Eigenstate Thermalization Hypothesis

Amir Shapour Mohammadi

PHY511 – Statistical Mechanics

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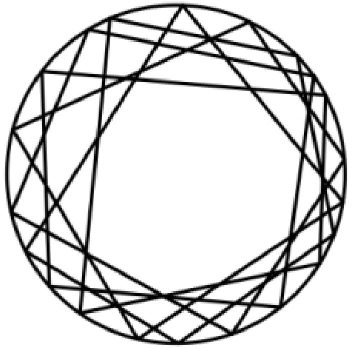
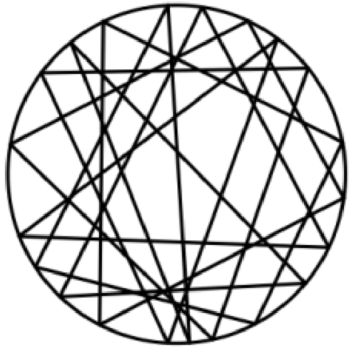
Ergodicity in Classical Statistical Mechanics

$$\langle O \rangle_t = \frac{1}{A(\Sigma)} \int_\Sigma d\sigma \cdot O(\sigma)$$



A. Non-ergodic

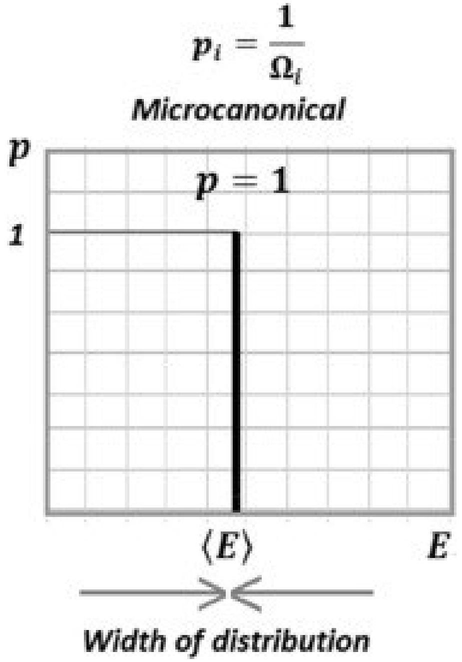


B. Ergodic

[T. Kong]

Microcanonical Ensemble (NVE)

$$\langle O \rangle_{mc} \equiv \sum_{m \in I_0} P(m) \cdot \langle O \rangle_m = \frac{1}{W} \sum_{m \in I_0} O_{mm}$$



[J. Stephanos]

