Image Formation & Geometric Camera Models

Adapted from: Computer Vision Lecture 2.2 (video)
and Geometric Image Formation notes

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Learning Objectives

- Understand the pinhole camera model and perspective projection.
- Represent points and transformations using homogeneous coordinates.
- Build the camera projection matrix and separate intrinsics/extrinsics.
- Recognize common lens distortions and basics of calibration.

Why geometric image formation?

- Relates 3D scene geometry to 2D images.
- Foundation for stereo, structure-from-motion, pose estimation, calibration.
- Mathematical models let us invert or reason about scene geometry.

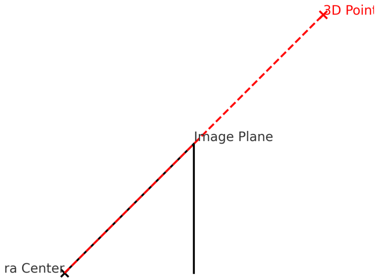
Pinhole camera model

Intuition: A pinhole camera projects 3D points onto an image plane through a single center of projection.

$$\text{3D point } \mathbf{X} = (X, Y, Z)^\top$$

$$\text{Image point } (x, y) = \left(\frac{fX}{Z}, \frac{fY}{Z}\right)$$

where f is the focal length.



Pinhole geometry (replace with own figure).

Homogeneous coordinates

- Represent Euclidean 2D/3D points with extra coordinate for linear transformations.
- 3D point: $\tilde{\mathbf{X}} = (X, Y, Z, 1)^\top$.
- 2D image (homogeneous): $\tilde{\mathbf{x}} = (x, y, 1)^\top$.
- Perspective projection becomes linear in homogeneous coordinates:

$$s\tilde{\mathbf{x}} = \mathbf{P}\,\tilde{\mathbf{X}}$$

where s is scale and $\mathbf{P}$ is the 3×4 projection matrix.

Camera projection matrix

- Full camera matrix: $\mathbf{P} = \mathbf{K}[\mathbf{R} \mid \mathbf{t}]$
- $\mathbf{K}$ (intrinsics) and $[\mathbf{R} \mid \mathbf{t}]$ (extrinsics):

$$\mathbf{K} = \begin{bmatrix} f_x & s & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix}$$

where f_x, f_y are focal lengths (pixels), s is skew, and c_x, c_y is principal point.

Interpretation of extrinsics

- World-to-camera: $\mathbf{X}_c = \mathbf{R}\mathbf{X}_w + \mathbf{t}$.
- $\mathbf{R}$ is a 3×3 rotation, $\mathbf{t}$ is translation — together they place the camera in the world.
- Combined into projection: $\tilde{\mathbf{x}} \sim \mathbf{K}[\mathbf{R} \mid \mathbf{t}]\tilde{\mathbf{X}}_w$.

Other camera approximations

- **Orthographic / scaled orthographic:** approximates perspective when depth variation is small.
- **Affine camera:** linear mapping, drops division by Z.
- **Omnidirectional / generalized cameras:** spherical, catadioptric or multi-view sensors.

Lens distortion (radial & tangential)

- Real lenses introduce radial distortion r (barrel/pincushion) and tangential terms due to misalignment.
- Simple radial correction (2–3 coefficients):

$$x_{\text{dist}} = x(1 + k_1 r^2 + k_2 r^4 + \dots)$$

where $r^2 = x^2 + y^2$ in normalized image coordinates.

Camera calibration

- Estimate **K**, **R**, **t** (and distortion) from known 3D–2D correspondences.
- Common toolboxes: Bouguet's Camera Calibration Toolbox, OpenCV calibrateCamera.
- Calibration yields parameters used to undistort images and recover metric geometry.

Example: projecting a 3D point

Given $\mathbf{X} = (X, Y, Z)^\top$, compute image point:

$$s \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & s & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{R} \mid \mathbf{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Solve for $u = \frac{s_u}{s_w},\ v = \frac{s_v}{s_w}$.

Takeaways

- The pinhole model and homogeneous coordinates give a compact, linear framework for projection.
- Intrinsics vs extrinsics: intrinsics describe the camera sensor/lens, extrinsics describe camera pose.
- Calibration and distortion modeling are essential for accurate geometric tasks.

References

- Lecture video: "Computer Vision - Lecture 2.2 (Image Formation: Geometric ...)" (YouTube).
- Lecture notes: Geometric Image Formation (example course notes).
- R. Szeliski, *Computer Vision: Algorithms and Applications* and Hartley & Zisserman, *Multiple View Geometry*.

Appendix: Useful formulas

- $\mathbf{P} = \mathbf{K}[\mathbf{R} \mid \mathbf{t}]$
- Projection (homogeneous): $s\tilde{\mathbf{x}} = \mathbf{P}\tilde{\mathbf{X}}$
- Intrinsics matrix $\mathbf{K}$ shown earlier.