r is called the ***absolute value*** of z. (The absolute value is also known as the *norm*, *modulus*, or *magnitude*.) θ is said to be the ***argument*** (or *phase angle*) of z. The absolute value of $a + bi$ is given by the following equation:

$$|a + bi| = \sqrt{a^2 + b^2}$$

For additional information about converting from Cartesian coordinates to polar coordinates and vice versa, see the Programming Projects at the end of the chapter.

Complex Arithmetic

The sum of two complex numbers is found by separately adding the real parts of the two numbers and the imaginary parts. For example,

$$(3 - 2i) + (1.5 + 3i) = (3 + 1.5) + (-2 + 3)i = 4.5 + i$$

The difference of two complex numbers is computed in a similar manner, by separately subtracting the real parts and the imaginary parts. For example,

$$(3 - 2i) - (1.5 + 3i) = (3 - 1.5) + (-2 - 3)i = 1.5 - 5i$$

Multiplying complex numbers is done by multiplying each term of the first number by each term of the second and then summing the products:

$$\begin{aligned}(3 - 2i) \times (1.5 + 3i) &= (3 \times 1.5) + (3 \times 3i) + (-2i \times 1.5) + (-2i \times 3i)\\ &= 4.5 + 9i - 3i - 6i^2 = 10.5 + 6i\end{aligned}$$

Note that the identity $i^2 = -1$ is used to simplify the result.

Dividing complex numbers is a bit harder. First, we need the concept of the ***complex conjugate*** of a number, which is found by switching the sign of the number's imaginary part. For example, $7 - 4i$ is the conjugate of $7 + 4i$, and $7 + 4i$ is the conjugate of $7 - 4i$. We'll use z^* to denote the conjugate of a complex number z.

The quotient of two complex numbers y and z is given by the formula

$$y/z = yz^*/zz^*$$

It turns out that zz^* is always a real number, so dividing zz^* into yz^* is easy (just divide both the real part and the imaginary part of yz^* separately). The following example shows how to divide $10.5 + 6i$ by $3 - 2i$:

$$\frac{10.5 + 6i}{3 - 2i} = \frac{(10.5 + 6i)(3 + 2i)}{(3 - 2i)(3 + 2i)} = \frac{19.5 + 39i}{13} = 1.5 + 3i$$

Complex Types in C99

C99 has considerable built-in support for complex numbers. Without including any library headers, we can declare variables that represent complex numbers and then perform arithmetic and other operations on these variables.