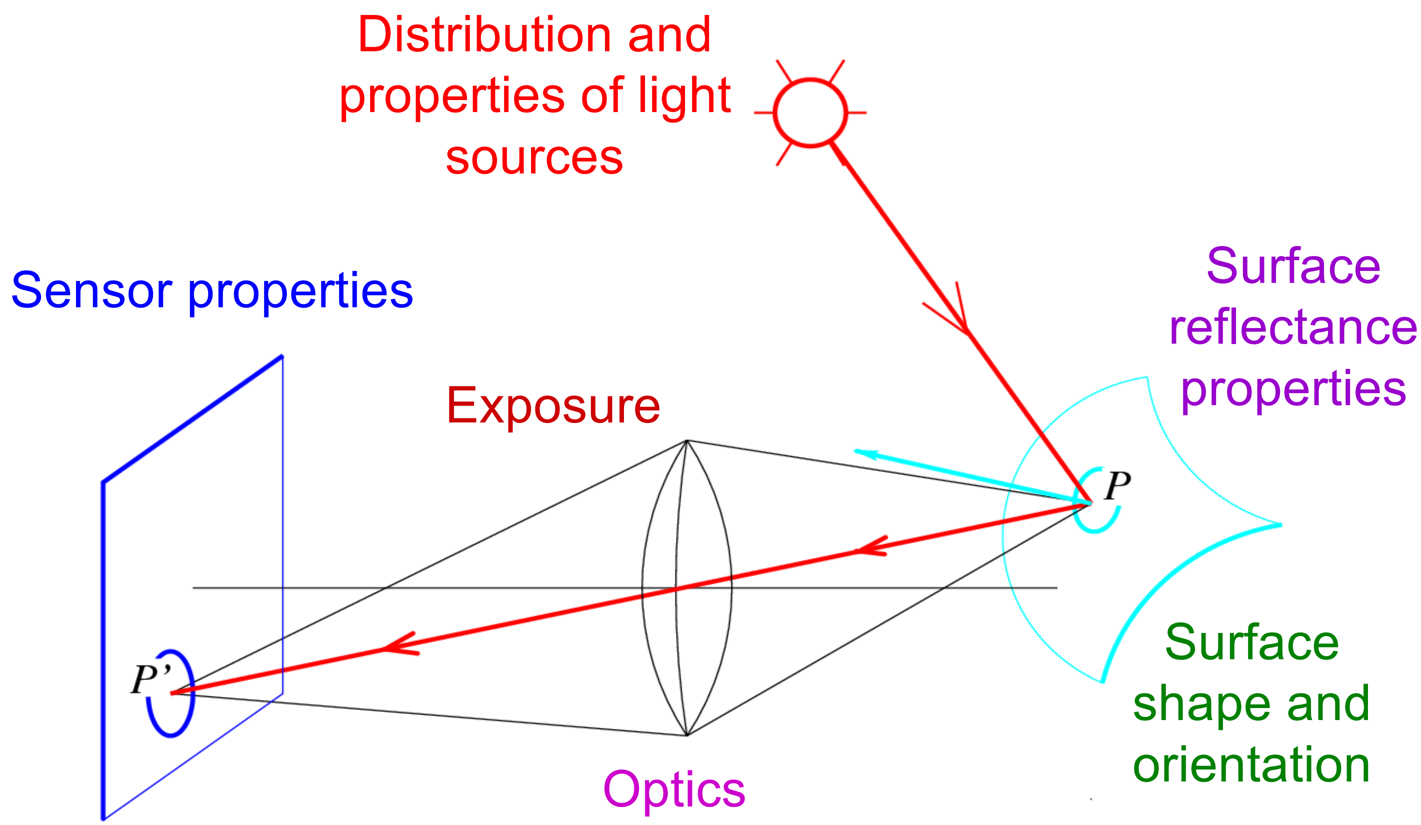
# Light and shading

P. Claesz, Still Life with a Skull and a Writing Quill, 1628

# Image formation

- What determines the *brightness* of an image pixel?

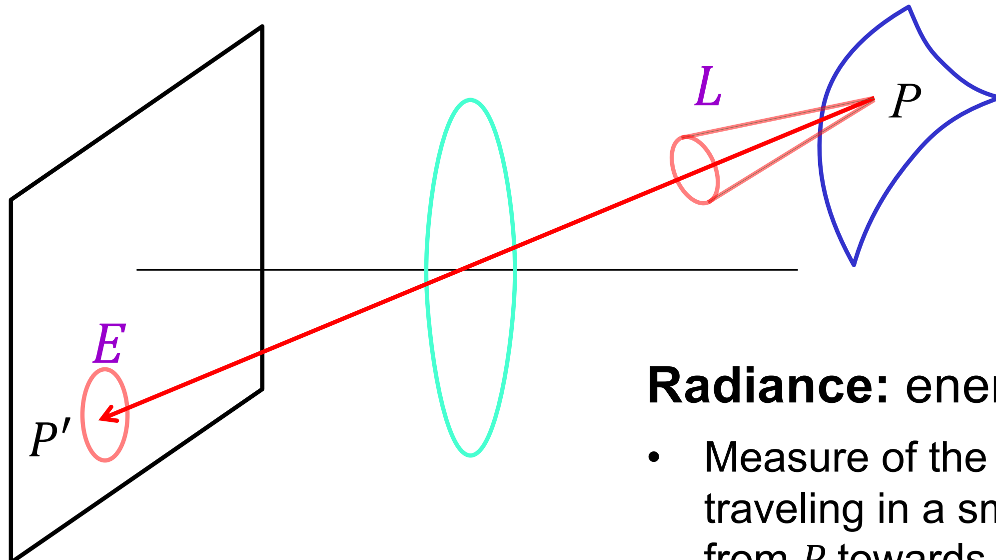



Slide by L. Fei-Fei

# Outline

- Small taste of radiometry
- In-camera transformation of light
- Reflectance properties of surfaces
- Diffuse and specular reflection
- Shape from shading
- Estimating direction of light sources

# Radiometry of image formation




**Radiance:** energy carried by a ray

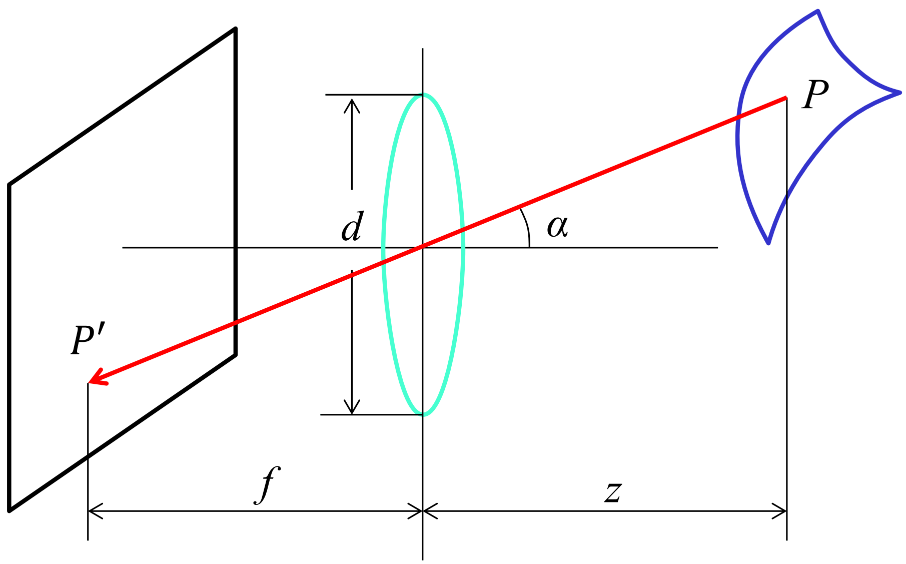
- Measure of the density of photons traveling in a small cone of directions from $P$ towards $P'$
- Power per unit area perpendicular to the direction of travel, per unit solid angle
- Units: Watts per square meter per steradian

**Irradiance:** energy arriving at a surface

- Incident power per unit area (not foreshortened)
- Units: Watts per square meter

What is the relationship between $E$ and $L$?

# Fundamental radiometric relation

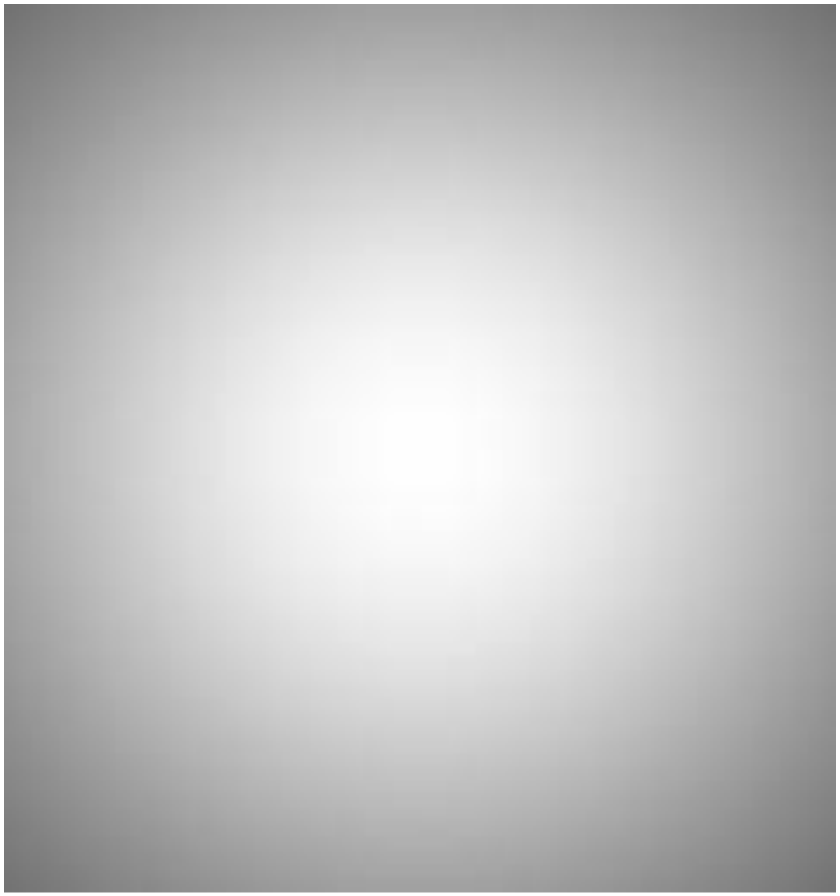



$$E = \left[\frac{\pi}{4}\left(\frac{d}{f}\right)^2 \cos^4\alpha\right] L$$

- Image irradiance ($E$) is linearly related to scene radiance ($L$)
- Irradiance is *directly* proportional to the area of the lens ($\frac{\pi d^2}{4}$) and *inversely* proportional to the squared distance between the lens and the image plane ($f$)
- The irradiance decreases as the angle between the viewing ray and the optical axis ($\alpha$) increases

