hypothesis testing normal mean with known variance

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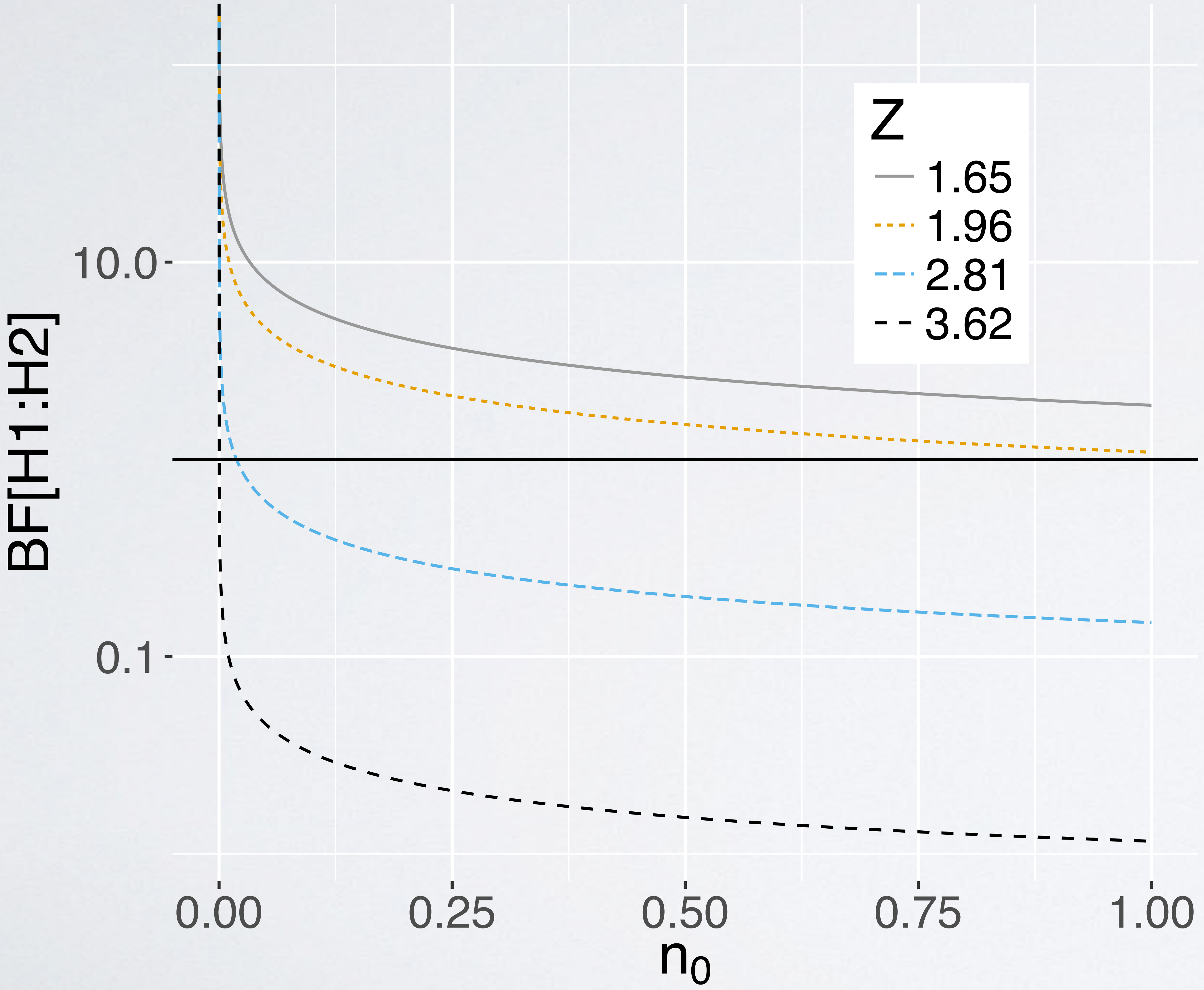

