Modeling Long- and Short-Term Temporal Patterns with Deep Neural Networks

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Abstract

Multivariate time series forecasting is an important machine learning problem across many domains, including predictions of solar plant energy output, electricity consumption, and traffic jam situation. Temporal data arise in these real-world applications often involves a mixture of long-term and short-term patterns, for which traditional approaches such as Autoregressive models and Gaussian Process may fail. In this paper, we proposed a novel deep learning framework, namely Long- and Short-term Time-series network (LSTNet), to address this open challenge. LSTNet uses the Convolution Neural Network (CNN) to extract short-term local dependency patterns among variables, and the Recurrent Neural Network (RNN) to discover long-term patterns and trends. In our evaluation on real-world data with complex mixtures of repetitive patterns, LSTNet achieved significant performance improvements over that of several state-of-the-art baseline methods.

Introduction

Multivariate time series data are ubiquitous in our everyday life ranging from the prices in stock markets, the traffic flows on highways, the outputs of solar power plants, the temperatures across different cities, and more. In such applications, users are often interested in forecasting new trends or potential hazardous events based on historical observations on time series signals.

A major research challenge in multivariate time series forecasting often faces is to capture and leverage the dynamics dependencies among multiple variables. Specifically, real-world applications often entail a mixture of short-term and long-term repeating patterns, as shown in Figure 1 which plots the hourly occupancy rate of a freeway. Apparently, there are two repeating patterns, daily and weekly. The former portraits the morning peaks vs. evening peaks, while the latter reflects the workday and weekend patterns. A successful time series forecasting model should capture both kinds of recurring patterns for accurate predictions. However, traditional approaches such as the large body of work in autoregressive methods (Hamilton 1994; Box et al. 2015; Zhang 2003; Yu, Rao, and Dhillon 2016; Li and Chen 2014) fall short in this aspect, as most of them do not distinguish the two kinds of patterns nor model their interactions explicitly and dynamically. Addressing such limitations of existing methods is the main focus of this paper, for which we propose a novel framework that takes advantages of recent developments in deep learning research.

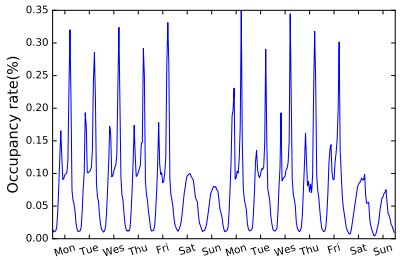


Figure 1: The hourly occupancy rate of a road in the bay area for 2 weeks

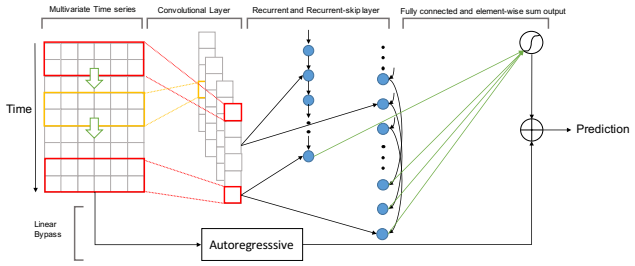


Figure 2: An overview of the Long- and Short-term Time-series network (LSTNet)

Deep neural networks have received an increasing amount of attention in time series analysis. A substantial portion of the previous work has been focusing on *time series classification*, i.e., the task of automated assignment of class labels to time series input. For instance, RNN architectures have been studied for extracting informative patterns from health-care sequential data (Lipton et al. 2015; Che et al. 2016) and classifying the data with respect diagnostic categories. RNN has also been applied to mobile data, for classifying the input sequences with respect to actions or activities (Hammerla, Halloran, and Ploetz 2016). CNN models have also been used in action/activity recognition (Lea et al. 2016; Yang et al. 2015; Hammerla, Halloran, and Ploetz 2016), for

the extraction of shift-invariant local patterns from input sequences as the features of classification models.

Deep neural networks have also been studied for *time series forecasting*, i.e., the task of using observed time series in the past to predict the unknown time series in a look-ahead horizon – the larger the horizon, the harder the problem. Efforts in this direction range from the early work using naive RNN models (Connor, Atlas, and Martin 1991) and the hybrid models (Zhang, Patuwo, and Hu 1998; Zhang 2003; Jain and Kumar 2007) combining the use of ARIMA (Box and Pierce 1970) and Multilayer Perceptron (MLP), to the recent combination of vanilla RNN and Dynamic Boltzmann Machines in time series forecasting (Dasgupta and Osogami 2016).

