laboratory. As an illustration, Figure 1 shows a test setup for applying vertical and horizontal forces on a specimen by vertical and horizontal actuators. First, the vertical actuators apply a constant vertical loads simulating the dead loads and then the horizontal actuator applies horizontal displacements according to a loading protocol at the top of the wall representing horizontal earthquake actions.

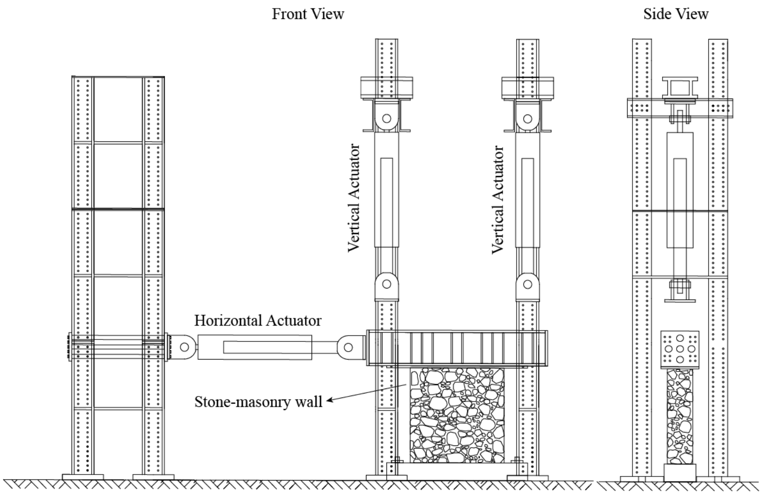


Figure 1: Test setup

Different parameters that are introduced in the next sections affect the load bearing capacity of walls. Vanin et al. [6] have gathered a database of quasi-static cyclic tests available in the literature conducted at different laboratories. The database has been published and is accessible in `https://zenodo.org/record/812146#.WuhHnYhuZaQ`. In this study, the same database is used to investigate the multiple linear regression models for predicting load capacities of the stone-masonry walls.

2 Regression Analysis

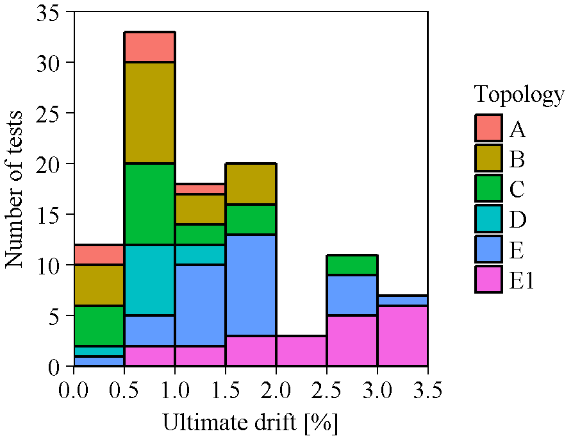
2.1 Statement of the problem

The Equivalent Frame Model (EFM) is one of the methods, in which the load bearing walls of buildings are divided into sub-elements called "piers" and "spandrels", to obtain the global response of stone-masonry structures to applied ground motions. To do so, the force-displacement response of each element (piers and spandrels) is required as an input for the modeling. The ultimate drift and force are two parameters that can be used for obtaining bi-linear force-displacement curves.

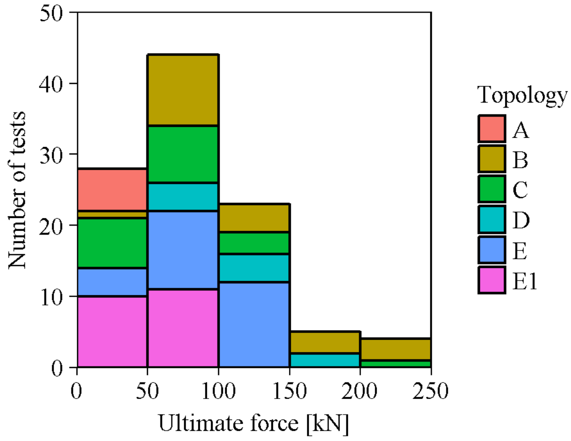
To the best of author's knowledge, a good mechanistic model has not yet been formulated for the prediction of ultimate force and drift of stone-masonry piers and spandrels. Thus, it is necessary to build a prediction model based on the available data obtained by conducting quasi-static tests.

2.2 Data collection

The database used in this study, has been gathered from experiments carried out in different laboratories. The number of replicates in the database is 104. Figure 2 illustrates the histogram plots of two response variables that are to be predicted by the multiple linear regression analysis. It should be mentioned that for each level of predictor variable, the number of replicates are not the same. As an illustration, the histograms in Figure 2 are color-coded with regard to the topology of the walls, which is to be used as one of the predictor variables. The whole set of predictors is introduced in the next sections.



(a) Histogram of ultimate drift



(b) Histogram of ultimate force

Figure 2: Histogram of response variables [6]

2.3 Model specification

It is of concern to find the simplest possible model; and to establish which variables are influential enough to be included in the model. To investigate whether the relationship between predictors and responses can be quantified by the multiple linear regression analysis, the general model is written as [7]:

$$\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X_1 + \hat{\beta}_2 X_2 + ... + \hat{\beta}_p X_p \tag{1}$$

where $\hat{Y}$ is the fitted value, $\hat{\beta}_0, \hat{\beta}_1, ..., \hat{\beta}_p$ are the estimates of the regression parameters and p is the number of explanatory variables. As shown in Table 1, there are two continuously varying responses, ultimate force and drift and eight predictors in the full model, among which X_1, X_2 and X_3 are categorical variables (polytomous) and the others are continuously varying. The possible values for the categorical variables are: X_1: [0,1,2,3]; X_2: [A,B,C,D,E,E1] and X_3: [1,2,3]. It must be pointed out that in this study, the multivariate regression analysis is regarded as two separate univariate regression analyses.

