

Topological Quantum Computation and the Kitaev Honeycomb Model

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Quantum statistics and anyons

$$P^2 = 1, \quad P_{ij}\psi[x] = e^{i\phi}\psi[x] \quad (1)$$

$$P^2 \neq 1, \quad P_{ij}\psi[x] = A\psi[x], \quad A \in \text{SU}(n) \quad (2)$$

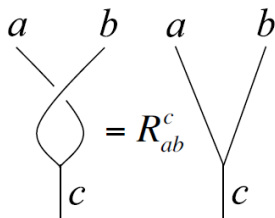
$$a \times b = \sum_c N_{ab}^c c \quad (3)$$

$$(a \times b) \times c = a \times (b \times c) \quad (4)$$

$$a \times b = i, \quad j = b \times c, \quad i \neq j \quad (5)$$

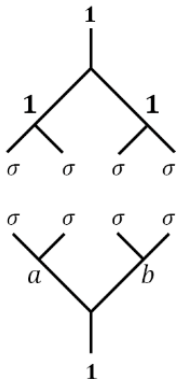
$$1 \text{ (vacuum)}, \quad \sigma \text{ (Ising anyon)}, \quad \psi \text{ (fermion)} \quad (6)$$

$$\sigma \times \sigma = 1 + \psi, \quad \sigma \times \psi = \sigma, \quad \psi \times \psi = 1 \quad (7)$$



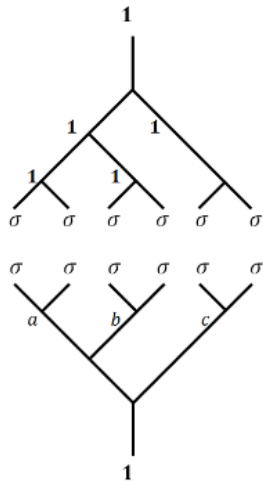
The diagram shows an equation between two string configurations. On the left, two lines labeled a and b enter from the top, cross each other, and then merge into a single line labeled c at the bottom. This configuration is equal to R_{ab}^c , which is then followed by a second diagram. In this second diagram, two lines labeled a and b enter from the top and merge into a single line labeled c at the bottom without crossing.

Ising anyons - 1 qubit



	a	b
$ 0\rangle$	1	1
$ 1\rangle$	ψ	ψ

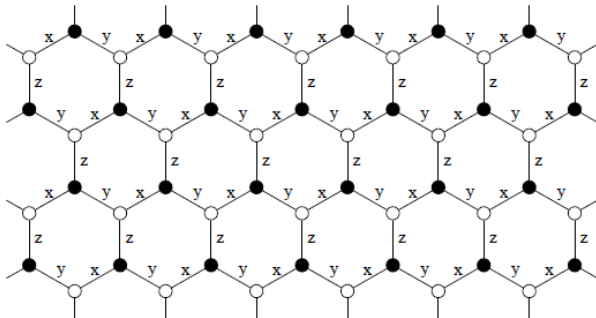
Ising anyons - 2 qubit



	a	b	c
$ 00\rangle$	1	1	1
$ 10\rangle$	ψ	ψ	1
$ 01\rangle$	1	ψ	ψ
$ 11\rangle$	ψ	1	ψ

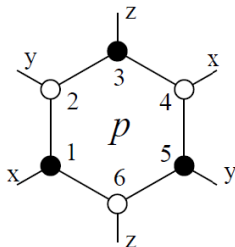
Kitaev honeycomb model

$$H = -J_x \sum_{x \text{ links}} \sigma_j^x \sigma_k^x - J_y \sum_{y \text{ links}} \sigma_j^y \sigma_k^y - J_z \sum_{z \text{ links}} \sigma_j^z \sigma_k^z \quad (8)$$



Plaquettes

$$W_p = \sigma_1^x \sigma_2^y \sigma_3^z \sigma_4^x \sigma_5^y \sigma_6^z \quad (9)$$

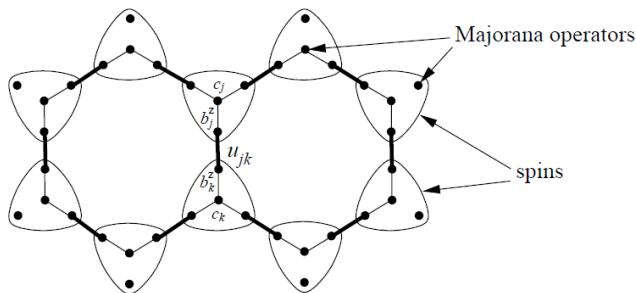


$$\mathcal{H} = \bigoplus_{[w_i]} \mathcal{H}_{[w_i]} \quad (10)$$

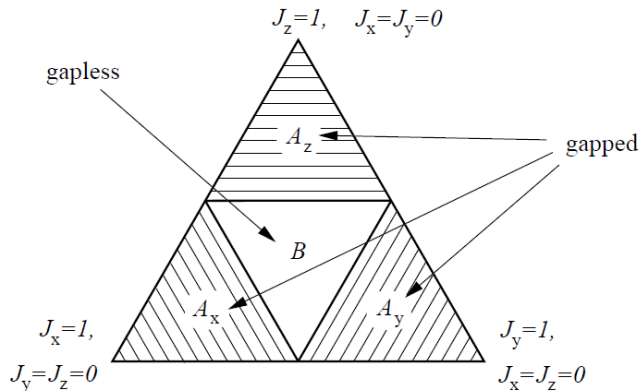
Transformation of spin operators into Majorana fermions

$$\gamma_{2j-1} = a_j + a_j^\dagger, \quad \gamma_{2j} = -i(a_j - a_j^\dagger) \quad (11)$$

$$\tilde{\sigma}^x = ib^x c, \quad \tilde{\sigma}^y = ib^y c, \quad \tilde{\sigma}^z = ib^z c. \quad (12)$$



Phases



Abelian anyons in A_z phase

$$|J_x|, |J_y| \ll |J_z| \quad (13)$$

$$1 \text{ (vacuum)}, \quad e \text{ (electric charge)}, \quad m \text{ (magnetic vortex)} \quad (14)$$

$$e \times e = m \times m = \varepsilon \times \varepsilon = 1, \quad e \times m = \varepsilon, \quad e \times \varepsilon = m, \quad m \times \varepsilon = e \quad (15)$$

Ising anyons in Kitaev model

$$V = - \sum_j (h_x \sigma_j^x + h_y \sigma_j^y + h_z \sigma_j^z) \quad (16)$$

$$1 \text{ (vacuum)}, \quad \sigma \text{ (vortex)}, \quad \psi \text{ (fermion)} \quad (17)$$

Works Cited

- [1] Steven M. Girvin and Kun Yang. *Modern Condensed Matter Physics*. Cambridge University press, 2019.
- [2] Alexei Kitaev. “Anyons in an exactly solved model and beyond”. In: *Annals of Physics* 321.1 (2006), pp. 2–111. DOI: 10.1016/j.aop.2005.10.005.
- [3] Yingkai Liu. *Introduction to topological Quantum Computation: Ising anyons case study*. May 2019. URL: <https://yk-liu.github.io/2019/Introduction-to-QC-and-TQC-Ising-Anyons/>.
- [4] Jiannis K Pachos. *Introduction to topological quantum computation*. Cambridge University Press, 2012.