NONLINEAR AND RATE-DEPENDENT HYSTERETIC ELECTRO MECHANICAL RESPONSES OF FERRO ELECTRIC MATERIALS

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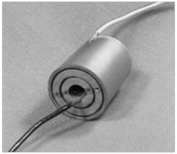
A Presentation for Tire Mechanics and Material Group in Cooper Tire

Outline

Outline

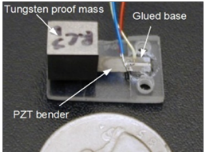
Application of Piezoelectric Materials

- Actuation [1]



- Sensing

- Energy Harvesting [5]

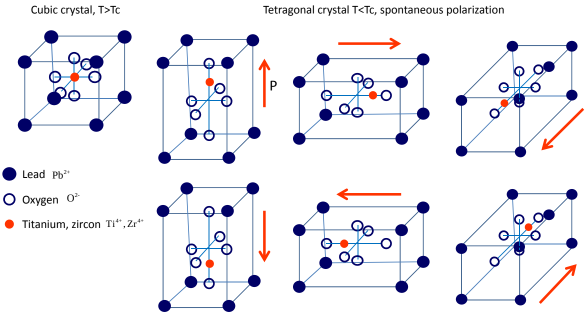


Outline

Crystal Structure of PZT

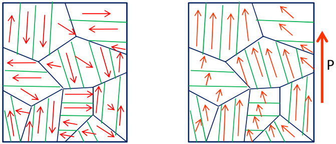
The origin of electromechanical coupling is polarization

Perovskit structure of ferroelectric crystals, i.e. PZT



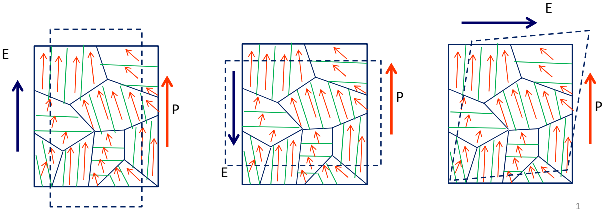
Polarization in the bulk material

Each grain contain crystals with same structures and polarization. Overall polarization is zero in virgin sample.



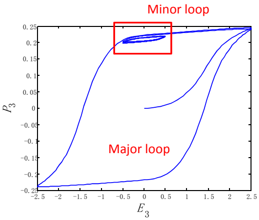
Non-polarized crystal

Polarized crystal

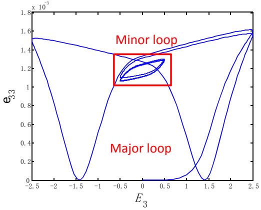


Typical behavior of bulk PZT under cyclic electric field

Input: cyclic electric field



Polarization response



Normal strain response

Response depends on the amplitude and frequency of loading

1

Outline

Constitutive Equation

The relationship between stress σ_{11} and strain ε_{11}, and electric field E_3 and electric flux D_3.

Electromechanical Constitutive Equation

$$\begin{aligned} \sigma_{11} &= C_{1111}\varepsilon_{11} - e_{311}E_3 \\ D_3 &= e_{311}\varepsilon_{11} + \kappa_{33}^{\varepsilon}E_3 \end{aligned} \tag{1}$$

C_{1111} is the elastic stiffness.
e_{311} is the piezoelectric constant.
$\kappa_{33}^{\varepsilon}$ is the permittivity at constant strain.

Constitutive Equation

Extending the constitutiv equation to 3D

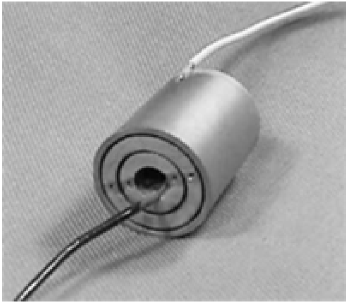
Electromechanical Constitutive Equation

$$
\begin{aligned}
\sigma_{ij} &= \sum_{k,l=1}^{3} C_{ijkl}\varepsilon_{kl} - \sum_{k=1}^{3} e_{kij}E_k \\
D_k &= \sum_{i,j=1}^{3} e_{kij}\varepsilon_{ij} + \sum_{j=1}^{3} \kappa_{kj}E_j \\
&(i, j, k = 1 \ldots 3)
\end{aligned}
\tag{2}
$$

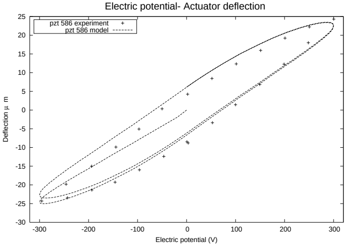
This is the standard constitutive equation presented in IEEE standard [4].