Although the aforementioned work have shed lights on how to use deep neural networks to improve time series analysis, none of them has offered good answers for the important questions below:

- How can RNN (LSTM and GRU) and CNN be effectively combined for dynamic modeling of short-term and long-term dependency patterns in multi-variate time series data?
- How much can the combined deep learning approach (CNN + RNN) improve the performance of representative autoregressive models?
- Can we further combine the deep learning models (CNN + RNN) and traditional autoregressive models to achieve better performance than that of using each type of the models alone?

Answering the above questions are crucial for improving the state-of-the-art in time series forecasting, and is the main contribution we aim in this paper. Specifically, we propose a novel deep learning framework, namely Long- and Short-term Time-series Network (LSTNet), as illustrated in Figure 2. It leverages the strengths of both the convolutional layer to discover the local dependency patterns among multi-dimensional input variables, and the recurrent layer that captures complex long-term dependencies. A particular recurrent structure, namely Recurrent-skip, is designed for capturing very long-term dependence patterns and making the optimization easier as it utilizes the periodic property of the input time series signals. Finally, the LSTNet incorporates a traditional autoregressive linear model in parallel to the non-linear neural network part; which is similar to a *highway component* (Srivastava, Greff, and Schmidhuber 2015).

The rest of this paper is organized as follows. Section outlines the related background, including representative autoregressive methods and Gaussian Process models. Section describe our proposed LSTNet. Section reports the evaluation results of our model in comparison with strong baselines on real-world datasets. Finally, we conclude our findings in Section .

Related Background

One of the most prominent univariate time series models is the autoregressive integrated moving average (ARIMA) model. The popularity of the ARIMA model is due to its statistical properties as well as the well-known Box-Jenkins methodology (Box et al. 2015) in the model selection procedure. ARIMA models are not only adaptive to various exponential smoothing techniques (McKenzie 1984) but also flexible enough to subsume other types of time series models including autoregression (AR), moving average (MA) and Autoregressive Moving Average (ARMA). However, ARIMA models, including their variants for modeling long-term temporal dependencies (Box et al. 2015), are rarely used in high dimensional multivariate time series forecasting due to their high computational cost.

On the other hand, vector autoregression (VAR) is arguably the most widely used models in multivariate time series (Hamilton 1994; Box et al. 2015; Lütkepohl 2005) due to its simplicity. VAR models naturally extend AR models to the multivariate setting, which ignores the dependencies between output variables. Significant progress has been made in recent years in a variety of VAR models, including the elliptical VAR model (Qiu et al. 2015) for heavy-tail time series and structured VAR model (Melnyk and Banerjee 2016) for better interpretations of the dependencies between high dimensional variables, and more. Nevertheless, the model capacity of VAR grows linearly over the temporal window size and quadratically over the number of variables. This implies, when dealing with long-term temporal patterns, the inherited large model is prone to overfitting. To alleviate this issue, (Yu, Rao, and Dhillon 2016) proposed to reduce the original high dimensional signals into lower dimensional hidden representations, then applied VAR for forecasting with a variety choice of regularization.

Time series forecasting problems can also be treated as standard regression problems with time-varying parameters. It is therefore not surprising that various regression models with different loss functions and regularization terms are applied to time series forecasting tasks. For example, linear support vector regression (SVR) (Kim 2003; Cao and Tay 2003) learns a max margin hyperplane based on the regression loss with a hyper-parameter ϵ controlling the threshold of prediction errors. Ridge regression is yet another example which can be recovered from SVR models by setting ϵ to zeros. Lastly, (Li and Chen 2014) applied LASSO models to encourage sparsity in the model parameters so that interesting patterns among different input signals could be manifest. These linear methods are practically more efficient for multivariate time series forecasting due to high-quality off-the-shelf solvers in the machine learning community. Nonetheless, like VARs, those linear models may fail to capture complex non-linear relationships of multivariate signals, resulting in an inferior performance at the cost of its efficiency.

Gaussian Processes (GP) is a non-parametric method for modeling distributions over a continuous domain of functions. This contrasts with models defined by a parameterized class of functions such as VARs and SVRs. GP can be applied to multivariate time series forecasting task as suggested in (Roberts et al. 2013), and can be used as a prior over the function space in Bayesian inference. For example, (Frigola et al. 2013) presented a fully Bayesian approach with the GP prior for nonlinear state-space models, which is capable of capturing complex dynamical phenomena. How-

ever, the power of Gaussian Process comes with the price of high computation complexity. A straightforward implementation of Gaussian Process for multivariate time-series forecasting has cubic complexity over the number of observations, due to the matrix inversion of the kernel matrix.

Framework

In this section, we first formulate the time series forecasting problem, and then discuss the details of the proposed LSTNet architecture (Figure 2) in the following part. Finally, we introduce the objective function and the optimization strategy.

