Machine learning and vision laboratory

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Pre-Lab 8

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Optical Flow Using a Pyramid

Soloution

Optical flow is the motion of pixels between two consecutive frames in a video, showing how much each pixel has moved. However, detecting large or complex motions can be challenging. A **pyramid** is used to make this process more efficient and accurate.

What is a Pyramid in Optical Flow?

A **pyramid** is a series of progressively smaller versions of the original image. The idea is to first calculate the optical flow on a lower-resolution image, where large movements are easier to detect. Then, we refine the flow as we move to higher-resolution images.

Why Use a Pyramid?

1. **Handling Large Movements**: Large motions are easier to track on low-resolution images.
2. **Faster Computation**: Lower-resolution images are faster to process, saving time.
3. **Better Detail**: We can detect small movements more accurately at higher resolutions.

How It Works

Each pyramid level is a downsampled version of the previous one. We first compute the optical flow at the coarsest level, then progressively refine it at higher levels for more detail. Here is an image for more details:

Additive Point Question

Soloution

We are tasked with minimizing the following expression with respect to u and v:

$$\min_{u,v} \sum_{i=-1}^{1} \sum_{j=-1}^{1} \left(f_x(i,j)u + f_y(i,j)v + f_t(i,j)\right)^2 \tag{1}$$

The objective function to minimize is:

$$E(u,v) = \sum_{i=-1}^{1}\sum_{j=-1}^{1} \left(f_x(i,j)u + f_y(i,j)v + f_t(i,j)\right)^2 \tag{2}$$

To minimize this function, we take the partial derivatives of $E(u,v)$ with respect to u and v and set them equal to zero:

$$\frac{\partial E(u,v)}{\partial u} = 0 \tag{3}$$

$$\rightarrow \frac{\partial}{\partial u}\sum_{i=-1}^{1}\sum_{j=-1}^{1} \left(f_x(i,j)u + f_y(i,j)v + f_t(i,j)\right)^2 = 0 \tag{4}$$

$$\rightarrow \sum_{i=-1}^{1}\sum_{j=-1}^{1} 2\left(f_x(i,j)u + f_y(i,j)v + f_t(i,j)\right) f_x(i,j) = 0 \tag{5}$$

And do the same for v. Then, we obtain the system of linear equations:

$$\sum_{i=-1}^{1}\sum_{j=-1}^{1} \left(f_x(i,j)u + f_y(i,j)v + f_t(i,j)\right) f_x(i,j) \tag{6}$$

$$= \sum_{i=-1}^{1}\sum_{j=-1}^{1} f_x(i,j)^2 u + f_x(i,j)f_y(i,j)v + f_x(i,j)f_t(i,j) = 0 \tag{7}$$

$$\sum_{i=-1}^{1}\sum_{j=-1}^{1} \left(f_x(i,j)u + f_y(i,j)v + f_t(i,j)\right) f_y(i,j) \tag{8}$$

$$= \sum_{i=-1}^{1}\sum_{j=-1}^{1} f_x(i,j)f_y(i,j)u + f_y(i,j)^2 v + f_y(i,j)f_t(i,j) = 0 \tag{9}$$

Note that It's 2 linear equations with 2 unknowns for scalar u and v. However, Since x and y are vectors, we can write the above equations in matrix form as:

$$\begin{bmatrix} f_{x_1} & f_{y_1} \\ f_{x_2} & f_{y_2} \\ \vdots & \vdots \\ f_{x_9} & f_{y_9} \end{bmatrix} \tag{10}$$

The above matrix is called A in instructions. The vector b is defined as:

$$b = \begin{bmatrix} f_{t_1} \\ f_{t_2} \\ \vdots \\ f_{t_9} \end{bmatrix} \tag{11}$$

Using basic matrix algebra, we can write the system of equations in matrix form as:

$$\begin{bmatrix} u \\ v \end{bmatrix} = (A^T A)^{-1} A^T b \tag{12}$$