The Non Linear and time Dependent Responce

Nonlinear and hysteretic response of piezoelectric structures under cyclic electric field input



Telescopic Actuator [1]



Deflection Response

Outline

Research Motivations

- Formulate nonlinear time-dependent electromechanical constitutive models of piezoelectric materials
- Develop finite element model for analyzing structures with proposed constitutive equation

Outline

Linear time dependent Electromechanical model

- Constitutive relation with normalized time functions

Nonlinear and time dependent constitutive equation

$$\sigma_{ij}(t) = \int_{0^-}^{t} \sum_{k=1}^{3}\sum_{l=1}^{3} K_{ijkl}^{C}(t-s)\frac{\partial \sigma_{ij}(s)}{\partial \varepsilon_{kl}(s)}\dot{\varepsilon}_{kl}(s)ds + \int_{0^-}^{t} \sum_{k=1}^{3} K_{kij}^{e}(t-s)\frac{\partial \sigma_{ij}(s)}{\partial E_k(s)}\dot{E}_k(s)ds$$

$$D_k(t) = \int_{0^-}^{t} \sum_{i=1}^{3}\sum_{j=1}^{3} K_{kij}^{e}(t-s)\frac{\partial D_k(s)}{\partial \varepsilon_{ij}(s)}\dot{\varepsilon}_{ij}(s)ds + \int_{0^-}^{t} \sum_{i=1}^{3} K_{ki}^{\kappa}(t-s)\frac{\partial D_k(s)}{\partial E_i(s)}\dot{E}_i(s)ds \tag{3}$$

Material Field Variables

Derivatives of the linear field variables

$$\begin{aligned}\frac{\partial \sigma_{ij}(s)}{\partial \varepsilon_{kl}(s)} &= C_{ijkl} \\ \frac{\partial \sigma_{ij}(s)}{\partial E_k(s)} &= -e_{kij} \\ \frac{\partial D_k(s)}{\partial E_i(s)} &= \kappa_{ki}\end{aligned} \tag{4}$$

Kernel Functions

Series of exponential functions, often called Prony series, is used for kernel function

$$
\begin{aligned}
K^{C}_{ijkl}(t) &= \sum_{l=0}^{NP} {}^{l}K^{C}_{ijkl} exp(-{}^{l}\lambda^{C}_{ijkl} t) \\
K^{e}_{ijk}(t) &= \sum_{l=0}^{NP} {}^{l}K^{e}_{ijk} exp(-{}^{l}\lambda^{e}_{ijk} t) \\
K^{\kappa}_{ki}(t) &= \sum_{l=0}^{NP} {}^{l}K^{\kappa}_{ki} exp(-{}^{l}\lambda^{\kappa}_{ki} t)
\end{aligned} \tag{5}
$$

Constitutive Equation

It is possible to use a developed nonlinear static constitutive equation in the time dependent framework.

Time Independent Non Linear Constitutive Equation [6]

$$\begin{aligned} \sigma_{ij} &= C_{ijkl}\varepsilon_{kl} - e_{kij}E_k - \frac{1}{2}\widehat{b}_{klij}E_kE_l \\ D_k &= e_{kij}\varepsilon_{ij} + \kappa_{ki}E_i + \frac{1}{2}\chi_{kij}E_iE_j \end{aligned} \tag{6}$$

Material Field Variables

Using the nonlinear electromechanical coupling constitutive equation.