Problem Formulation

In this paper, we are interested in the task of multivariate time series forecasting. More formally, given a series of fully observed time series signals $\boldsymbol{Y} = \{\boldsymbol{y}_1, \boldsymbol{y}_2, \ldots, \boldsymbol{y}_T\}$ where $\boldsymbol{y}_t \in \mathbb{R}^n$, and n is the variable dimension, we aim at predicting a series of future signals in a rolling forecasting fashion. That being said, to predict $\boldsymbol{y}_{T+h}$ where h is the desirable horizon ahead of the current time stamp, we assume $\{\boldsymbol{y}_1, \boldsymbol{y}_2, \ldots, \boldsymbol{y}_T\}$ are available. Likewise, to predict the value of the next time stamp $\boldsymbol{y}_{T+h+1}$, we assume $\{\boldsymbol{y}_1, \boldsymbol{y}_2, \ldots, \boldsymbol{y}_T, \boldsymbol{y}_{T+1}\}$ are available. We hence formulate the input matrix at time stamp T as $X_T = \{\boldsymbol{y}_1, \boldsymbol{y}_2, \ldots, \boldsymbol{y}_T\} \in \mathbb{R}^{n \times T}$.

In the most of cases, the horizon of the forecasting task is chosen according to the demands of the environmental settings, e.g. for the traffic usage, the horizon of interest ranges from hours to a day; for the stock market data, even seconds/minutes-ahead forecast can be meaningful for generating returns.

Figure 2 presents an overview of the proposed LSTnet architecture. The LSTNet is a deep learning framework specifically designed for multivariate time series forecasting tasks with a mixture of long- and short-term patterns. In following sections, we introduce the building blocks for the LSTNet in detail.

Convolutional Component

The first layer of LSTNet is a convolutional network without pooling, which aims to extract short-term patterns in the time dimension as well as local dependencies between variables. The convolutional layer consists of multiple filters of width ω and height n (the height is set to be the same as the number of variables). The k-th filter sweeps through the input matrix X and produces

$$h_k = RELU(W_k * X + b_k) \tag{1}$$

where $*$ denotes the convolution operation and the output h_k would be a vector, and the $RELU$ function is $RELU(x) = \max(0, x)$. We make each vector h_k of length T by zero-padding on the left of input matrix X. The output matrix of the convolutional layer is of size $d_c \times T$ where d_c denotes the number of filters.

Recurrent Component

The output of the convolutional layer is simultaneously fed into the Recurrent component and Recurrent-skip component (to be described in subsection). The Recurrent component is a recurrent layer with the Gated Recurrent Unit (GRU) (Chung et al. 2014) and uses the $RELU$ function as the hidden update activation function. The hidden state of recurrent units at time t is computed as,

$$\begin{aligned} r_t &= \sigma(x_t W_{xr} + h_{t-1} W_{hr} + b_r) \\ u_t &= \sigma(x_t W_{xu} + h_{t-1} W_{hu} + b_u) \\ c_t &= RELU(x_t W_{xc} + r_t \odot (h_{t-1} W_{hc}) + b_c) \\ h_t &= (1 - u_t) \odot h_{t-1} + u_t \odot c_t \end{aligned} \tag{2}$$

where $\odot$ is the element-wise product, σ is the sigmoid function and x_t is the input of this layer at time t. The output of this layer is the hidden state at each time stamp. While researchers are accustomed to using $\tanh$ function as hidden update activation function, we empirically found $RELU$ leads to more reliable performance, through which the gradient is easier to back propagate.

Recurrent-skip Component

The Recurrent layers with GRU (Chung et al. 2014) and LSTM (Hochreiter and Schmidhuber 1997) unit are carefully designed to memorize the historical information and hence to be aware of relatively long-term dependencies. Due to gradient vanishing, however, GRU and LSTM usually fail to capture very long-term correlation in practice. We propose to alleviate this issue via a novel recurrent-skip component which leverages the periodic pattern in real-world sets. For instance, both the electricity consumption and traffic usage exhibit clear pattern on a daily basis. If we want to predict the electricity consumption at t o'clock for today, a classical trick in the seasonal forecasting model is to leverage the records at t o'clock in historical days, besides the most recent records. This type of dependencies can hardly be captured by off-the-shelf recurrent units due to the extremely long length of one period (24 hours) and the subsequent optimization issues. Inspired by the effectiveness of this trick, we develop a recurrent structure with temporal skip-connections to extend the temporal span of the information flow and hence to ease the optimization process. Specifically, skip-links are added between the current hidden cell and the hidden cells in the same phase in adjacent periods. The updating process can be formulated as,