Table 1: Variables in the regression analysis

Variable		Definition
Response variables	Y_1	Ultimate drift [%]: d_u
	Y_2	Ultimate force [kN]:V_u
Predictor variables	X_1	Number of quasi-static test cycles: N_c
	X_2	Stone topology: ST
	X_3	Number of leaves: N_{Leaves}
	X_4	Height of the wall [mm]: H
	X_5	Length of the wall [mm]: L
	X_6	Thickness of the wall [mm]: t
	X_7	Shear span ratio: H_0/H
	X_8	Axial load ratio: $\sigma_{0\,tot}/f_c$

3 Model fitting

3.1 Ultimate drift prediction

Full model

In this section, all of the explanatory variables are used as the predictors for fitting a model. To predict the ultimate drift ($\hat{d_u}$), the form of the linear regression model in R package is assumed to be

$$\hat{d_u} = \hat{\beta}_0 + \hat{\beta}_1 N_{c1} + \hat{\beta}_2 N_{c2} + \hat{\beta}_3 N_{c3} + \hat{\beta}_4 N_B + \hat{\beta}_5 N_C + \hat{\beta}_6 N_D + \hat{\beta}_7 N_E + \hat{\beta}_8 N_{E1} + \hat{\beta}_9 N_{Leaves=2} + \hat{\beta_{10}} N_{Leaves=3} + \hat{\beta_{11}} H + \hat{\beta_{12}} L + \hat{\beta_{13}} t + \hat{\beta_{14}} \frac{H_0}{H} + \hat{\beta_{15}} \frac{\sigma_{0\,tot}}{f_c}. \quad (2)$$

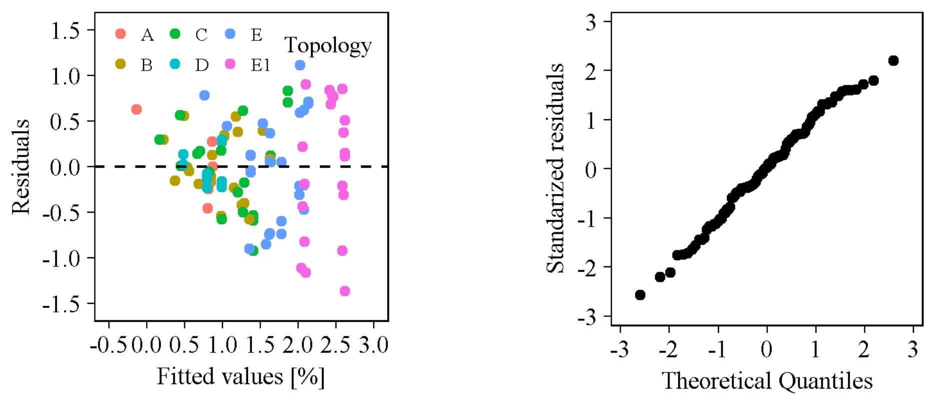
The references for the categorical variables in the model are: N_{c0}: the number of cycles is zero (monotonic test); N_A: the wall has the topology of A and $N_{Leaves=1}$: the number of leaves is one. Thus, the predictors N_{c1}, N_{c2}, N_{c3}, N_B, N_C, N_D, N_E, N_{E1}, $N_{Leaves=2}$ and $N_{Leaves=3}$ become two-level categorical variables, which can only take 0 or 1. Moreover, it must be pointed out that all of the variables are completely independent.

The results of the multiple linear regression analysis are summarized in Table 2. The axial load ratio, the height of the wall and the number of cycles (=3) are statistically significant at the level $\alpha = 0.05$. The next step is to check the linearity assumptions. To do so, diagnostics plots are depicted in Figure 3. It is evident that the Q-Q plot has an almost linear trend and no obvious outlier is detected. The plot of residuals versus fitted values has a cone shape, meaning that as the fitted value increases the variance of the residuals also increases. The reason for having such a shape in the plot (heteroscedasticity) might be due to the fact that not all necessary predictors, the ones that thoroughly describe the behavior of the system and are unknown, are not included in the model. Thus, new variables are needed to be measured and a new model must be built. On the other hand, if the parameters that are considered in the model, are assumed to be enough for building the model; two ways to fix the heteroscedasticity are to either transform data or using the weighted least squares. Additionally, according to the color-coded plot (Figure 3(a)), it seems that for topologies E and E1, it is better to build a separate model. In other words, data should be classified into different groups based on the topology and one model should be obtained for the topologies E and E1 and the other for the rest.

Table 2: Regression output of the study of ultimate drift: Full model

Variable	Estimate	Std. Error	t-value	p-value
Constant	6.23e-01	1.14e+00	0.55	0.59
N_{c1}	1.66e-01	2.34e-01	0.71	0.48
N_{c2}	-6.47e-01	4.18e-01	-1.55	0.12
N_{c3}	-9.84e-01	4.44e-01	-2.21	0.03 *
N_B	1.578e-01	5.44e-01	0.29	0.77
N_C	-7.55e-03	5.41e-01	-0.01	0.99
N_D	-6.03e-01	5.33e-01	-1.13	0.26
N_E	3.70e-02	5.73e-01	0.07	0.95
N_{E1}	4.51e-01	6.16e-01	0.73	0.47
$N_{Leaves=2}$	-6.44e-01	5.54e-01	-1.16	0.25
$N_{Leaves=3}$	6.96e-01	3.71e-01	1.88	0.06 .
H	1.22e-03	5.95e-04	2.05	0.04 *
L	-6.61e-04	3.58e-04	-1.85	0.07 .
t	7.81e-05	1.27e-03	0.06	0.95
H_0/H	5.25e-01	3.86e-01	1.36	0.18
$\sigma_{0\,tot}/f_c$	-3.04e+00	8.49e-01	-3.58	< 0.001***
$n = 104$	$R^2 = 0.64$	Adjusted $R^2 = 0.58$	Residual Std. Error (df=88) =0.56	

Note: Signif. codes: p<0.001 '***'; p<0.01 '**'; p< 0.05 '*'; p<0.1 '.'

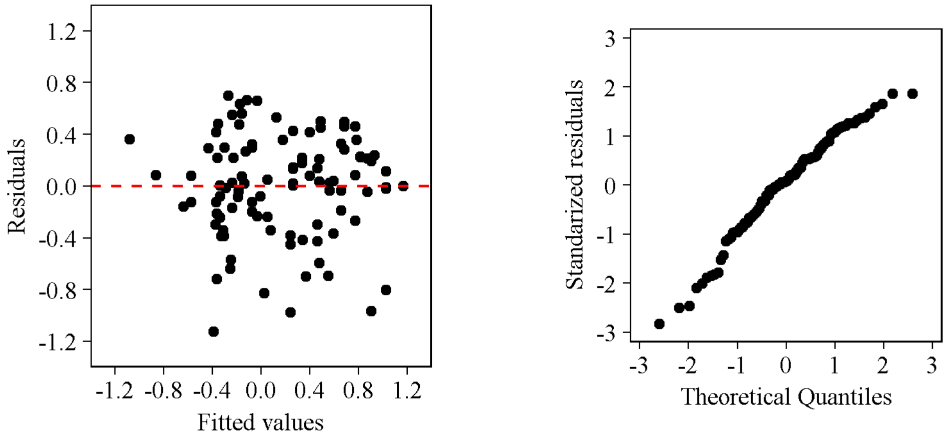


(a) graph of residuals vs. fitted values

(b) quantile-quantile plot

Figure 3: Diagnostic plots for the full model: ultimate drift

To correct for the heteroscedasticity, the response variable and the predictor variable "axial load ratio" are transformed using logarithmic function. The reason for transforming this predictor is that the trend observed in the scatter plot of ultimate drift vs. axial load ratio is exponential (Appendix A). It can be observed from Figure 4 that the dependency of variance on the mean of the response variable is rather removed and residuals are randomly distributed and no heteroscedasticity can be detected. In addition, the q-q plot shows that the assumption of normality of residuals is rather reasonable; and no significant outlier is observed.