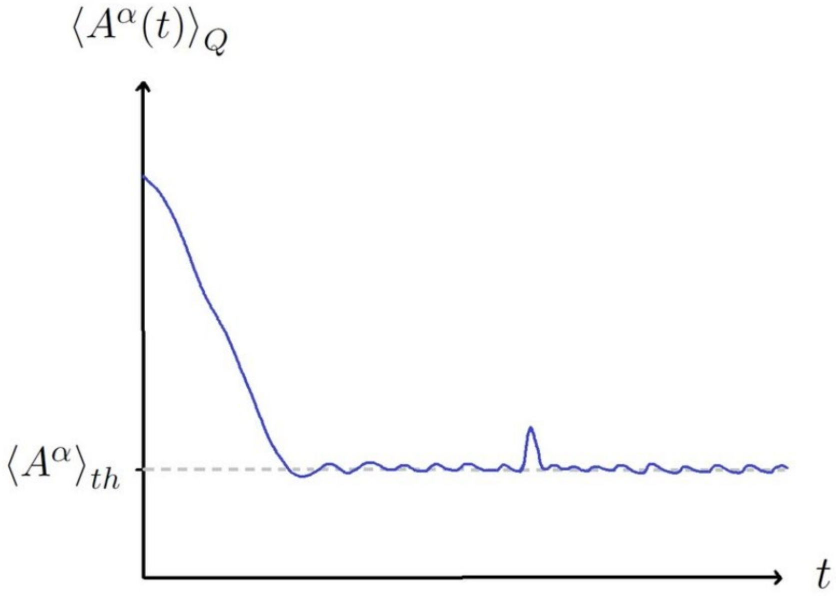
Thermalization in Quantum Mechanics

$$O_\infty \equiv \lim_{T\to\infty} \overline{\langle O\rangle_{\psi(t)}} \equiv \lim_{T\to\infty} \frac{1}{T}\int_0^T dt \cdot \langle O\rangle_{\psi(t)}$$
$$= \sum_n |c_n|^2 O_{nn} + i\hbar \lim_{T\to\infty}\left[\sum_{n\neq m} \frac{c_n^* c_m O_{nm}}{\varepsilon_m - \varepsilon_n}\left(\frac{e^{-i(\varepsilon_m-\varepsilon_n)T/\hbar} - 1}{T}\right)\right]$$

$$\sigma_\infty^2 \equiv \lim_{T\to\infty} \overline{(\langle O\rangle_{\psi(t)} - O_\infty)^2} \equiv \lim_{T\to\infty} \frac{1}{T}\int_0^\infty dt \cdot (\langle O\rangle_{\psi(t)} - O_\infty)^2$$
$$= \sum_{m\neq n} |c_m|^2 |c_n|^2 |O_{mn}|^2.$$

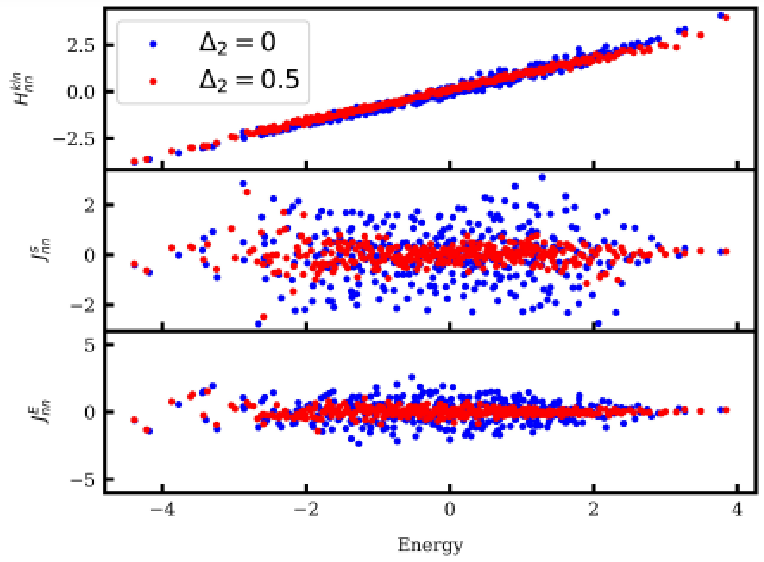
ETH Assumption

$$O_{mn} \approx \overline{O}(m) \cdot \delta_{mn} + \sqrt{\frac{\overline{O^2}(m,n)}{D}} \cdot R_{mn}$$

