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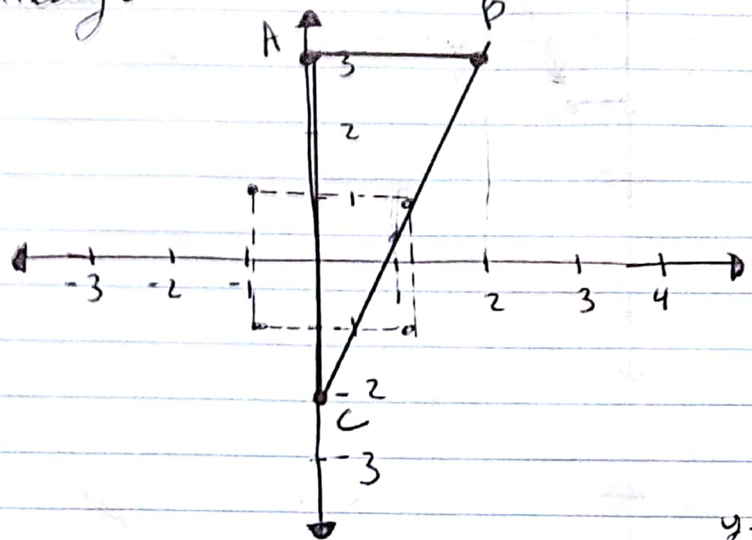
HW#3

1.) Clipping

~~a.)~~ Initially:

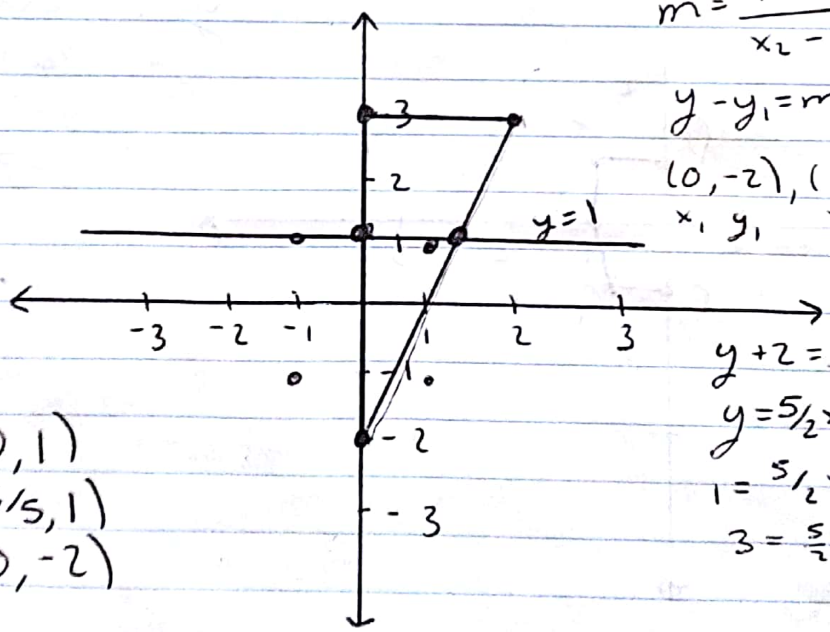


$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3+2}{2-0} = \frac{5}{2}$$

$$y - y_1 = m(x - x_1)$$

$(0,-2), (2,3)$
$x_1\ y_1 \quad x_2\ y_2$

a.)



$$y + 2 = \frac{5}{2}(x - 0) \Rightarrow y + 2 = \tfrac{5}{2}x - 2$$

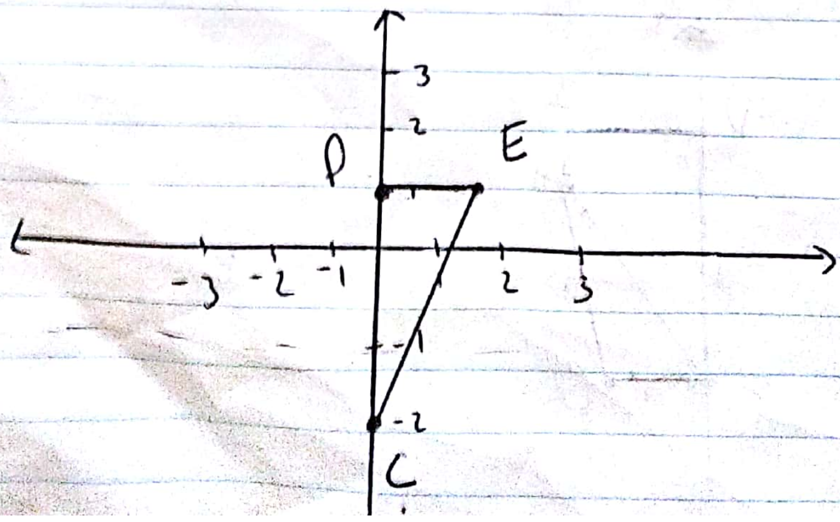
$$y = \tfrac{5}{2}x - 2$$

$$1 = \tfrac{5}{2}x - 2$$

$$3 = \tfrac{5}{2}x \rightarrow \frac{6}{5} = \frac{\cancel{5}x}{\cancel{5}}$$

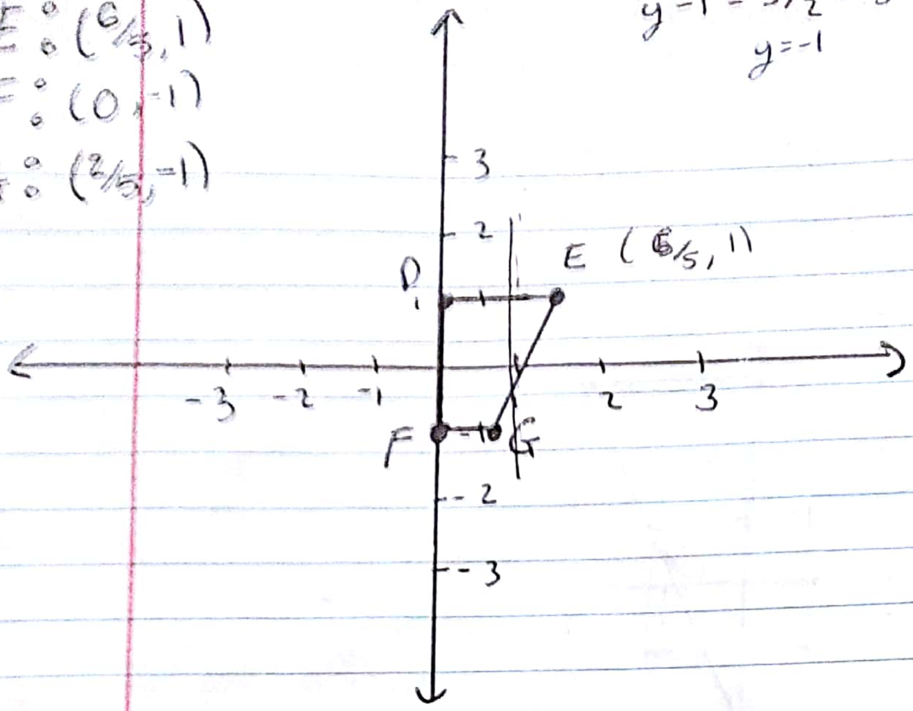
$$x = 6/5$$

D: $(0,1)$
E: $(6/5,1)$
C: $(0,-2)$



b.) D: (0,1)
E: (6/5,1)
F: (0,-1)
G: (2/5,-1)

