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Department of Computer Engineering

Embedded System Design

Feedback Control (10)

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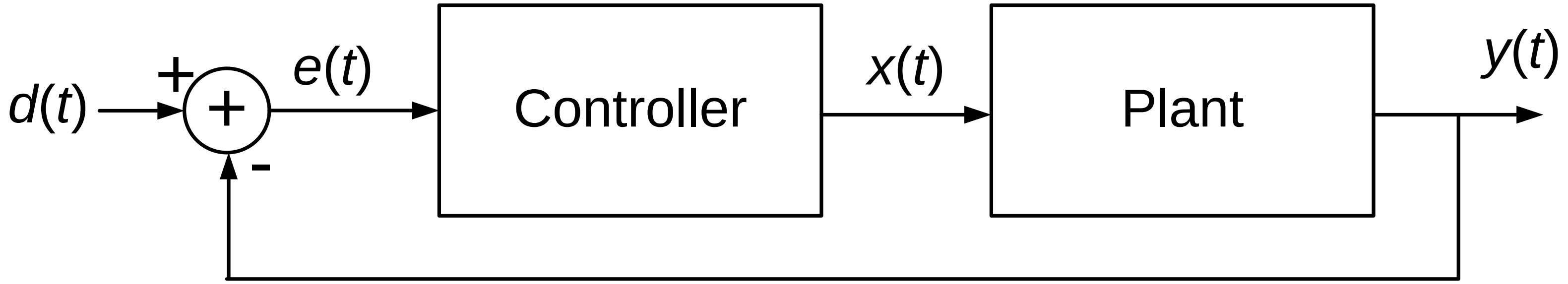
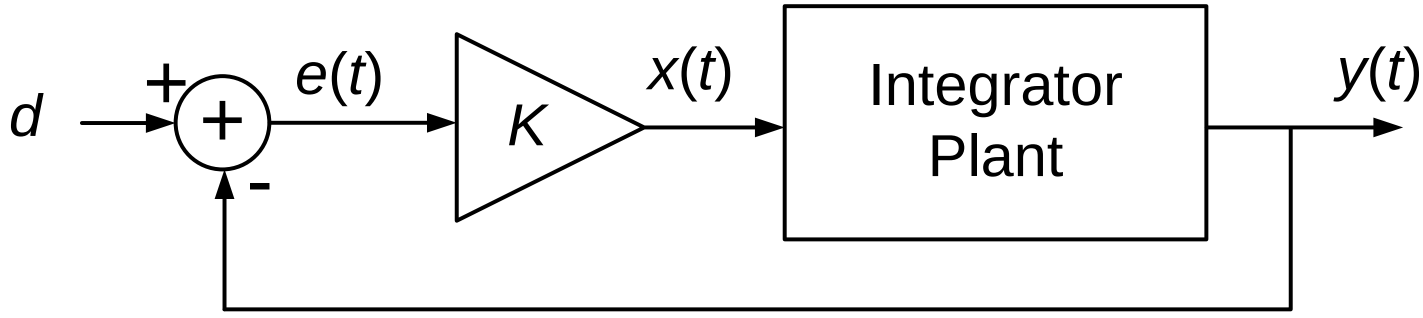
Feedback Control

- Feedback control is a sophisticated topic, easily occupying complete courses.
- We barely touch on the subject, just to analyse the interactions between embedded and physical systems.

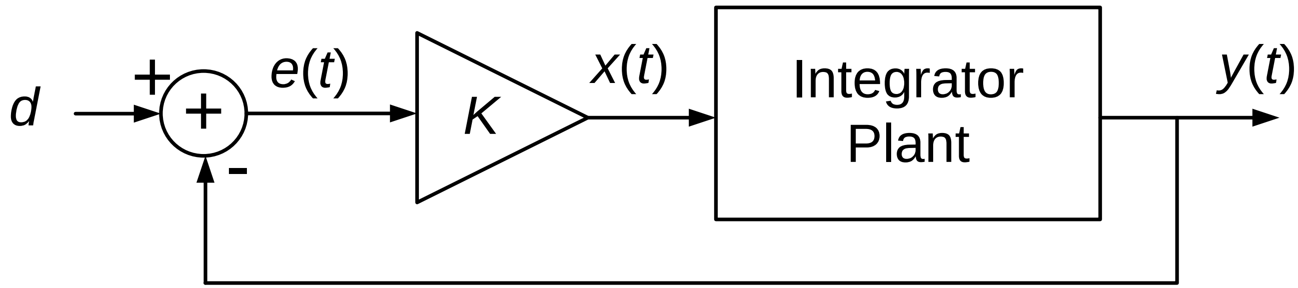


Control of an integrator plant

- **A scalar can be used for this purpose.**
  - Proportional control system to maintain a setpoint.
- **Intuition:**
  - **The force that you apply to a moving object is proportional to the difference between the actual and desired speeds.**