For derivation, see, e.g., Szeliski 2.2.3

# Fundamental radiometric relation

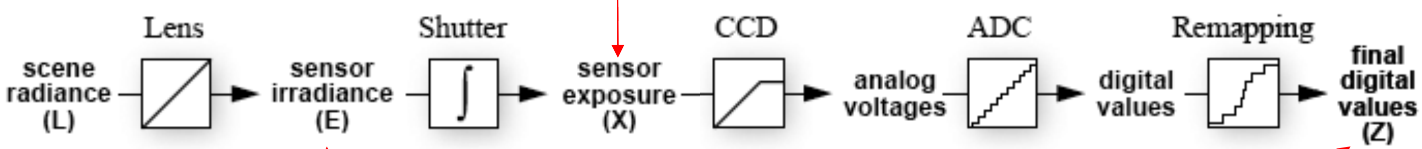


$$E = \left[\frac{\pi}{4}\left(\frac{d}{f}\right)^2 \cos^4\alpha\right] L$$

S. B. Kang and R. Weiss. [Can we calibrate a camera using an image of a flat, textureless Lambertian surface?]() ECCV 2000

# From light rays to pixel values

$$X = E \cdot \Delta t$$




$$E = \left[\frac{\pi}{4}\left(\frac{d}{f}\right)^2 \cos^4\alpha\right] L$$

$$Z = f(E \cdot \Delta t)$$

- **Camera response function**: the mapping $f$ from irradiance to pixel values
  - Needed for applications like estimation of scene reflectance properties, creating high dynamic range (HDR) images
  - For further reading: M. Brown, Understanding the In-Camera Image Processing Pipeline for Computer Vision, CVPR 2016 Tutorial

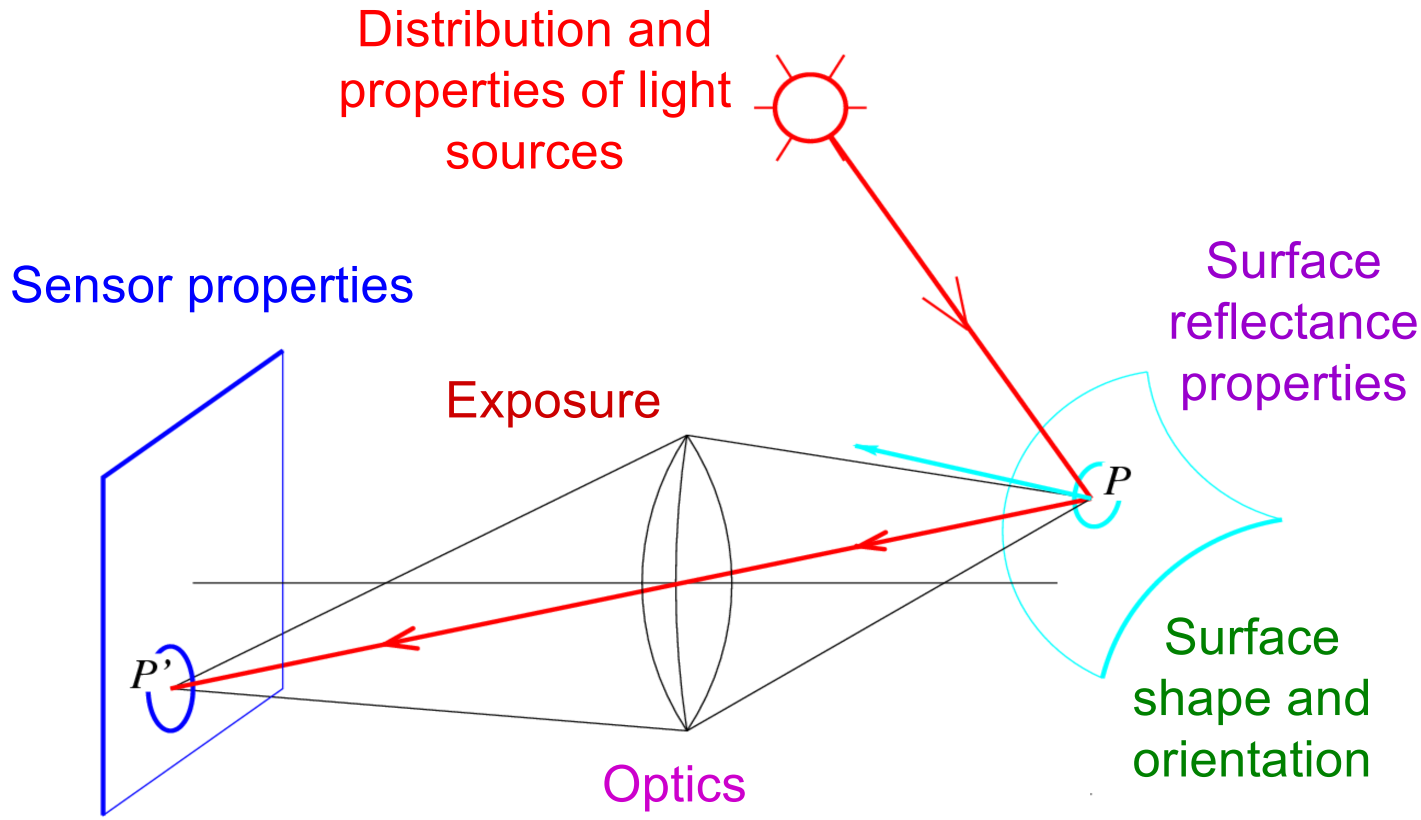
Figure source: P. Debevec and J. Malik. Recovering High Dynamic Range Radiance Maps from Photographs. SIGGRAPH 1997

# Outline

- Small taste of radiometry
- In-camera transformation of light
- Reflectance properties of surfaces

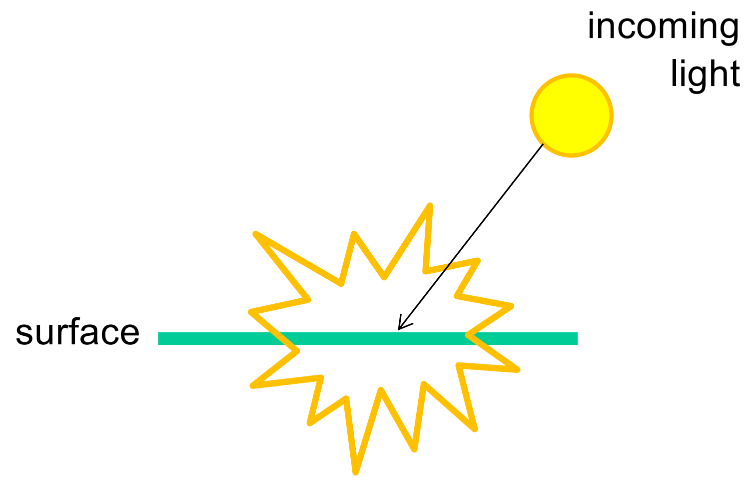
# Recall: Image formation

- What determines the brightness of an image pixel?



