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HW #2

1.) a.) $model\_transform = A \cdot C \cdot D$

b.) $B = (K \cdot L \cdot M)^{-1} = M^{-1} \cdot L^{-1} \cdot K^{-1}$

2.) For explicit Euler, it is important to note that acceleration is dependent on force, velocity is dependent on acceleration, and position is dependent on velocity, hence their interrelation. Essentially, explicit Euler is calculated by iterating over a constant time period and computing the positions, velocities, and accelerations until the desired time/location is reached. Essentially, the Explicit Euler is computed using the following algorithm:

① We find the next position of a specified object by utilizing the equation: $x_i(t+\Delta t) = x_i(t) + \Delta t v_i(t+\Delta t)$. Essentially, this equation describes that the next position is dependent on time and velocity and increments by the next time step of velocity ($\Delta t v_i(t+\Delta t)$).

② Next, velocity is calculated using the formula: $v_i(t+\Delta t) = v_i(t) + \Delta t a_i(t)$. Essentially, the next velocity is computed by incrementing the current velocity by one time step of the current acceleration.

③ Lastly, acceleration is calculated by utilizing/examining total force and dividing it by total mass:

$$a_i(t) = \frac{f_{i\,total}}{m_i}$$

3.) In order to calculate the total forces acting upon on a node, we utilize the equation: $F_{i,total} = -y_i \dot{x}_i + g_i + f_i$,

the force spring $g_{ij} = k_{ij} \cdot e_{ij} \cdot \frac{d_{ij}}{||d_{ij}||}$, where g_i is the total force on the node, due to the springs connecting ~~to~~ it to neighboring nodes and $j \in N_i$.

① $g_i(t) = \sum_{j \in N_i} g_{ij}$

② Damping force is represented by y_i

③ External force is represented by $\vec{f}_{i,ext}$

The functions that are called as specified in the lecture slides are:

```
Void computeforces() {
    zeroize_forces();
    spring_forces();
    damping_forces();
    external_forces();
}
```

4.) a.) For a mass-spring model with a non-zero length, we would utilize the equation: $g_i(t) = \sum_{j \in N_i} g_{ij}$ and the force spring ij exerts on node i: $g_{ij} = k_{ij} \cdot e_{ij} \cdot \dfrac{d_{ij}}{\|d_{ij}\|}$

Essentially, we examine the internal spring force acting on a node.

b.) Cloth-Viscoelasticity-Mass-Springs Model

We utilize viscoelastic forces as specified by the equation: $m_i \ddot{x}_i + y_i \dot{x}_i + C_{ij} r_{ij} = f$

as well as effective stiffness: $C_{ij}(x_i, x_j) = \dfrac{k_{ij} \cdot e_{ij} + y_{ij} \cdot \dot{e}_{ij}}{\|r_{ij}\|}$

which, in matrix form, it is as follows:

$$\begin{bmatrix} m_i & 0 \\ 0 & m_j \end{bmatrix} \begin{bmatrix} \ddot{x}_i \\ \ddot{x}_j \end{bmatrix} + \begin{bmatrix} y_i & 0 \\ 0 & y_j \end{bmatrix} \begin{bmatrix} \dot{x}_i \\ \dot{x}_j \end{bmatrix} + \begin{bmatrix} -C_{ij} & C_{ij} \\ C_{ij} & -C_{ij} \end{bmatrix} \begin{bmatrix} x_i \\ x_j \end{bmatrix} = \begin{bmatrix} f_i \\ f_j \end{bmatrix}$$

$$M \cdot \ddot{X} + G \cdot \dot{X} + k(x) \cdot X = f$$

c.) Heating and Melting Deformable Models-Mass-Springs Model

We would observe thermoelasticity governed by the equation: → thermoelastic forces

$$k_{ij} = \begin{cases} k_{ij}^{0}, & \text{if } \theta^{a} \le \theta^{s} \\ k_{ij}^{0} - v(\theta^{a} - \theta^{s}), & \text{if } \theta^{s} < \theta^{a} < \theta^{m} \\ 0, & \text{if } \theta^{a} \ge \theta^{m} \end{cases}$$

d.) Liquids- Particle Models

We examine the total forces acting on particle i, due to all the other particles ~~j~~ ⟹ this includes both attraction and repulsion forces as governed by the equation:

$$g_i(t) = \sum_{j \neq i} g_{ij}(t)$$

$$g_{ij}(t) = m_i m_j (x_i - x_j)\left(\underbrace{\frac{-\tau}{(d_{ij} + C_i)^a}}_{\text{attraction}} + \underbrace{\frac{\beta}{(d_{ij})^b}}_{\text{repulsion}} \right), \quad a = 2 \ \& \ b = 4.$$

- Essentially, α and β determine the strengths of the attraction and repulsion forces.
- $d_{ij} = \|x_j - x_i\|$
- C minimum required separation between particles.