(a) graph of residuals vs. fitted values (b) quantile-quantile plot

Figure 4: Diagnostic plots for the full model (transformed data): ultimate drift

Table 3: Regression output of the study of ultimate drift (transformed data): Full model

Variable	Estimate	Std. Error	t-value	p-value
Constant	-1.79e+00	0.79e+00	-2.27	0.025 *
N_{c1}	0.09e+00	0.18e+00	0.51	0.610
N_{c2}	-0.47e+00	0.30e+00	-1.56	0.120
N_{c3}	-0.72e+00	0.34e+00	-2.14	0.040 *
N_B	0.60e+00	0.36e+00	1.65	0.100
N_C	0.79e+00	0.34e+00	2.31	0.020 *
N_D	0.15e+00	0.38e+00	0.40	0.690
N_E	0.75e+00	0.34e+00	2.19	0.030 *
N_{E1}	0.57e+00	0.44e+00	1.30	0.200
$N_{Leaves=2}$	-0.62e+00	0.42e+00	-1.47	0.140
$N_{Leaves=3}$	0.73e+00	0.28e+00	2.64	< 0.01**
H	0.09e-02	0.04e-02	2.01	0.050 *
L	-0.05e-02	0.03e-02	-1.80	0.080 .
t	-0.06e-02	0.09e-02	-0.64	0.520
H_0/H	0.26e+00	0.29e+00	0.90	0.370
$\sigma_{0\,tot}/f_c$	-0.28e+00	0.09e+00	-3.24	0.002 **
$n = 104$	$R^2 = 0.62$	Adjusted $R^2 = 0.55$	Residual Std. Error (df=88) =0.42	

Note: Signif. codes: p<0.001 '***'; p<0.01 '**'; p< 0.05 '*'; p<0.1 '.'

Reduced model

For practical purposes, it is desirable to only include the numerical variables in the linear regression model since some categorical variables like topology of the stone-masonry walls and number of leaves are not yet defined uniquely in a same manner by different experts. Moreover, there is not any quantitative measurement describing the topology of the stone-masonry walls and they are categorized subjectively based on the experience of experts. Therefore, the reduced model is

$$\hat{d_u} = \hat{\beta_0} + \hat{\beta_1} H + \hat{\beta_2} L + \hat{\beta_3} t + \hat{\beta_4} \frac{H_0}{H} + \hat{\beta_5} \frac{\sigma_{0\,tot}}{f_c}. \quad (3)$$

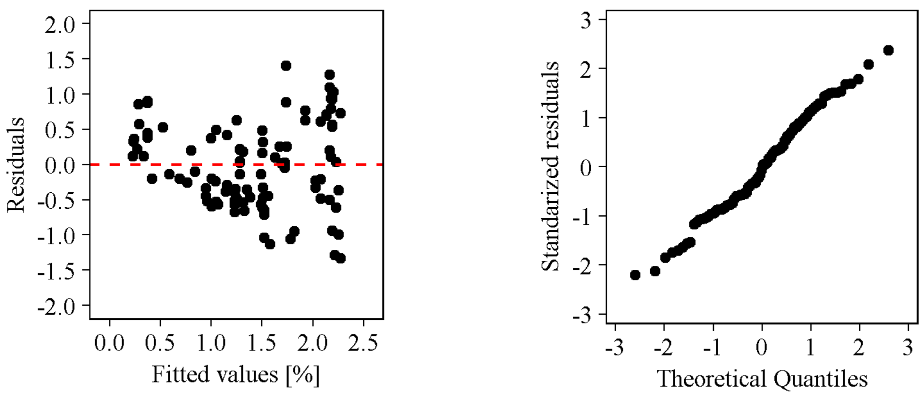
The results of the multiple regression analysis obtained using R are shown in Table 4.

Table 4: Regression output of the study of ultimate drift: Reduced model

Variable	Estimate	Std. Error	t-value	p-value
Constant	2.95e+00	0.63e+00	4.65	$< 0.001^{***}$
H	-0.02e-02	0.02e-02	-0.93	0.36
L	-0.05e-02	0.03e-02	-1.66	0.10
t	-0.14e-02	0.07e-02	-1.89	0.06 .
H_0/H	0.30e+00	0.32e+00	0.93	0.35
$\sigma_{0\,tot}/f_c$	-2.71e+0	0.46e+00	-5.89	$< 0.001^{***}$
$n = 104$	$R^2 = 0.50$	Adjusted $R^2 = 0.48$	Residual Std. Error (df=98) =0.62	

Note: Signif. codes: p<0.001 '***'; p<0.01 '**'; p< 0.05 '*'; p<0.1 '.'

Examining the p-values in the Table 4, with the significance level $\alpha = 0.05$, the constant term and axial load ratio are statistically significant. In addition, the diagnostic plots in Figure 5, show the same trend as the plots for the full model. It can be seen that the variance is dependent on the mean values. Additionally, according to the Figure 5(a), the model is largely biased for the small levels of drift as the residuals are all above the horizontal line. The cone shape in the Figure 5 suggests that some covariates must be transformed.

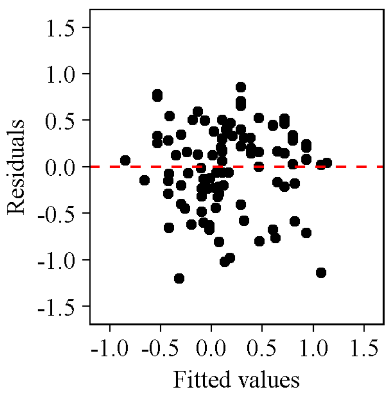


(a) Graph of residuals vs. fitted values

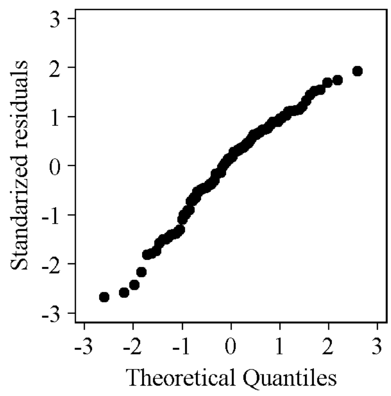
(b) quantile-quantile plot

Figure 5: Diagnostic plots for the reduced model: ultimate drift

To correct for the change of variance with respect to the mean, the response variable and axial load ratio are transformed (logarithmic). The diagnostic plots after transformation of these variables are shown in Figure 6.



