Topological Quantum Computation using Bilayer Graphene

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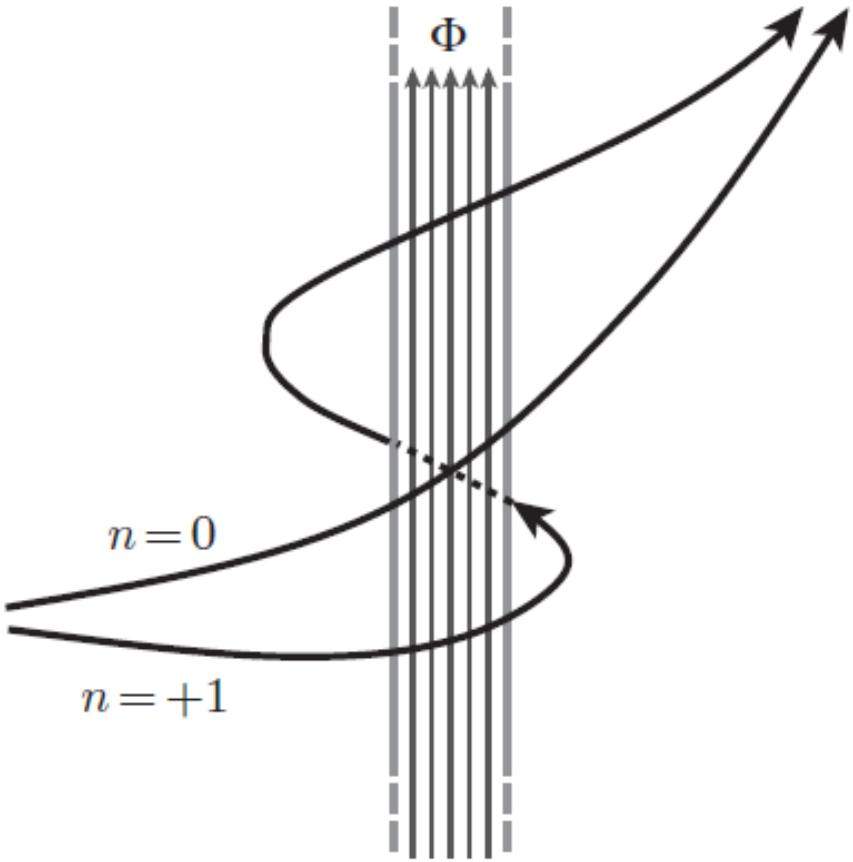
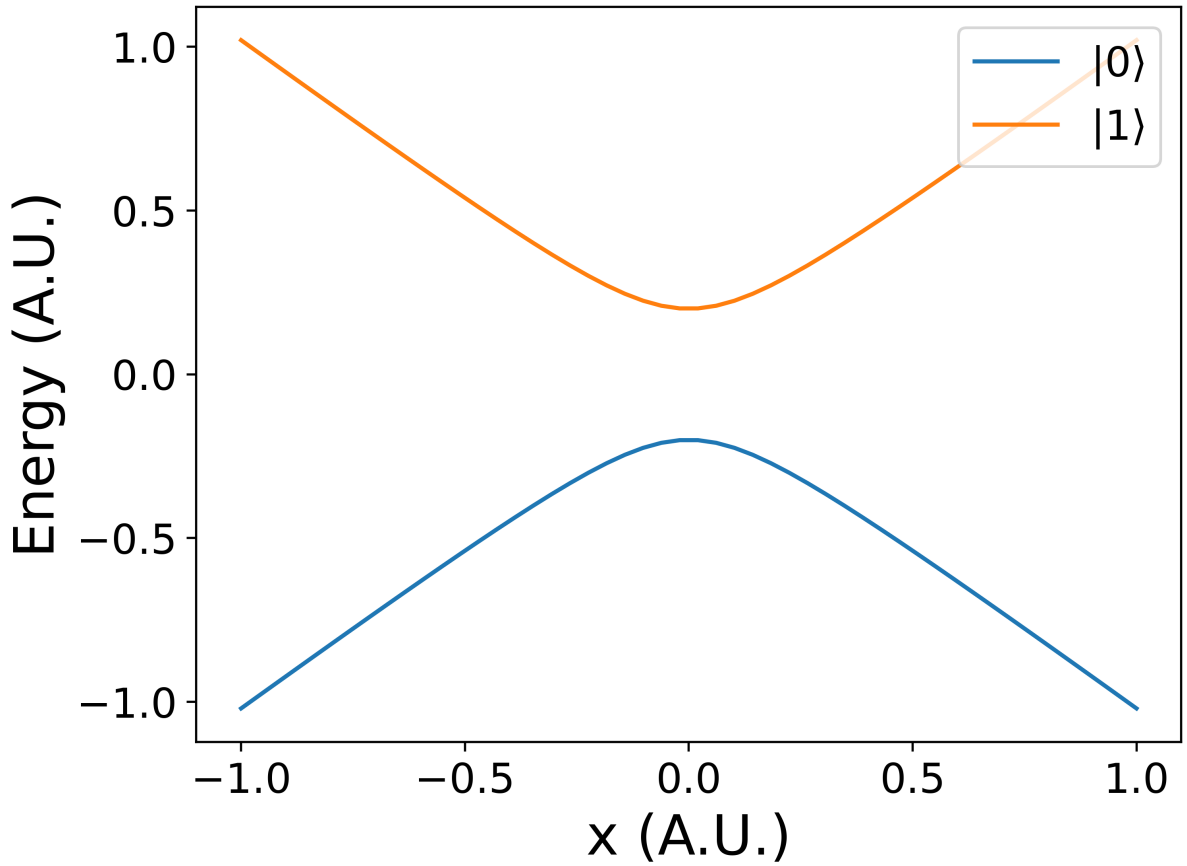
Aharonov-Bohm Effect