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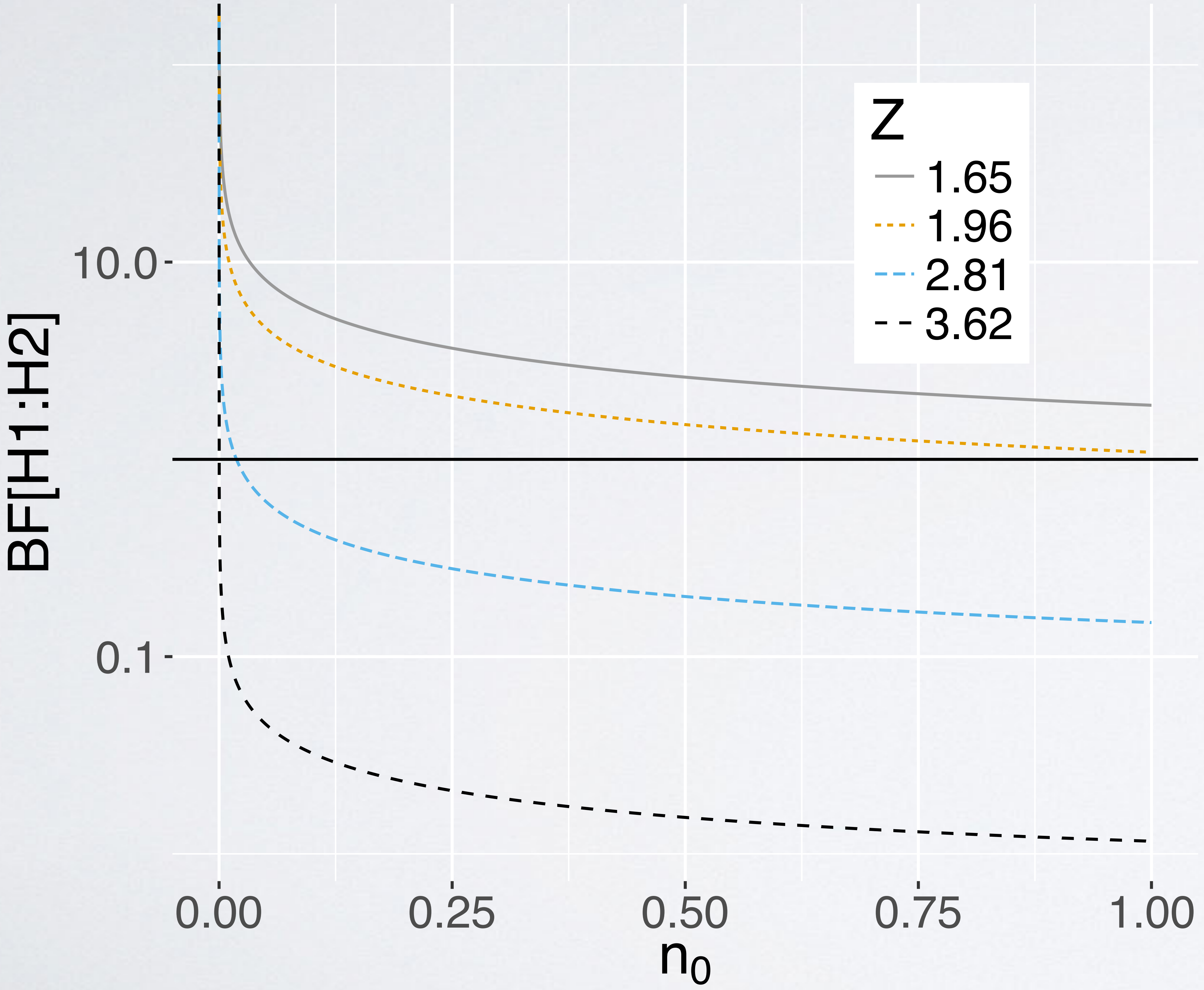

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