(a) Graph of residuals vs. fitted values



(b) quantile-quantile plot

Figure 6: Diagnostic plots for the reduced model (transformed data): ultimate drift

Table 5: Regression output of the study of ultimate drift (transformed data): Reduced model

Variable	Estimate	Std. Error	t-value	p-value
Constant	0.47e+00	0.54e+00	0.86	0.40e+00
H	0.02e-02	0.02e-02	1.06	0.29e+00
L	-0.08e-02	0.02e-02	-3.25	0.16e-02 **
t	-0.08e-02	0.06e-02	-1.34	0.18e+00
H_0/H	-0.02e-01	0.23e+00	-0.01	0.99e+00
$\sigma_{0\,tot}/f_c$	-0.24e+00	0.04e+00	-5.68	1.36e-07 ***
$n = 104$	$R^2 = 0.49$	Adjusted $R^2 = 0.46$	Residual Std. Error (df=98) =0.46	

Note: Signif. codes: p<0.001 '***'; p<0.01 '**'; p< 0.05 '*'; p<0.1 '.'

For variable selection, two methods are used: partial F-test and step-wise regression. The partial F-test is used to decide whether adding a variable to the model make the model significantly better. The model including all variables, investigated in previous section is referred to as "full model", and the model with less predictors, which are a subset of variables in the full model, is denoted as the "reduced" or "nested" model [7]. In the partial F-test, the null and alternative hypotheses are:

H_0 : Reduced model is adequate. (no significant difference between two models)

H_1 : Full model is significantly better than the reduced model.

To test null hypothesis against the alternative hypothesis, the test statistic (F-test) is calculated as [7]:

$$F = \frac{[SSE(RM) - SSE(FM)/(p + 1 - k)]}{SSE(FM)/(n - p - 1)} \tag{4}$$

where $SSE(RM)$ and $SSE(FM)$ are the sums of the squared residuals for the reduced and full model, respectively. The n is the number of replicates, the p and k are the numbers of coefficients in the full and reduced models, respectively. The observed F-ratio in the Equation (4), has F-distribution with $p + 1 - k$ and $n - p - 1$ degrees of freedom. Evidently, $SSE(RM)$ is greater than $SSE(FM)$, because as the number of variables increases the sum of squared residuals decreases. By F-test, it is possible to evaluate whether or not the difference between the sums of squared residuals is statistically significant.

According to Table 6, the null hypothesis is rejected with the significance level $\alpha = 0.05$. In other words, the full model is significantly better than the reduced model.

Table 6: Analysis of variance: variable selection for the ultimate drift prediction

Model	Res.Df	RSS	Df	Sum of Square	F	Pr(>F)
Reduced model	98	20.74				
Full model	88	15.64	10	5.10	2.87	0.004 ***

Note: Signif. codes: p<0.001 '***'; p< 0.01 '**'; p<0.05 '*'

For selecting the most influential predictor variables, a value must be defined that can help decide among various models which one is better. There are various measurements in the literature making this comparison such as the Akaike Information Criterion (AIC), Bayes Information Criterion (BIC) and Mallows C_p [7]. In this study, AIC, which is defined as follows [7], is used as the value of the model through a step-wise algorithm to find the best model. The results of the analysis are shown in Table 7, in which, the length, the height, the number of cycles and leaves and axial load ratio are the most influential predictors.

$$AIC_p = n \ln(SSE_p/n) + 2p \tag{5}$$

Table 7: Step-wise variable selection for the ultimate drift model

Coefficients:								
Constant	N_{c1}	N_{c2}	N_{c3}	$N_{Leaves=2}$	$N_{Leaves=3}$	H	L	$\sigma_{0\,tot}/f_c$
-0.77	0.33	-0.56	-0.21	-0.52	0.31	0.00066	-0.00065	-0.30

Table 8: Results of the regression analysis for the model with the predictors selected by the step-wise method

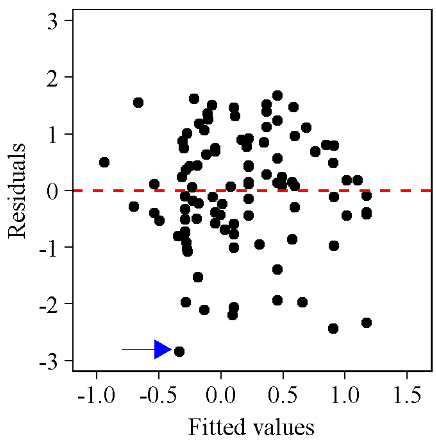
Variable	Estimate	Std. Error	t-value	p-value
Constant	-0.77e+00	0.39e+00	-1.98	0.05e+00 .
N_{c1}	0.33e+00	0.15e+00	2.25	0.03e+00 *
N_{c2}	-0.56e+00	0.20e+00	-2.83	0.6e-02 **
N_{c3}	-0.21e+00	0.15e+00	-1.42	0.16e+00
$N_{Leaves=2}$	-0.52e+00	0.27e+00	-1.94	0.06e+00 .
$N_{Leaves=3}$	0.31e+00	0.15e+00	2.13	0.04e+00 *
H	0.07e-02	0.02e-02	2.76	0.07e-01 **
L	-0.06e-02	0.03e-02	-2.48	0.02e+00 *
$\sigma_{0\,tot}/f_c$	-0.30e+00	0.04e+00	-7.00	3.66e-10 ***
$n = 104$	$R^2 = 0.58$	Adjusted $R^2 = 0.42$	Residual Std. Error (df=95) =0.42	

Note: Signif. codes: p<0.001 '***'; p<0.01 '**'; p< 0.05 '*'; p<0.1 '.'

