A comparative study of Transformation Functions for Non-rigid Image Registration

Control points
- → corners
- → region centroids
- → line intersections

Correspondence b/w Control Points:
- → chamfer matching
- → graph matching
- → random sample consensus
- → probabilistic relaxation labeling
- → matching of minimum-spanning-tree edges
- → matching of convex-hull edges
- → Hausdorff distance
- → hierarchical attribute matching
- → clustering
- → template matching
- → Hough transform
- → geometric hashing

→ When noise and inaccuracies in the CP correspondences exist, transformation fns that are based on approximation are preferred over transformation fns. that are based on interpolation.

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Thin-Plate Spline (TPS)	Multiquadric (MQ)	Weighted Mean (WM)	Piecewise Linear (PL)
- Most widely used - Interpolating f^n. i.e. maps corresponding CPs to each other exactly. - Performs well when images have global geometric differences - Performs poorly when the images have local geometric differences.	- Interpolating f^n. - Performs worse (global geometric differences case) than TPS ~~and~~; performs similar to TPS ~~when~~ (local geometric difference case)	- More stable than TPS and MQ. - Approximation based tform. f^n. - Use of rational Gaussians (weights) makes them weight f^n. stretch towards the gaps. - Registration error can be controlled by the standard dev. (σ) of the Gaussians. - ~~SD~~. std. dev. of Gaussians $\propto \dfrac{1}{\text{density of CPs}}$. - Suitable when large no. of CPs (~ 1000) & sharp geometric differences exist; also when the correspondences are noisy	- PL mapping is continuous, but it is not smooth - When the regions are small or local geometric differences between images are small, PL works well

When a large number of control point correspondences is given and the correspondences are noisy, WM is preferred over PL, TPS, and MQ for four main reasons. First, WM does not require the solution of a system of equations. A transformation is immediately obtained from the coordinates of corresponding control points in the images. Second, a transformation is obtained from a weighted average of the control points and the averaging process smoothes noise in the correspondences. Third, the rational weights adapt to the density and organization of the control points by automatically stretching toward large gaps and their widths change with the density of the control points. Fourth, the width of all weight functions can be increased or decreased together by controlling parameter n of the transformation.

Some applications require transformations that are spatially differentiable up to some degree. Among the four transformation functions discussed in this paper, TPS, MQ, and WM are C^{∞} and PL is C^0; that is, all derivative of TPS, MQ, and WM are continuous, while PL is continuous as a function, but its derivatives are all discontinuous across the image domain.

Use of TPS, MQ, WM, and PL transformations in nonrigid registration was explored. Experiments show that among the four transformations, TPS and MQ are least suitable for the registration of images with local geometric differences. This is because of three main factors. First, the basis functions from which a component of a transformation is determined are radially symmetric. When spacing between the control points is very irregular, large errors may be obtained in areas where large gaps exist between the control points. Second, because the radial basis functions used in TPS and MQ are monotonically increasing, the transformation becomes global and cannot adapt well to local geometric differences between images. Third, a system of equations has to be solved to find each component of a transformation, and when spacing between the control points varies greatly the system of equations to be solved may become ill-conditioned. When a small set of widely spread and accurate control points is given and local geometric differences between the images are not large, TPS and MQ are actually preferred over WM and PL because they are interpolating functions and map corresponding control points to each other exactly and also because they extend beyond the convex hull of the control points, registering entire images.

PL interpolation is most suitable when the images to be registered have local geometric differences because a control point affects registration in a small neighborhood of the control point. This property not only makes the computations very efficient, it keeps inaccuracies in the correspondences local without spreading them over the entire image domain. Recent subdivision schemes [38, 50] provide efficient means of fitting piecewise smooth surfaces to triangular meshes, creating continuous and smooth transformation functions.