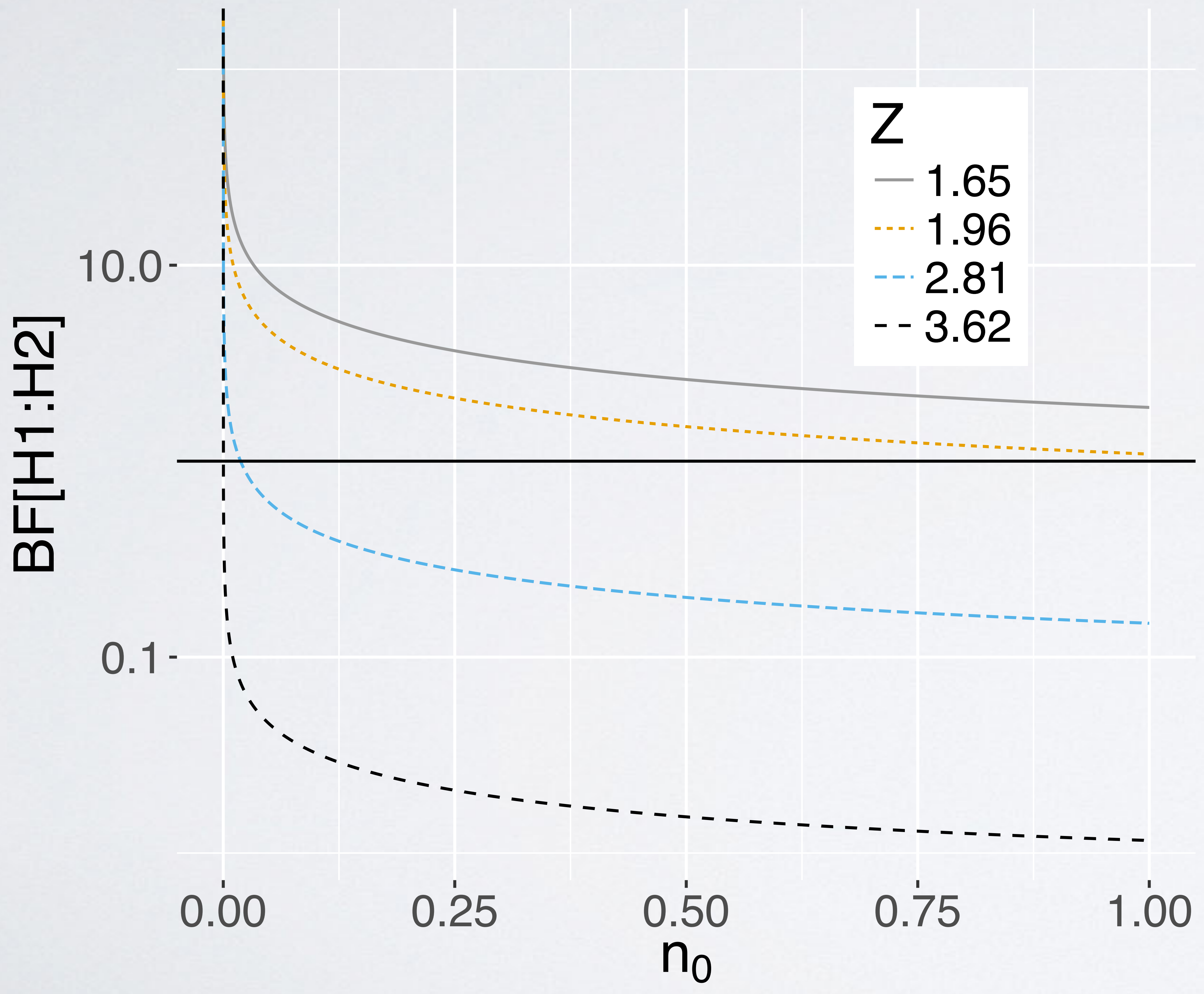

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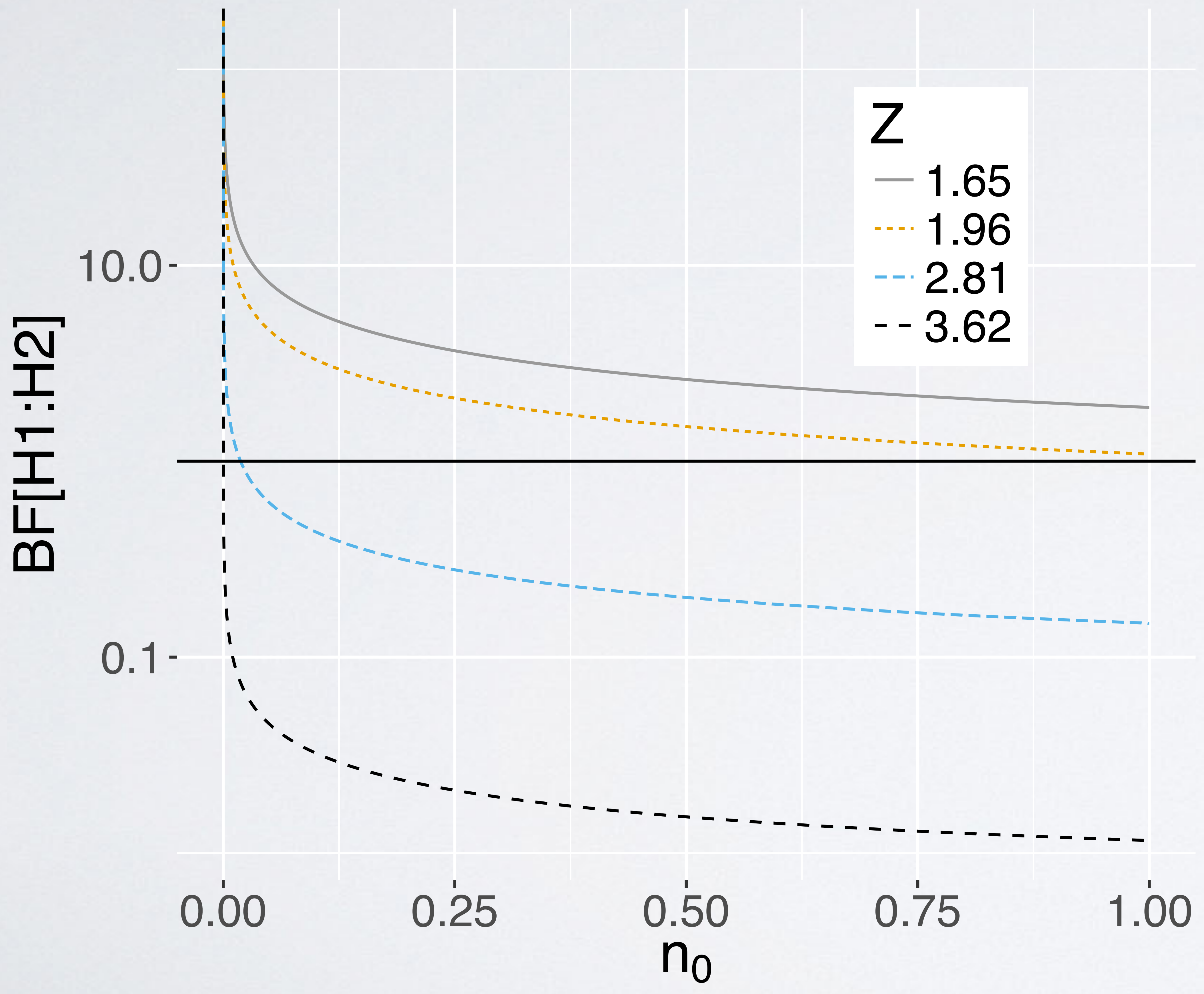

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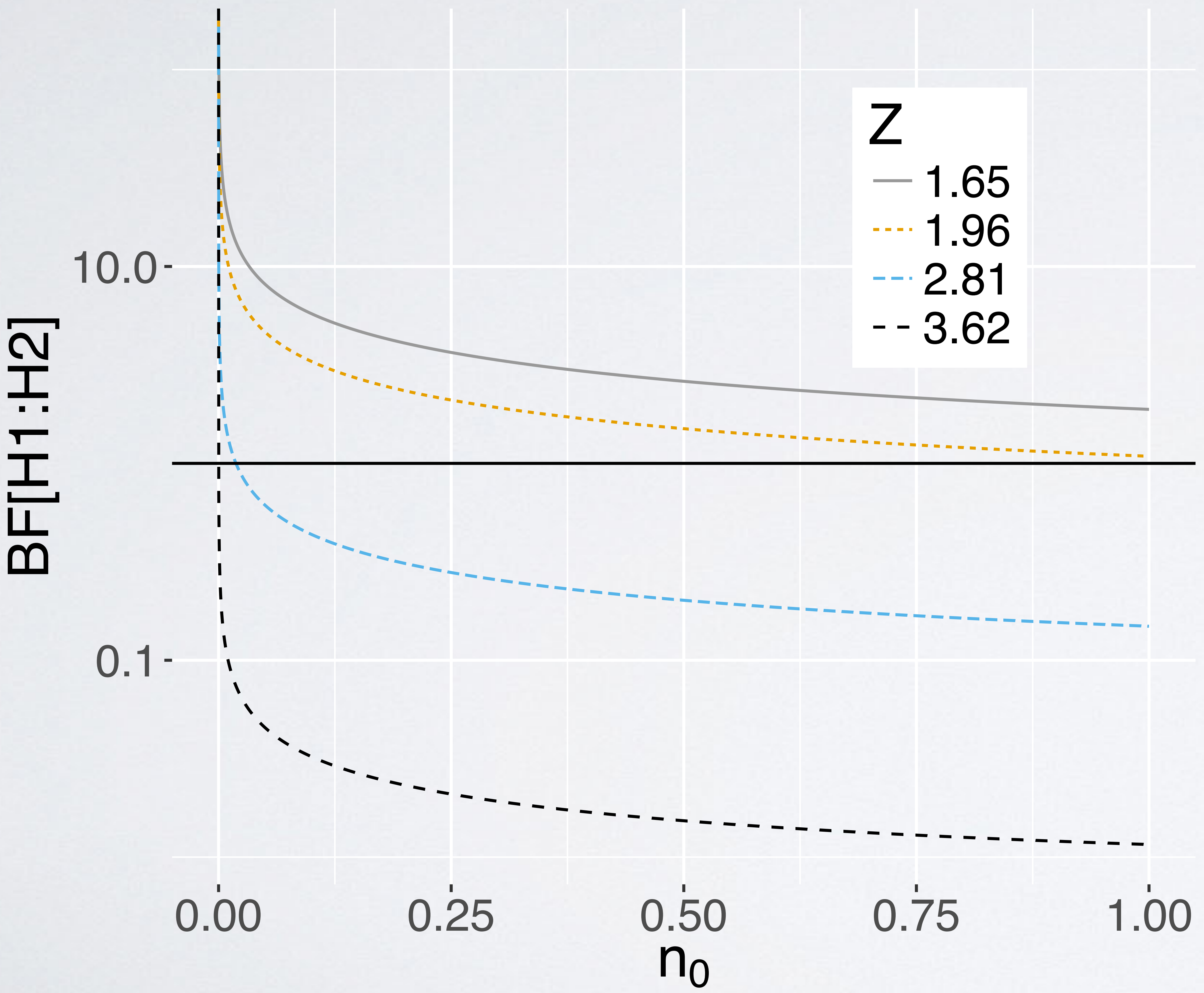

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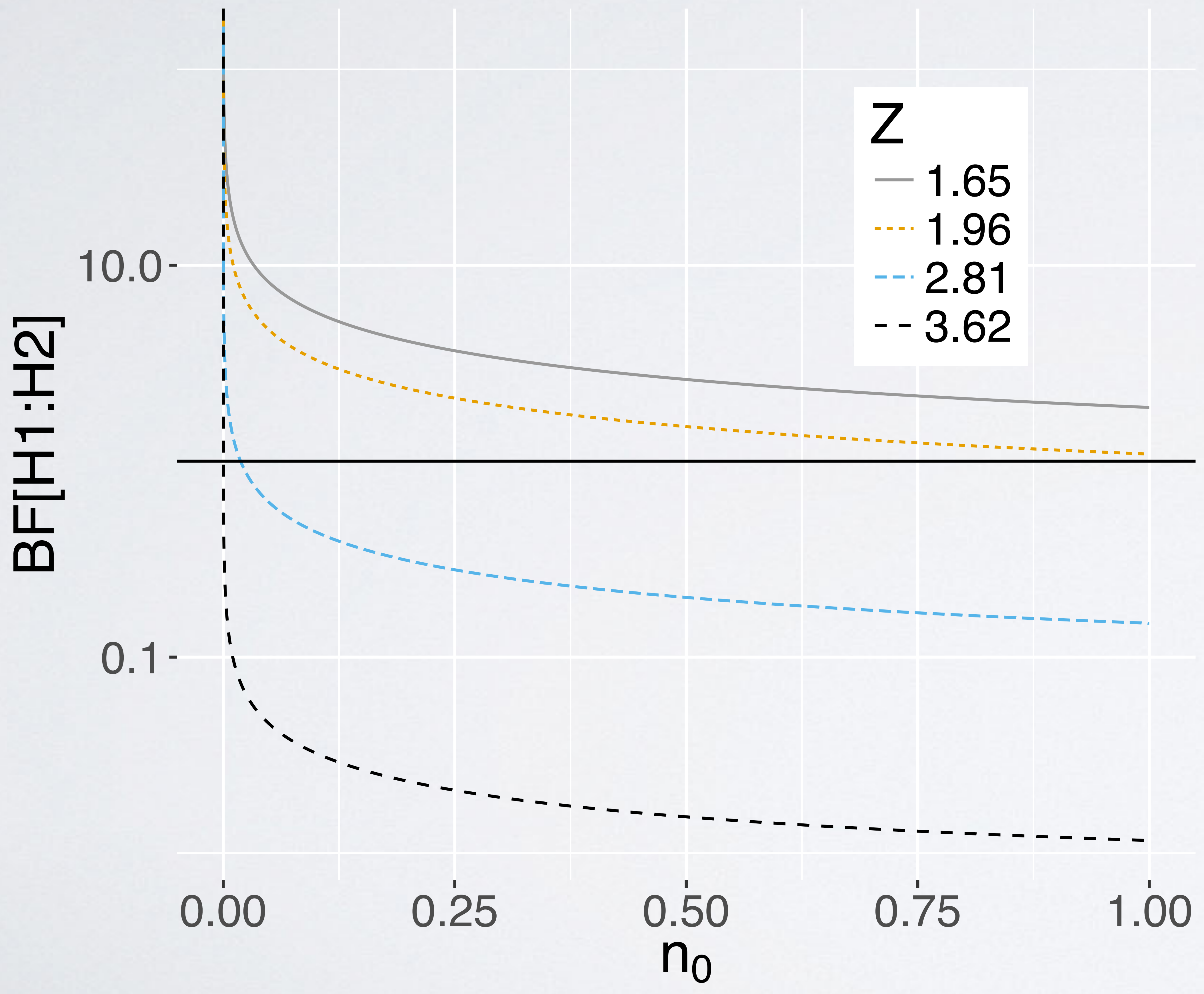

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- Bartlett's paradox or Jeffreys-Lindley's paradox

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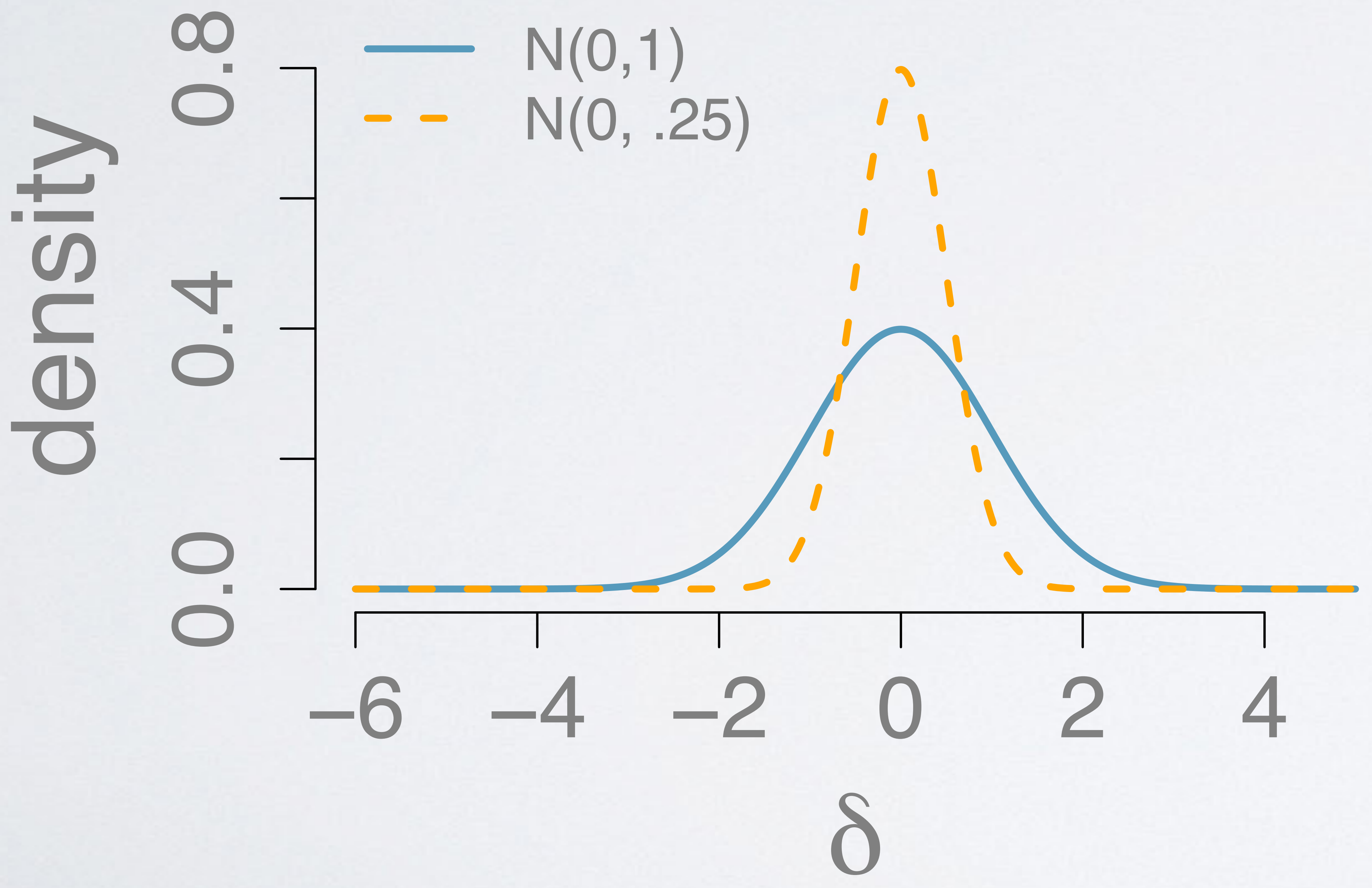

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