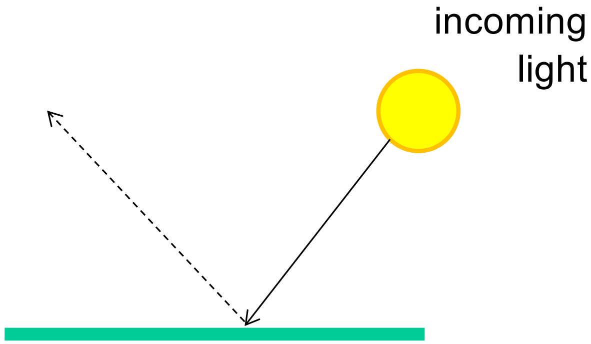

Slide by L. Fei-Fei

# What can happen to light when it hits a surface?

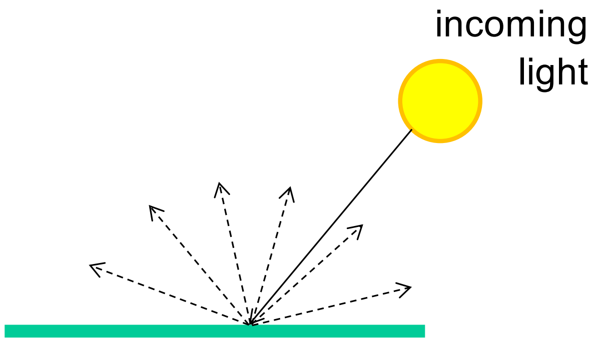


# Basic models of reflection

- **Specular reflection:** light is reflected about the surface normal



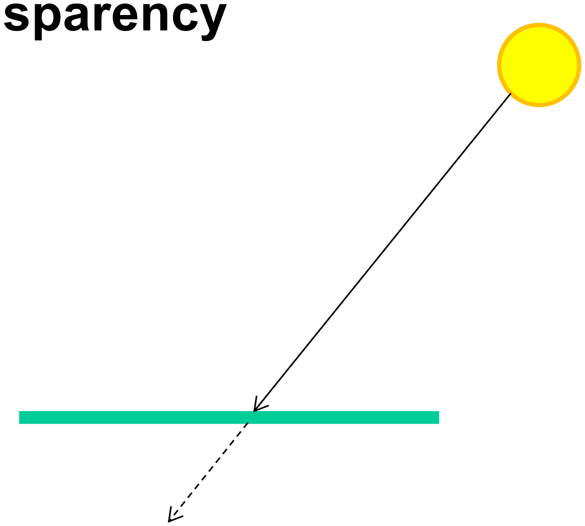

- **Diffuse reflection:** light scatters equally in all directions



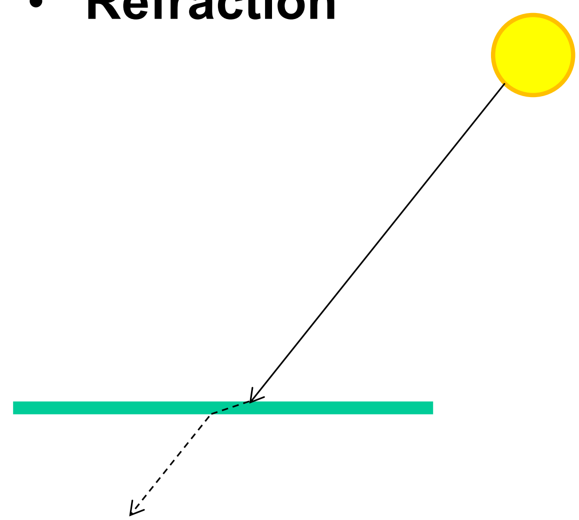

Slide from D. Hoiem

# Other possible effects

- **Transparency**



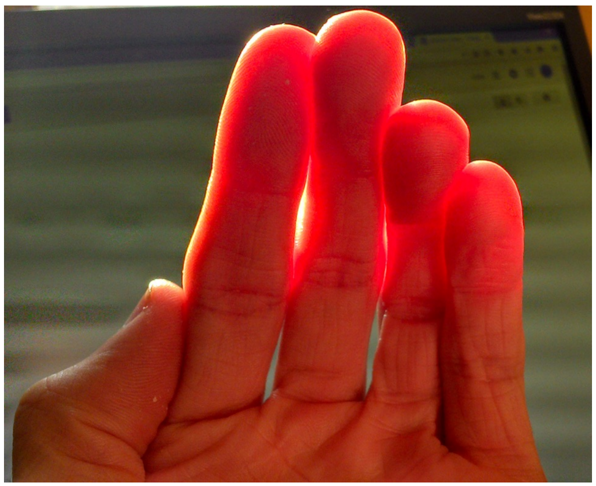
- **Refraction**

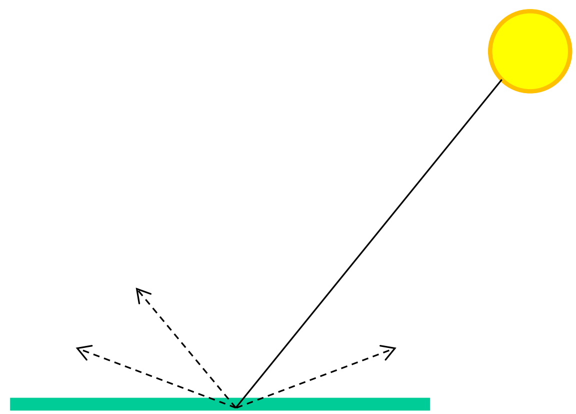


Slide from D. Hoiem

# Other possible effects

- **Subsurface scattering**



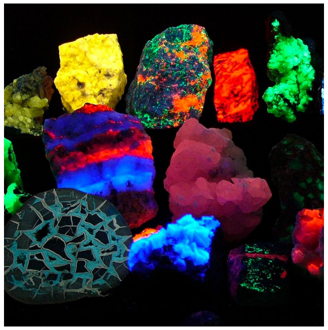


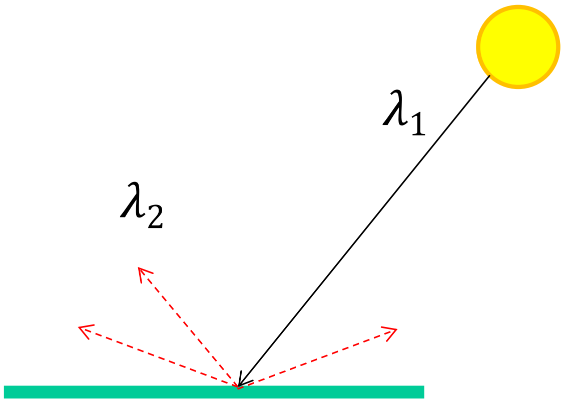
Slide from D. Hoiem

Image source

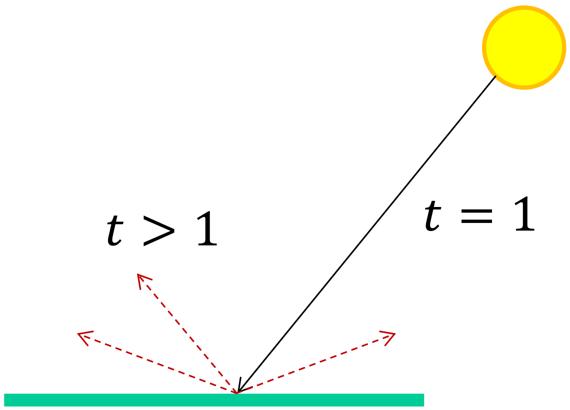
# Other possible effects

- **Fluorescence**






- **Phosphorescence**

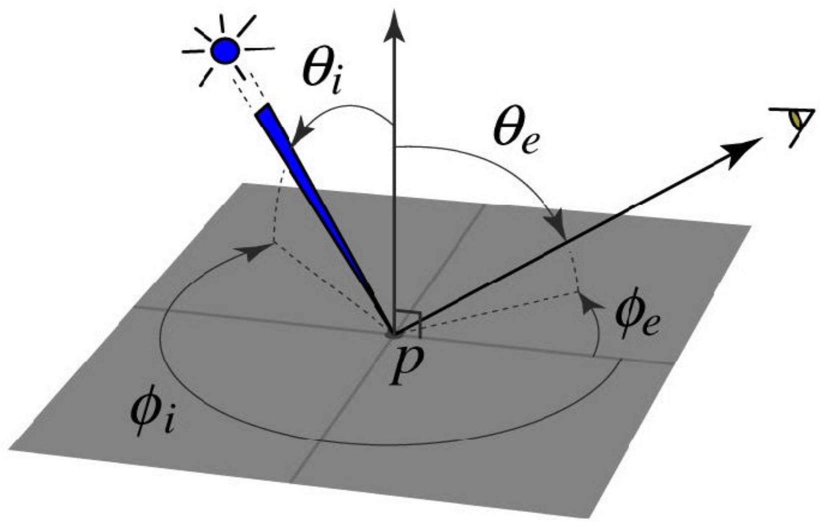



Slide from D. Hoiem

Image source

# Bidirectional reflectance distribution function (BRDF)

- How bright a surface appears when viewed from one direction when light falls on it from another
- Definition: ratio of the radiance in the emitted direction to irradiance in the incident direction




Function of (at least) four parameters: incident and outgoing $\theta, \phi$

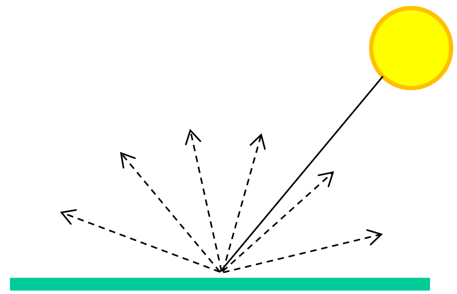
Source: Steve Seitz

# Bidirectional reflectance distribution function (BRDF)

- How bright a surface appears when viewed from one direction when light falls on it from another
- Definition: ratio of the radiance in the emitted direction to irradiance in the incident direction
- Can be incredibly complicated!

# Diffuse reflectance

- Light scatters equally in all directions
  - E.g., brick, matte plastic, rough wood



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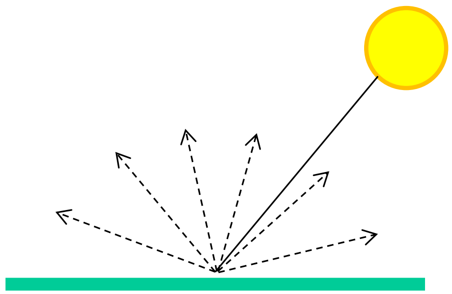


- This happens because of *microfacets* that scatter incoming light randomly

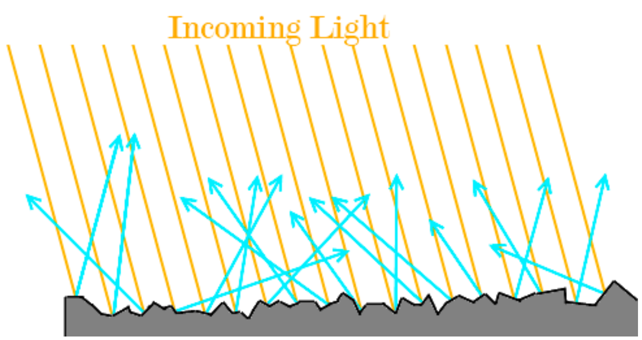
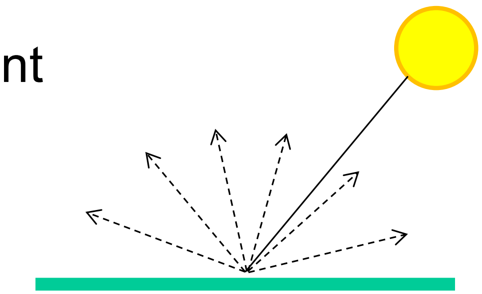


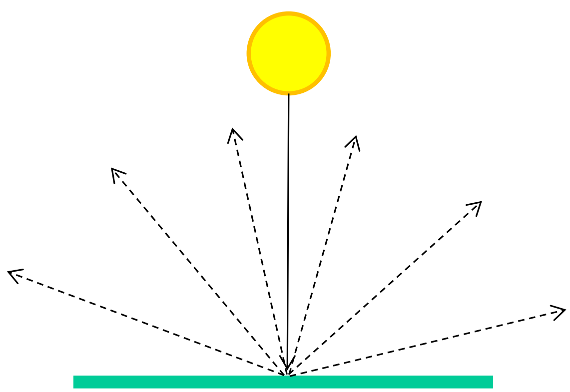

Image source

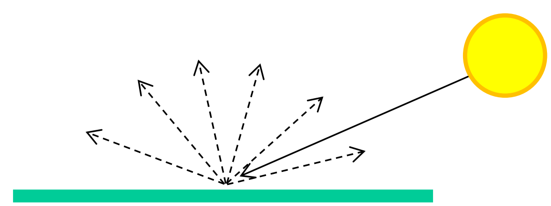
# Diffuse reflectance

- Light scatters equally in all directions
  - For a fixed incidence angle, BRDF is constant



- What if we change the incidence angle?