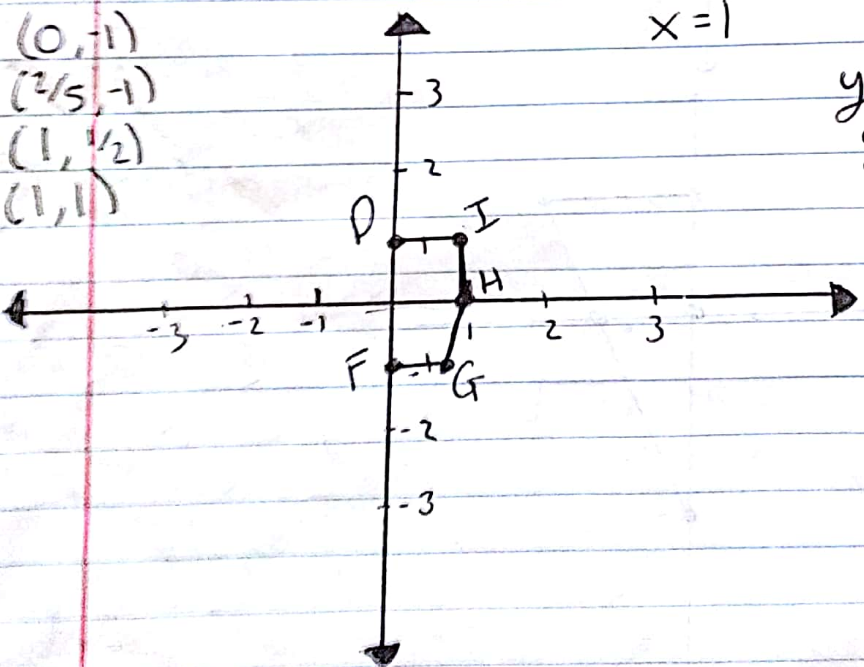
$y-1=\cancel{6/5}\ 5/2(x-6/5)$
$y-1=5/2x-3 \Rightarrow y=5/2x-2$
$y=-1$
$-1=5/2x-2$
$1=5/2x$
$x=\frac{2}{5}$



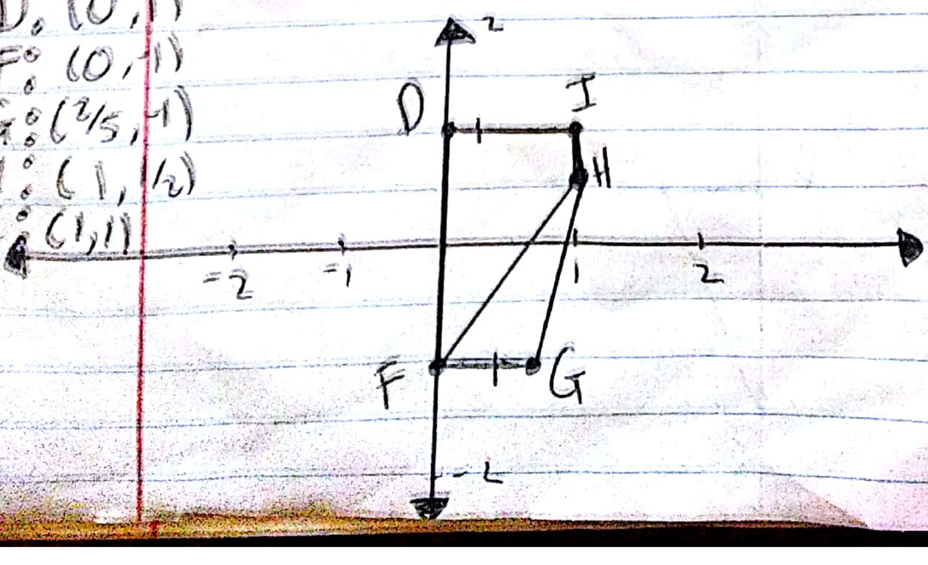
c.) D: (0,1)
F: (0,-1)
G: (2/5,-1)
H: (1,1/2)
I: (1,1)

$x=1$

$y+1=5/2(x-\frac{2}{5})$
$y+1=5/2x-1$
$y=5/2x-2$
$y=5/2-4/2=1/2$



d.) D: (0,1)
F: (0,-1)
G: (2/5,-1)
H: (1,1/2)
I: (1,1)



2.) Rastorization / Scan - Conversion

a.) $\cancel{(x-x_0)^2+(y-y_0)^2=r^2}$ → $\boxed{(x-x_0)^2+(y-y_0)^2+r^2=0}$

b.) Naive Scan-Converting Algorithm

```
For x = -r to r
    y = sqrt(r^2 - x^2)
    PlotPixel(x, round(y))
    PlotPixel(x, -round(y))
end;
```

c.) 1.) Input radius r and circle center (x_0, y_0) and obtain the first point on the circumference of a circle centered on the origin as

$$(x_0, y_0) = (0, r)$$

2.) Calculate the initial value of the decision parameter as

$$p_0 = 5/4 - r$$

3.) At each x_k position, starting at $k=0$, perform the following test: If $p_k < 0$, the next point along the circle centered on $(0,0)$ is (x_{k+1}, y_k) and

$$p_{k+1} = p_k + 2x_{k+1} + 1$$

Otherwise, the next point along the circle is $(x_k + 1, y_k - 1)$ and

$$p_{k+1} = p_k + 2x_{k+1} + 1 - 2y_{k+1}$$

where $2x_{k+1} = 2x_k + 2$ and $2y_{k+1} = 2y_k - 2$

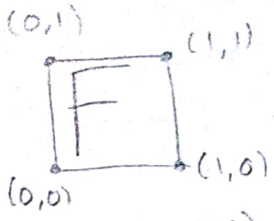
4.) Determine symmetry points in the other seven octants

5.) Move each calculated pixel position (x, y) onto the circular path centered on (x_c, y_c) and plot the coordinate values:

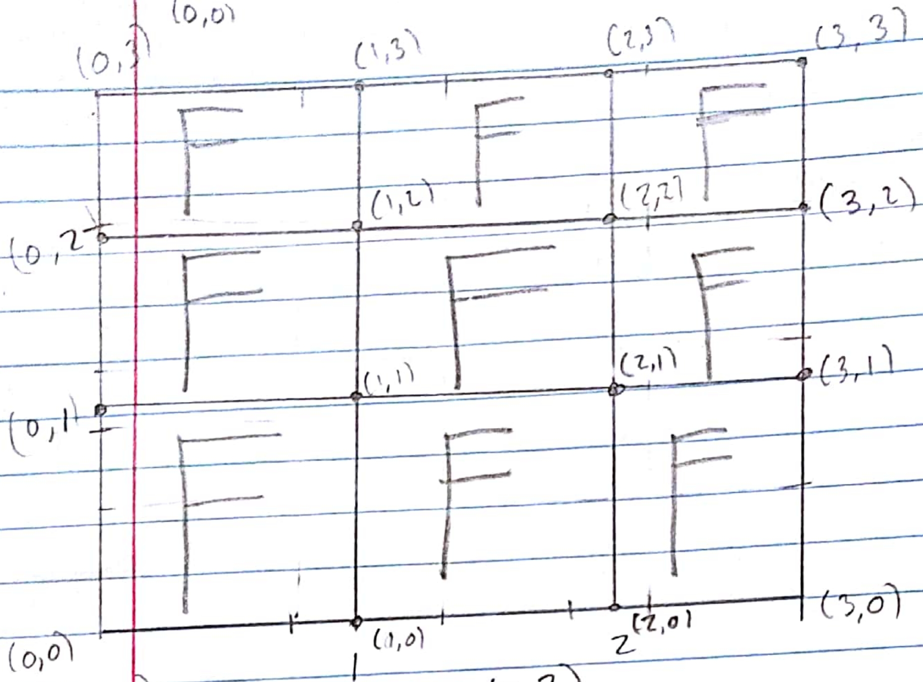
$$x = x + x_c \qquad y = y + y_c$$

6.) Repeat steps 3 through 5 until $x \geq y$.