$$\begin{aligned} r_t &= \sigma(x_t W_{xr} + h_{t-p} W_{hr} + b_r) \\ u_t &= \sigma(x_t W_{xu} + h_{t-p} W_{hu} + b_u) \\ c_t &= RELU(x_t W_{xc} + r_t \odot (h_{t-p} W_{hc}) + b_c) \\ h_t &= (1 - u_t) \odot h_{t-p} + u_t \odot c_t \end{aligned} \tag{3}$$

where the input of this layer is the output of the convolutional layer, and p is the number of hidden cells skipped through. The value of p can be easily determined for datasets with clear periodic patterns (e.g. $p = 24$ for the hourly electricity consumption and traffic usage datasets), and has to be

tuned otherwise. In our experiments, we empirically found that a well-tuned p can considerably boost the model performance even for the latter case. Furthermore, the LSTNet could be easily extended to contain variants of the skip length p.

We use a dense layer to combine the outputs of the Recurrent and Recurrent-skip components. The inputs to the dense layer include the hidden state of Recurrent component at time stamp t, denoted by h_t^R, and p hidden states of Recurrent-skip component from time stamp $t-p+1$ to t denoted by $h_{t-p+1}^S, h_{t-p+2}^S \ldots, h_t^S$. The output of the dense layer is computed as,

$$h_t^D = W^R h_t^R + \sum_{i=0}^{p-1} W_i^S h_{t-i}^S + b \tag{4}$$

where h_t^D is the prediction result of the neural network (upper) part in the Fig.2 at time stamp t.

Autoregressive Component

Due to the non-linear nature of the Convolutional and Recurrent components, one major drawback of the neural network model is that the scale of outputs is not sensitive to the scale of inputs. Unfortunately, in specific real datasets, the scale of input signals constantly changes in a non-periodic manner, which significantly lowers the forecasting accuracy of the neural network model. A concrete example of this failure is given in Section . To address this deficiency, similar in spirit to the highway network (Srivastava, Greff, and Schmidhuber 2015), we decompose the final prediction of LSTNet into a linear part, which primarily focuses on the local scaling issue, plus a non-linear part containing recurring patterns. In the LSTNet architecture, we adopt the classical Autoregressive (AR) model as the linear component. Denote the forecasting result of the AR component as $h_t^L \in \mathbb{R}^n$, and the coefficients of the AR model as $W^{ar} \in \mathbb{R}^{q^{ar}}$ and $b^{ar} \in \mathbb{R}$, where q^{ar} is the size of input window over the input matrix. Note that in our model, all dimensions share the same set of linear parameters. The AR model is formulated as follows,

$$h_{t,i}^L = \sum_{k=0}^{q^{ar}-1} W_k^{ar} \boldsymbol{y}_{t-k,i} + b^{ar} \tag{5}$$

The final prediction of LSTNet is then obtained by by integrating the outputs of the neural network part and the AR component:

$$\hat{\boldsymbol{Y}}_t = h_t^D + h_t^L \tag{6}$$

where $\hat{\boldsymbol{Y}}_t$ denotes the model's final prediction at time stamp t.

Objective function

The squared error is the default loss function for many forecasting tasks, the corresponding optimization objective is formulated as,

$$\underset{\Theta}{\text{minimize}} \quad \sum_{t \in \Omega_{Train}} ||\boldsymbol{Y}_t - \hat{\boldsymbol{Y}}_{t-h}||_F^2 \tag{7}$$

where Θ denotes the parameter set of our model, Ω_{Train} is the set of time stamps used for training, $||\cdot||_F$ is the Frobenius norm, and h is the horizon as mentioned in Section . The traditional linear regression model with the square loss function is named as Linear Ridge, which is equivalent to the vector autoregressive model with ridge regularization. However, experiments show that the Linear Support Vector Regression (Linear SVR) (Vapnik et al. 1997) dominates the Linear Ridge model in certain datasets. The only difference between Linear SVR and Linear Ridge is the objective function. The objective function for Linear SVR is,

$$\begin{aligned} \underset{\Theta}{\text{minimize}} \quad & \frac{1}{2}||\Theta||_F^2 + C \sum_{t \in \Omega_{Train}} \sum_{i=0}^{n-1} \xi_{t,i} \\ \text{subject to} \quad & |\hat{\boldsymbol{Y}}_{t-h,i} - \boldsymbol{Y}_{t,i}| \leq \xi_{t,i} + \epsilon, t \in \Omega_{Train} \\ & \xi_{t,i} \geq 0 \end{aligned} \tag{8}$$

where C and ϵ are hyper-parameters. Motivated by the remarkable performance of the Linear SVR model, we incorporate its objective function in the LSTNet model as an alternative of the squared loss. For simplicity, we assume $\epsilon = 0$[1], and the objective function above reduces to absolute loss (L1-loss) function as follows:

$$\underset{\Theta}{\text{minimize}} \quad \sum_{t \in \Omega_{Train}} \sum_{i=0}^{n-1} |\boldsymbol{Y}_{t,i} - \hat{\boldsymbol{Y}}_{t-h,i}| \tag{9}$$

In the experiment section, we carefully examine the effectiveness of both objective functions defined in Equation 7 and Equation 9.