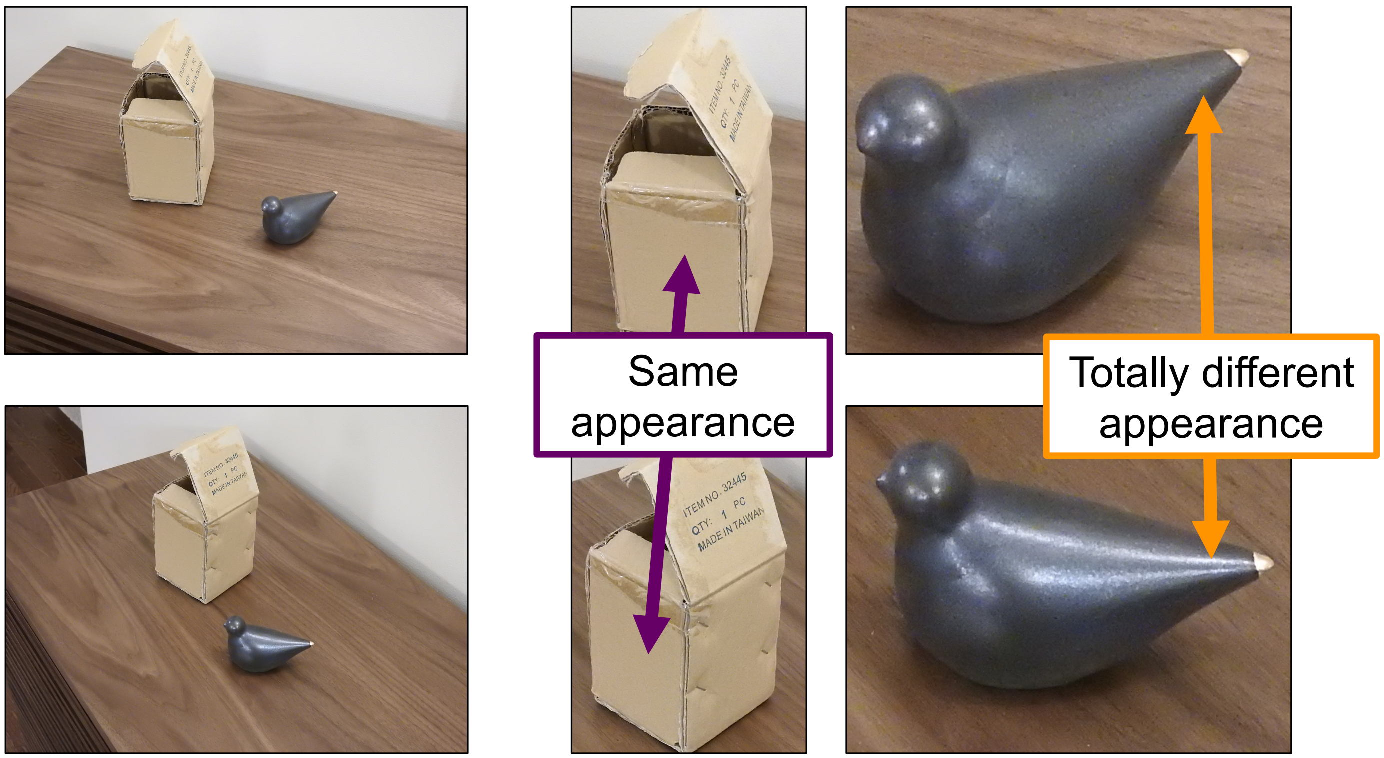


# Why do we care about diffuse reflectance?

Same lighting, as close as possible camera settings, but different **camera position**

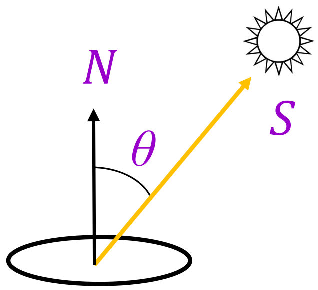
Diffuse

Specular

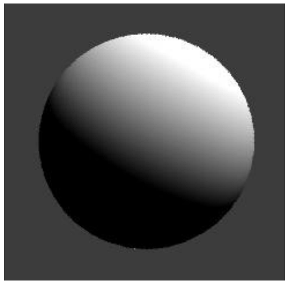



Source: [J. Johnson and D. Fouhey]

# Diffuse reflectance: Lambert's law




$$
\begin{aligned}
I &= \rho\,(S \cdot N) \\
&= \rho \|S\| \cos\theta
\end{aligned}
$$



$I$: reflected intensity (technically: *radiosity*, or total power leaving the surface per unit area)

$\rho$: albedo (fraction of incident irradiance reflected by the surface)

$S$: direction of light source (magnitude proportional to intensity of the source)

$N$: unit surface normal

# Outline

- Small taste of radiometry
- In-camera transformation of light
- Reflectance properties of surfaces
- Diffuse and specular reflection
- Shape from shading

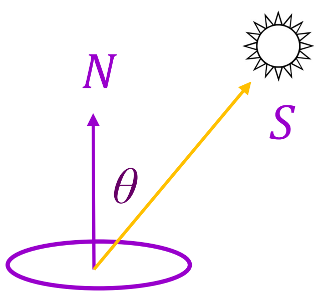
# Photometric stereo, or shape from shading

- Can we reconstruct the shape of an object based on shading cues?

Luca della Robbia,
*Cantoria*, 1438

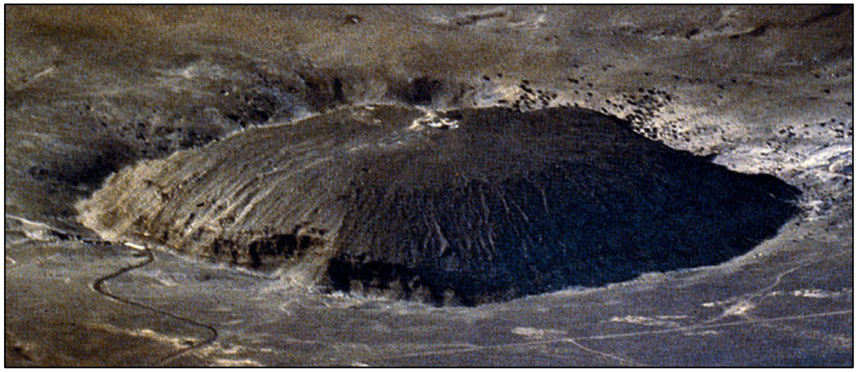
# Photometric stereo, or shape from shading

- Can we reconstruct the shape of an object based on shading cues?
- Assuming a Lambertian object, given the image intensity ($I$), can we recover the light source direction ($S$) and the surface normal ($N$)?
- Can we do this from a single image?




$$\begin{aligned} I &= \rho\,(S \cdot N) \\ &= \rho \|S\| \cos\theta \end{aligned}$$

# Shape from shading ambiguity



Source: J. Johnson and D. Fouhey

Image source

# Shape from shading ambiguity

- Humans assume light from above (and the blueness also tells you distance)

Source: [J. Johnson and D. Fouhey]

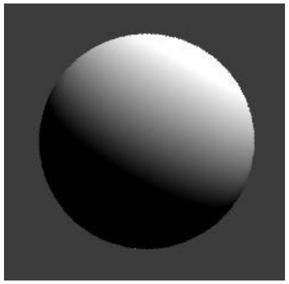
[Image source]

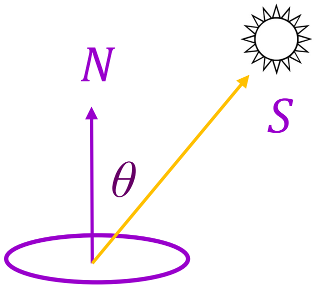
# Last time: Light and shading

- Small taste of radiometry
- In-camera transformation of light
- Reflectance properties of surfaces
- Diffuse and specular reflection
- Shape from shading

# Review: Lambert's law

- What determines the observed brightness at a point on the surface?