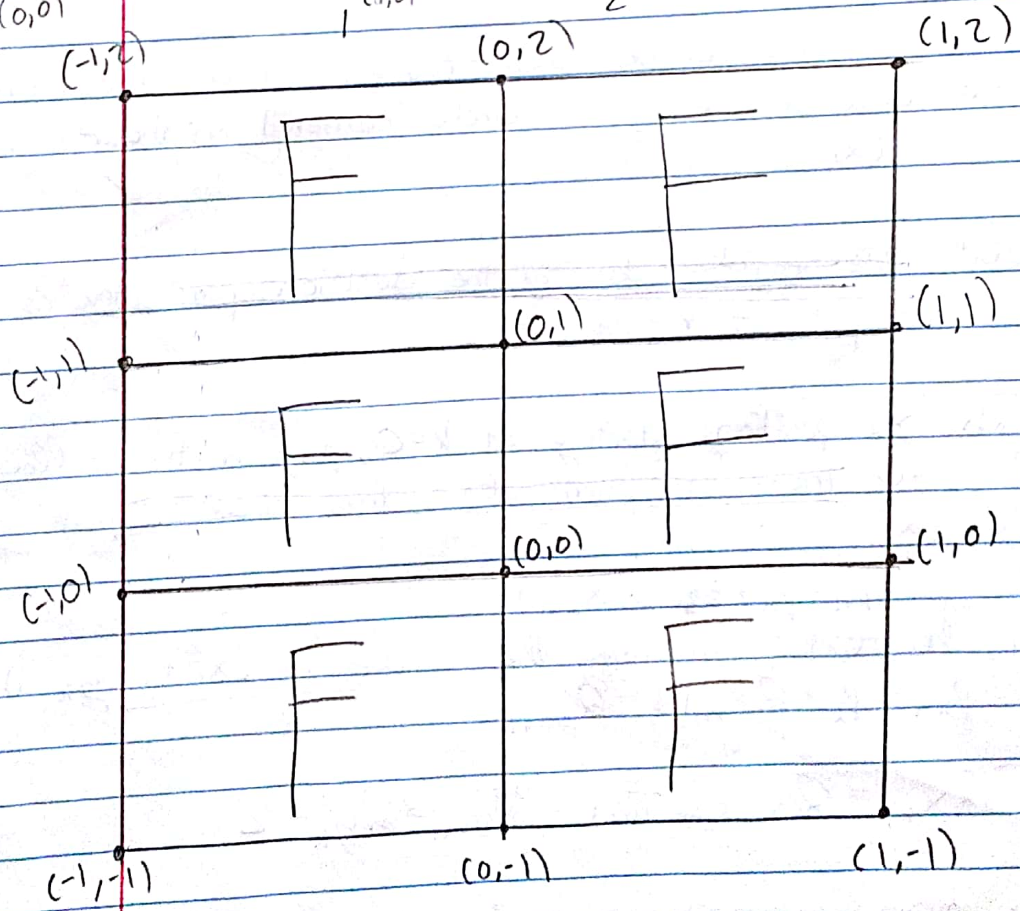
3.)


(0,1)
(1,1)
F
(0,0)
(1,0)

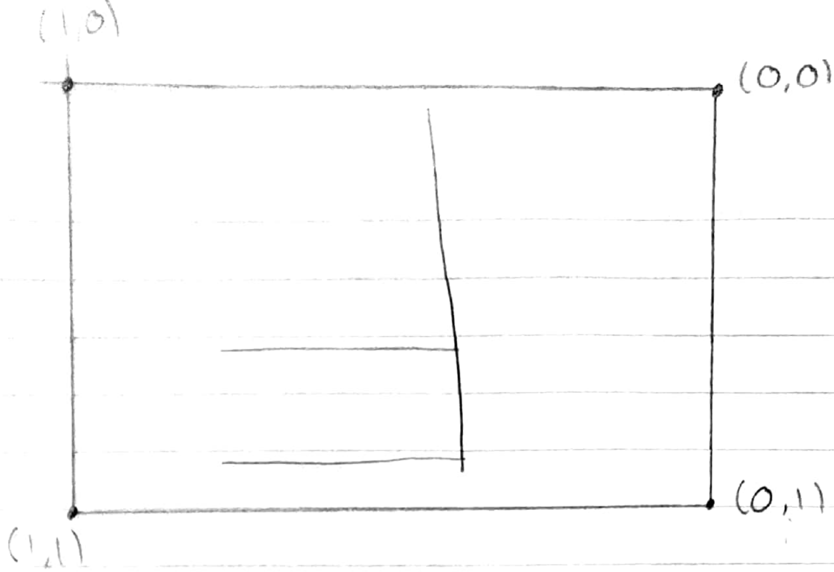
a.)


(0,3)
(1,3)
(2,3)
(3,3)
(1,2)
(2,2)
(3,2)
(0,2)
(1,1)
(2,1)
(3,1)
(0,1)
(0,0)
(1,0)
(2,0)
(3,0)
1
2

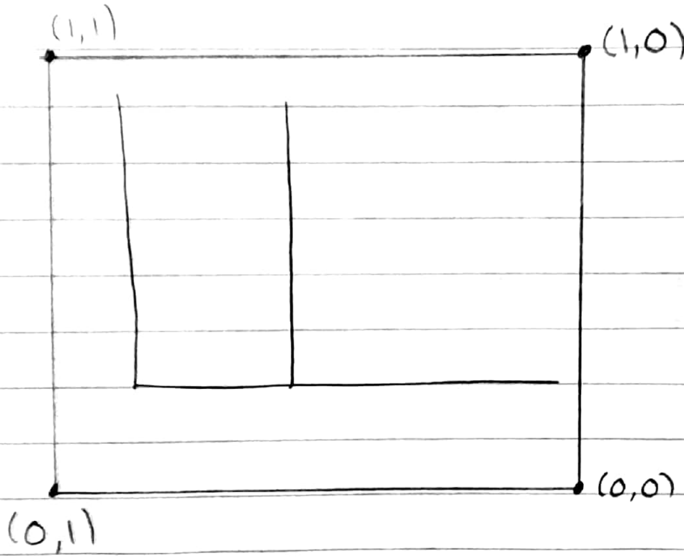
b.)


(-1,2)
(0,2)
(1,2)
(0,1)
(1,1)
(-1,1)
(0,0)
(1,0)
(-1,0)
(-1,-1)
(0,-1)
(1,-1)

C.)

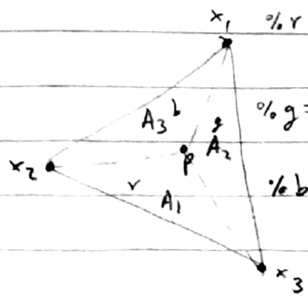


d.)



4.) Interpolation

Point	Color
$P_1 = [3 \quad 6 \quad 0]$	$C_1 = [.3 \quad .2 \quad .9]$
$P_2 = [5 \quad 8 \quad 0]$	$C_2 = [.5 \quad .5 \quad .9]$
$P_3 = [8 \quad 3 \quad 0]$	$C_3 = [.7 \quad .2 \quad .7]$



$\% r = \frac{A_1}{A} = \lambda_1$

$\% g = \frac{A_2}{A} = \lambda_2$

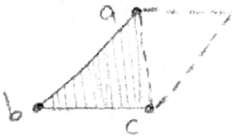
$\% b = \frac{A_3}{A} = \lambda_3$

$P = [6 \quad 6 \quad 0]$

$$\alpha = \frac{Area(P, P_2, P_3)}{Area(P_1, P_2, P_3)} \qquad \beta = \frac{Area(P, P_1, P_3)}{Area(P_1, P_2, P_3)} \qquad \gamma = \frac{Area(P, P_1, P_2)}{Area(P_1, P_2, P_3)}$$

$$\text{Area}(a,b,c) = \frac{1}{2}\sqrt{\|a-b\|^2\|b-c\|^2 - ((a-b)\cdot(b-c))^2}$$

$$= \frac{1}{2}\|(a-b)\times(b-c)\|$$ "cross product"



$$\text{Area}(P,P_2,P_3) = \frac{1}{2}\left\|\left(\begin{bmatrix}6-5\\6-8\\0-0\end{bmatrix}\times\begin{bmatrix}5-8\\8-3\\0-0\end{bmatrix}\right)\right\| = \frac{1}{2}\left\|\left(\begin{bmatrix}1\\-2\\0\end{bmatrix}\times\begin{bmatrix}-3\\5\\0\end{bmatrix}\right)\right\|$$

$$\begin{vmatrix}i & j & k\\1 & -2 & 0\\-3 & 5 & 0\end{vmatrix} = 0i-(0)j+(5-6)k = \begin{bmatrix}0\\0\\-1\end{bmatrix} \Rightarrow \sqrt{0^2+0^2+(-1)^2} = 1\cdot\frac{1}{2} = \frac{1}{2} = 0.5$$

