Camera Geometry: Slide-by-Slide Narration and Notes

Comprehensive Narrative with Technical Trade-offs and SO(3)

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December 21, 2025

Slide 1: Title Page

Welcome to this lecture on Camera Geometry. [cite_start] We will be exploring the mathematical bridge between the 3D world and 2D image measurements. Our goal today is to understand how physical objects in space are projected onto a digital sensor through a series of geometric transformations. [cite_end]

Slide 3: Extrinsic Camera Parameters

Extrinsics define the camera's "pose." As seen in the figure, the transformation $\mathbf{X}_c = \mathbf{R}\mathbf{X}_w + \mathbf{t}$ re-centers the world around the camera's lens.

- **Advantages:** This modularity allows us to move the camera freely in the world without changing its internal calibration.
- **Disadvantages:** Small errors in $\mathbf{R}$ (rotation) lead to massive pixel shifts as the distance from the camera increases.

Slide 5: Why Camera Geometry?

Geometry is foundational because it separates position from appearance.

- **Advantages:** Geometry is invariant to lighting, color, and texture changes. It allows for millimeter-level precision.
- **Disadvantages:** Geometry alone cannot identify *what* an object is—it only tells us *where* it is. It requires clear edges or features to solve for.

Slide 9: Defining $SO(3)$

The rotation matrix must belong to $SO(3)$ (Special Orthogonal group).

- **Advantages:** Guarantees a "rigid" transformation where distances and angles are preserved.
- **Disadvantages:** Statically enforcing these 6 constraints on 9 matrix elements makes optimization (like bundle adjustment) more complex than using unconstrained vectors.

Slide 10: Rotation Representations

- **Euler Angles:**
  - *Advantage:* Highly intuitive for humans (Yaw, Pitch, Roll).
  - *Disadvantage:* Suffers from *Gimbal Lock* (loss of a degree of freedom).
- **Quaternions:**
  - *Advantage:* No gimbal lock; very efficient for smooth interpolation between rotations.
  - *Disadvantage:* Mathematically abstract and difficult to visualize.

Slide 12: Homogeneous Coordinates

On this slide, we define Homogeneous Coordinates. Essentially, we are embedding our n-dimensional Euclidean points into an $(n+1)$-dimensional projective space.

Advantages

- **Linearization of Projection:** The primary advantage is that the non-linear "perspective division" by Z is deferred. We can treat the entire imaging pipeline as a sequence of linear matrix multiplications.
- **Composability:** We can multiply several matrices (Rotation, Translation, Projection) into a single 3×4 matrix before applying it to any points, which is computationally efficient.
- **Handling Infinity:** It allows us to mathematically model vanishing points. In Euclidean math, parallel lines never meet; in projective space, they meet at a point where the last coordinate w is zero.

Disadvantages

- **Redundancy:** Because $(x, y, 1)$ and $(2x, 2y, 2)$ represent the same physical point, the representation is not unique. This "scale ambiguity" must be handled during optimization.
- **Coordinate Overhead:** Adding an extra dimension increases the memory footprint and the number of floating-point operations slightly, though the benefits of linearization usually outweigh this cost.

Slide 12: Why Homogeneous Coordinates?

Homogeneous coordinates add a scale factor w to our vectors.

- **Advantages:** Converts non-linear perspective division into linear matrix multiplication. It can represent points at infinity (vanishing points).
- **Disadvantages:** Increases the dimensionality of the data, which can be memory-intensive in massive point-cloud computations.

Slide 16: Focal Length Interpretation

- **Short Focal Length (Wide Angle):**
  - *Advantage:* Captures a large scene; great for navigation.
  - *Disadvantage:* Significant radial distortion at the edges.
- **Long Focal Length (Telephoto):**
  - *Advantage:* High resolution on distant objects.
  - *Disadvantage:* Very sensitive to camera shake and vibration.

Slide 19: Lens Distortion Models

We use polynomial models to correct for Barrel and Pincushion effects.

- **Advantages:** Allows us to use cheap, wide-angle lenses and still get mathematically "straight" lines.
- **Disadvantages:** Undistorting an image requires resampling pixels, which can introduce interpolation artifacts or blurriness.

Slide 20: Camera Calibration Overview

- **Advantages:** Zhang's method is low-cost; you only need a printed checkerboard and a few photos.
- **Disadvantages:** Quality is highly dependent on the user capturing a wide variety of angles and covering the corners of the image sensor.

Slide 25: Big Picture

We conclude with the realization that camera geometry is where physics and math intersect. By mastering these trade-offs, you can design vision systems that aren't just "guessing" based on pixels, but are calculating the 3D truth of the world.