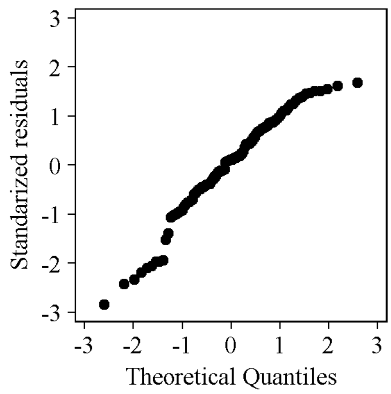
Comparing the results of the model consisting of the predictors selected by the step-wise method based on the AIC value with the results of the full model, it is observed that there is a slight decrease in the value of R^2, from 0.62 for the full model to 0.58 for the reduced model. Thus, the reduced model (obtained by the step-wise method) is superior over the full model, since the number of independent variables are only 8 compared with 15 in the full model; and moreover, the accuracy of the prediction is satisfactory. The validity of the assumptions for this model has also been checked according to the diagnostic plots in Figure 7. The arrow depicted in Figure 7(a) shows an outlier in the data, which must be removed for building the regression model. In addition, the distribution of residuals is slightly left-skewed, thus the normality of the residuals is fairly acceptable. The selected model for the ultimate drift capacity is chosen as:

$$\log(\hat{d_u}) = -0.77 + 0.33N_{c1} - 0.56N_{c2} - 0.21N_{c3} \\ - 0.52N_{Leaves=2} + 0.31N_{Leaves=3} + 0.00066H - 0.00065L - 0.30\log(\frac{\sigma_{0\,tot}}{f_c}). \quad (6)$$

The standard error of prediction of the ultimate drift is around 0.42 that is satisfactory.



(a) Graph of residuals vs. fitted values



(b) quantile-quantile plot

Figure 7: Diagnostic plots for the proposed model: ultimate drift

3.2 Ultimate force prediction

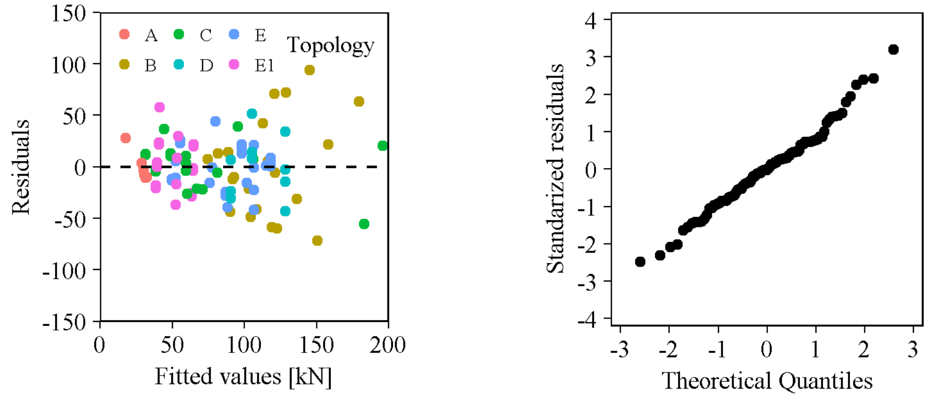
Full model

In this section, the same procedures for obtaining the linear regression model for predicting the ultimate force are followed. Firstly, the full model with all measured variables is built. Table 9 shows the results of the regression analysis. In this model, all two-level categorical variables describing topology and number of leaves are statistically significant at the level $\alpha = 0.05$. Also, from continuously varying variables, the length of the wall and axial load ratio are statistically significant at the level $\alpha = 0.05$. Figure 8 depicts the diagnostic plots for the ultimate force full model. The points in the residual plot are not randomly distributed suggesting transformation of response and some predictor variables.

Table 9: Regression output of the study of ultimate force: Full model

Variable	Estimate	Std. Error	t-value	p-value
Constant	-2.67e+02	6.44e+01	-4.14	< 0.001***
N_{c1}	1.09e+01	1.32e+01	0.824	0.410
N_{c2}	3.20e+01	2.36e+01	1.36	0.180
N_{c3}	-1.26e+01	2.51e+01	-0.50	0.620
N_B	1.67e+02	3.08e+01	5.42	< 0.001***
N_C	1.33e+02	3.06e+01	4.36	< 0.001***
N_D	6.78e+01	3.02e+01	2.25	0.030 *
N_E	1.55e+02	3.24e+01	4.79	< 0.001***
N_{E1}	1.72e+02	3.49e+01	4.94	< 0.001***
$N_{Leaves=2}$	-8.26e+01	3.13e+01	-2.64	0.009 **
$N_{Leaves=3}$	4.56e+01	2.10e+01	2.18	0.032 *
H	4.40e-02	3.36e-02	1.31	0.194
L	1.21e-01	2.03e-02	5.99	< 0.001***
t	2.19e-03	7.17e-02	0.03	0.980
H_0/H	-2.57e+01	2.18e+01	-1.18	0.240
$\sigma_{0\,tot}/f_c$	1.39e+02	4.80e+01	2.90	0.005 **
$n = 104$	$R^2 = 0.62$	Adjusted $R^2 = 0.55$	Residual Std. Error (df=88) =31.48	

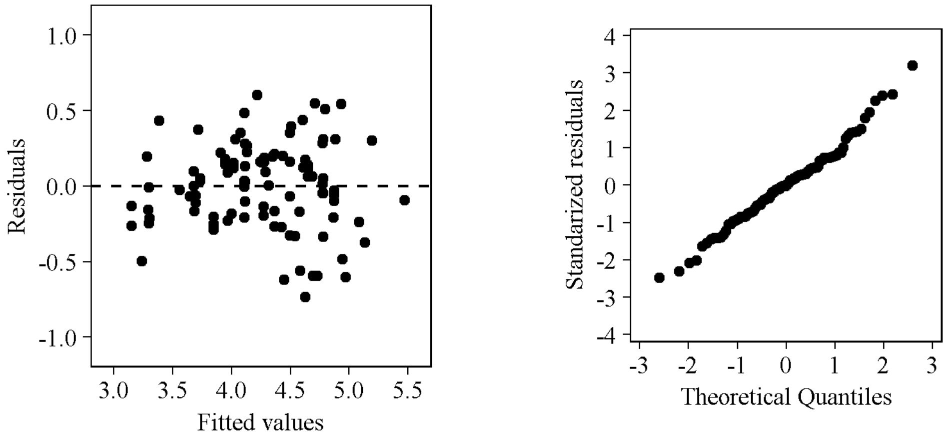
Note: Signif. codes: p<0.001 '***'; p<0.01 '**'; p< 0.05 '*'; p<0.1 '.'



(a) graph of residuals vs. fitted values (b) quantile-quantile plot

Figure 8: Diagnostic plots for the full model: ultimate force

As the variance of residuals varies with regard to the response variable, the data is transformed (axial load ratio and response variable are transformed using log function). The results of diagnostic plots (Figure 9) after transformation of axial load ratio and response variable validate the assumptions of linear regression model. The value of the coefficient of determination is higher than the model for ultimate drift which suggests that the ultimate force can be better described as a linear function of the concerned covariates than ultimate drift. According to the results in Table 3, the topology, length, number of cycles and axial load ratio are statistically significant at the level $\alpha = 0.001$.