$$\text{Area}(P,P_1,P_3) = \frac{1}{2}\left\|\left(\begin{bmatrix}6-3\\6-6\\0-0\end{bmatrix}\times\begin{bmatrix}3-8\\6-3\\0-0\end{bmatrix}\right)\right\| = \frac{1}{2}\left\|\left(\begin{bmatrix}3\\0\\0\end{bmatrix}\times\begin{bmatrix}-5\\3\\0\end{bmatrix}\right)\right\| =$$

$$\begin{vmatrix}i & j & k\\3 & 0 & 0\\-5 & 3 & 0\end{vmatrix} = 0i-0j+9k = \begin{bmatrix}0\\0\\9\end{bmatrix} \Rightarrow \sqrt{0^2+0^2+9^2} = \sqrt{81} = 9 = \frac{1}{2}\cdot 9 = 4.5$$

$$\text{Area}(P,P_1,P_2) = \frac{1}{2}\left\|\left(\begin{bmatrix}6-3\\6-6\\0-0\end{bmatrix}\times\begin{bmatrix}3-5\\6-8\\0-0\end{bmatrix}\right)\right\| = \frac{1}{2}\left\|\left(\begin{bmatrix}3\\0\\0\end{bmatrix}\times\begin{bmatrix}-2\\-2\\0\end{bmatrix}\right)\right\| =$$

$$\begin{vmatrix}i & j & k\\3 & 0 & 0\\-2 & -2 & 0\end{vmatrix} = 0i-0j+(-6)k = \sqrt{36} = 6\cdot\frac{1}{2} = 3$$

$$\text{Area}(P_1,P_2,P_3) = \frac{1}{2}\left\|\left(\begin{bmatrix}3-5\\6-8\\0-0\end{bmatrix}\times\begin{bmatrix}5-8\\8-3\\0-0\end{bmatrix}\right)\right\| = \frac{1}{2}\left\|\left(\begin{bmatrix}-2\\-2\\0\end{bmatrix}\times\begin{bmatrix}-3\\5\\0\end{bmatrix}\right)\right\| =$$

$$\begin{vmatrix}i & j & k\\-2 & -2 & 0\\-3 & 5 & 0\end{vmatrix} = 0i-0j+(-10-6)k = \sqrt{(-16)^2} = 16\cdot\frac{1}{2} = 8$$

$$\alpha = \frac{0.5}{8} = 0.0625 \quad \beta = \frac{4.5}{8} = 0.5625 \quad \gamma = \frac{3}{8} = 0.375$$

Barycentric Coordinates

$0.0625 + 0.5625 + 0.375 = 1$ ✓

$\alpha + \beta + \gamma = 1$

Color at P:

$$C_p = \alpha C_1 + \beta C_2 + \gamma C_3 = 0.0625 \cdot \begin{bmatrix} .3 \\ .2 \\ .9 \end{bmatrix}^T + 0.5625 \begin{bmatrix} .5 \\ .5 \\ .9 \end{bmatrix}^T + 0.375 \begin{bmatrix} .7 \\ .2 \\ .7 \end{bmatrix}^T =$$

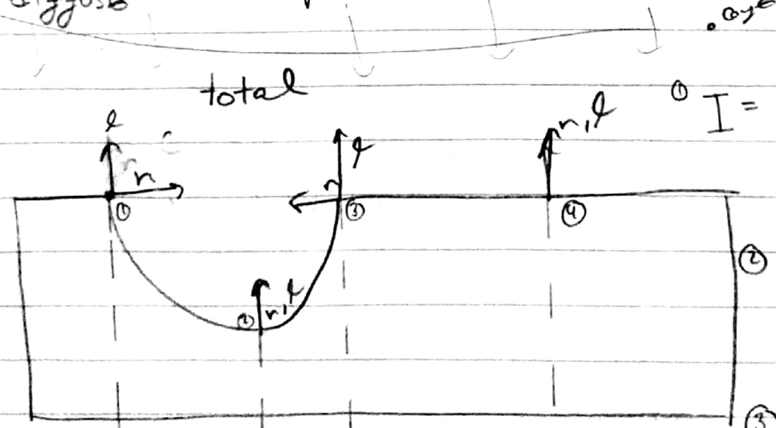
$$C_p = \begin{bmatrix} 0.01875 \\ 0.0125 \\ 0.05625 \end{bmatrix} + \begin{bmatrix} 0.28125 \\ 0.28125 \\ 0.50625 \end{bmatrix} + \begin{bmatrix} 0.2625 \\ 0.075 \\ 0.2625 \end{bmatrix} = \begin{bmatrix} 0.5625 \\ 0.36875 \\ 0.825 \end{bmatrix}^T$$

← Color at P

5.) $I = \underbrace{I_d k_d (n \cdot l)}_{\text{diffuse}} + \underbrace{I_s k_s (r \cdot v)^n}_{\text{specular}} + \underbrace{I_a k_a}_{\text{ambient}}$

a.) total

Diffuse:



① $I = (1.0)(0.8) \cdot (1 \cdot 1 \cdot \cos(90))$

$I = 0$

② $I = (1.0)(0.8) \cdot (1 \cdot 1 \cdot \cos(0))$

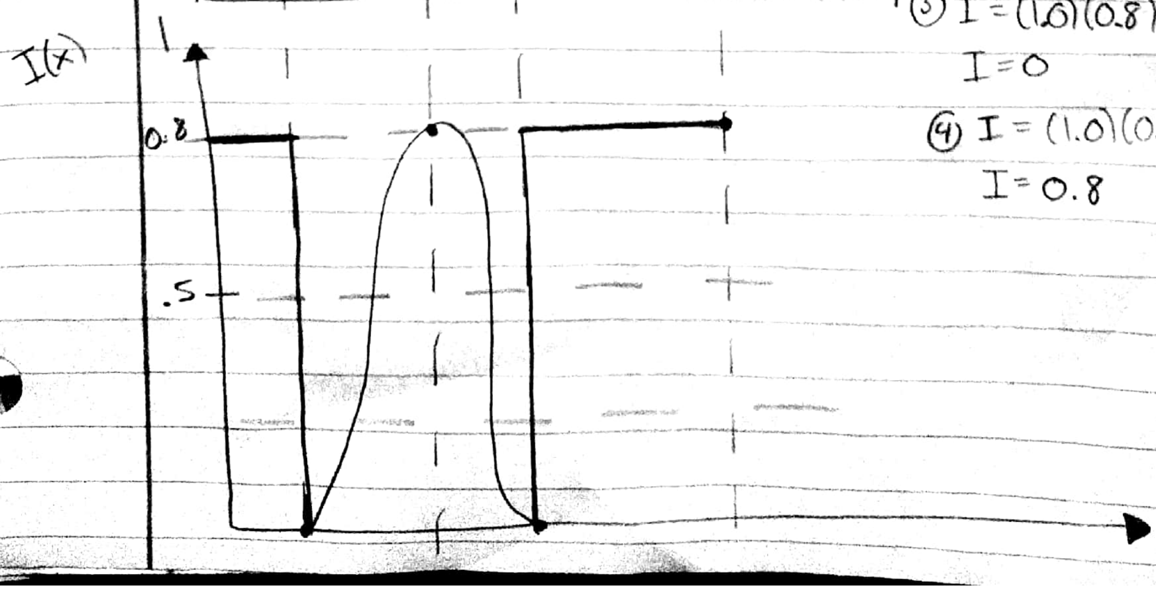
$I = 0.8$

③ $I = (1.0)(0.8) \cdot (1 \cdot 1 \cdot \cos(90))$

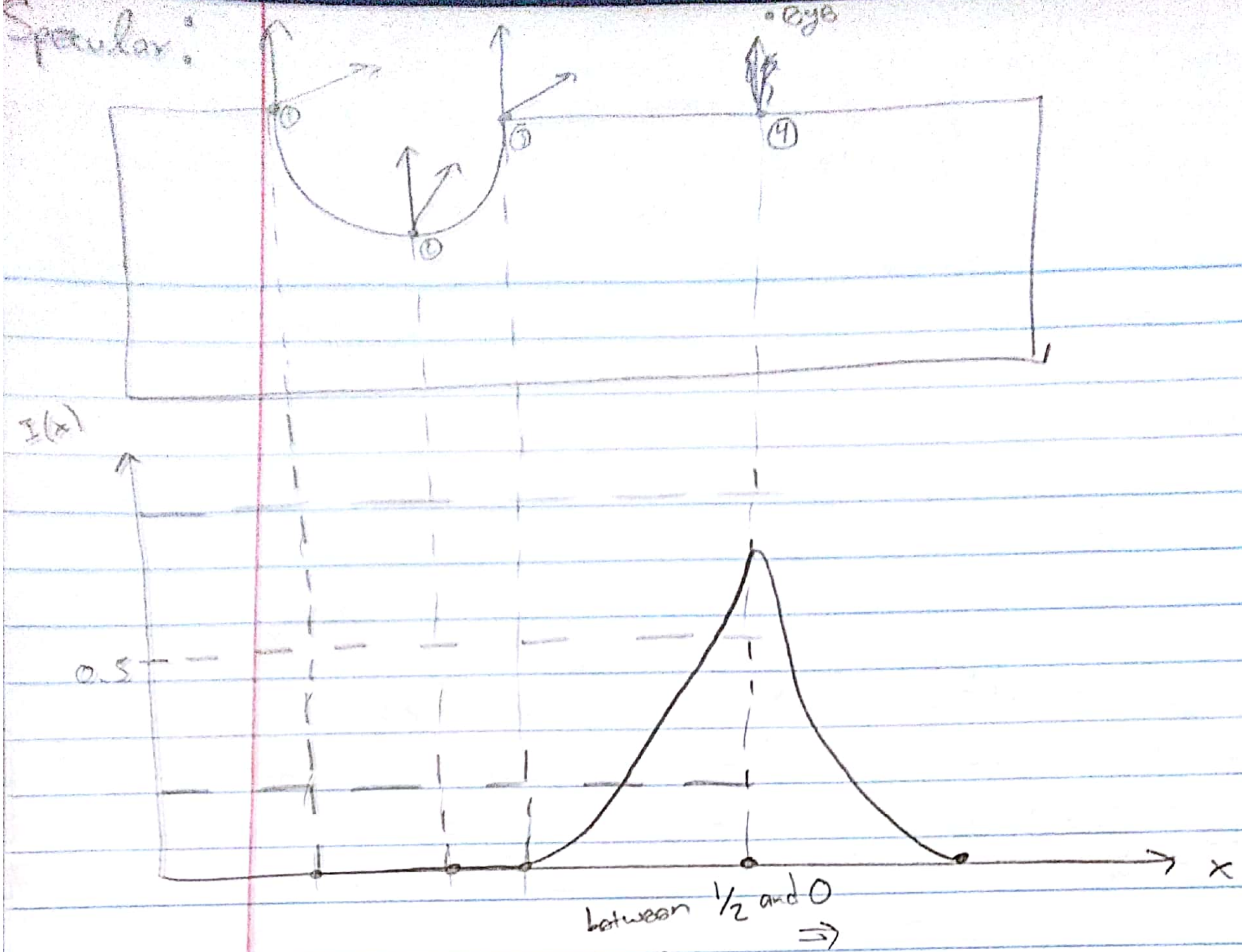
$I = 0$

④ $I = (1.0)(0.8) \cdot (1 \cdot 1 \cdot \cos(0))$

$I = 0.8$



Specular:



$I_s k_s (r \cdot v)^n \rightarrow I_s k_s (|r||v| \cos(\theta))^n$

$60 \leq \theta \leq 90$

$n = 100$

$(1 \cdot 1 \cdot \cos(\theta))^n \rightarrow$ θ is between 1/2 and 0, $\therefore$ taking any $\cos(\theta)$ to the 100^{th} power will be a value near $0 \rightarrow$ take it to be zero.

① $(1.0)(0.7)((1)(1)\cos(\theta))^{100}$, $60 \leq \theta \leq 90$

$= 0$

② $(1.0)(0.7)((1)(1)\cos(\theta))^{100}$, $45 \leq \theta \leq 60$ Same reasoning

$= 0$

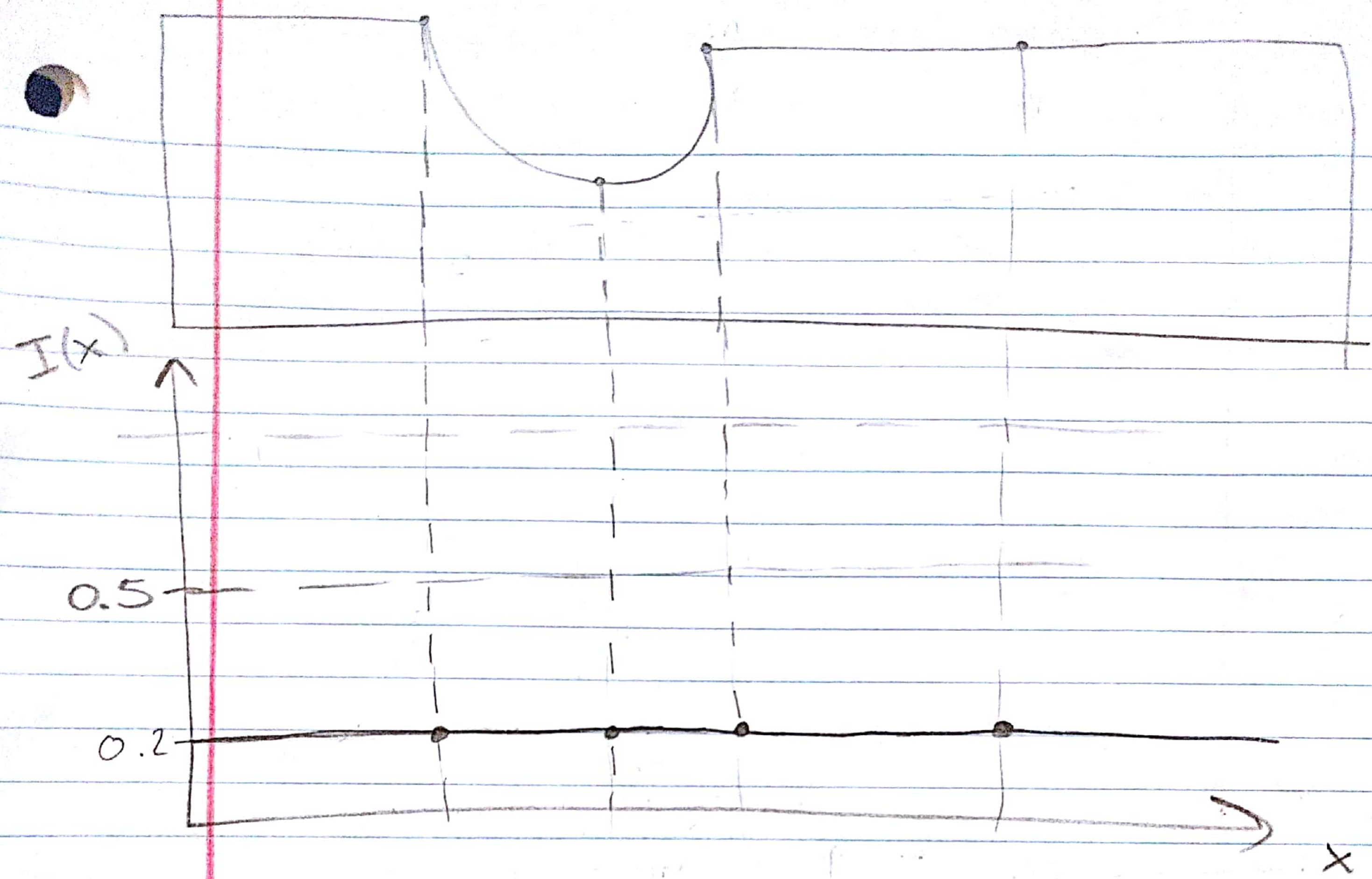
③ $(1.0)(0.7)((1)(1)\cos(\theta))^{100}$, $60 \leq \theta \leq 90$ " "