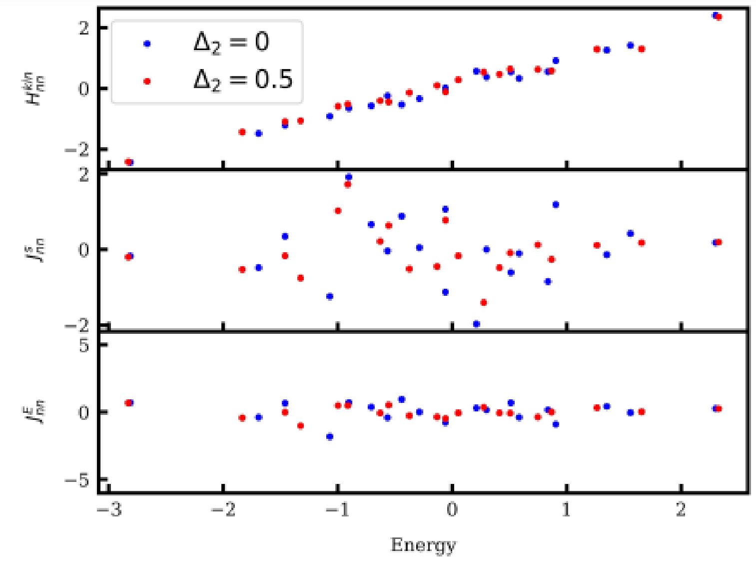


Anisotropic S=1/2 Heisenberg Spin Chain

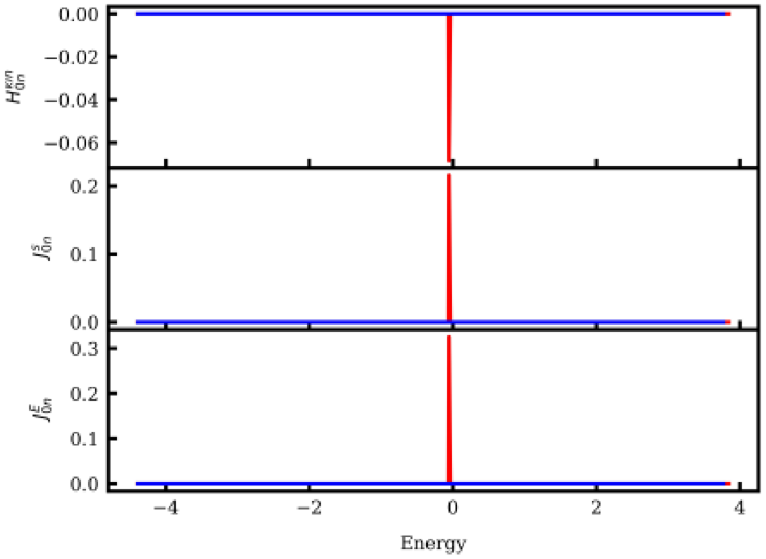
$$H = J \sum_{i=1}^{L} (S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta S_i^z S_{i+1}^z + \Delta_2 S_i^z S_{i+2}^z)$$