Optimization Strategy

Our optimization strategy is the same as that in the traditional time series forecasting model. Supposing the input time series is $\boldsymbol{Y}_t = \{\boldsymbol{y}_1, \boldsymbol{y}_2, \ldots, \boldsymbol{y}_t\}$, we define a tunable window size q, and reformulate the input at time stamp t as $\boldsymbol{X}_t = \{\boldsymbol{y}_{t-q+1}, \boldsymbol{y}_{t-q+2}, \ldots, \boldsymbol{y}_t\}$. The problem then becomes a regression task with a set of feature-value pairs $\{\boldsymbol{X}_t, \boldsymbol{Y}_{t+h}\}$, and can be solved by Stochastic Gradient Decent (SGD) or its variants such as Adam (Kingma and Ba 2014).

Evaluation

We conducted extensive experiments with 9 methods (including our new methods) on 4 benchmark datasets for time series forecasting tasks.

Methods for Comparison

The methods in our comparative evaluation are the follows.

- AR stands for the autoregressive model, which is equivalent to the one dimensional VAR model.
- LRidge is the vector autoregression (VAR) model with L2-regularization

[1] One could keep ϵ to make the objective function more faithful to the Linear SVR model without modifying the optimization strategy. We leave this for future study.

- LSVR is the vector autoregression (VAR) model with Support Vector Regression loss (Vapnik et al. 1997) .
- GP is the Gaussian Process for time series modeling. (Frigola-Alcade 2015; Roberts et al. 2013)
- VAR-MLP combines Multilayer Perception (MLP) and autoregressive model (Zhang 2003).
- RNN is the Recurrent Neural Network model using GRU cell.
- LST-L1 is our proposed LSTNet model with the absolute loss objective function.
- LST-L2 is our proposed LSTNet model with the square loss objective function.

For the single output methods above such as AR, LRidge, LSVR and GP, we just trained n models independently, i.e., one model for each of the n output variables.

Metrics

We consider three evaluation metrics defined as:

- Root Relative Squared Error (RSE):

$$RSE = \frac{\sqrt{\sum_{(i,t)\in\Omega_{Test}}(Y_{it} - \hat{Y}_{it})^2}}{\sqrt{\sum_{(i,t)\in\Omega_{Test}}(Y_{it} - \mu(\boldsymbol{Y}))^2}}$$

- Empirical Correlation Coefficient (CORR)

$$CORR = \frac{1}{n}\sum_{i=1}^{n}\frac{\sum_t \left(Y_{it} - \mu(\boldsymbol{Y}_i)\right)\left(\hat{Y}_{it} - \mu(\hat{\boldsymbol{Y}}_i)\right)}{\sqrt{\sum_t \left(Y_{it} - \mu(\boldsymbol{Y}_i)\right)^2\left(\hat{Y}_{it} - \mu(\hat{\boldsymbol{Y}}_i)\right)^2}}$$

where $\mu(\cdot)$ is the mean function and $\boldsymbol{Y}, \hat{\boldsymbol{Y}} \in \mathbb{R}^{n\times T}$ are ground true signals and system prediction signals, respectively. RSE, the scaled version of Root Mean Square Error(RMSE), is designed as more interpretable evaluations across different datasets. RSE is lower value the better, while CORR higher value the better.

Data

We used four benchmark datasets which are publicly available. We denote time length T, number of variables D, and frequency between two samples F.

- Traffic[2]: A collection of 48 months (2015-2016) hourly data from the California Department of Transportation. The data describes the road occupancy rates (between 0 and 1) measured by different sensors on San Francisco Bay area freeways. $T = 17544$, $D = 682$, F is 1 hour.
- Solar-Energy[3] : the solar power production records in the year of 2006, which is sampled every 10 minutes from 137 PV plants in Alabama State. $T = 52560$, $D = 137$, F is 10 minutes.
- Electricity[4]: The electricity consumption in kWh was recorded every 15 minutes from 2012 to 2014, for $n = 321$ clients. We converted the data to reflect hourly consumption. $T = 26304$, $D = 321$, F is 1 hour.
- Exchange-Rate: the collection of the daily exchange rates of eight foreign countries including Australia, British, Canada, Switzerland, China, Japan, New Zealand and Singapore ranging from 1990 to 2016. $T = 7588$, $D = 8$, F is 1 day.

