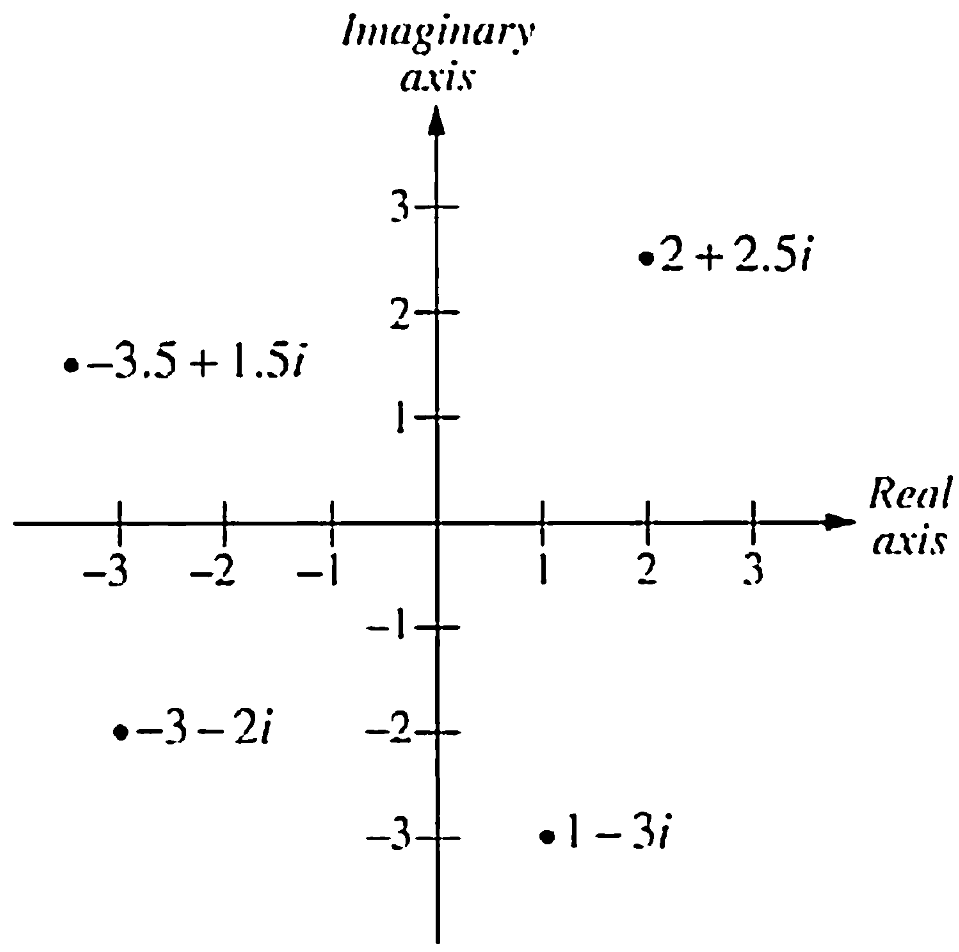
Definition of Complex Numbers

Let i be the square root of -1 (a number such that $i^2 = -1$). i is known as the ***imaginary unit;*** engineers often represent it by the symbol j instead of i. A ***complex number*** has the form $a + bi$, where a and b are real numbers. a is said to be the ***real part*** of the number, and b is the ***imaginary part.*** Note that the complex numbers include the real numbers as a special case (when $b = 0$).

Why are complex numbers useful? For one thing, they allow solutions to problems that are otherwise unsolvable. Consider the equation $x^2 + 1 = 0$, which has no solution if x is restricted to the real numbers. If complex numbers are allowed, there are two solutions: $x = i$ and $x = -i$.

Complex numbers can be thought of as points in a two-dimensional space known as the ***complex plane.*** Each complex number—a point in the complex plane—is represented by Cartesian coordinates, where the real part of the number corresponds to the x-coordinate of the point, and the imaginary part corresponds to the y-coordinate. For example, the complex numbers $2 + 2.5i$, $1 - 3i$, $-3 - 2i$, and $-3.5 + 1.5i$ can be plotted as follows:



An alternative system known as ***polar coordinates*** can also be used to specify a point on the complex plane. With polar coordinates, a complex number z is represented by the values r and θ, where r is the length of a line segment from the origin to z, and θ is the angle between this segment and the real axis: