Topological Quantum Computation and the Kitaev Honeycomb Model

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Quantum statistics and anyons

$$P^2 = 1, \qquad P_{ij}\psi[x] = e^{i\phi}\psi[x] \tag{1}$$

$$P^2 \neq 1, \qquad P_{ij}\psi[x] = A\psi[x], \qquad A \in \mathrm{SU}(n) \tag{2}$$

Fusion

$$a \times b = \sum_c N_{ab}^c c \tag{3}$$

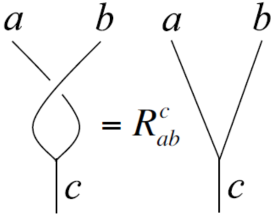
$$(a \times b) \times c = a \times (b \times c) \tag{4}$$

$$a \times b = i, \qquad j = b \times c, \qquad i \neq j \tag{5}$$

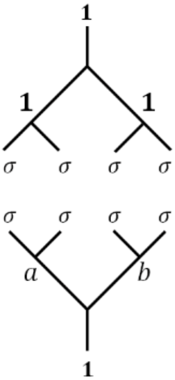
Ising anyons

$$1 \text{ (vacuum)}, \quad \sigma \text{ (Ising anyon)}, \quad \psi \text{ (fermion)} \tag{6}$$

$$\sigma \times \sigma = 1 + \psi, \qquad \sigma \times \psi = \sigma, \qquad \psi \times \psi = 1 \tag{7}$$

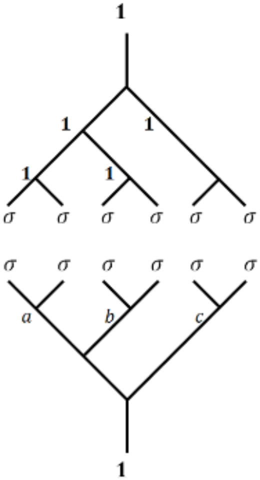


Ising anyons - 1 qubit



	a	b
$\|\mathbf{0}\rangle$	$\mathbf{1}$	$\mathbf{1}$
$\|\mathbf{1}\rangle$	ψ	ψ

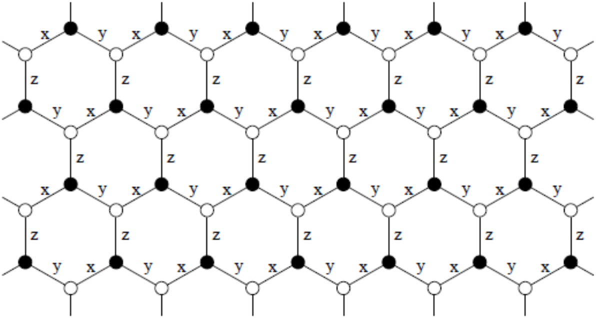
Ising anyons - 2 qubit



	a	b	c
$\|00\rangle$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$\|10\rangle$	ψ	ψ	$\mathbf{1}$
$\|01\rangle$	$\mathbf{1}$	ψ	ψ
$\|11\rangle$	ψ	$\mathbf{1}$	ψ

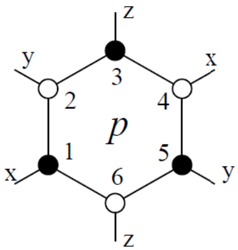
Kitaev honeycomb model

$$H = -J_x \sum_{x \text{ links}} \sigma_j^x \sigma_k^x - J_y \sum_{y \text{ links}} \sigma_j^y \sigma_k^y - J_z \sum_{z \text{ links}} \sigma_j^z \sigma_k^z \quad (8)$$



Plaquettes

$$W_p = \sigma_1^x \sigma_2^y \sigma_3^z \sigma_4^x \sigma_5^y \sigma_6^z \tag{9}$$

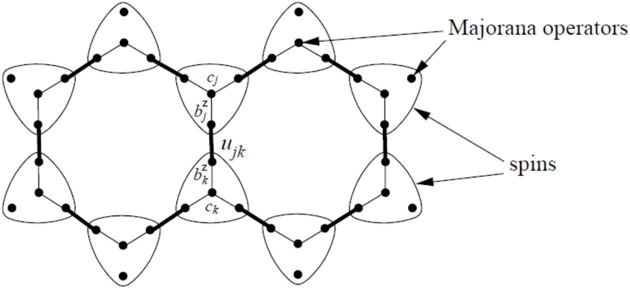


$$\mathcal{H} = \bigoplus_{[w_i]} \mathcal{H}_{[w_i]} \tag{10}$$

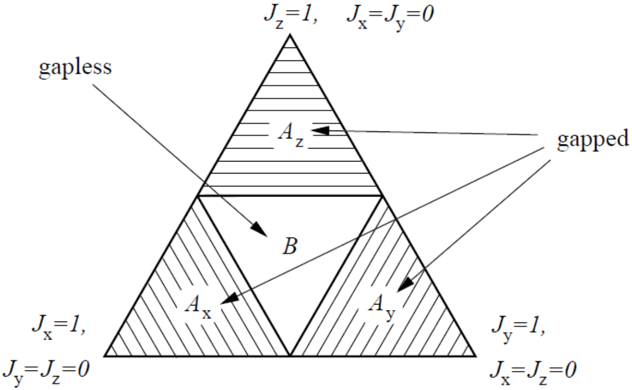
Transformation of spin operators into Majorana fermions

$$\gamma_{2j-1} = a_j + a_j^\dagger, \qquad \gamma_{2j} = -i(a_j - a_j^\dagger) \tag{11}$$

$$\tilde{\sigma}^x = ib^x c, \qquad \tilde{\sigma}^y = ib^y c, \qquad \tilde{\sigma}^z = ib^z c. \tag{12}$$



Phases



Abelian anyons in A_z phase

$$|J_x|, |J_y| \ll |J_z| \tag{13}$$

$$1 \text{ (vacuum)}, \quad e \text{ (electric charge)}, \quad m \text{ (magnetic vortex)} \tag{14}$$

$$e \times e = m \times m = \varepsilon \times \varepsilon = 1, \quad e \times m = \varepsilon, e \times \varepsilon = m, \quad m \times \varepsilon = e \tag{15}$$

Ising anyons in Kitaev model

$$V = -\sum_j (h_x \sigma_j^x + h_y \sigma_j^y + h_z \sigma_j^z) \tag{16}$$

$$1 \text{ (vacuum)}, \quad \sigma \text{ (vortex)}, \quad \psi \text{ (fermion)} \tag{17}$$

Works Cited

[1] Steven M. Girvin and Kun Yang. *Modern Condensed Matter Physics*. Cambridge University press, 2019.

[2] Alexei Kitaev. “Anyons in an exactly solved model and beyond”. In: *Annals of Physics* 321.1 (2006), pp. 2–111. DOI: 10.1016/j.aop.2005.10.005.

[3] Yingkai Liu. *Introduction to topological Quantum Computation: Ising anyons case study*. May 2019. URL: https://yk-liu.github.io/2019/Introduction-to-QC-and-TQC-Ising-Anyons/.

[4] Jiannis K Pachos. *Introduction to topological quantum computation*. Cambridge University Press, 2012.