After one has obtained the above information, one can compute the total forces acting on the liquid-particle system.

5.) In order to find the position of the particle at the time it hits the ground, we need to use Explicit Euler with $dt = 1$.

Essentially, I wrote a Python script that iteratively solved this for me. The result I obtained is that the position at the time it hits the ground is : $\underset{x}{(8},\underset{y}{0},\underset{z}{-20)}$, and that time $t = 3$ when $y = 0$.

$$\begin{array}{r} 16 \\ +\ 9 \\ 64 \\ \hline \sqrt{89} \end{array}$$

6.) $(2,10,3)^T = P_{eye}$, $P_e = (-2,2,0)^T$, $V_{up} = (-1,-1,0)^T$

$$k = \frac{P_{eye} - P_e}{|P_{eye} - P_e|} = \begin{pmatrix} 2-(-2) \\ 10-2 \\ 3-0 \end{pmatrix} = \frac{\begin{pmatrix} 4 \\ 8 \\ 3 \end{pmatrix}}{\sqrt{89}} = \begin{pmatrix} 4/\sqrt{89} \\ 8/\sqrt{89} \\ 3/\sqrt{89} \end{pmatrix}$$

$$i = \frac{(-1,-1,0)^T \times (4/\sqrt{89}, 8/\sqrt{89}, 3/\sqrt{89})^T}{|V_{up} \times k|} = \frac{(-3/\sqrt{89}, 3/\sqrt{89}, -4/\sqrt{89})^T}{\sqrt{34}/\sqrt{89}}$$

$$\begin{vmatrix} i & j & k \\ -1 & -1 & 0 \\ 4/\sqrt{89} & 8/\sqrt{89} & 3/\sqrt{89} \end{vmatrix} = \left(\frac{-3}{\sqrt{89}} - 0\right)i + \left(0 - \left(-\frac{3}{\sqrt{89}}\right)\right)j + \left(-\frac{8}{\sqrt{89}} - \left(-\frac{4}{\sqrt{89}}\right)\right)k$$

$$\begin{array}{r} 18 \\ +\ 16 \\ \hline 34 \end{array}$$

$$= \frac{-3}{\sqrt{89}}i + \frac{3}{\sqrt{89}}j - \frac{4}{\sqrt{89}}k \Rightarrow \left(\frac{-3}{\sqrt{89}}, \frac{3}{\sqrt{89}}, \frac{-4}{\sqrt{89}}\right)^T$$

$$\sqrt{\frac{9}{89} + \frac{9}{89} + \frac{16}{89}} = \sqrt{\frac{34}{89}} = \sqrt{\frac{34}{89}}$$

$$i = \left(-3/\sqrt{34}, 3/\sqrt{34}, -4/\sqrt{34}\right)^T$$

$$j = k \times i = \begin{vmatrix} i & j & k \\ 4/\sqrt{89} & 8/\sqrt{89} & 3/\sqrt{89} \\ -3/\sqrt{34} & 3/\sqrt{34} & -4/\sqrt{34} \end{vmatrix} = \left(\frac{-32}{\sqrt{89\cdot 34}} - \frac{9}{\sqrt{89\cdot 34}}\right)i + \left(\frac{-9}{\sqrt{89\cdot 34}} + \frac{16}{\sqrt{89\cdot 34}}\right)j + \left(\frac{12}{\sqrt{89\cdot 34}} + \frac{24}{\sqrt{89\cdot 34}}\right)k \Rightarrow$$

$$\Rightarrow \frac{-41}{\sqrt{89\cdot 34}}i + \frac{7}{\sqrt{89\cdot 34}}j + \frac{36}{\sqrt{89\cdot 34}}k$$

$$\left(\frac{-32-9}{\sqrt{89\cdot 34}}\right)i+\left(\frac{-9-(-16)}{\sqrt{89\cdot 34}}\right)j+\left(\frac{12-(-24)}{\sqrt{89\cdot 34}}\right)k$$

$$\left(\frac{-41}{\sqrt{89\cdot 34}},\ \frac{7}{\sqrt{89\cdot 34}},\ \frac{36}{\sqrt{89\cdot 39}}\right)$$

$$M_{cam}^{-1}=\left(\begin{bmatrix}1&0&0&P_{eye_x}\\0&1&0&P_{eye_y}\\0&0&1&P_{eye_z}\\0&0&0&1\end{bmatrix}\begin{bmatrix}i_x&j_x&k_x&0\\i_y&j_y&k_y&0\\i_z&j_z&k_z&0\\0&0&0&1\end{bmatrix}\right)^{-1}$$