(a) graph of residuals vs. fitted values (b) quantile-quantile plot

Figure 9: Diagnostic plots for the full model (transformed data): ultimate force

Table 10: Regression output of the study of ultimate force (transformed data): Full model

Variable	Estimate	Std. Error	t-value	p-value
Constant	2.01e+00	5.62e-01	3.57	6e-04 ***
N_{c1}	2.01e-01	1.27e-01	1.58	0.12e+00
N_{c2}	5.50e-01	2.16e-01	2.55	0.01e+00 *
N_{c3}	8.90e-02	2.40e-01	0.370	0.71e+00
N_B	1.87e+00	2.60e-01	7.18	0.02e-9 ***
N_C	1.45e+00	2.45e-01	5.90	0.06e-07 ***
N_D	1.18e+00	2.68e-01	4.40	0.03e-04 ***
N_E	2.10e+00	2.45e-01	8.56	0.03e-12 ***
N_{E1}	2.92e+00	3.12e-01	9.37	0.07e-14 ***
$N_{Leaves=2}$	-5.18e-01	3.02e-01	-1.72	0.09e+00 .
$N_{Leaves=3}$	1.92e-01	1.99e-01	0.96	0.34e+00
H	2.90e-04	3.21e-04	0.91	0.37e+00
L	1.08e-03	1.94e-04	5.60	0.02e-06 ***
t	8.05e-05	6.59e-04	0.12	0.90e+00
H_0/H	-2.95e-01	2.09e-01	-1.41	0.16e+00
$\sigma_{0\,tot}/f_c$	4.84e-01	6.18e-02	7.82	0.01e-10 ***
$n = 104$	$R^2 = 0.77$	Adjusted $R^2 = 0.72$	Residual Std. Error (df=88) =0.30	

Note: Signif. codes: p<0.001 '***'; p<0.01 '**'; p< 0.05 '*'; p<0.1 '.'

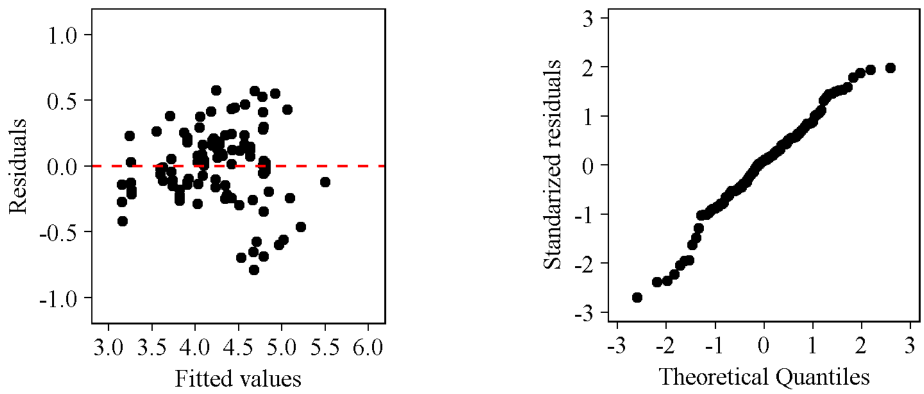
Reduced model

Using the step-wise method based on AIC value, the topology of the wall, number of cycles, length of the wall, shear span ratio and axial load ratio are selected as the predictor variables. Table 11 shows the result of linear regression analysis with the predictor variables selected by AIC. In both Figures (9(a)-10(a)), it can be seen that small and large values of fitted response variable, points are below the horizontal line. In fact, the graph has a quadratic polynomial trend with down concave. The same trend can be observed in the plot of ultimate force vs. axial load ratio (Appendix A).

Table 11: Results of the regression analysis for the ultimate force model with the predictors selected by step-wise method

Variable	Estimate	Std. Error	t-value	p-value
Constant	2.66	0.31	8.46	3.91e-13 ***
N_B	1.67	0.21	8.11	2.10e-12 ***
N_C	1.22	0.19	6.27	1.15e-08 ***
N_D	1.44	0.21	6.89	6.93e-10 ***
N_E	2.00	0.23	8.85	5.92e-14 ***
N_{E1}	2.80	0.29	9.79	6.37e-16 ***
N_{c1}	0.23	0.12	1.90	0.06e+00 .
N_{c2}	0.44	0.20	2.43	0.02e+00 *
N_{c3}	0.29	0.15	2.00	0.05e+00 *
L	0.001	0.0002	6.07	2.77e-08 ***
H_0/H	-0.23	0.17	-1.35	0.20e+00
$\sigma_{0\,tot}/f_c$	0.51	0.06	8.76	9.05e-14 ***
$n = 104$	$R^2 = 0.76$	Adjusted $R^2 = 0.73$	Residual Std. Error (df=92) =0.30	

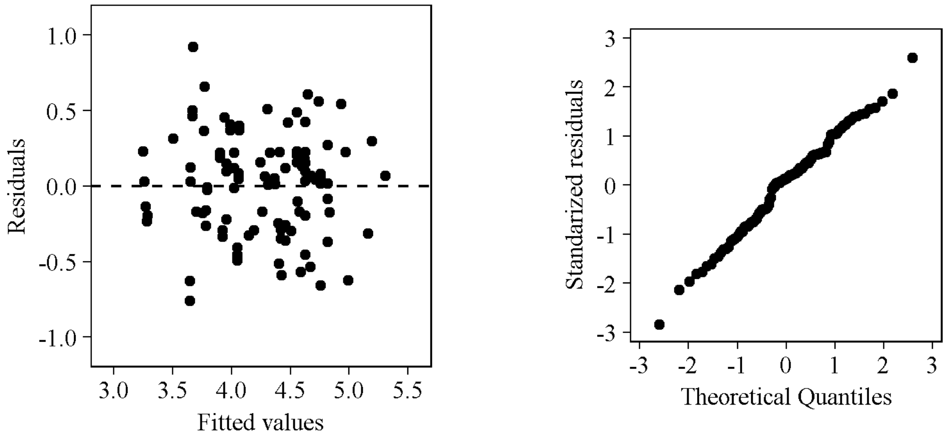
Note: Signif. codes: p<0.001 '***'; p<0.01 '**'; p< 0.05 '*'; p<0.1 '.'



(a) graph of residuals vs. fitted values (b) quantile-quantile plot

Figure 10: Diagnostic plots for the reduced model: ultimate force

To tackle this issue, the predictor variable "axial load ratio" is transformed using quadratic polynomial function while the response variable is mapped by logarithmic function. The diagnostic plots for this model are depicted in Figure 11. The q-q plot illustrates that there are two points that can be regarded as outliers. As residuals are randomly distributed, after removing outliers from the data, the final multiple linear model is created. The results are shown in the Table 12. The standard error of this model (0.35) can be acceptable for engineering application. It should be mentioned that for the ultimate force model with transformed variables, AIC criterion has been used and only the variables number of leaves, thickness of the wall and height of the wall were removed from the full model. However, partial F-test showed that the full model is significantly better than the reduced model. Therefore, here only the results of full model are mentioned.



