

CS174A → Assignment 1

1.)

a.) A: $p = 0 + 2i_A - j_A$
 B: $p = 0 + 3i_B + j_B$
 C: $p = 0 - 3i_C + 7/3j_C$

b.) 2 points needed to describe a vector.

A: $v = 2i_A - j_A$
 B: $v = 2i_B - 3/2j_B$
 C: $v = i_C - 0j_C$

c.) A coordinate frame is represented by basis vectors and origin.

d.) $M_A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

$M_B = \begin{bmatrix} -1 & 0 & 6 \\ -1 & 2 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

$M_C = \begin{bmatrix} 2 & 3 & 2 \\ -1 & -3 & 5 \\ 0 & 0 & 1 \end{bmatrix}$

$w = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$

e.) $P_w = M_A P_A$

$P_A = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$

$M_A \quad P_A \quad P_w \checkmark$
 $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2+1 \\ -1+2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$

$$P_W =$$

$$P_W = M_B P_B$$

$$P_B = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 6 \\ -1 & 2 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -3+6 \\ -3+2+2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} \quad \checkmark$$

$$P_W = M_C P_C$$

$$P_C = \begin{bmatrix} -3 \\ -7/3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 & 2 \\ -1 & -3 & 5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -3 \\ -7/3 \\ 1 \end{bmatrix} = \begin{bmatrix} -6+7+2 \\ 3-7+5 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} \quad \checkmark$$

2.)

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3.) $\text{Scale}(1,1,2)$ $\text{Trans}(1,1,1)$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

modelMatrix.setAsIdentity
 $M = \text{modelMatrix} * \text{Scale}(1,1,2);$
 $M = M * \text{Translate}(1,1,1);$

$$\therefore M = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4.) $[2 \ 10 \ 8 \ 4]^T \Rightarrow [2/4 \ 10/4 \ 8/4 \ 4/4]^T \Rightarrow$

$$\Rightarrow [0.5 \ 2.5 \ 2 \ 1]^T$$

5.)

//A

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

//B

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 90 & 0 & \sin 90 & 0 \\ 0 & 1 & 0 & 0 \\ -\sin 90 & 0 & \cos 90 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow$$

Source

//B

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 3 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

//C

$$\begin{bmatrix} 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 3 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 2 \\ 0 & 0.5 & 0 & 3 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 & 2 \\ 0 & 0.5 & 0 & 3 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 2 \\ 0 & 0.5 & 0 & 3.5 \\ -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

//D

$$\begin{bmatrix} 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 3 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 3 \\ -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

6.

Let a be the y-intercept

Let θ be the angle of the tilted line back at the origin

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1-b \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix}$$

Composite Matrix

OpenGL Commands:

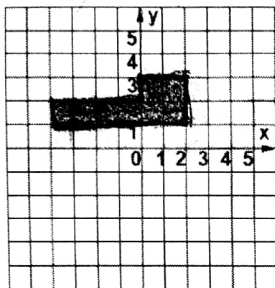
```
modelMatrix = setAsIdentity();  
modelMatrix = modelMatrix * Translate(0, -b);  
modelMatrix = modelMatrix * RotateY(-θ);  
modelMatrix = modelMatrix * Scale(1, -1);  
modelMatrix = modelMatrix * RotateY(θ);  
modelMatrix = modelMatrix * Translate(0, b);
```

7.) On Last Page →

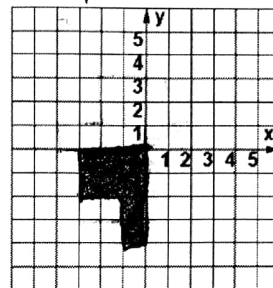


$$\begin{array}{l}
 \text{Scale} \\
 A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
 \end{array}
 \begin{array}{l}
 \text{Trans} \\
 B = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
 \end{array}
 \begin{array}{l}
 \text{Rotation } 90^\circ \text{ cc} \\
 C = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
 \end{array}
 \begin{array}{l}
 \text{Ref. abt. } y \\
 D = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
 \end{array}$$

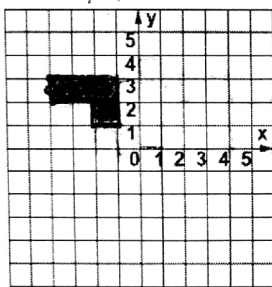
a) $L' = ABC L$



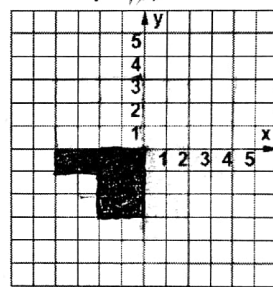
b) $L' = \cancel{C} \cancel{B} L$



c) $L' = \cancel{C} \cancel{B} L$



d) $L' = \cancel{D} \cancel{C} \cancel{B} L$



a.) `modelMatrix.setAsIdentity();
modelMatrix = modelMatrix * RotateZ(90);
modelMatrix = modelMatrix * Translate(1,1,1);
modelMatrix = modelMatrix * Scale(2,1,1);
drawL(modelMatrix);`

b.) `modelMatrix.setAsIdentity();
modelMatrix = modelMatrix * Scale(-1,1,1);
modelMatrix = modelMatrix * Scale(2,1,1);
modelMatrix = modelMatrix * RotateZ(90);
drawL(modelMatrix);`

c.) `modelMatrix.setAsIdentity();
modelMatrix = modelMatrix * Scale(-1,1,1);
modelMatrix = modelMatrix * Translate(1,1,1);
modelMatrix = modelMatrix * RotateZ(90);
drawL(modelMatrix);`

d.) `modelMatrix.setAsIdentity();
modelMatrix = modelMatrix * Scale(-1,1,1);
modelMatrix = modelMatrix * Scale(2,1,1);
modelMatrix = modelMatrix * RotateZ(90);
modelMatrix = modelMatrix * RotateZ(90);
modelMatrix = modelMatrix * Scale(-1,1,1);
drawL(modelMatrix);`