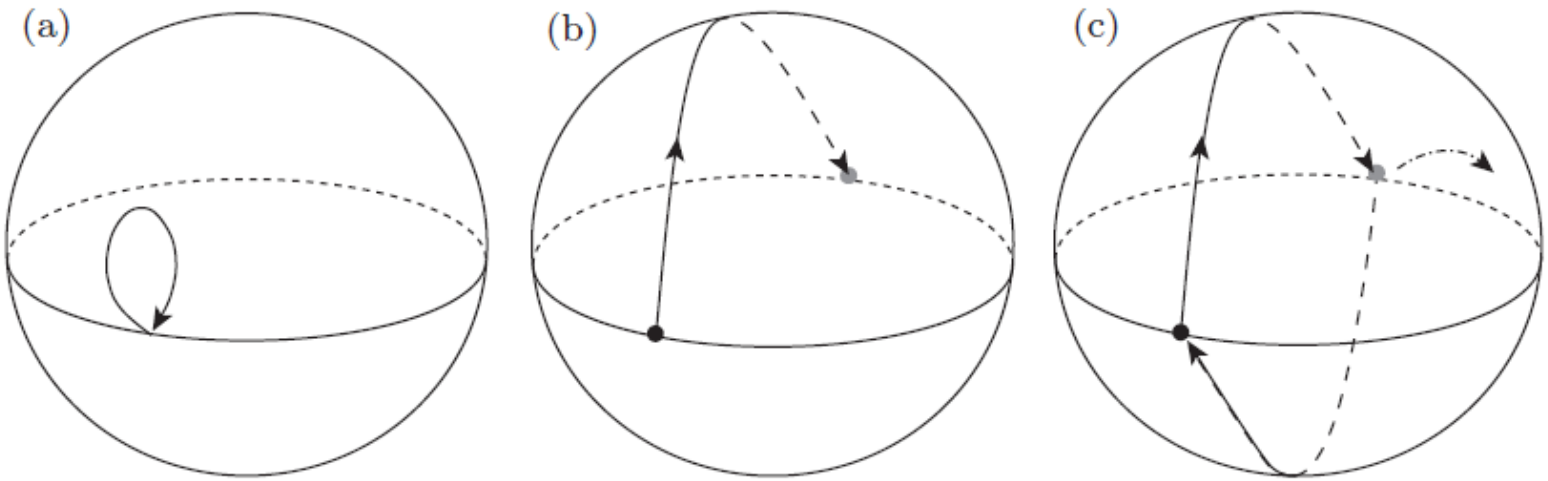


Modern Condensed Matter Physics, Girvin

Adiabatic Theorem



Quantum statistics and anyons



Modern Condensed Matter Physics, Girvin

$$\text{Trivial: } P^2 = 1, \quad P_{ij}\psi[\mathbf{x}] = e^{i\phi}\psi[\mathbf{x}], \quad e^{i\phi} \in \mathrm{SU}(1)$$

$$\text{Anyons: } P^2 \neq 1, \quad P_{ij}\psi[\mathbf{x}] = A\psi[\mathbf{x}], \quad A \in \mathrm{SU}(n)$$