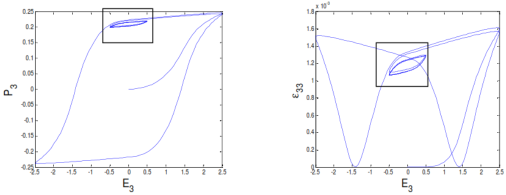
Derivatives of the field variables

$$
\begin{aligned}
\frac{\partial \sigma_{ij}(s)}{\partial \varepsilon_{kl}(s)} &= C_{ijkl} \\
\frac{\partial \sigma_{ij}(s)}{\partial E_k(s)} &= -e_{kij} - \widehat{b}_{klij} E_l \\
\frac{\partial D_k(s)}{\partial E_i(s)} &= \kappa_{ki} + \chi_{kij} E_j \\
\frac{\partial D_k(s)}{\partial \varepsilon_{kl}(s)} &= e_{kij}
\end{aligned}
\tag{7}
$$

Outline

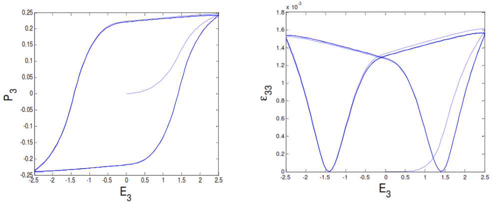
On polarization switching

- Minor Loop



(a) (b)

- Major Loop



(a) (b)

Additive Decomposition of Polarization

Consider an electric field input in the x_3 direction $E_3(s)$, and $E_3(s) = 0, \forall s < 0$, where s is the time history. The corresponding polarization response at the current time t is:

Additive decomposition of Polarization

$$P_3^t \equiv P_3[E_3(t-s), t] = R[E_3(t-s), t] + Q[E_3(t-s), t] \tag{8}$$

Time Dependent Reversible Polarization

The reversible polarization response is expressed as:

$$R^t \equiv R[E_3(t-s),t] = R[E_3^0,t] + \int_{0^+}^{t} \frac{\partial R}{\partial E_3}[E_3^t, t-s]\frac{dE_3^s}{ds}ds, t \geq 0 \tag{9}$$

$$R[E_3^0,t] = R_0(E_3^0) + R_1(E_3^0)\left(1 - exp\left[-\frac{t}{\tau_1}\right]\right) \tag{10}$$

$$\begin{aligned} R_0(E_3^s) &= \kappa_0 E_3^s \\ R_1(E_3^s) &= \kappa_1 E_3^s \end{aligned} \tag{11}$$

Evolution Law for Irreversible Polarization

- Irreversible polarization is formed when $f(P_3^t, P_c) = 0$ and $E_3^t P_3^t > 0$

$$Q^t \equiv Q[E_3^t] = \int_{0^+}^{E_3^t} \frac{dQ^s}{dE_3} dE_3 \tag{12}$$

$$\frac{dQ^t}{dE_3} = \begin{cases} \lambda \left| \frac{E_3^t}{E_c} \right|^n & \text{if } |E_3^t| \leq E_c, f = 0 \\ \mu . exp[-\omega(\frac{|E_3^t|}{E_c} - 1)] & \text{if } |E_3^t| > E_c, f = 0 \\ 0 & \text{if } f \leqslant 0 \end{cases} \tag{13}$$

$$f(P_3^t, P_c) = {P_3^t}^2 - P_c^2 \tag{14}$$

Constitutive Equation Beyond Coercive Limit

- A nonlinear polarization switching electromechanical coupling constitutive model

$$\begin{aligned} \varepsilon_{ij}^{t} &= S_{ijkl}\sigma_{kl}^{t} + 4g_{nij}^{t}\kappa_{nm}g_{mkl}^{t}\sigma_{kl}^{t} + 2g_{kij}^{t}P_{k}^{t} \\ D_{i}^{t} &= 2\kappa_{im}g_{mkl}^{t}\sigma_{kl}^{t} + P_{i}^{t} \end{aligned} \tag{15}$$

$$P_{1}^{t} = \kappa_{11}E_{1}^{t}\ ;\ P_{2}^{t} = \kappa_{22}E_{2}^{t} \tag{16}$$

$$g_{ijk}^{t} \equiv g_{ijk}(P_{3}^{t}) = \frac{P_{3}^{t}}{P_{r}}e^{-\left|P_{3}^{t}\right|/c_{1}}g_{ijk}^{r} \tag{17}$$

Outline

The Virtual Work

Variational formulation

$$\delta\pi = \int_V \sigma_{ij}\delta\varepsilon_{ij}dV - \int_V D_i\delta E_i dV - \int_{\partial V} T_i\delta u_i d\partial V + \int_{\partial V} Q\delta\phi d\partial V \tag{18}$$