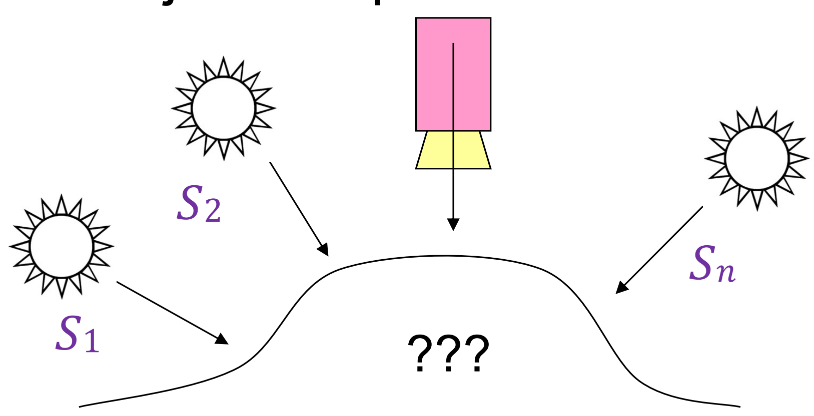





$$I = \rho\,(S \cdot N)$$
$$= \rho \|S\| \cos\theta$$

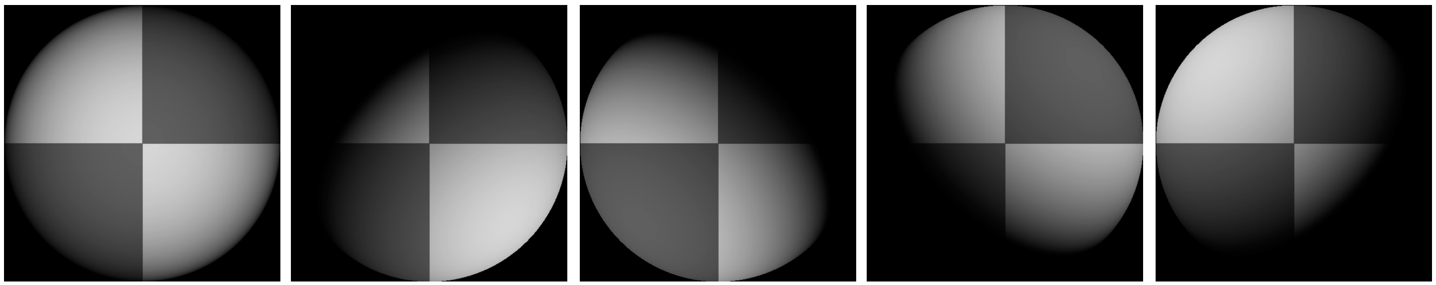
# Photometric stereo

- Assume:
  - A Lambertian object
  - A *local shading model* (each point on a surface receives light only from sources visible at that point)
  - A set of *known* light source directions
  - A set of pictures of an object, obtained in exactly the same camera/object configuration but using different sources
  - Orthographic projection
- Goal: reconstruct object shape and albedo

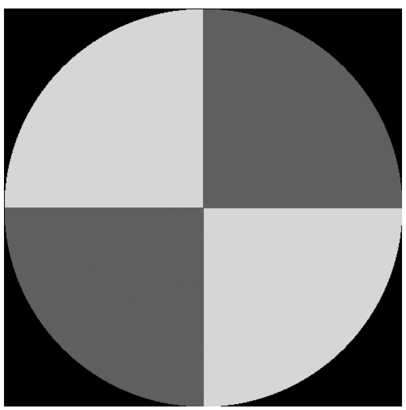



F&P 2nd ed., sec. 2.2.4

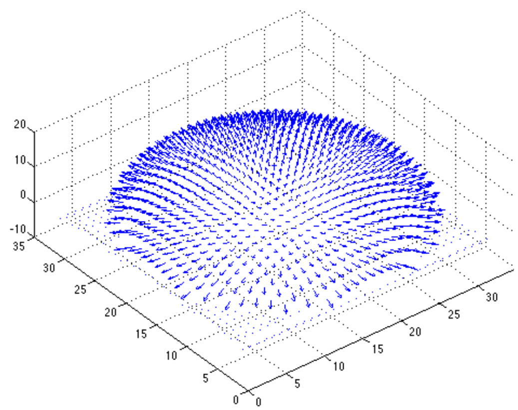
# Synthetic example



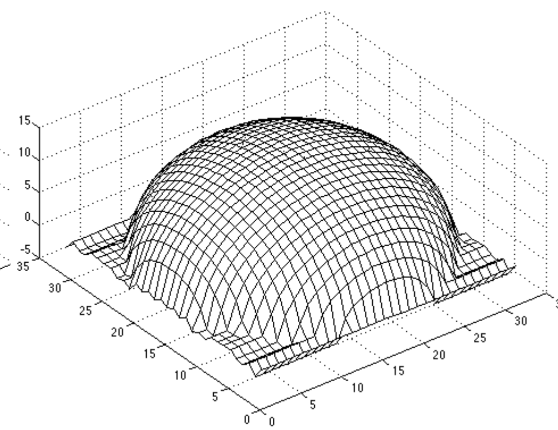
Recovered albedo



Recovered normal field



Recovered surface model



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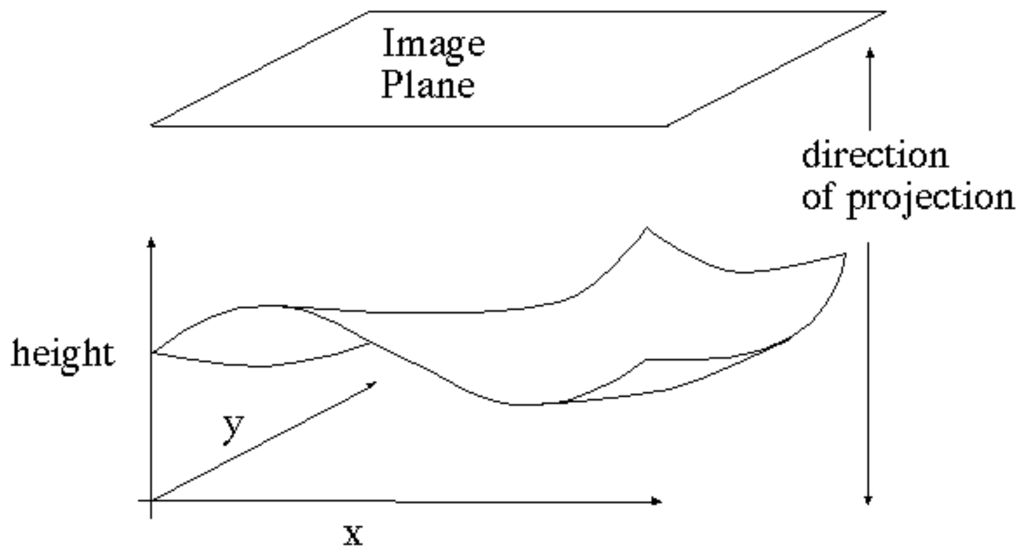
# Image model

- **Known:** source vectors $S_j$ and pixel values $I_j(x, y)$
- **Unknown:** surface normal $N(x, y)$ and albedo $\rho(x, y)$




F&P 2nd ed., sec. 2.2.4

# Image model

- **Known:** source vectors $S_j$ and pixel values $I_j(x, y)$
- **Unknown:** surface normal $N(x, y)$ and albedo $\rho(x, y)$
- Assume that the response function of the camera is a linear scaling by a factor of $k$
- Lambert's law:

$$\begin{aligned} I_j(x, y) &= k\,\rho(x, y)\big(N(x, y) \cdot S_j\big) \\ &= (\rho(x, y) N(x, y)) \cdot \big(k\, S_j\big) \\ &= g(x, y) \cdot V_j \end{aligned}$$

F&P 2nd ed., sec. 2.2.4

# Least squares problem

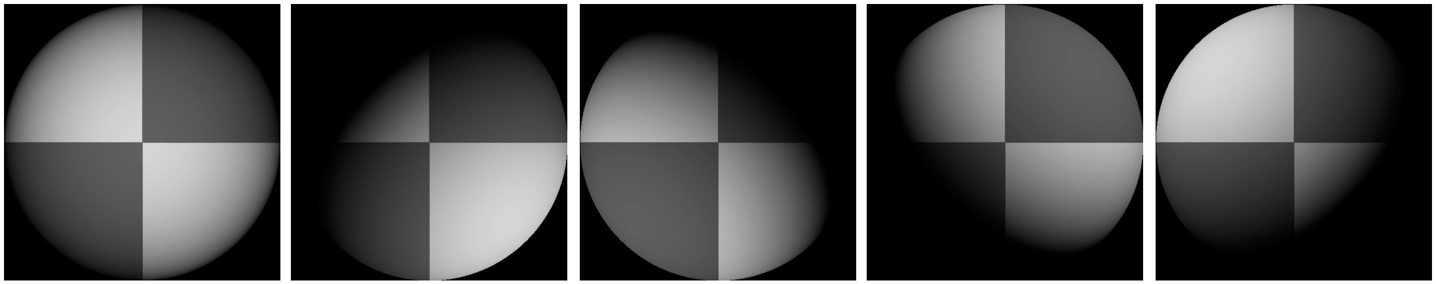
- For each pixel, set up a linear system:

$$\underset{\substack{n \times 3 \\ \text{known}}}{\begin{bmatrix} V_1^T \\ V_2^T \\ \vdots \\ V_n^T \end{bmatrix}} \underset{\substack{3 \times 1 \\ \text{unknown}}}{g(x,y)} = \underset{\substack{n \times 1 \\ \text{known}}}{\begin{bmatrix} I_1(x,y) \\ I_2(x,y) \\ \vdots \\ I_n(x,y) \end{bmatrix}}$$

- Obtain least-squares solution for $g(x,y)$, which we defined as $\rho(x,y)N(x,y)$
- Since $N(x,y)$ is the *unit* normal, $\rho(x,y)$ is given by the magnitude of $g(x,y)$
- Finally, $N(x,y) = \frac{1}{\rho(x,y)} g(x,y)$