All datasets are split into training (60%), validation (20%) and test (20%) set in chronological order. To facilitate future research in multivariate time series forecasting, we publicize all raw datasets and the one after preprocessing in the website [5].

To examine the existence of long-term and/or short-term repetitive patterns in time series data, we plot autocorrelation graph for some randomly selected variables from the four datasets in Figure 3. Autocorrelation is the correlation of a signal with a delayed copy of itself as a function of delay, defined as $R(\tau) = \mathbb{E}[(X_t - \mu)(X_{t+\tau} - \mu)]/\sigma^2$ where X_t is the time series signals, μ is mean and σ^2 is variance.

In Figure 3, there are repetitive patterns with high autocorrelation in the Traffic, Solar-Energy and Electricity datasets, but not in the Exchange-Rate dataset. Furthermore, we observe a short-term daily pattern (in every 24 hours) and long-term weekly pattern (in every 7 days) in the Traffic and Electricity dataset, reflecting the expected regularity in highway traffics and electricity consumptions. On the other hand, we hardly see any repetitive long-term patterns in the Exchange-Rate dataset, expect some short-term local continuity. These observations suggest for methods capable of leveraging both short-term and long-term repetitive patterns should outperforms on Electricity, Traffic and Solar-Energy dataset. In contrast, if the dataset does not contain such patterns (like in Exchange-Rate), the advantageous power of those methods may not lead a better performance.

Experimental Details

We conduct grid search over all tunable hyper-parameters on the validation set for each method and dataset. Specifically, all methods share the same grid search range of the window size q ranging from $\{2^0, 2^1, \ldots, 2^9\}$ if applied. For LRidge and LSVR, the regularization coefficient λ is chosen from $\{2^{-10}, 2^{-8}, \ldots, 2^8, 2^{10}\}$. For GP, the RBF kernel bandwidth σ and the noise level α are chosen from $\{2^{-10}, 2^{-8}, \ldots, 2^8, 2^{10}\}$. For TRMF, the hidden dimension is chosen from $\{2^2, \ldots, 2^6\}$ and the regularization coefficient λ is chosen from $\{0.1, 1, 10\}$. For LSTNet, we adopted the hidden dimension of the Recurrent and Convolutional layer is chosen from $\{50, 100, 200\}$, and $\{20, 50, 100\}$ for Recurrent-skip layer. The skip-length p of Recurrent-skip layer is set as 24 for the Traffic and Electricity dataset,

[2] http://pems.dot.ca.gov

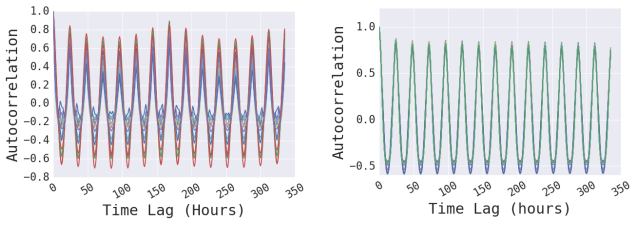
[3] http://www.nrel.gov/grid/solar-power-data.html