The equilibrium is achived when virtual work in stationary.

$$R_k = \frac{\partial\delta\pi}{\partial\delta d_k} = 0, (k = 1\ldots NDF) \tag{19}$$

The Finite Element Approximation

- Degrees of freedom in each point of 3D continuum electro mechanical element

$$[u]^T = [u_1, u_2, u_3, \phi]^T \tag{20}$$

- The element degrees of freedom

$$[d]^T = [u_1^1, u_2^1, u_3^1, \phi^1 \ldots u_1^{ND}, u_2^{ND}, u_3^{ND}, \phi^{ND}] \tag{21}$$

- The approximation function

$$u_i = \varphi_{ip} d_p, (i = 1 \ldots 4), (p = 1 \ldots NDF) \tag{22}$$

Defining Field Variables

- The linear strain and electric field vector are defined as:

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \quad E_i = -\phi_{,i} \tag{23}$$

- Variation of field functions

$$\delta u_i = \varphi_{ip} \delta d_p, (i = 1 \ldots 4), (p = 1 \ldots NDF) \tag{24}$$

- The tangent matrix is defined as:

$$\mathcal{K}_{kl} = \frac{\partial R_k}{\partial d_l} = \frac{\partial}{\partial d_l} \frac{\partial \delta \pi}{\partial \delta d_k}; (l, k = 1 \ldots NDF) \tag{25}$$

Explicit Expression for Residual and Tangent

- Explicit form for residual vector:

$$R_k = \int_V \sigma_{ij} \frac{\partial \delta \varepsilon_{ij}}{\partial \delta d_k} dV - \int_S T_i \frac{\delta u_i}{\partial \delta d_k} ds - \int_V D_i \frac{\delta E_i}{\partial \delta d_k} dV + \int_S Q \frac{\partial \phi}{\partial \delta d_k} ds, (k = 1 \ldots NDF) \quad (26)$$

- The tangent matrix is defined with respect to field derivatives:

$$\mathcal{K}_{kl} = \frac{\partial R_k}{\partial d_l} = \frac{\partial R_k}{\partial \varepsilon_{ij}} \frac{\partial \varepsilon_{ij}}{\partial d_l} + \frac{\partial R_k}{\partial E_i} \frac{\partial E_i}{\partial d_l} \quad (27)$$

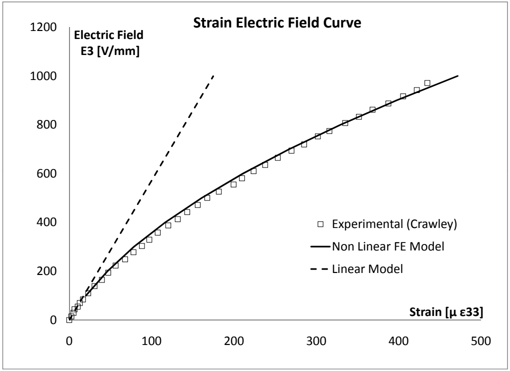
- The tangent matrix is explicitly defined as:

$$\begin{aligned} \frac{\partial R_k}{\partial \varepsilon_{ij}} &= \int_V \frac{\partial \sigma_{mn}}{\partial \varepsilon_{ij}} \frac{\partial \delta \varepsilon_{mn}}{\partial \delta d_k} dV - \int_V \frac{\partial D_p}{\partial \varepsilon_{ij}} \frac{\delta E_p}{\partial \delta d_k} dV, (k = 1 \ldots NDF) \\ \frac{\partial R_k}{\partial E_i} &= \int_V \frac{\partial \sigma_{mn}}{\partial E_i} \frac{\partial \delta \varepsilon_{mn}}{\partial \delta d_k} dV - \int_V \frac{\partial D_p}{\partial E_i} \frac{\delta E_p}{\partial \delta d_k} dV, (k = 1 \ldots NDF) \end{aligned} \quad (28)$$

Outline

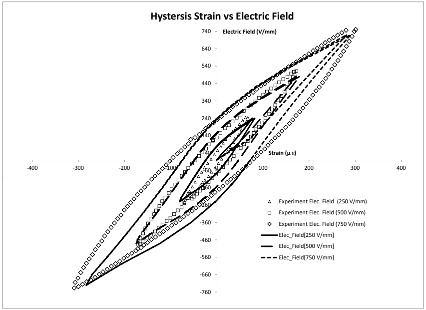
Nonlinear electromechanical coupling response