Fusion

Fusing particles a and b together

$$a \times b = \sum_c N_{ab}^c c$$

The total outcomes are associative

$$(a \times b) \times c = a \times (b \times c)$$

but intermediate particles need not be identical

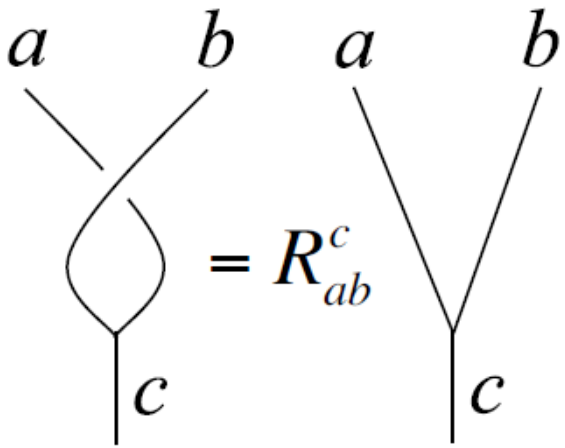
$$a \times b = i, \qquad j = b \times c, \qquad i \neq j$$

Central idea of topological quantum computing: encode information in intermediate particles

Ising anyons

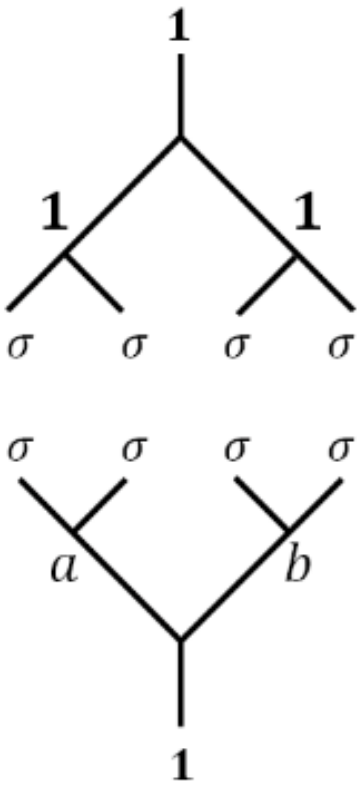
1 (vacuum), $\quad \sigma$ (Ising anyon), $\quad \psi$ (fermion)

$$\sigma \times \sigma = 1 + \psi, \qquad \sigma \times \psi = \sigma, \qquad \psi \times \psi = 1$$