$= 0$

④ $(1.0)(0.7)(\underbrace{(1)(1)\cos(\theta)}_{0})^{100}$, $0 \leq \theta \leq 30 \rightarrow$ θ is really close to 0, $\therefore \cos(\theta) \approx 0.9$

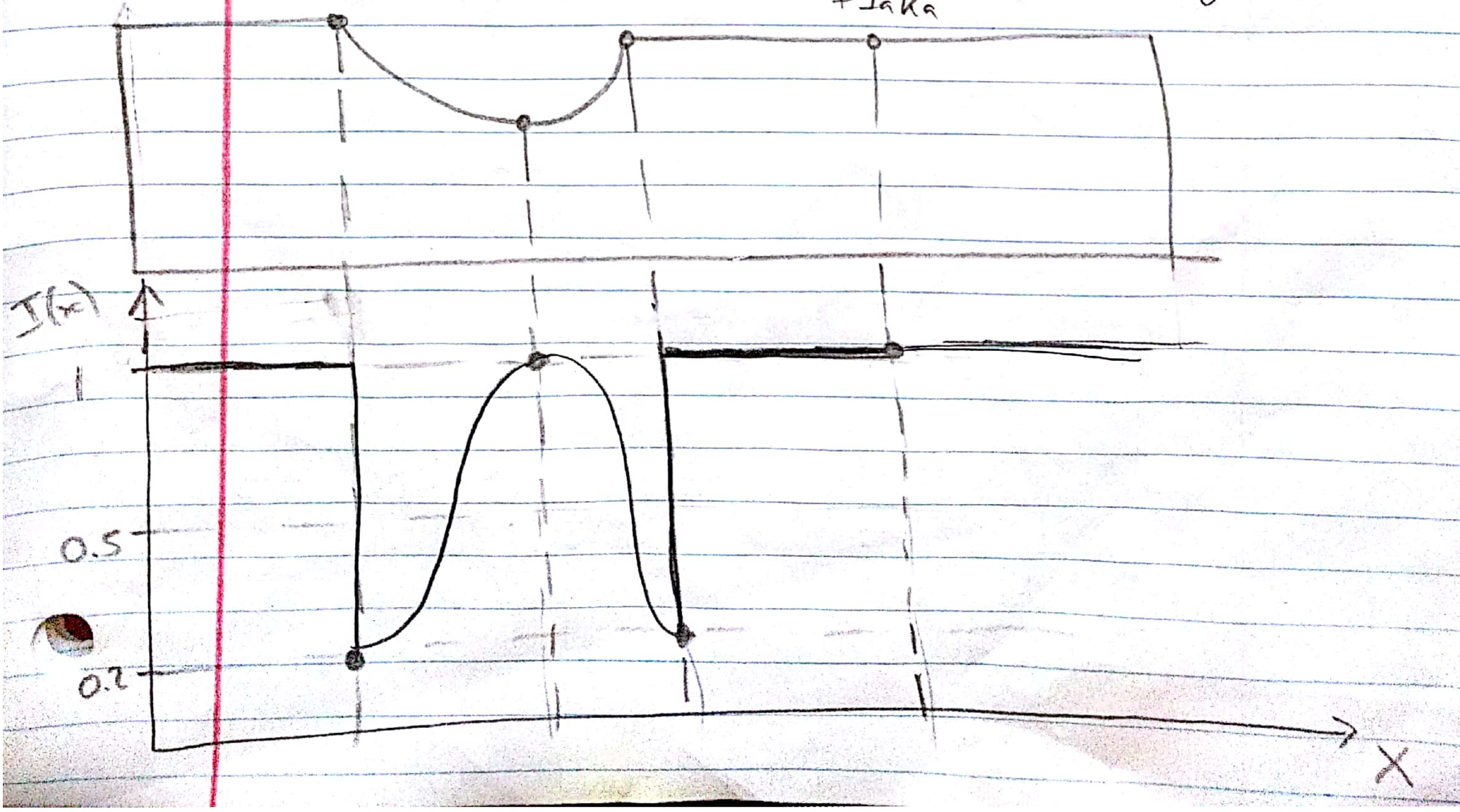
$\therefore 0$

Ambient:



$I_A = I_a k_a = (1.0)(0.2) = 0.2 \rightarrow$ Same at all points $\rightarrow$ "ambient" glow everywhere

Total Illumination: $I_{total} = I_d k_d (n \cdot l) + I_s k_s (r \cdot v)^n + I_a k_a$



6.) The local illumination models are evaluated in the Viewing Coordinate System since it is imperative to know where the location of the eye is in order to carry out the calculations and we cannot perform the illumination after because perspective space is not suitable for lighting.

6.)

6.) a.) $\overrightarrow{AB} = B - A = \begin{pmatrix} 6 \\ 3 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \\ 0 \end{pmatrix}$

$$\begin{pmatrix} 5 \\ 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$5a + b + 0 \cdot c = 0$

$a = 1,\ b = -5,\ c = 0 \rightarrow$ a possible solution and is a vector orthogonal to $\overrightarrow{AB}$.

$$\begin{pmatrix} 1 \\ -5 \\ 0 \end{pmatrix}$$

$$\vec{N}_{AB} = \frac{\langle 1,-5,0 \rangle}{\|\langle 1,-5,0 \rangle\|} \Rightarrow \frac{1}{\sqrt{26}} \langle 1,-5,0 \rangle \approx \langle .1961, -0.9806, 0 \rangle$$

$$\overrightarrow{BC} = C - B = \begin{pmatrix} 8 \\ 5 \\ 0 \end{pmatrix} - \begin{pmatrix} 6 \\ 3 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} = 0 \Rightarrow 2a + 2b + 0 \cdot c = 0$$

$a = 1,\ b = -1,\ c = 0 \rightarrow$ a possible solution and is a vector orthogonal to $\overrightarrow{BC}$.

$$\vec{N}_{BC} = \frac{\langle 1,-1,0 \rangle}{\|\langle 1,-1,0 \rangle\|} \Rightarrow \frac{1}{\sqrt{2}} \langle 1,-1,0 \rangle$$

$$\Rightarrow \langle 0.7071, -0.7071, 0 \rangle$$

Interpolate: $\vec{N}_B = \dfrac{\vec{N}_{AB} + \vec{N}_{BC}}{\|\vec{N}_{AB} + \vec{N}_{BC}\|} \Rightarrow \dfrac{\langle .1961 + .7071, -.9806 - .7071 \rangle}{\|\langle .1961 + .7071, -.9806 - .7071 \rangle\|}$

a.)

$$\boxed{\vec{N}_B = \langle .4719, -.8817, 0 \rangle}$$

Scanned by CamScanner

b.) Bling-Phong Lighting Model

$$I = k_A I_A + \underbrace{k_D I_D (\vec{N}\cdot\vec{L})}_{\text{Diffuse}} + \underbrace{k_S I_S (\vec{H}\cdot\vec{V})^{\alpha}}_{\text{Specular}}$$

Ambient

$I_A = \langle .1, .2, .1 \rangle$

$I_L = \langle 1.0, 1.0, .9 \rangle$

$k_d = \langle .3, .8, .9 \rangle$

$k_a = \langle .1, .1, .1 \rangle$

$k_s = \langle 1, 1, 1 \rangle$

$\alpha = 20$

$I_D = I_S = I_L = \langle 1.0, 1.0, .9 \rangle$

Ambient: $k_A I_A = \langle (.1)(.1), (.1)(.2), (.1)(.1) \rangle = \langle .01, .02, .01 \rangle$

Diffuse component: $k_D I_D = \langle (.3)(1), (.8)(1), (.9)(.9) \rangle = \langle .3, .8, .81 \rangle$

Specular component: $k_S I_S = \langle (1)(1), (1)(1), (1)(.9) \rangle = \langle 1, 1, .9 \rangle$

