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Question 1

Let $(X_1, X_2, X_3 -- X_n)$ be random sample of size taken from a normal population with parameters, mean θ_1, and variance θ_2. Find the maximum likelihood estimates of these 2 parameters.

Answer// given that $X_1, X_2 ... X_n$ is a Random sample from a normal distribution with mean θ_1 and variance θ_2 the, likelihood funcⁿ is

$$L\left(\theta_1 \theta_2 / x_1 x_2 -- x_n\right) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi \theta_2}} e^{-\frac{(x-\theta_1)^2}{2\theta_2}}$$

Taking log on both sides

$$\ln L\left(\frac{\theta_1 \theta_2}{x_1 x_2 -- x_n}\right) = n/2 \log(2\pi\theta_2) - \frac{1}{2\theta_2}\sum_{i=1}^{n}(x_i - \theta_1)^2$$

To find MLE, we will Differentiate the log likelihood with respect to θ_1 and θ_2, set derivative equal to zero.

For θ_1,

$$\frac{\partial}{\partial \theta_1} \ln L\left(\frac{\theta_1 \theta_2}{x_1 x_2 -- x_n}\right) = \frac{1}{\theta_2}\sum_{i=1}^{n}(x_i - \theta_1)$$

Setting this equal to zero,

$$\frac{1}{\theta_2}\sum_{i=1}^{n}(x_i-\hat{\theta}_1) = 0 \Rightarrow \sum_{i=1}^{n}(x_i-\hat{\theta}_1) = 0$$

$$\hat{\theta}_1 = \frac{1}{n}\sum_{i=1}^{n} x_i$$

So, MLE for θ_1 is sample Mean

For θ_2

$$\frac{\partial}{\partial\theta_2}\ln L\left(\frac{\theta_1, \theta_2}{x_1 x_2 \ldots x_n}\right) = \frac{-n}{2\theta_2} + \frac{1}{2\theta_2^2}\sum_{i=1}^{n}(x_i-\theta_1)^2$$

setting this equal to zero.

$$\frac{-n}{2\hat{\theta}_2} + \frac{1}{2\hat{\theta}_2^2}\sum_{i=1}^{n}(x_i-\hat{\theta}_1)^2 = 0$$

$$\frac{n}{2\hat{\theta}_2} - \frac{1}{2\hat{\theta}_2^2}\sum_{i=1}^{n}(x_i-\hat{\theta}_1)^2$$

$$\hat{\theta}_2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\hat{\theta}_1)^2$$

So, MLE for θ_2 is Sample variance.

Question 2

Let $x_1, x_2 \ldots x_n$ be Random sample from $B(m, \theta)$ distribution, where $\theta \in \Theta = (0,1)$ is unknown and m is known +ve ~~Integ~~ integer, compute values of θ using MLE.

Answer

To find MLE of θ for Random sample $x_1, x_2 \ldots x_n$ from a Bernaulli Distribution with parameter θ and θ known in the likelihood for this scenario is,

$$L\left(\frac{\theta,}{x_1 x_2 \ldots x_n}\right) = \prod_{i=1}^{n} P\left(\frac{x_i = x_i}{\theta}\right)$$

Since X_i follows a Bernoulli Distribution

$$P\left(X_i = \frac{x_i}{\theta}\right)$$

$$= \theta^{x_i}(1-\theta)^{m-x_i} \text{ for Each } i$$

Taking log on both sides

$$\ln\left(\frac{L\theta,}{x_1 x_2 \ldots x_n}\right) = \sum_{i=1}^{n} \ln\left(\theta^{x_i}(1-\theta)^{m-x_i}\right)$$

$$= \sum_{i=1}^{n}\left(x_i \ln\theta + (m-x_i)\ln(1-\theta)\right)$$

No differentiate with respect to θ and set to zero.

$$\frac{d}{d\theta}\left(\ln L\left(\frac{\theta}{x_1 x_2 \ldots x_n}\right)\right) = 0$$

$$\sum_{i=1}^{n}\left(\frac{x_i}{\theta} - \frac{m - x_i}{1-\theta}\right) = 0$$

$$\sum_{i=1}^{n}\frac{x_i}{\theta} = \frac{n \cdot m - \sum_{i=1}^{n} x_i}{1-\theta}$$

$$\therefore \quad \theta = \sum_{i=1}^{n}\frac{x_i}{n \cdot m}$$

So, Max likelihood estimator for θ

$$\hat{\theta}_{MLE} = \sum_{i=1}^{n}\frac{x_i}{n \cdot m}$$