Pachos, 2012

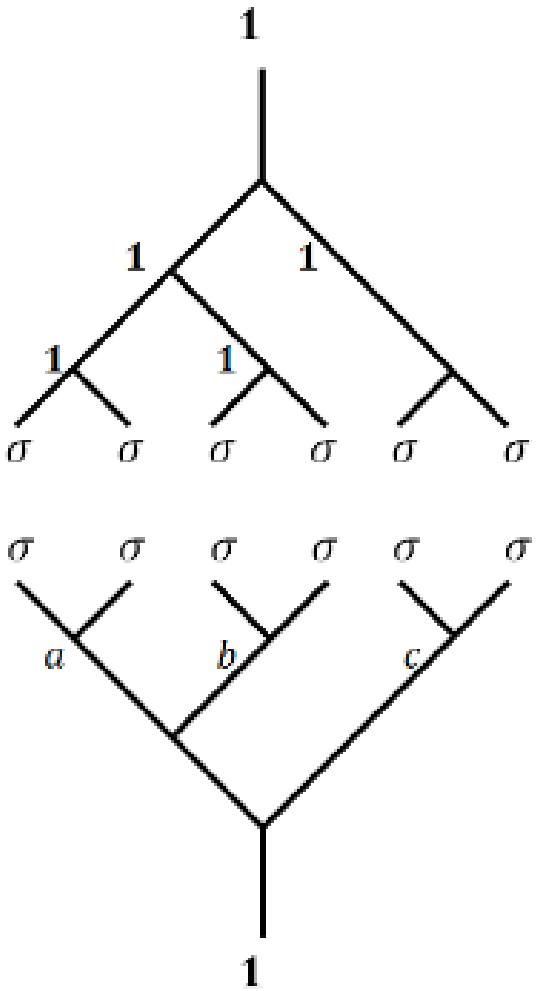
Ising anyons - encoding 1 qubit



	a	b
$\|\mathbf{0}\rangle$	**1**	**1**
$\|\mathbf{1}\rangle$	ψ	ψ

Pachos, 2012

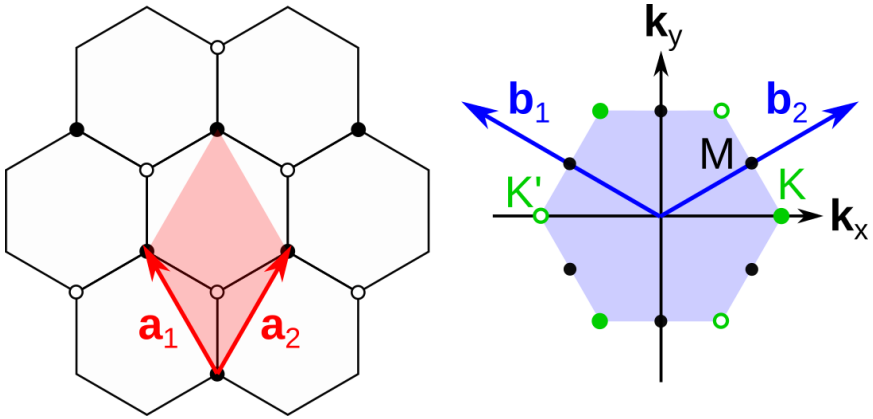
Ising anyons - enconding 2 qubits



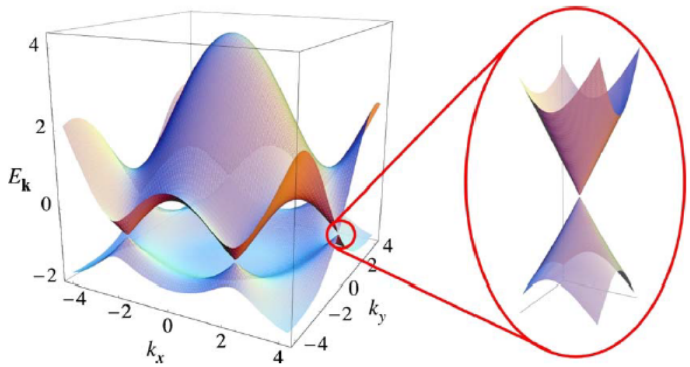
	a	b	c
$\|00\rangle$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$\|10\rangle$	ψ	ψ	$\mathbf{1}$
$\|01\rangle$	$\mathbf{1}$	ψ	ψ
$\|11\rangle$	ψ	$\mathbf{1}$	ψ