Proportional Control



$$x(t) = Ke(t) = Kd - Ky(t)$$

$$y(t) = c\int_0^t x(\tau)d\tau + y_0 \Rightarrow y'(t) = cx(t) \Rightarrow y'(t) = cKd - cKy(t)$$

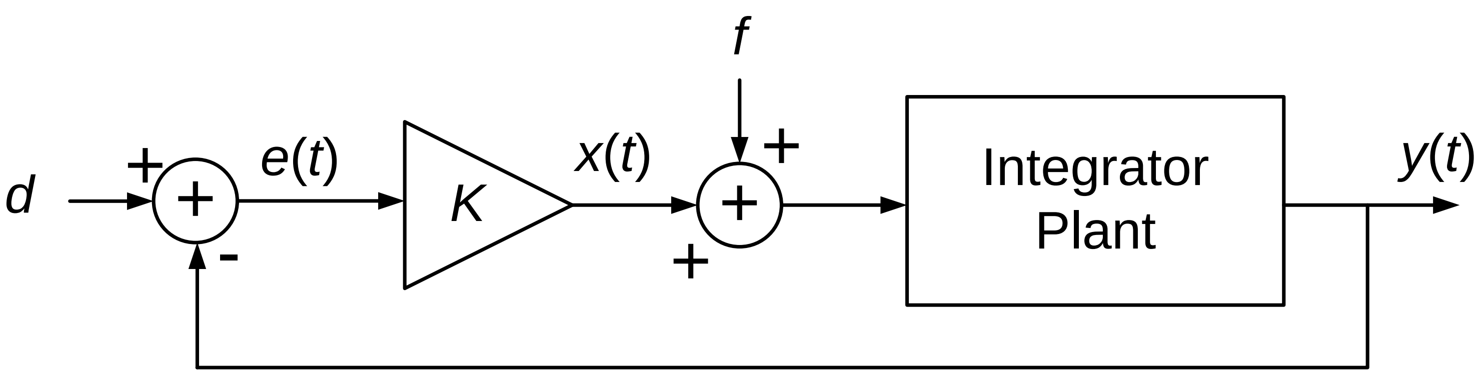
$$sY(s) - y_0 = \frac{cKd}{s} - cKY(s)$$

$$Y(s) = \frac{cKd + y_0 s}{s(s + cK)} = \frac{d}{s} + \frac{-d + y_0}{s + cK}$$

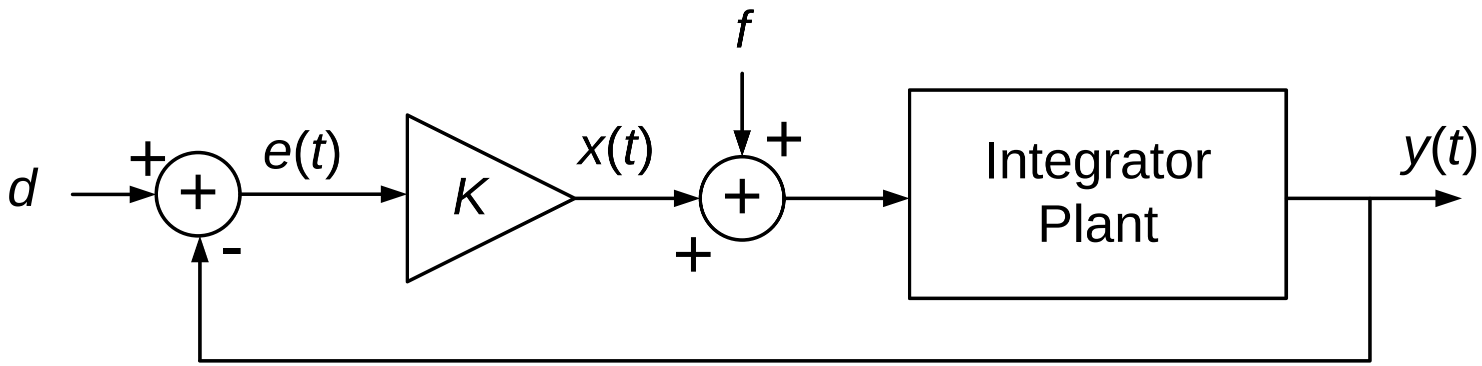
$$y(t) = d + (-d + y_0)e^{-cKt}$$

Steady-State Error

- **When undesired plant inputs exist proportional control has a steady-state error.**
  - **Example of undesired plant inputs:**
    - **Moving object: Friction force**
    - **Air conditioning: External heat source**