Point B

$\boxed{\text{Ambient: } \langle .01, .02, .01 \rangle}$

Diffuse: $\vec{N}_B = \langle .4719, -.8817, 0 \rangle$ $\quad \vec{L}_B = \langle 1, -1, 0 \rangle \frac{1}{\sqrt{2}} = \langle 0.7071, -0.7071, 0 \rangle$

$\vec{N}_B \cdot \vec{L}_B = 0.9571$

$$\vec{L}_B = \begin{pmatrix} 8-6 \\ 1-3 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix} = \frac{\langle 2, -2, 0 \rangle}{2\sqrt{2}} = \langle \tfrac{1}{\sqrt{2}}, -\tfrac{1}{\sqrt{2}}, 0 \rangle = \tfrac{1}{\sqrt{2}}\langle 1, -1, 0 \rangle$$

$= \langle$

Diffuse

$k_d I_d (\vec{N}_B \cdot \vec{L}_B) = (\langle .3, .8, .81 \rangle)(0.9571) = \boxed{\langle .2772, .7657, .7752 \rangle}$

Specular: $\overline{BE} = E - B$

$$\vec{V}_B = \begin{pmatrix} 8-6 \\ -1-3 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ 0 \end{pmatrix} = \langle \tfrac{2}{\sqrt{20}}, \tfrac{-4}{\sqrt{20}}, \tfrac{0}{\sqrt{20}} \rangle = \langle .4472, -.8944, 0 \rangle$$

$\sqrt{4+16} = \sqrt{20}$

$$\vec{H}_B = \frac{\vec{V}_B + \vec{L}_B}{\|\vec{V}_B + \vec{L}_B\|} = \frac{\langle 1.1543, -1.6015, 0 \rangle}{1.9741} = \langle .5847, -.8112, 0 \rangle$$

$$(\vec{H}_B \cdot \vec{V}_B)^{\alpha} = \left[(.5847)(.4472) + (-.8112)(-.8944)\right]^{20} = 0.8372$$

Specular

$k_s I_s (\vec{H}_B \cdot \vec{V}_B)^{\alpha} = \langle 1, 1, .9 \rangle (.8372) = \boxed{\langle .8372, .8372, .7534 \rangle}$

Total Illumination:

$I_B = \langle .01, .02, .01 \rangle + \langle .2772, .7657, .7752 \rangle + \langle .8372, .8372, .7534 \rangle = \langle 1.134, 1.623, 1.53$

$\boxed{I_B = \langle 1, 1, 1 \rangle}$ ✓ clamp down to 1.

Point C: Ambient: $\boxed{\langle .01, .02, .01 \rangle}$

Diffuse: $\vec{N}_C = \langle 1, 0, 0 \rangle$

$$\vec{L}_C = \begin{pmatrix} 8-8 \\ 1-5 \\ 0-0 \end{pmatrix} = \begin{pmatrix} 0 \\ -4 \\ 0 \end{pmatrix} = \frac{\langle 0, -4, 0 \rangle}{4} = \langle 0, -1, 0 \rangle$$

$\vec{L}_c = \langle 0, -1, 0 \rangle$

$\vec{N}_C \cdot \vec{L}_C = (1)(0) + (0)(-1) + (0)(0) = 0$

$\therefore k_d I_d (\vec{N}_C \cdot \vec{L}_C) = \boxed{\langle 0, 0, 0 \rangle}$

Specular: I-L

$$\vec{V}_C = \begin{pmatrix} 8-8 \\ -1-5 \\ 0-0 \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \\ 0 \end{pmatrix} = \frac{\langle 0, -6, 0 \rangle}{6} = \langle 0, -1, 0 \rangle$$

$$\vec{H}_C = \frac{\vec{V}_C + \vec{L}_C}{\|\vec{V}_C + \vec{L}_C\|} = \frac{\langle 0, -2, 0 \rangle}{2} = \langle 0, -1, 0 \rangle$$

$(\vec{H}_C \cdot \vec{V}_C)^{20} = 0$ $\quad \boxed{\therefore k_s I_s (\vec{H}_C \cdot \vec{V}_C)^{20} = \langle 0, 0, 0 \rangle}$

Total Illumination:

$I = \langle .01, .02, .01 \rangle + \langle 0, 0, 0 \rangle + \langle 0, 0, 0 \rangle = \boxed{\langle .01, .02, .01 \rangle}$

Point D: Ambient: $\boxed{\langle .01, .02, .01 \rangle}$

Since we are utilizing the flat shading model,

$I_D = I_B$ " " at B $= \langle 1, 1, 1 \rangle$

Illumination at D

Scanned by CamScanner

Diffuse: $\boxed{\langle .2772, .7657, .7752 \rangle}$

Specular: $\boxed{\langle .8372, .8372, .7534 \rangle}$

Total Illumination: $\boxed{I_D = \langle 1, 1, 1 \rangle}$

C.) Using Gourad Shading Model

Point B → Same as in 5b.) for point B
Point C → Same as in 5b.) for point C

Point D → linearly interpolate between B & C.

$$I_D = \tfrac{1}{2} I_B + \tfrac{1}{2} I_C = k_A I_A + \tfrac{1}{2} k_D I_D (\vec{N}_B \cdot \vec{L}_B) + \tfrac{1}{2} k_s I_s (\vec{H}_B \cdot \vec{V}_B)^{\alpha}$$

Ambient: $\boxed{\langle .01, .02, .01 \rangle}$

Diffuse: $\tfrac{1}{2} k_D I_D (\vec{N}_B \cdot \vec{L}_B) = \tfrac{1}{2} \langle .2772, .7657, .7752 \rangle$

$= \boxed{\langle .1436, .3828, .3876 \rangle}$

Specular: $\tfrac{1}{2} k_s I_s (\vec{H}_B \cdot \vec{V}_B)^{\alpha} = \tfrac{1}{2} \langle .8372, .8372, .7534 \rangle$
$\boxed{= \langle .4186, .4186, .3767 \rangle}$

Total Illumination:
$I_D = \langle .01, .02, .01 \rangle + \langle .1436, .3828, .3876 \rangle + \langle .4186, .4186, .3767 \rangle$
$\boxed{I_D = \langle .5721, .8214, .7743 \rangle}$

Using Phong Shading

D.) Point B → Same as in 5b.) for point B
Point C → Same as in 5b.) for point C

Point D: ⇒ Interpolate surface normal over polygon

∴ We need to linearly interpolate $\vec{N}_B$ & $\vec{N}_C$ to obtain $\vec{N}_D$.

Scanned by CamScanner

$$\vec{N}_D = \frac{\vec{N}_B + \vec{N}_C}{||\vec{N}_B + \vec{N}_C||} = \frac{\langle 1.4719, -.8817, 0 \rangle}{\sqrt{2.9435}} = \langle .8579, -.5139, 0 \rangle$$

$= 1.7164$

$E - D$ $$\vec{V}_D = \begin{pmatrix} 8-7 \\ -1-4 \\ 0-0 \end{pmatrix} = \begin{pmatrix} 1 \\ -5 \\ 0 \end{pmatrix} = \frac{\langle 1, -5, 0 \rangle}{\sqrt{26}} = \langle .1961, -.9806, 0 \rangle$$

$$\vec{L}_D = \begin{pmatrix} 8-7 \\ 1-4 \\ 0-0 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \\ 0 \end{pmatrix} = \frac{\langle 1, -3, 0 \rangle}{\sqrt{10}} = \langle .3162, -.9806, 0 \rangle$$

Ambient: $\boxed{\langle .01, .02, .01 \rangle}$

$$\vec{H}_D = \frac{\vec{V}_D + \vec{L}_D}{||\vec{V}_D + \vec{L}_D||} = \langle .2567, -.9665, 0 \rangle$$

Diffuse: $k_D I_D (\vec{N}_D \cdot \vec{L}_D) = \boxed{\langle .2276, .6070, .6196 \rangle}$