Pachos, 2012

Graphene

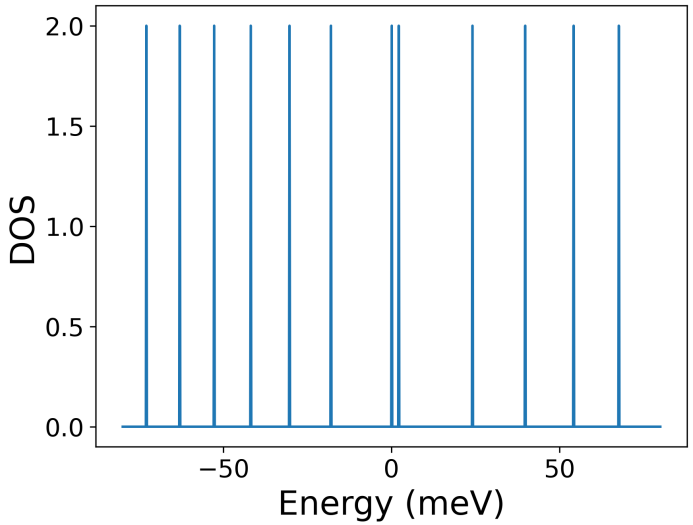


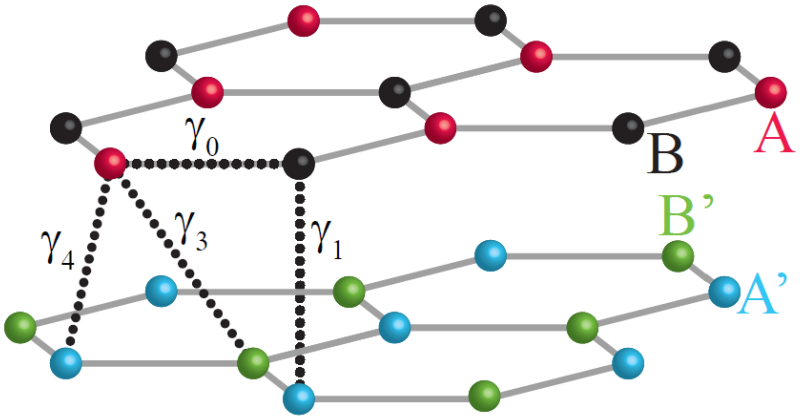
Pela, 2019



Gurrieri, 2020

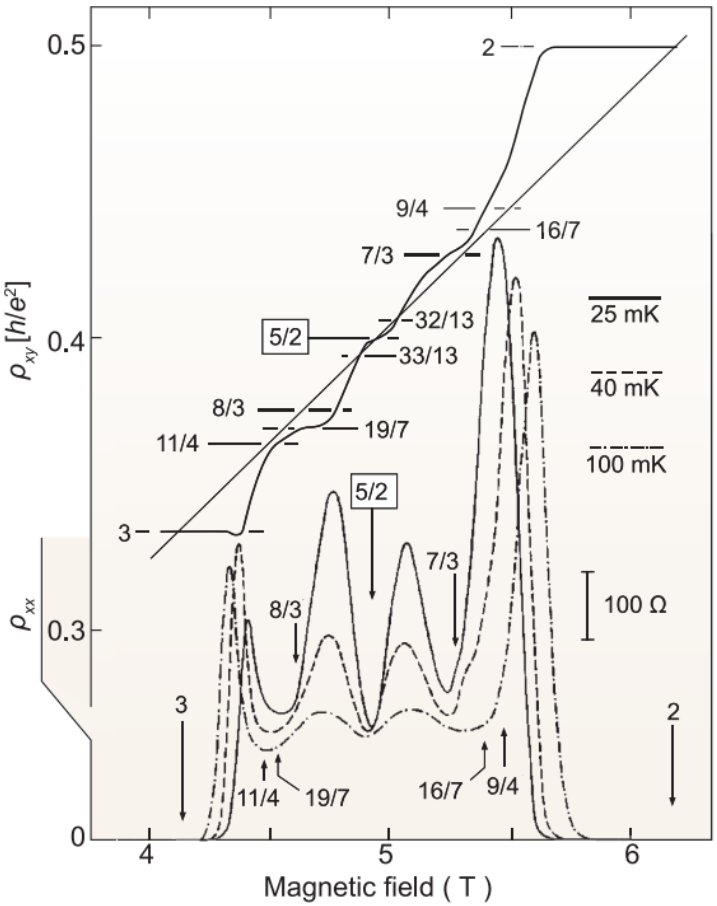
Bilayer graphene in magnetic field





Cheng, 2019

Nonabelian anyons in $\nu = 5/2$ fractional quantum Hall phase



GaAs/AlGaAs, Willet, 1987

Important works for self-study

[1] Cory Dean et al. "Fractional Quantum Hall Effects in Graphene". In: *Fractional Quantum Hall Effects* (2020), pp. 317–375. DOI: 10.1142/9789811217494_0007.

[2] Steven M. Girvin and Kun Yang. *Modern Condensed Matter Physics*. Cambridge University press, 2019.

[3] Alexei Kitaev. "Anyons in an exactly solved model and beyond". In: *Annals of Physics* 321.1 (2006), pp. 2–111. DOI: 10.1016/j.aop.2005.10.005.

[4] Yingkai Liu. *Introduction to topological Quantum Computation: Ising anyons case study*. May 2019. URL: https://yk-liu.github.io/2019/Introduction-to-QC-and-TQC-Ising-Anyons/.

[5] Jiannis K Pachos. *Introduction to topological quantum computation*. Cambridge University Press, 2012.

My own notes

https://github.com/amirsm02/squid_2023