Steady-State Error (Cont.)



$$x(t) = Ke(t) = Kd - Ky(t)$$

$$y(t) = c\int_0^t [x(\tau) + f]d\tau + y_0 \Rightarrow y'(t) = cx(t) + cf \Rightarrow y'(t) = cKd - cKy(t) + cf$$

$$sY(s) - y_0 = \frac{cKd + cf}{s} - cKY(s)$$

$$Y(s) = \frac{cKd + cf + y_0 s}{s(s + cK)} = \frac{d + f/K}{s} + \frac{-d - f/K + y_0}{s + cK}$$

$$y(t) = d + f/K + (-d - f/K + y_0)e^{-cKt}$$

f/K = Steady-State Error

Useful Laplace Transforms

$$1 \leftrightarrow \frac{1}{s}$$

$$e^{-at} \leftrightarrow \frac{1}{s+a}$$

$$y'(t) \leftrightarrow sY(s) - y(0)$$

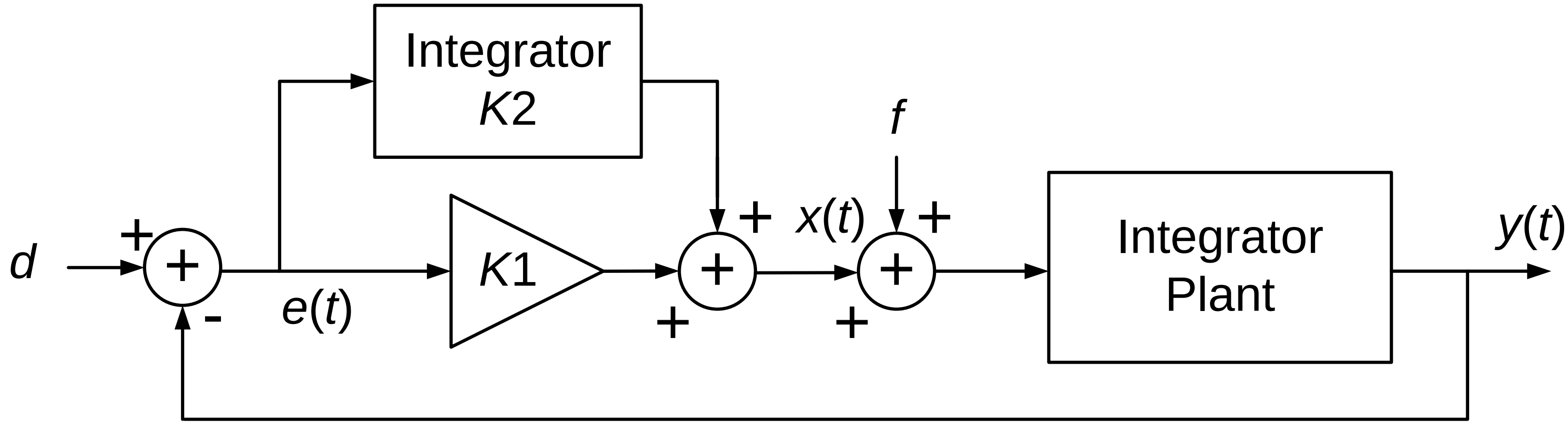
$$y''(t) \leftrightarrow s^2Y(s) - sy(0) - y'(0)$$

$$e^{-at}\sin(\omega t) \leftrightarrow \frac{\omega}{(s+a)^2+\omega^2}$$

$$e^{-at}\cos(\omega t) \leftrightarrow \frac{s+a}{(s+a)^2+\omega^2}$$

$$\lim_{t \longrightarrow \infty} y(t) = \lim_{s \longrightarrow 0} sY(s)$$

PI Controller



$$Y(s) = \frac{y_0 s^2 + \left(K_1 y_0 + y'_0\right)s + K_2 d}{(s^2 + K_1 s + K_2)s}$$