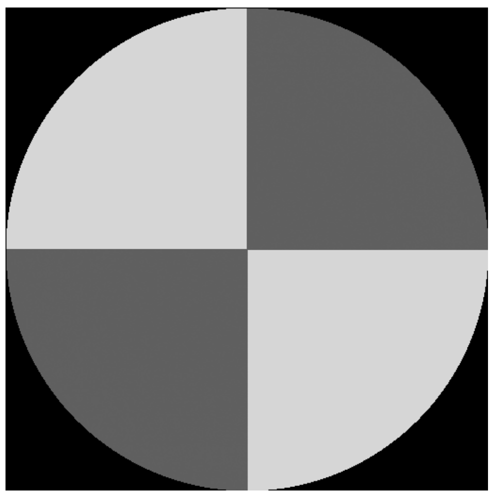
F&P 2nd ed., sec. 2.2.4

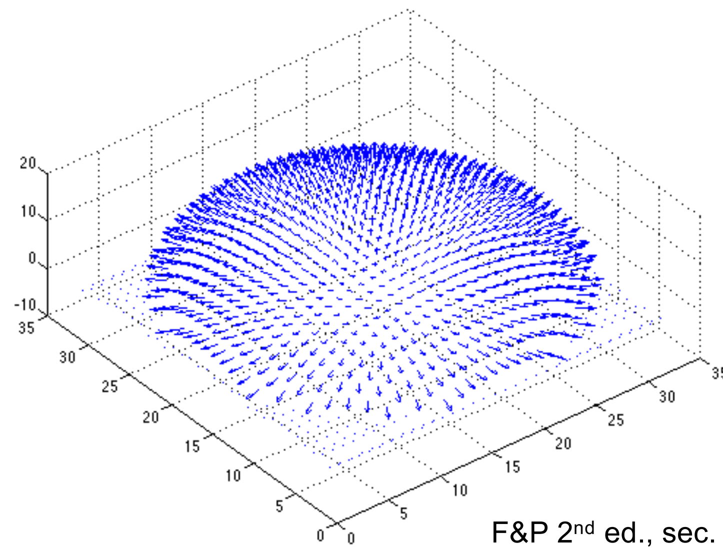
# Synthetic example



Recovered albedo

Recovered normal field






F&P 2nd ed., sec. 2.2.4

# Recovering a surface from normals

- Recall: the surface is written as

$$(x, y, f(x, y))$$

- This means the unit normal has the following form:

$$N(x, y) = \frac{1}{\sqrt{f_x^2 + f_y^2 + 1}} \begin{bmatrix} f_x \\ f_y \\ 1 \end{bmatrix}$$

- Write the estimated vector $g$ as

$$g(x, y) = \begin{bmatrix} g_1(x, y) \\ g_2(x, y) \\ g_3(x, y) \end{bmatrix}$$

- Then we obtain values for the partial derivatives of the surface:

$$f_x(x, y) = \frac{g_1(x, y)}{g_3(x, y)}$$

$$f_y(x, y) = \frac{g_2(x, y)}{g_3(x, y)}$$

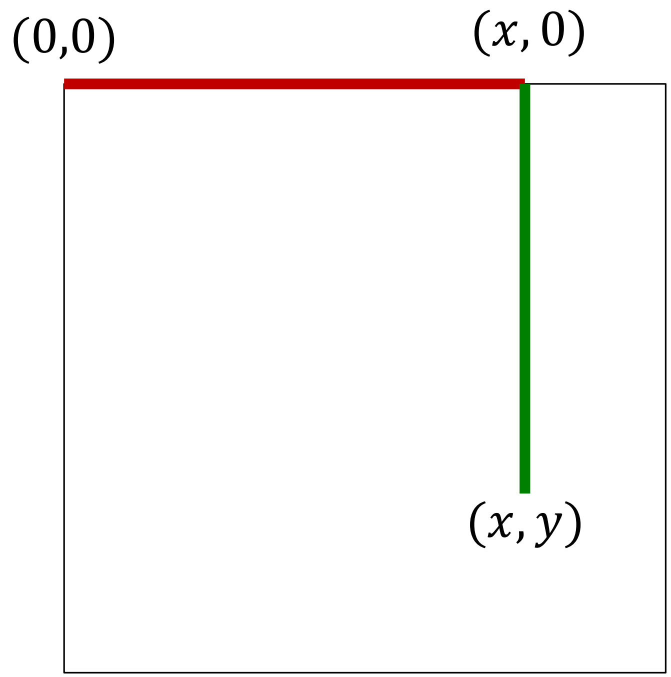
F&P 2nd ed., sec. 2.2.4

# Recovering a surface from normals

- We can now recover the surface height at any point by integration along some path, e.g.

$$f(x, y) = \int_0^x f_x(s, 0)ds + \int_0^y f_y(x, t)dt + C$$

- For robustness, it is better to take integrals over many different paths and average the results




F&P 2nd ed., sec. 2.2.4

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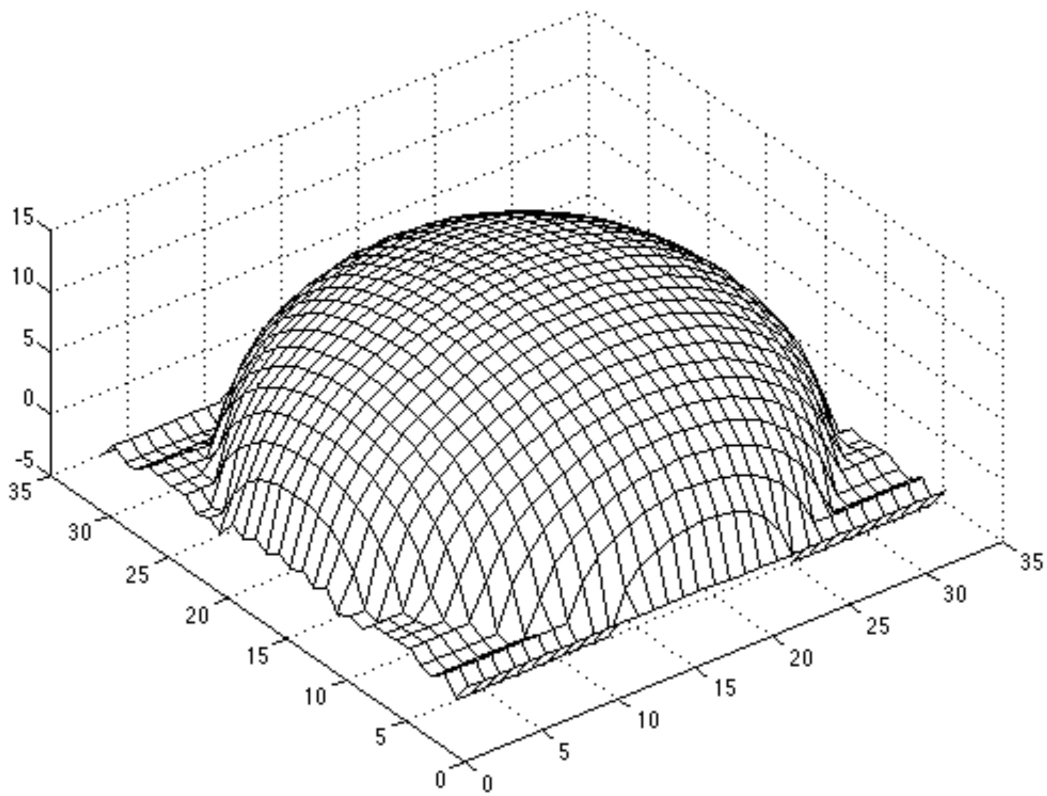
- For robustness, it is better to take integrals over many different paths and average the results

- Note: *integrability* must be satisfied: for the surface $f$ to exist, the mixed second partial derivatives must be equal (or at least similar in practice):

$$\frac{\partial}{\partial y}\left(\frac{g_1(x,y)}{g_3(x,y)}\right) = \frac{\partial}{\partial x}\left(\frac{g_2(x,y)}{g_3(x,y)}\right)$$