[4] https://archive.ics.uci.edu/ml/datasets/ElectricityLoadDiagrams20112014

[5] https://github.com/laiguokun/multivariate-time-series-data

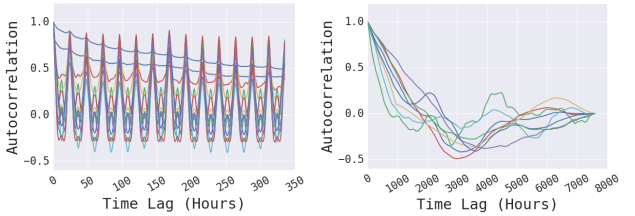
Dataset		Solar-Energy				Traffic				Electricity				Exchange-Rate			
		Horizon				Horizon				Horizon				Horizon			
Methods	Metrics	3	6	12	24	3	6	12	24	3	6	12	24	3	6	12	24
AR	RSE	0.2435	0.3790	0.5911	0.8699	0.5991	0.6218	0.6252	0.6293	0.0995	0.1035	0.1050	0.1054	0.0228	0.0279	0.0353	0.0445
(0)	CORR	0.9710	0.9263	0.8107	0.5314	0.7752	0.7568	0.7544	0.7519	0.8845	0.8632	0.8591	0.8595	0.9734	0.9656	0.9526	0.9357
LRidge	RSE	0.2019	0.2954	0.4832	0.7287	0.5833	0.5920	0.6148	0.6025	0.1467	0.1419	0.2129	0.1280	0.0184	0.0274	0.0419	0.0675
(0)	CORR	0.9807	0.9568	0.8765	0.6803	0.8038	0.8051	0.7879	0.7862	0.8890	0.8594	0.8003	0.8806	0.9784	0.9702	0.9543	0.9305
LSVR	RSE	0.2021	0.2999	0.4846	0.7300	0.5740	0.6580	0.7714	0.5909	0.1523	0.1372	0.1333	0.1180	0.0189	0.0284	0.0425	0.0662
(0)	CORR	0.9807	0.9562	0.8764	0.6789	0.7993	0.7267	0.6711	0.7850	0.8888	0.8861	0.8961	0.8891	0.9782	0.9697	0.9546	0.9370
GP	RSE	0.2259	0.3286	0.5200	0.7973	0.6082	0.6772	0.6406	0.5995	0.1500	0.1907	0.1621	0.1273	0.0239	0.0272	0.0394	0.0580
(0)	CORR	0.9751	0.9448	0.8518	0.5971	0.7831	0.7406	0.7671	0.7909	0.8670	0.8334	0.8394	0.8818	0.8713	0.8193	0.8484	0.8278
VARMLP	RSE	0.1922	0.2679	0.4244	0.6841	0.5582	0.6579	0.6023	0.6146	0.1393	0.1620	0.1557	0.1274	0.0265	0.0304	0.0407	0.0578
(0)	CORR	0.9829	0.9655	0.9058	0.7149	0.8245	0.7695	0.7929	0.7891	0.8708	0.8389	0.8192	0.8679	0.8609	0.8725	0.8280	0.7675
RNN-GRU	RSE	0.1932	0.2628	0.4163	0.4852	0.5358	0.5522	0.5562	0.5633	0.1102	0.1144	0.1183	0.1295	0.0192	0.0264	0.0408	0.0626
(0)	CORR	0.9823	0.9675	0.9150	0.9823	0.8511	0.8405	0.8345	0.8300	0.8597	0.8625	0.8472	0.8651	0.9786	0.9712	0.9531	0.9223
LSTNet-Skip	RSE	0.1843	0.2559	0.3254	0.4636	0.4849	0.4995	0.5110	0.5221	0.0864	0.0931	0.1007	0.1060	0.0232	0.0302	0.0382	0.0570
(0)	CORR	0.9843	0.9690	0.9467	0.8870	0.8721	0.8645	0.8576	0.8552	0.9283	0.9135	0.9077	0.9119	0.9746	0.9670	0.9517	0.9314

Table 1: Results summary: 1) bold face indicates the best result of each column in a particular metric; and 2) the total number of bold-faced results of each method is listed under the method name within parentheses.



(a) Traffic dataset

(b) Solar-Energy dataset



(c) Electricity dataset

(d) Exchange-Rate dataset

Figure 3: Autocorrelation graphs of some sampled variables.

and tuned range from 2^1 to 2^6 for the Solar-Energy and Exchange-Rate datasets. The regularization coefficient of the AR component is chosen from $\{0.1, 1, 10\}$ to achieve the best performance. The Adam(Kingma and Ba 2014) algorithm is utilized to optimize the parameters of our model.

Main Results

Table 1 summarizes the evaluation results. We set $horizon = \{3, 6, 12, 24\}$, which means the horizons was set from 3 to 24 hours for the forecasting over the Electricity and Traffic data, from 30 to 240 minutes over the Solar-Energy data, and from 3 to 24 days over the Exchange-Rate data. The larger the horizons, the harder the prediction tasks. The best result for each (data, metric) pair is highlighted in bold face in this table. The total count of the bold-faced results is 28 for LST-L1 (one version of the proposed LSTNet), 10 for LST-L2 (the other version of our LSTNet), 5 for AR, 4 for LRidge, and between 0 to 2 for the rest of the methods.

Clearly, LSTNet has the dominating performance on the first three datasets (Electricity, Solar-Energy and Traffic), especially in the settings with the large horizon. In the $horizon = 24$ settings, LSTNet model improves the best state-of-the-art result by 35% in Solar-Energy, 20% in Traffic and 5% in Electricity dataset in RSE evaluation metric. But it is slightly worse than AR and LRidge on the Exchange-Rate dataset. Why? Recall that in Section and Figure 3 we used the autocorrelation curves of these datasets to show the existence of repetitive patterns in the Solar-Energy, Traffic and Electricity datasets but not in Exchange-Rate. The current results provide empirical evidence for the success of LSTNet models in modeling long-term and short-term dependency patterns when they do occur in data. Otherwise, LSTNet performed comparably with the better ones (AR and LRidge) among the representative baselines. The robustness of LSTNet is also due to its inclusion of the autoregressive model as a component, which we will discuss further in next section.