$$=\begin{bmatrix}i_x&i_y&i_z&0\\j_x&j_y&j_z&0\\k_x&k_y&k_z&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&-P_{eye_x}\\0&1&0&-P_{eye_y}\\0&0&1&-P_{eye_z}\\0&0&0&1\end{bmatrix}=\begin{bmatrix}-3/\sqrt{34}&3/\sqrt{34}&-4/\sqrt{34}&0\\-41/\sqrt{3026}&7/\sqrt{3026}&36/\sqrt{3026}&0\\4/\sqrt{89}&8/\sqrt{89}&3/\sqrt{89}&0\\0&0&0&1\end{bmatrix}\cdot\begin{bmatrix}1&0&0&-2\\0&1&0&-10\\0&0&1&-3\\0&0&0&1\end{bmatrix}$$

$$\boxed{M_{cam}^{-1}=\begin{pmatrix}-0.51&0.51&-0.69&-2.1\\-0.75&0.13&0.65&-1.75\\0.42&0.85&0.32&-10.28\\0&0&0&1\end{pmatrix}}$$

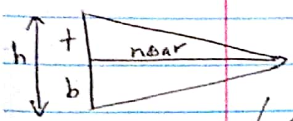
near = 1 $\quad$ horiz: 30°
far = 100 $\quad$ aspect ratio: 1:2

7.) $\tan\left(\frac{30}{2}\right) = \frac{r}{near} \Rightarrow r = \tan(15) = 0.268$

$l = -r = -0.268$

- width of image at near: $w = 2 \cdot r = 0.5356$
- height of image plane at near: $\frac{w}{h} = aspect = \frac{1}{2}$

$\therefore\ h = 2(0.5356)$



$t = \frac{h}{2} = 0.5356 \qquad b = -t = -0.5356$

$$M_{proj} = \begin{pmatrix} \frac{2(1)}{0.268+0.268} & 0 & \frac{0.268+(-0.268)}{0.268+(0.268} & 0 \\ 0 & \frac{2(1)}{0.5356+0.5356} & \frac{0.5356-0.5356}{0.5356+0.5356} & 0 \\ 0 & 0 & -\left(\frac{100+1}{100-1}\right) & -2\left(\frac{100+1}{100-1}\right) \\ 0 & 0 & -1 & 0 \end{pmatrix} =$$

$$\Rightarrow \boxed{M_{proj} = \begin{pmatrix} 3.734 & 0 & 0 & 0 \\ 0 & 1.867 & 0 & 0 \\ 0 & 0 & -1.02 & -2.02 \\ 0 & 0 & -1 & 0 \end{pmatrix}}$$

200px. wide → width
200px. high → height

6-12, 1

8.)

$$M_{N2P} = \begin{pmatrix} 1 & 0 & 0 & \frac{200-1}{2} \\ 0 & 1 & 0 & \frac{200-1}{2} \\ 0 & 0 & 1 & 1/2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{200}{2} & 0 & 0 & 0 \\ 0 & \frac{200}{2} & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 & \frac{199}{2} \\ 0 & 1 & 0 & 199/2 \\ 0 & 0 & 1 & 1/2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 100 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \Rightarrow$$

$$\boxed{= \begin{pmatrix} 100 & 0 & 0 & 99.5 \\ 0 & -100 & 0 & 99.5 \\ 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 1 \end{pmatrix}}$$

9.)

$$\begin{pmatrix} 3 \\ 2 \\ 1 \\ 1 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 0 \\ -3 \\ 1 \end{pmatrix} \quad \begin{pmatrix} -2 \\ -1 \\ 2 \\ 1 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 5 \\ -1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} -0.51 & 0.51 & -0.69 & -2.1 \\ -0.75 & 0.13 & 0.65 & -1.75 \\ 0.42 & 0.85 & 0.32 & -10.28 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 1 \\ 1 \end{pmatrix} = \boxed{\begin{pmatrix} -3.26 \\ -3.07 \\ -6.7 \\ 1 \end{pmatrix}}$$

$$\begin{pmatrix} -0.51 & 0.51 & -0.69 & -2.1 \\ -0.75 & 0.13 & 0.65 & -1.75 \\ 0.42 & 0.85 & 0.32 & -10.28 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ -3 \\ 1 \end{pmatrix} = \boxed{\begin{pmatrix} 0 \\ -3.71 \\ -11.24 \\ 1 \end{pmatrix}}$$

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$$\begin{pmatrix} -0.51 & 0.51 & -0.69 & -2.1 \\ -0.75 & 0.13 & 0.65 & -1.75 \\ 0.42 & 0.85 & 0.32 & -10.28 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -2 \\ -1 \\ 2 \\ 1 \end{pmatrix} = \boxed{\begin{pmatrix} -2.92 \\ 0.93 \\ -11.34 \\ 1 \end{pmatrix}}$$

$$\begin{pmatrix} -0.51 & 0.51 & -0.69 & -2.1 \\ -0.75 & 0.13 & 0.65 & -1.75 \\ 0.42 & 0.85 & 0.32 & -10.28 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 5 \\ -1 \\ 1 \end{pmatrix} = \boxed{\begin{pmatrix} 0.69 \\ -2.51 \\ -5.94 \\ 1 \end{pmatrix}}$$