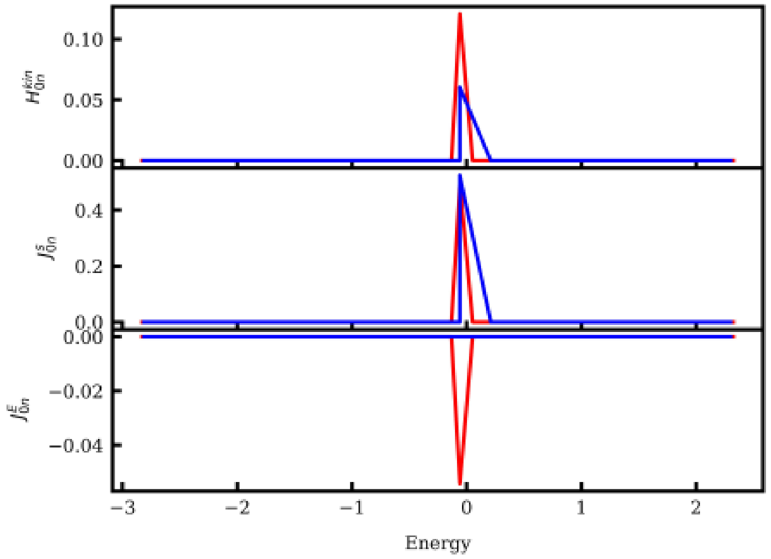
(a) $L = 15$



(b) $L = 10$

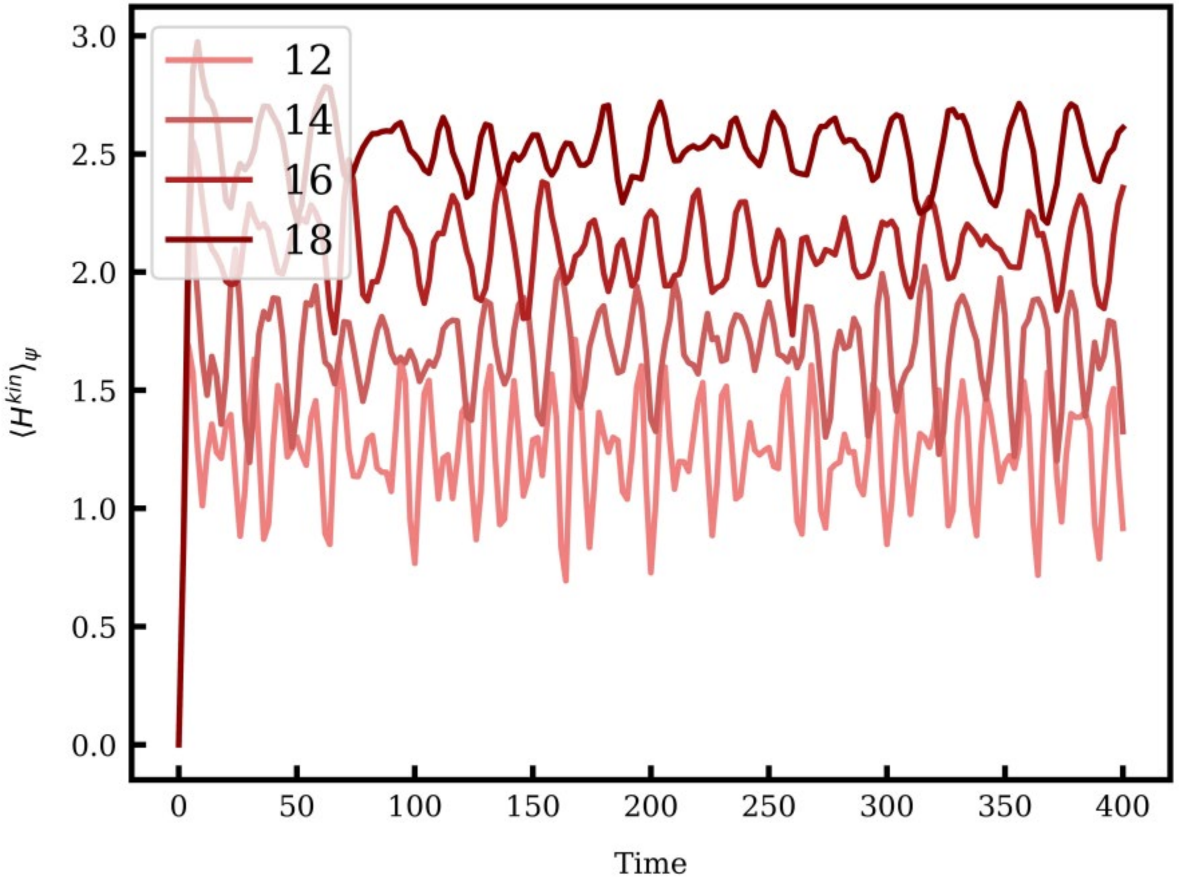


(c) $L = 15$

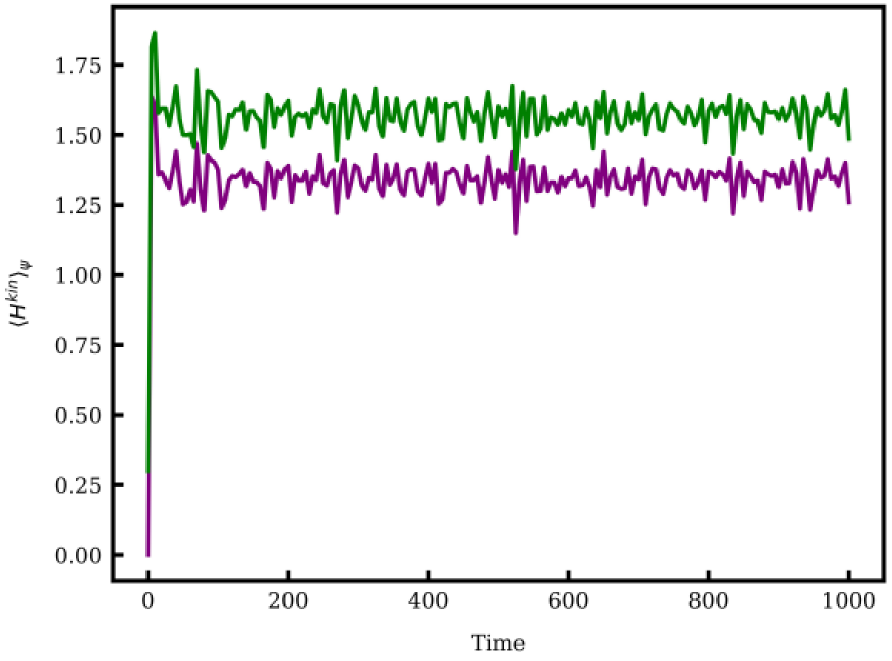


(d) $L = 10$

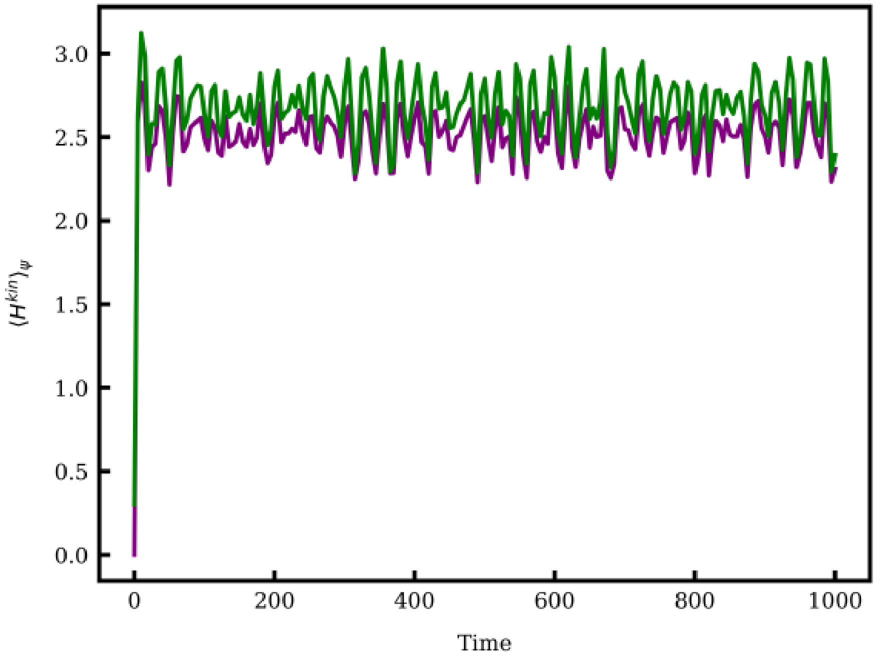
Thermalization dependence on length



Thermalization dependence on initial condition



(a) $L = 18,\ D_2 = 0$



(b) $L = 18,\ D_2 = 0.5$

Citations

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- Deutsch, Joshua M. “Eigenstate thermalization hypothesis.” *Reports on Progress in Physics*, vol. 81, no. 8, 2018, p. 082001, https://doi.org/10.1088/1361-6633/aac9f1.
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- Klein, Martin J. “The ergodic theorem in quantum statistical mechanics.” *Physical Review*, vol. 87, no. 1, 1952, pp. 111–115, https://doi.org/10.1103/physrev.87.111.