- The static nonlinear response of PZT-5A results compared with experiments [2]



Nonlinear electromechanical coupling response

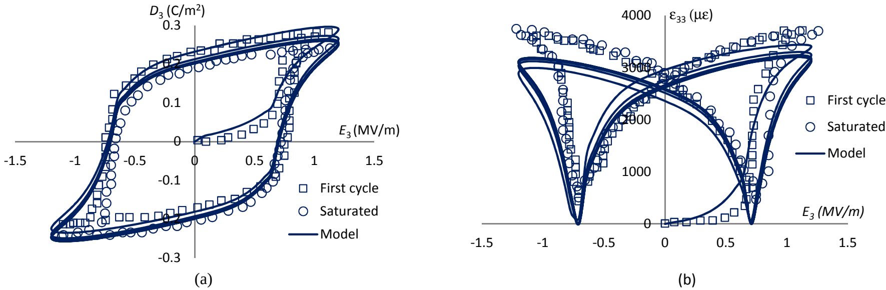
- Nonlinear time dependent strain response under electric field 0.1 [Hz] compared with experiment [2]



Outline

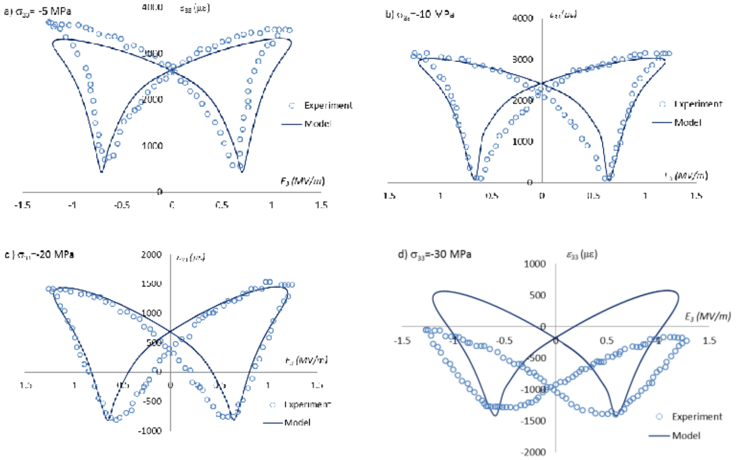
Nonlinear electromechanical coupling response

- Hysteresis polarization (a) and butterfly strain (b) responses for PZT51 at zero stress compared with experiment [3]



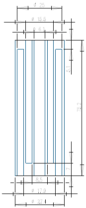
Nonlinear electromechanical coupling response

- Butterfly strain responses under constant compressive stresses for PZT51 compared with experiment [3]