Compared the results of univariate AR with that of the multivariate baseline methods, we see that in some datasets, i.e. Solar-Energy and Traffic, the multivariate is stronger, but weaker otherwise, which means that the richer input information would causes overfitting in the traditional multivariate approaches. In contrast, the LSTNet has robust performance in different situations, partly due to its autoregressive component, which we will discuss further in Section .

Ablation Study

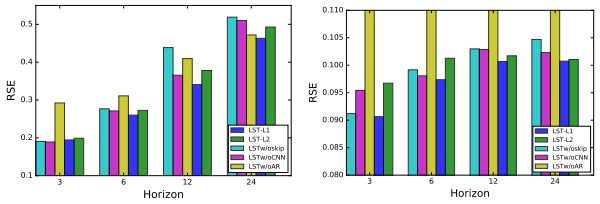
To examine the importance of each component in our framework, we conducted a set of ablation tests. We denote **LSTw/oskip**, **LSTw/oCNN**, **LSTw/oAR** as the LSTNet models without the Recurrent-skip, the Convolution, the AR component, respectively.

For different baselines, we tune the hidden dimension of models such that they have the similar numbers of model parameters to the LSTNet model, which eliminates the per-

formance enhance introduced by the number of the model parameters.

The results are shown in Figure 4 [6]. We observe that removing the Skip and CNN components in (**LSTw/oCNN** or **LSTw/oskip**) caused certain performance drops. All the components of LSTNet together leads to the robust performance of our approach on all the datasets. Also note that removing the AR component (in LSTw/oAR) caused the most significant performance drops on Electricity, but not on Solar-Energy dataset.

The conclusion is that our architecture design is most robust across all experiment settings, especially with the large horizons.



(a) Solar-Energy dataset (b) Electricity dataset

Figure 4: Results of LSTNet in the ablation tests.

Mixture of long- and short-term patterns

To illustrate the success of LSTNet in modeling the mixture of short-term and long-term recurring patterns, Figure 5 compares the performance of LSTNet and VAR on an specific time series (one of the output variables) in the Traffic dataset. Note that the true signal (in blue) of traffic occupancy are very different on Fridays and Saturdays, and another on Sunday and Monday.

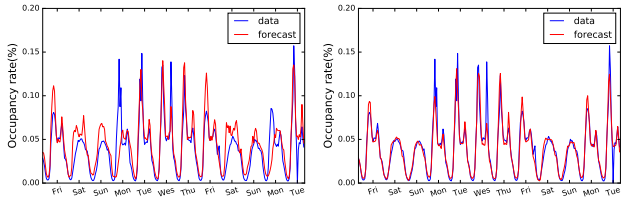
The figure shows that the VAR model is only capable to deal with the short-term patterns. The pattern of prediction results of the VAR model only depend on the day before the predictions. We can clearly see that the results of it in Saturday (2rd and 9th peaks) and Monday (4th and 11th peaks) is different from the ground truth, where the ground truth of Monday (weekday) has two peaks, one peak for Saturday (weekend). In the contrary, our proposed LSTNet model performs two patterns for weekdays and weekends respectfully. This example proves the ability of LSTNet model to memorize short-term and long-term recurring patterns simultaneously, which the traditional forecasting model is not equipped, and it is crucial in the prediction task of the real world time series signals.

Conclusion

In this paper, we presented a novel deep learning framework (LSTNet) for the task of multivariate time series forecasting. By combining the strengths of convolutional and recurrent neural networks and an autoregressive component, the proposed approach significantly improved the state-of-the-art results in time series forecasting on multiple benchmark datasets. With in-depth analysis and empirical evidence we show that LSTNet indeed successfully captures both short-term and long-term repeating patterns in data, and combines both linear and non-linear models for robust prediction.

For future research, there are several promising directions in extending the work. Firstly, the skip length p of the skip-recurrent layer is a crucial hyper-parameter. Currently, we manually tune it based on the validation dataset. How to automatically choose p according to data is an interesting problem. Secondly, in the convolution layer we treat each variable dimension equally, but in the real world dataset, we usually have rich attribute information. Integrating them into the LSTNet model is another challenging problem.

[6] We omit the results on Traffic and Exchange-Rate dataset due to the space limits as it shows similar trends in Figure 4



(a) VAR (b) LSTNet

Figure 5: The true time series (blue) and the predicted ones (red) for one variable in the Traffic dataset. The x axis indicates the week days and the forecasting $horizon = 24$.

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