(a) graph of residuals vs. fitted values (b) quantile-quantile plot

Figure 11: Diagnostic plots for the proposed model: ultimate force

Table 12: Results of the regression analysis for the proposed ultimate force model

Variable	Estimate	Std. Error	t-value	p-value
Constant	0.12	0.72	0.16	0.87
N_B	2.02	0.35	5.85	8.68e-08 ***
N_C	1.71	0.34	5.01	2.91e-06 ***
N_D	1.28	0.34	3.80	0.00027 ***
N_E	1.97	0.36	5.47	4.30e-07 ***
N_{E1}	2.16	0.39	5.57	2.89e-07 ***
N_{c1}	0.25	0.15	1.64	0.11
N_{c2}	0.44	0.26	1.68	0.10 .
N_{c3}	0.03	0.28	0.11	0.91
$N_{Leaves=2}$	-0.95	0.35	-2.70	0.008 **
$N_{Leaves=3}$	0.57	0.23	2.44	0.017 *
H	0.0007	0.0004	1.79	0.08 .
L	0.0010	0.0002	4.65	1.18e-05 ***
t	-0.0003	0.0008	-0.41	0.68
H_0/H	-0.25	0.24	-1.05	0.30
$\sigma_{0\,tot}/f_c$	1.64	0.53	3.08	0.003 **
$n = 104$	$R^2 = 0.67$	Adjusted $R^2 = 0.62$	Residual Std. Error (df=86) =0.35	

Note: Signif. codes: p<0.001 '***'; p< 0.01 '**'; p<0.05 '*', p<0.1 '.'

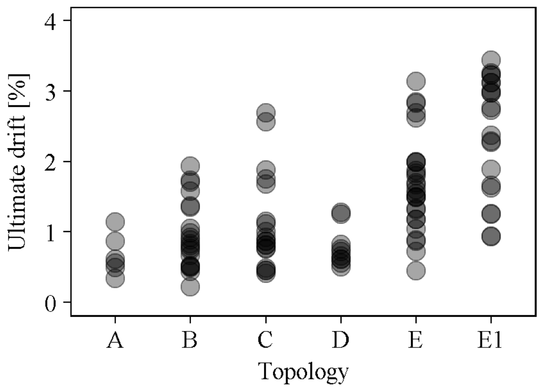
4 Conclusions

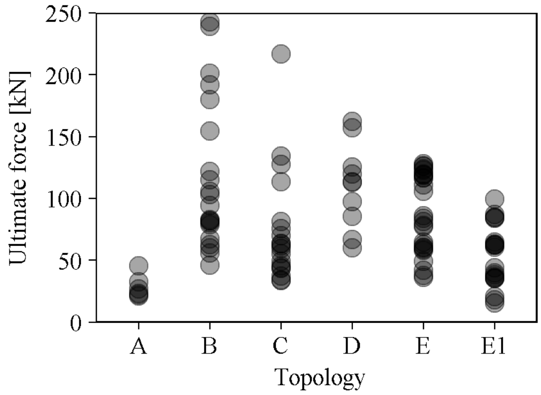
In this study, the applicability of using linear regression analysis to predict ultimate force and drift capacity of stone-masonry walls is evaluated using a database of quasi-static tests consisting of 104 replicates. To do so, firstly, all of the variables (categorical and numerical) in the tests are included in the linear regression model and compared with the models including just numerical predictors. The validity of linear regression assumptions is then examined and finally, the partial F-test is used to investigate which model is adequate. Moreover, based on the AIC criterion, the most influential parameters are selected. The results show that the response variables and the predictor variable "axial load ratio" must be transformed to correct for the effect of heteroscedasticity. The higher value of the coefficient of determination in the ultimate force prediction model ($R^2 = 0.67$) shows that the variability in the ultimate force capacity can be better described using linear model compared to ultimate drift model ($R^2 = 0.58$). However, it should be mentioned that the linearity assumptions for both prediction model are rather satisfied as the residuals are not completely random. Furthermore, the most effective explanatory variable is axial load ratio since in all of the analyses, the estimate of regression parameter corresponding to this variable is statistically significant at level $\alpha = 0.01$.

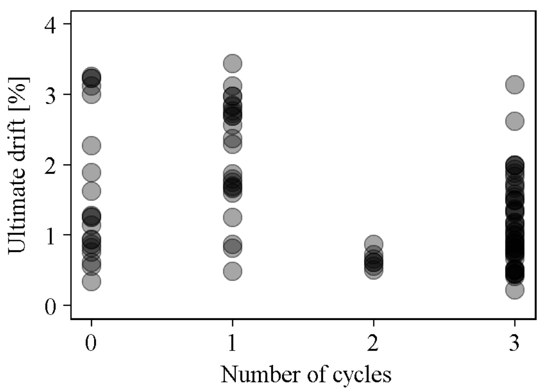
Appendix A

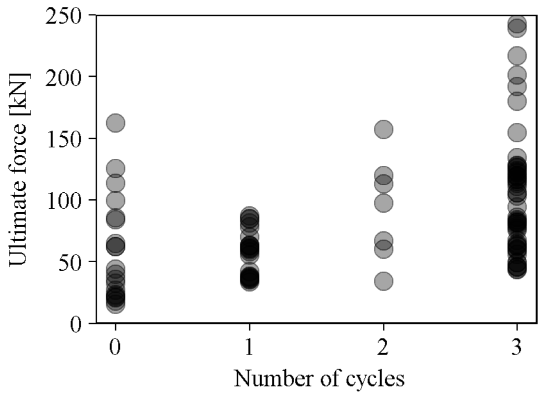
Data Visualization: Scatter plots

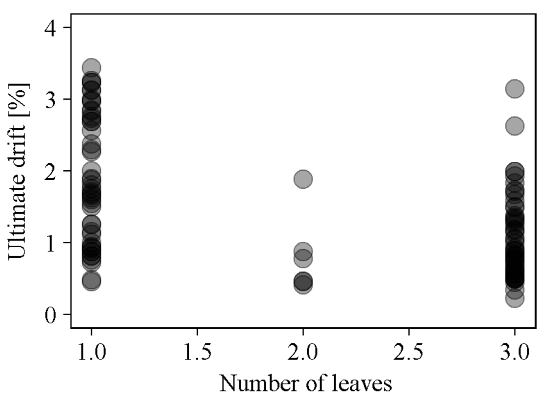
In this section, the scatter plots of every predictor variables are plotted against the response variables.