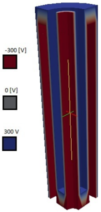


Outline

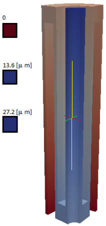
Telescopic Actuator Geometry



Geometry



Electric Potential



Axial Deformation

Patched Beam

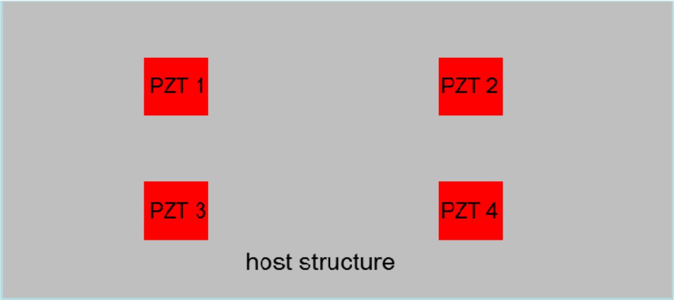


Figure: Geometry of plate with PZT patches

Tip Deflection of Patched Beam

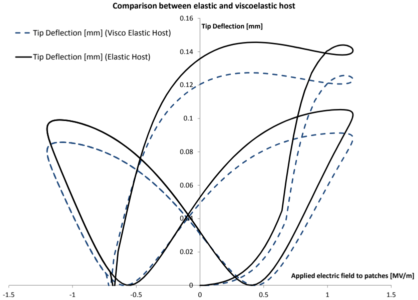
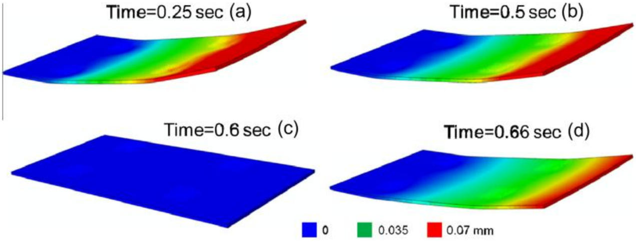


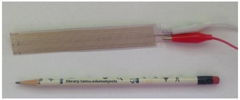
Figure: Tip deflection of uniform cyclic electric fields applied patches

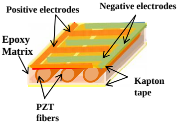
Tip Deflection of Patched Beam

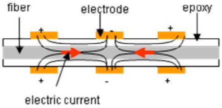


Active Fiber Composite

Active Fiber Composite (AFC) is an actuation architecture that has been introduced to amplify the deflection cased by piezo-electric effect. It is formed by placing PZT fibers in epoxy matrix.







The smallest possible unite cell of AFC is considered in order to simulate the overall performance of AFC.

Active Fiber Composite (AFC) is an actuation architecture that has been introduced to amplify the deflection cased by piezo-electric effect. It is formed by placing PZT fibers in epoxy matrix.

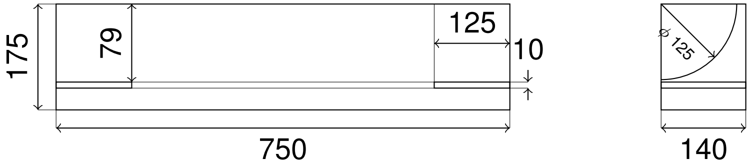
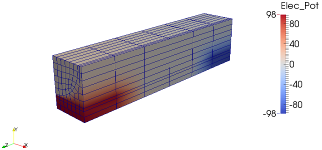
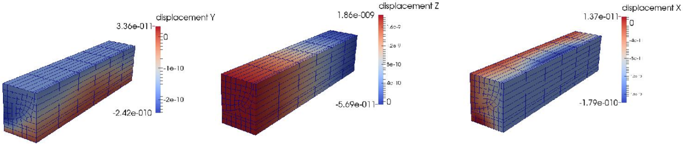


Figure: Geometry of Unit Cell (dimensions are in μm)

Result for AFC Unit Cell

The finite element model for AFC unit cell





Large Strain in Elecro Active Polymers

Electro Active Polymers offering large deformation under large electric fields. The total free energy for the EAPs, Φ, is:

Total Elergy of EAP

$$\Phi = \Phi_m + \Phi_e + \Phi_{em} \tag{29}$$

where Φ_m is the mechanical part and defined as:

Mechnical Part of Energy Φ_m

$$\Phi_m = \frac{\kappa}{2}(J-1) + \frac{\mu}{2}(\overline{\mathbf{C}} : \mathbb{I} - dim) \tag{30}$$

κ, μ, are material constants,
J, $\overline{\mathbf{C}}$ are Jacobian of the deformation gradient tensor and isochoric part of right Cauchy Green strain tensor

Large Strain in Elecro Active Polymers

Electromechanical part is defined with respect to electric feild vector **E** and a constant C_m

ElectroMechnical Part of Energy Φ_{em}

$$\Phi_{em} = C_m(\overline{\mathbf{C}} : \mathbf{E} \otimes \mathbf{E}) \tag{31}$$

Purely Electric Part of Energy Φ_e

$$\Phi_e = C_e(\mathbb{I} : \mathbf{E} \otimes \mathbf{E}) - \epsilon_0 J \mathbf{C}^{-1} : (\mathbf{E} \otimes \mathbf{E}) \tag{32}$$

C_e,ϵ_0 are the electric permitivity of material and permitivity of free space respectively.