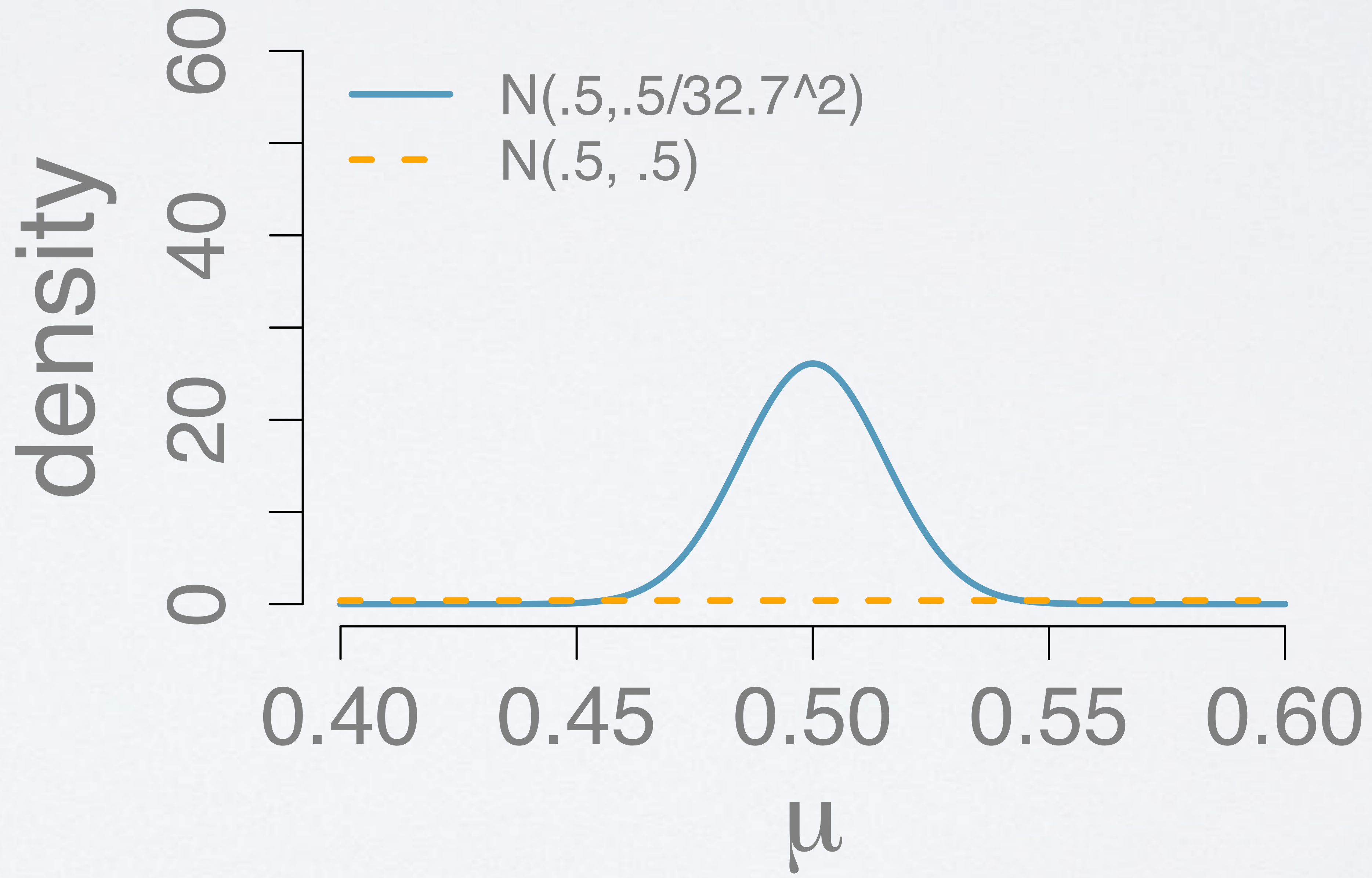

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- $Z = 3.61$
- informative $BF[H1 : H2] = 0.46$
- $BF[H2 : H1] = 1/BF[H1 : H2] = 2.19$

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next: testing with unknown variance