10.) Clipping Coordinate System

$P_{clip} = P \cdot P_{camera}$

$$\begin{pmatrix} 3.734 & 0 & 0 & 0 \\ 0 & 1.867 & 0 & 0 \\ 0 & 0 & -1.02 & -2.02 \\ 0 & 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 1 \\ 1 \end{pmatrix} = \boxed{\begin{pmatrix} -12.17 \\ -5.74 \\ 5.12 \\ 6.7 \end{pmatrix}}$$

$$\begin{pmatrix} 3.734 & 0 & 0 & 0 \\ 0 & 1.867 & 0 & 0 \\ 0 & 0 & -1.02 & -2.02 \\ 0 & 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ -3 \\ 1 \end{pmatrix} = \boxed{\begin{pmatrix} 0 \\ -6.92 \\ 9.44 \\ 11.24 \end{pmatrix}}$$

$$\begin{pmatrix} 3.734 & 0 & 0 & 0 \\ 0 & 1.867 & 0 & 0 \\ 0 & 0 & -1.02 & -2.02 \\ 0 & 0 & -1 & 0 \end{pmatrix}\begin{pmatrix} -2 \\ -1 \\ 2 \\ 1 \end{pmatrix} = \boxed{\begin{pmatrix} -10.89 \\ 1.73 \\ 9.55 \\ 11.34 \end{pmatrix}}$$

$$\begin{pmatrix} 3.734 & 0 & 0 & 0 \\ 0 & 1.867 & 0 & 0 \\ 0 & 0 & -1.02 & -2.02 \\ 0 & 0 & -1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 5 \\ -1 \\ 1 \end{pmatrix} = \boxed{\begin{pmatrix} 2.56 \\ -4.68 \\ 4.03 \\ 5.94 \end{pmatrix}}$$

11.)

$$\begin{pmatrix} \frac{-12.17}{6.7} \\ \frac{-5.74}{6.7} \\ \frac{5.12}{6.7} \\ \frac{6.7}{6.7} \end{pmatrix} = \boxed{\begin{pmatrix} -1.74 \\ -0.82 \\ 0.73 \\ 1 \end{pmatrix}} \qquad \begin{pmatrix} 0/11.24 \\ -6.92/11.24 \\ 9.44/11.24 \\ 11.24/11.24 \end{pmatrix} = \boxed{\begin{pmatrix} 0 \\ -0.62 \\ 0.84 \\ 1 \end{pmatrix}}$$

$$\begin{pmatrix} -10.89/11.34 \\ 1.73/11.34 \\ 9.55/11.34 \\ 11.34/11.34 \end{pmatrix} = \boxed{\begin{pmatrix} -0.96 \\ 0.15 \\ 0.84 \\ 1 \end{pmatrix}} \qquad \begin{pmatrix} 2.56/5.94 \\ -4.68/5.94 \\ 4.03/5.94 \\ 5.94/5.94 \end{pmatrix} = \boxed{\begin{pmatrix} 0.43 \\ -0.79 \\ 0.68 \\ 1 \end{pmatrix}}$$

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Display Coordinate System

12.) $$\begin{pmatrix} 100 & 0 & 0 & 99.5 \\ 0 & -100 & 0 & 99.5 \\ 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1.74 \\ -0.82 \\ 0.73 \\ 1 \end{pmatrix} = \boxed{\begin{pmatrix} -74.42 \\ 181.49 \\ 0.87 \\ 1 \end{pmatrix}}$$

$$\begin{pmatrix} 100 & 0 & 0 & 99.5 \\ 0 & -100 & 0 & 99.5 \\ 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ -0.62 \\ 0.84 \\ 1 \end{pmatrix} = \boxed{\begin{pmatrix} 99.5 \\ 161.12 \\ 0.92 \\ 1 \end{pmatrix}}$$

$$\begin{pmatrix} 100 & 0 & 0 & 99.5 \\ 0 & -100 & 0 & 99.5 \\ 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -0.96 \\ 0.15 \\ 0.84 \\ 1 \end{pmatrix} = \boxed{\begin{pmatrix} 3.52 \\ 84.24 \\ 0.92 \\ 1 \end{pmatrix}}$$

$$\begin{pmatrix} 100 & 0 & 0 & 99.5 \\ 0 & -100 & 0 & 99.5 \\ 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0.43 \\ -0.79 \\ 0.68 \\ 1 \end{pmatrix} = \boxed{\begin{pmatrix} 142.65 \\ 178.40 \\ 0.84 \\ 1 \end{pmatrix}}$$