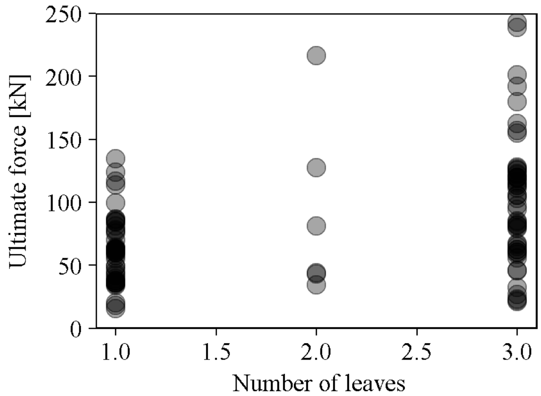


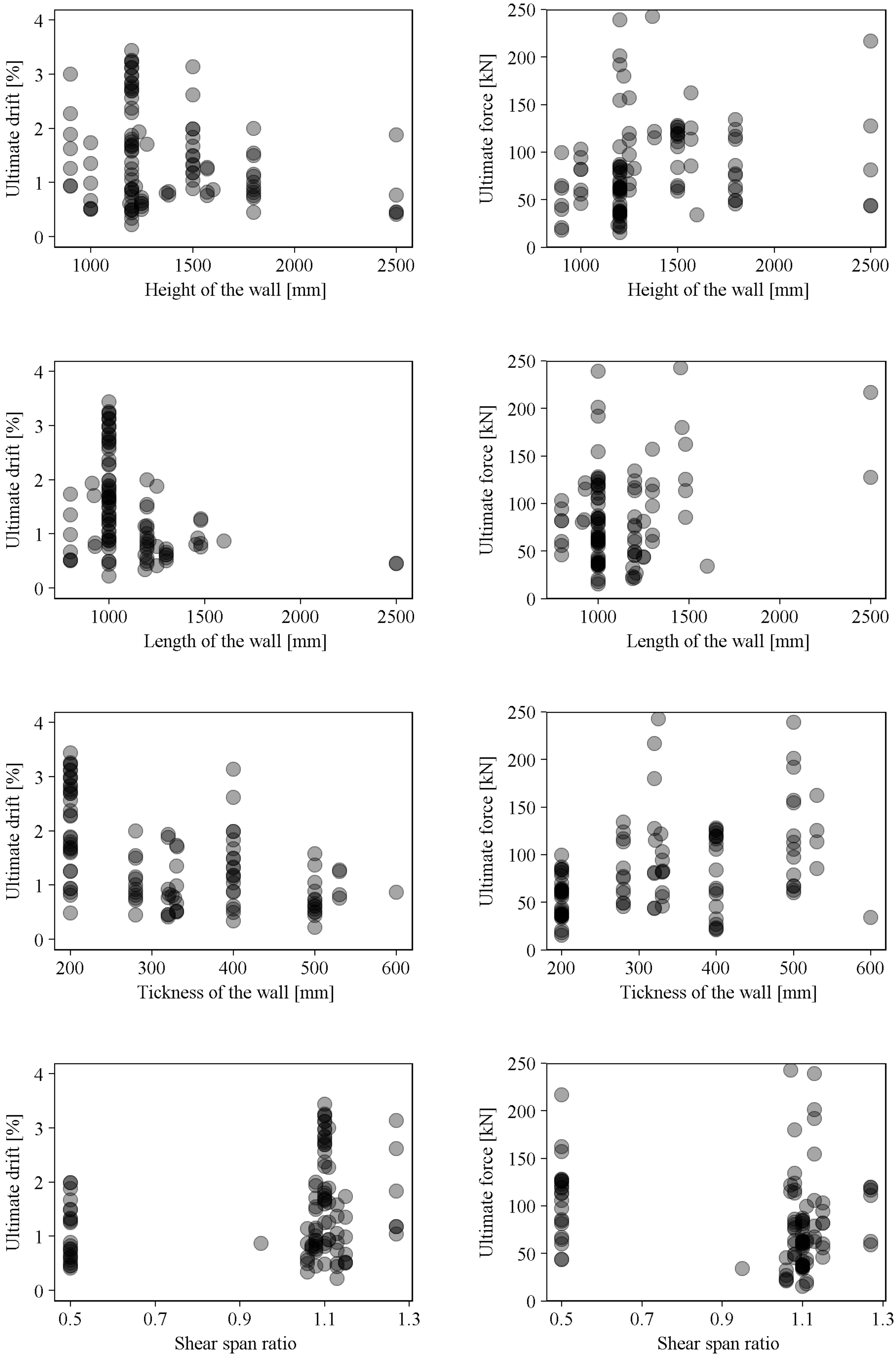


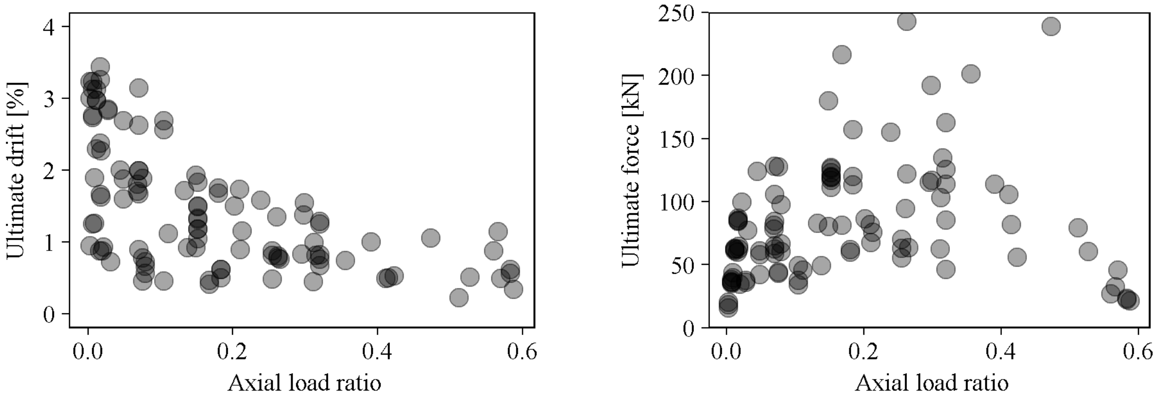









Ultimate drift [%]
Height of the wall [mm]
Ultimate force [kN]
Height of the wall [mm]
Ultimate drift [%]
Length of the wall [mm]
Ultimate force [kN]
Length of the wall [mm]
Ultimate drift [%]
Tickness of the wall [mm]
Ultimate force [kN]
Tickness of the wall [mm]
Ultimate drift [%]
Shear span ratio
Ultimate force [kN]
Shear span ratio


Ultimate drift [%]
Axial load ratio
Ultimate force [kN]
Axial load ratio

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