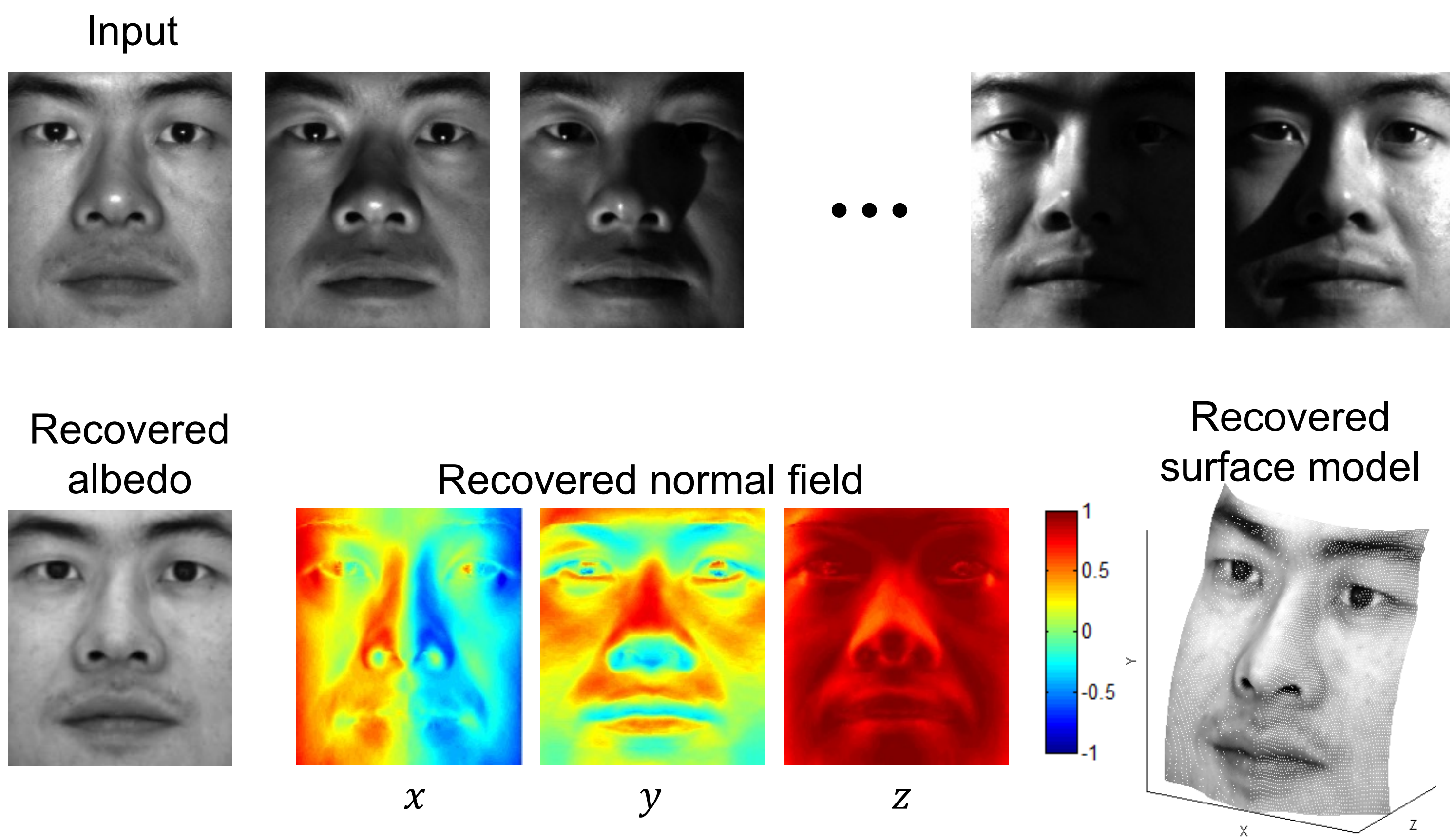
F&P 2nd ed., sec. 2.2.4

# Surface recovered by integration




F&P 2nd ed., sec. 2.2.4

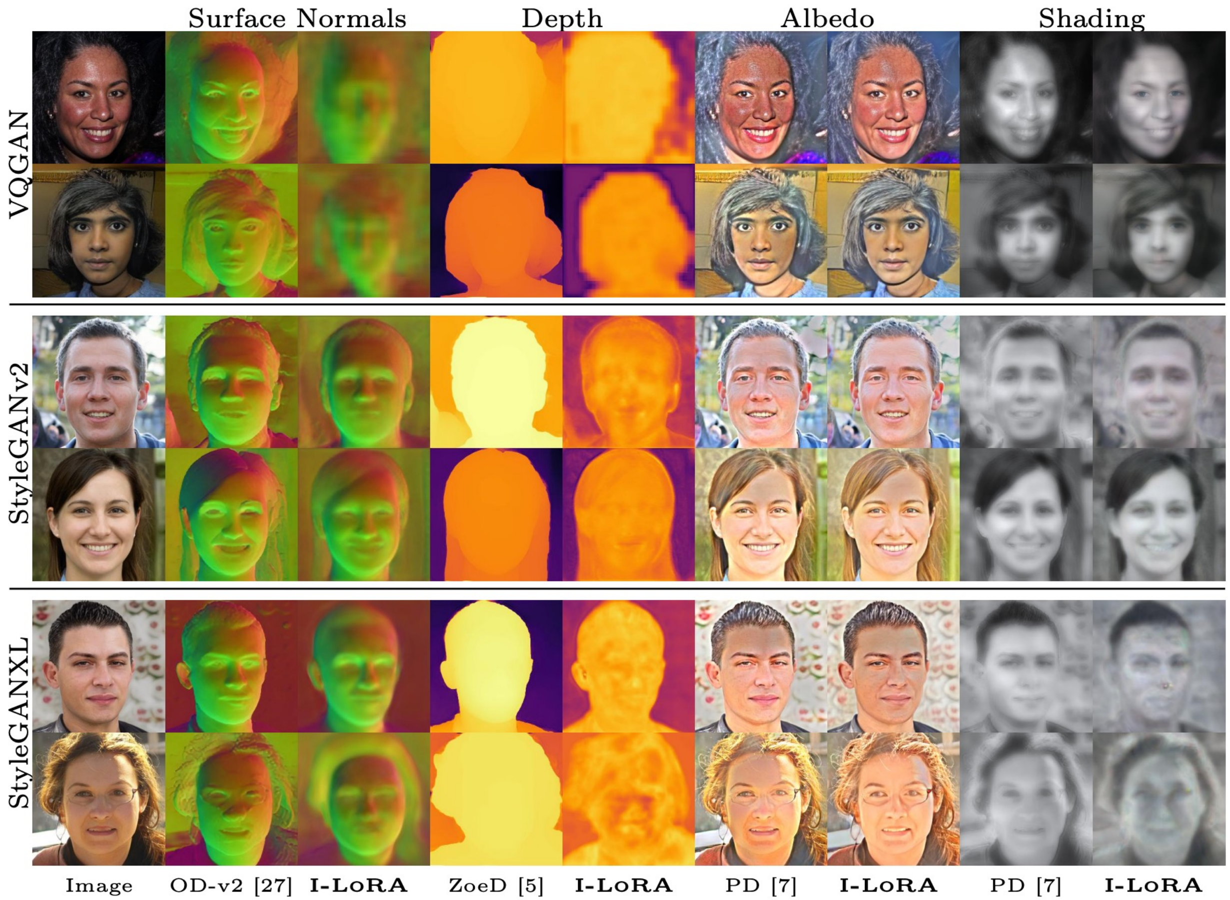
# MP4 preview


Input
• • •
Recovered
albedo
Recovered normal field
$x$
$y$
$z$
1
0.5
0
-0.5
-1
Recovered
surface model
Y
X
Z

# Limitations of basic shape from shading

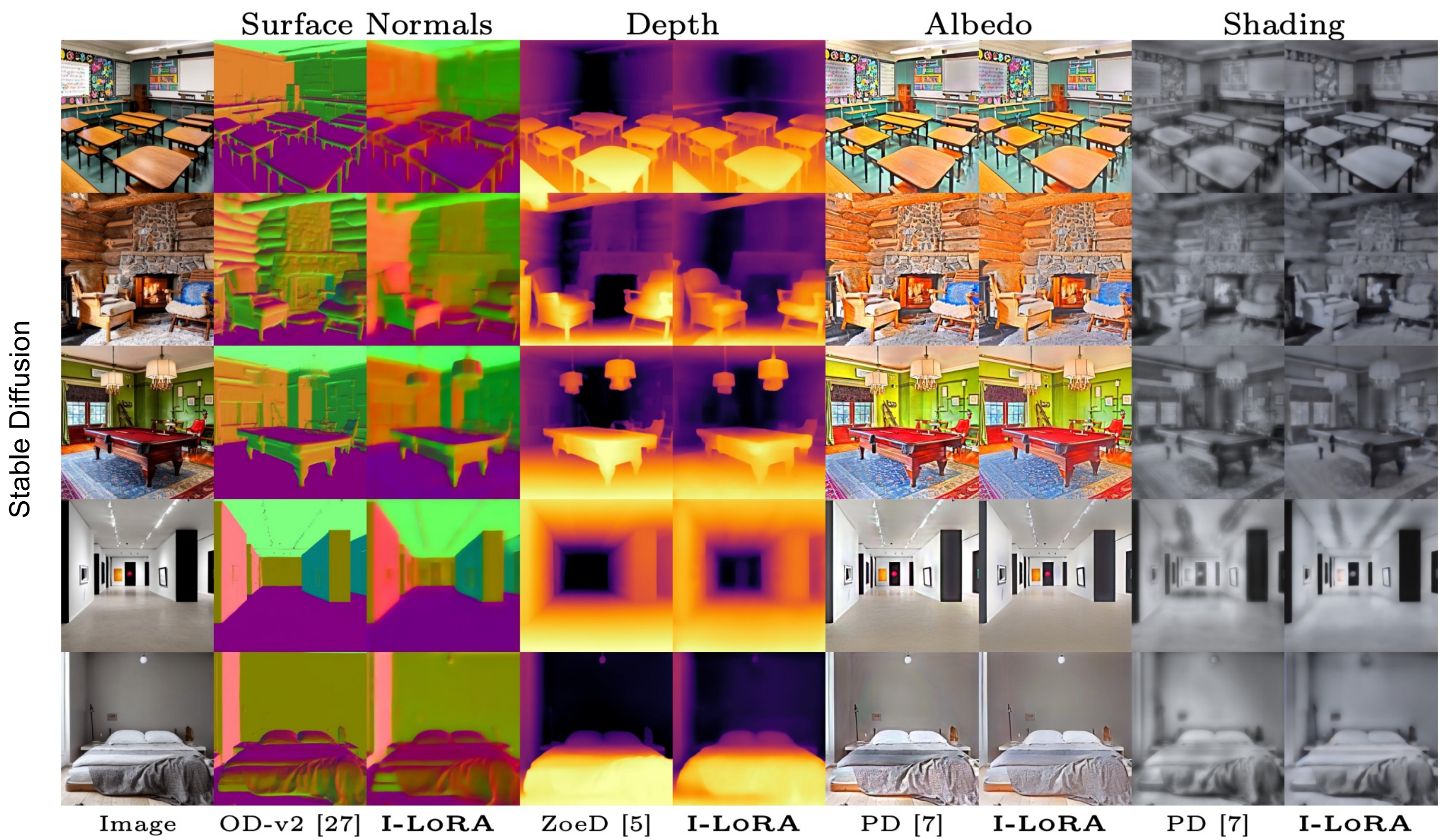
- Orthographic camera model
- Simplistic reflectance and lighting model
- No shadows
- No interreflections
- No missing data
- Integration is tricky