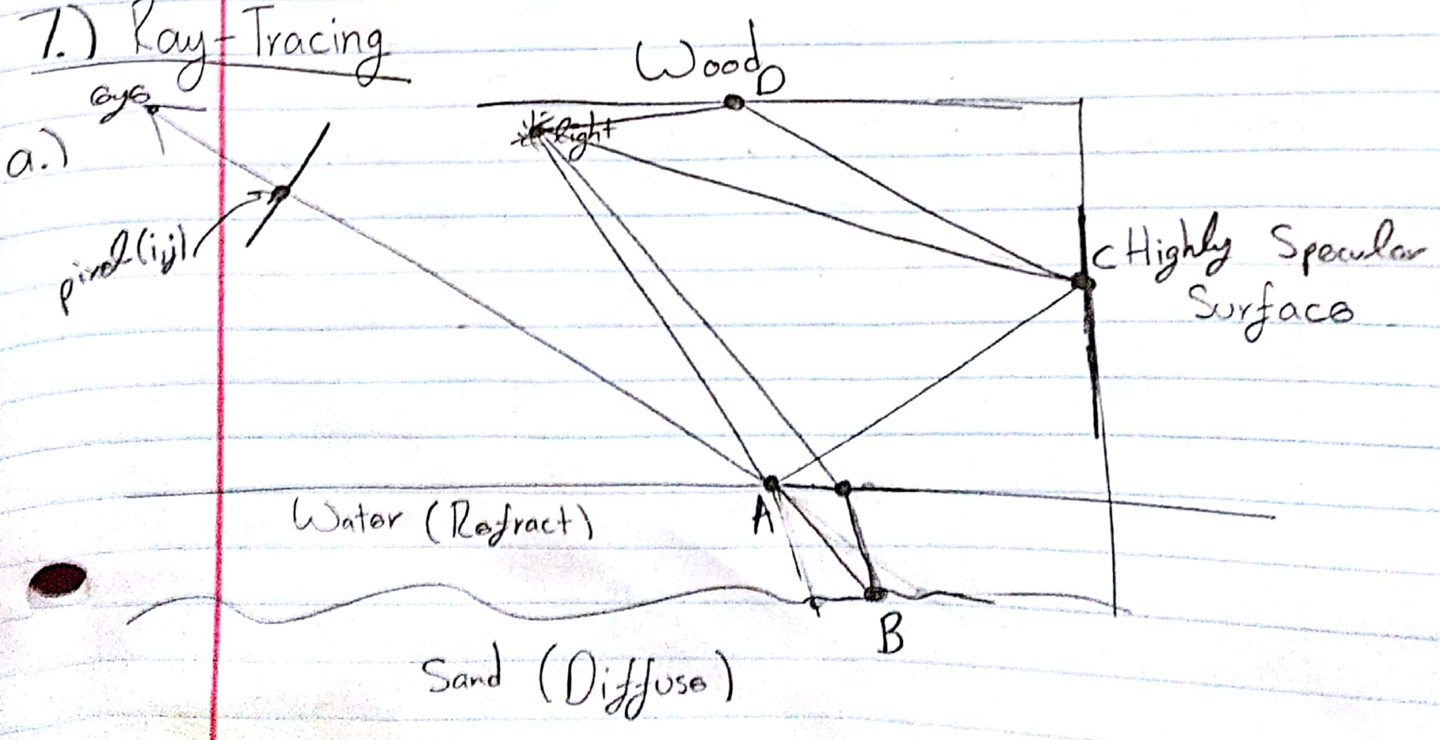
Specular: $k_S I_S (\vec{H}_D \cdot \vec{V}_D)^\alpha = \boxed{\langle .0013, .0013, .0012 \rangle}$

Total Illumination: $I_D = \langle .01, .02, .01 \rangle + \langle .2276, .6070, .6196 \rangle$
$+ \langle .0013, .0013, .0012 \rangle$

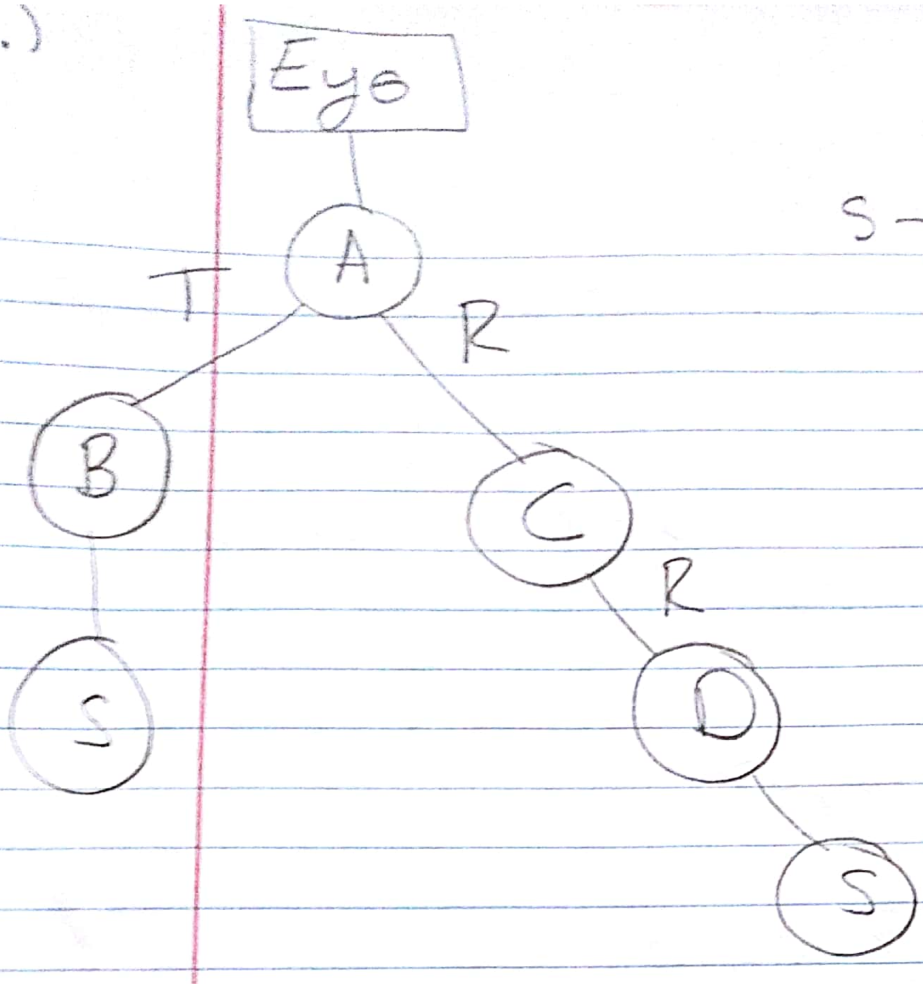
$I_D = \boxed{\langle .2389, .6283, .6287 \rangle}$

7.) Ray-Tracing

a.)



b.)



S → shadow ray

8.) a.) $p(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$

b.) $p'(t) = a_1 + 2a_2 t + 3a_3 t^2$

$p''(t) = 2a_2 + 6a_3 t$

c.) $p_0 = P(0) = a_0 + a_1(0) + a_2(0)^2 + a_3(0)^3$

$T_0 = P'(0) = a_1 + 2a_2(0) + 3a_3(0)^2$

$A_0 = P''(0) = 2a_2 + 6a_3(0)$

$P_1 = P(1) = a_0 + a_1(1) + a_2(1)^2 + a_3(1)^3$

$$\begin{bmatrix} P_0^T \\ T_0^T \\ A_0^T \\ P_1^T \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}}_{C} \begin{bmatrix} a_0^T \\ a_1^T \\ a_2^T \\ a_3^T \end{bmatrix}$$

$$p^T(t) = [1 \; t \; t^2 \; t^3] \begin{bmatrix} a_0^T \\ a_1^T \\ a_2^T \\ a_3^T \end{bmatrix}$$

$$p^T(t) = [1 \; t \; t^2 \; t^3] C^{-1} \begin{bmatrix} a_0^T \\ a_1^T \\ a_2^T \\ a_3^T \end{bmatrix}$$

$$\begin{bmatrix} a_0^T \\ a_1^T \\ a_2^T \\ a_3^T \end{bmatrix} = C^{-1} \begin{bmatrix} P_0^T \\ T_0^T \\ A_0^T \\ P_1^T \end{bmatrix} \Rightarrow \boxed{C^{-1} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}^{-1}}_{\text{Basis Matrix}}}$$