Finite Element Model for Electro Active Polymers

The Virtual Work Statement

$$\delta\pi = \int_V P_{ij}\delta F_{ij} dV - \int_V D_i \delta E_i dV - \int_{\partial V} T_i \delta u_i d\partial V - \int_{\partial V} Q\delta\phi d\partial V \quad (33)$$

The 1st Piola Kirchhoff stress and electric displacement

$$P_{ij} = \frac{\partial \Phi}{\partial F_{ij}}; D_i = -\frac{\partial \Phi}{\partial E_i} \quad (34)$$

Finite Element Model for Electro Active Polymers

Gradient of Deformation and Electric Field

$$
\begin{gathered}
F_{ij} = \frac{\partial u_i}{\partial X_j} + \delta_{ij} \\
\delta F_{ij} = \frac{\delta \partial u_i}{\delta \partial X_j} \qquad E_i = \frac{\partial \psi}{\partial X_i}
\end{gathered} \tag{35}
$$

where $\partial\psi$ is the electric potential. The finite element model will be developed based on

$$
R_k = \frac{\partial \delta \pi}{\partial \delta d_k} = 0, (k = 1 \ldots NDF) \tag{36}
$$

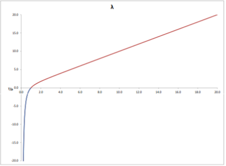


Figure: Mechanical Response

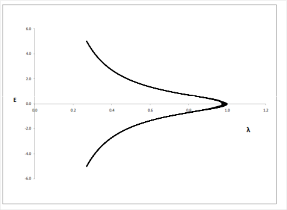


Figure: Electromechanical Response

Conclusion

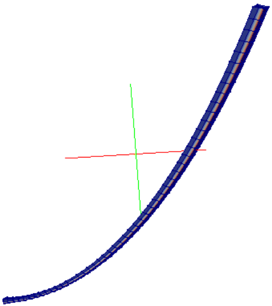


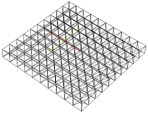
Figure: Bimorph Electro Active Beam

Outline

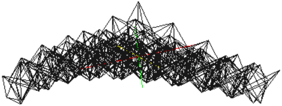
Active Truss Structure

In this part of research we will try to find a way to control shape of an active truss system. This system can be embedded into a matrix and offer desired shape for plate or other type of structures

- Reference Configuration



- Deformed Configuration

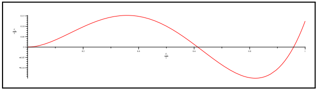


Deformation gradient

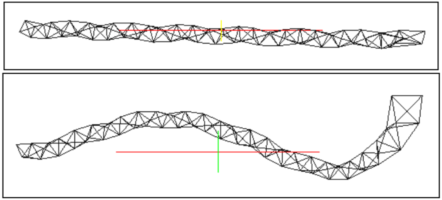
- Refrence configuration is defined as X_i
- Deformed configuration is defined as x_i
- The gradient of deformation will be $F_{ij} = \frac{\partial x_j}{\partial X_j}$
- The strain induced due to this change in configuration is $E_{KL} = \frac{1}{2}\left(\frac{\partial x_j}{\partial X_K}\frac{\partial x_j}{\partial X_L} - \delta_{KL}\right)$
- Components of direction vector of truss element are n_i^{ele}
- The constitutiv equation between two point of the truss $C_{ijkl} = n_i^{ele} n_j^{ele} n_k^{ele} n_l^{ele} E_Y$

Tetrahedral truss

- Desired shape

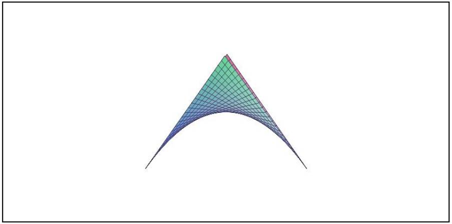


- The applying the shape change stress

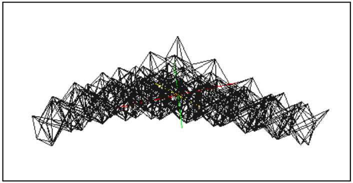


Plane Truss

- Desired shape $z = xy$



- The applying the shape change stress



Conclusion

- The nonlinear time dependent constitutive equation is incorporated into FE model
- The developed FE model is validated by several computational close form solutions and also experimental data
- Performance of developed model is shown by simulating active structures

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Questions ?

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