# Shape from shading today




X. Du et al. [Generative Models: What do they know? Do they know things? Let's find out!](#) CVPR 2024 workshops

# Shape from shading today

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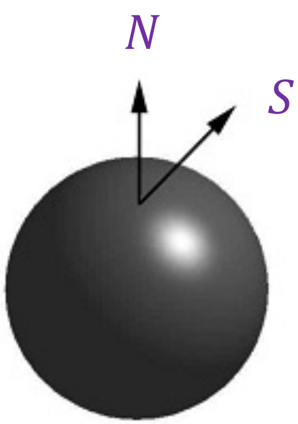
# Finding the direction of the light source

$$\boxed{I(x,y) = N(x,y) \cdot S(x,y)}$$



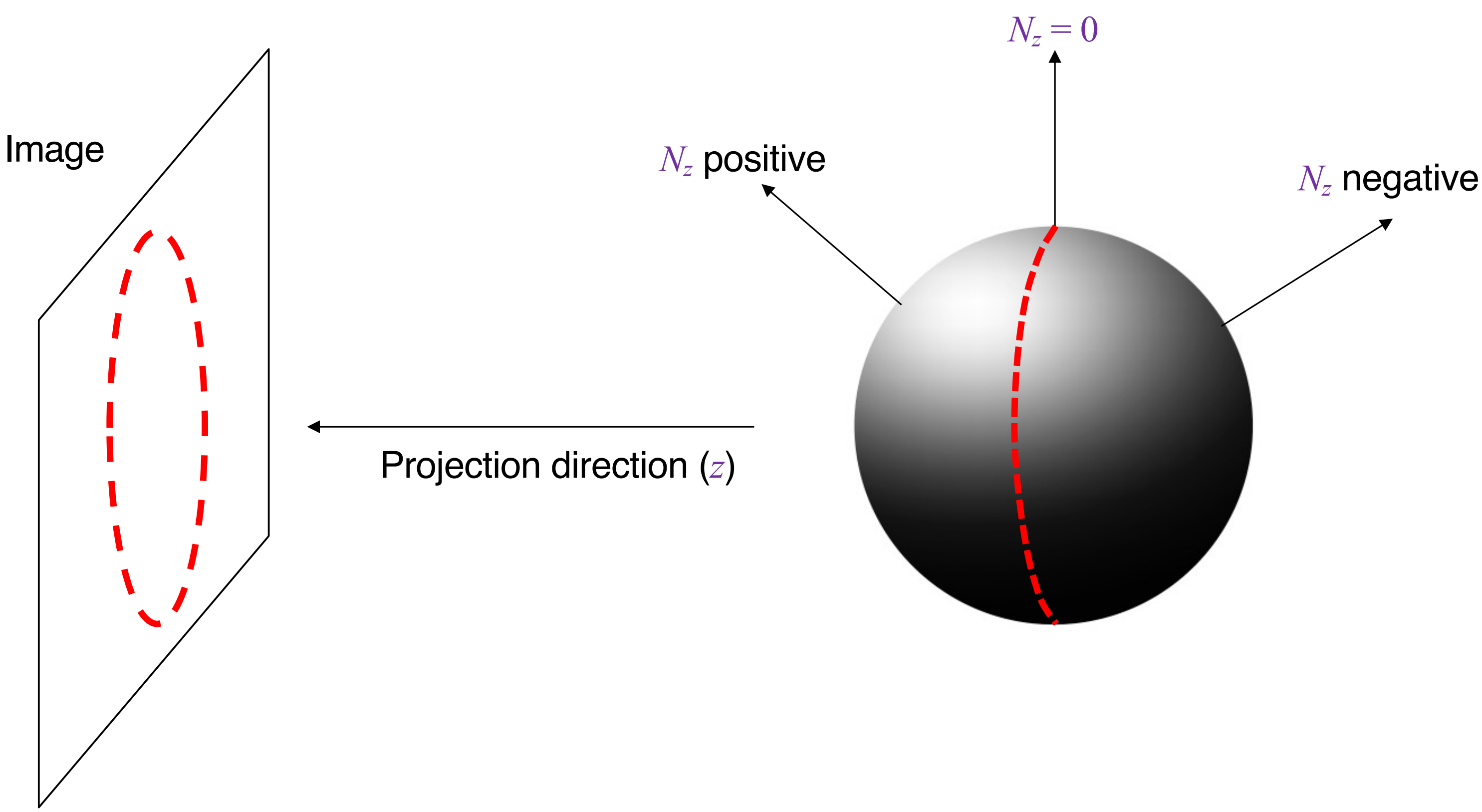

- Full 3D case:

$$\begin{bmatrix} N_x(x_1,y_1) & N_y(x_1,y_1) & N_z(x_1,y_1) \\ N_x(x_2,y_2) & N_y(x_2,y_2) & N_z(x_2,y_2) \\ \vdots & \vdots & \vdots \\ N_x(x_n,y_n) & N_y(x_n,y_n) & N_z(x_n,y_n) \end{bmatrix} \begin{bmatrix} S_x \\ S_y \\ S_z \end{bmatrix} = \begin{bmatrix} I(x_1,y_1) \\ I(x_2,y_2) \\ \vdots \\ I(x_n,y_n) \end{bmatrix}$$

P. Nillius and J.-O. Eklundh. Automatic estimation of the projected light source direction. CVPR 2001

# Finding the direction of the light source

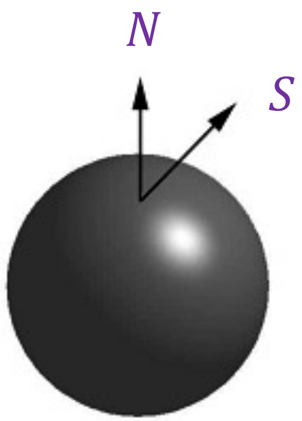
Consider points on the *occluding contour*:




P. Nillius and J.-O. Eklundh. [Automatic estimation of the projected light source direction.](#) CVPR 2001

# Finding the direction of the light source

$$\boxed{I(x,y) = N(x,y) \cdot S(x,y)}$$




- Full 3D case:

$$\begin{bmatrix} N_x(x_1,y_1) & N_y(x_1,y_1) & N_z(x_1,y_1) \\ N_x(x_2,y_2) & N_y(x_2,y_2) & N_z(x_2,y_2) \\ \vdots & \vdots & \vdots \\ N_x(x_n,y_n) & N_y(x_n,y_n) & N_z(x_n,y_n) \end{bmatrix} \begin{bmatrix} S_x \\ S_y \\ S_z \end{bmatrix} = \begin{bmatrix} I(x_1,y_1) \\ I(x_2,y_2) \\ \vdots \\ I(x_n,y_n) \end{bmatrix}$$

- For points on the occluding contour ($N_z = 0$):

$$\begin{bmatrix} N_x(x_1,y_1) & N_y(x_1,y_1) \\ N_x(x_2,y_2) & N_y(x_2,y_2) \\ \vdots & \vdots \\ N_x(x_n,y_n) & N_y(x_n,y_n) \end{bmatrix} \begin{bmatrix} S_x \\ S_y \end{bmatrix} = \begin{bmatrix} I(x_1,y_1) \\ I(x_2,y_2) \\ \vdots \\ I(x_n,y_n) \end{bmatrix}$$

P. Nillius and J.-O. Eklundh. Automatic estimation of the projected light source direction. CVPR 2001

# Finding the direction of the light source


P. Nillius and J.-O. Eklundh. Automatic estimation of the projected light source direction. CVPR 2001

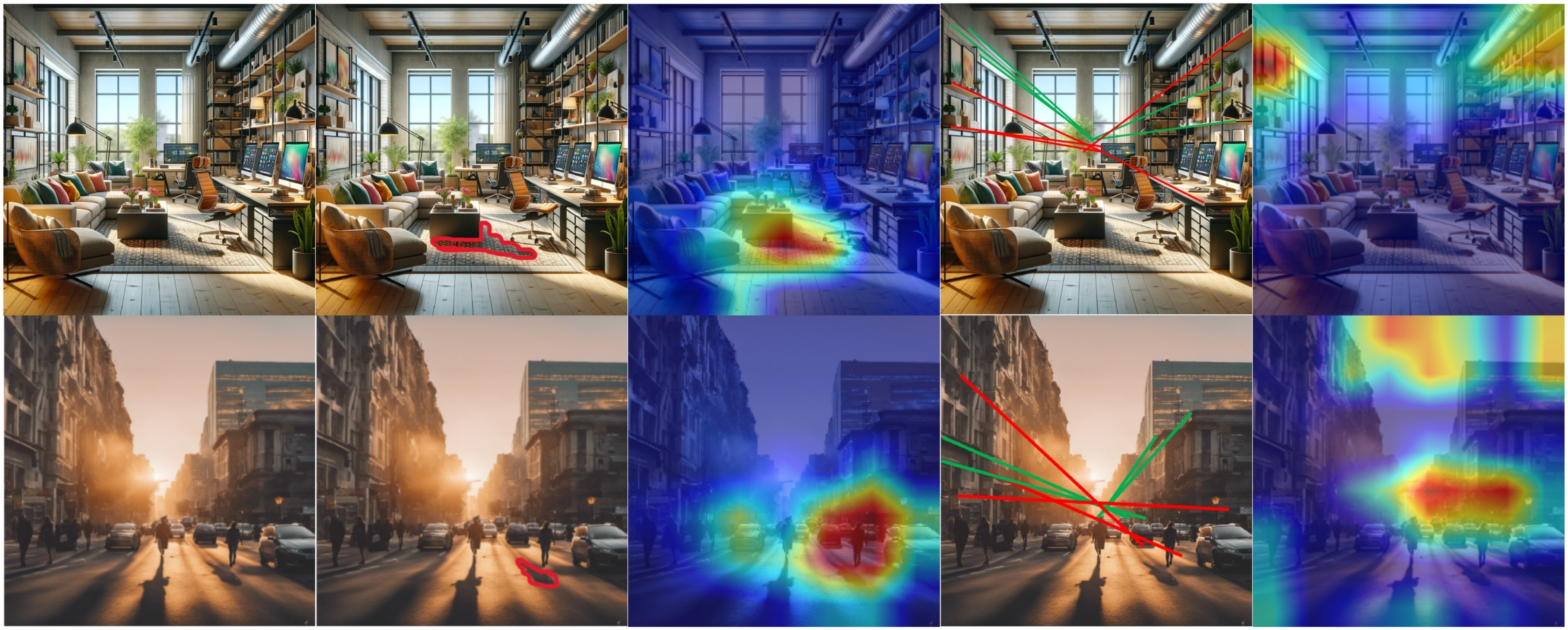
# Application: Detecting composite photos

Fake photo

Real photo

M. K. Johnson and H. Farid. [Exposing Digital Forgeries by Detecting Inconsistencies in Lighting](). ACM Multimedia and Security Workshop, 2005

# DeepFake detection today



Generated Image | Shadow Errors | Detected Shadow Errors | Vanishing Point Errors | Detected Perspective Errors

A. Sarkar et al. [Shadows Don't Lie and Lines Can't Bend! Generative Models don't know Projective Geometry ... for now.]
CVPR 2024