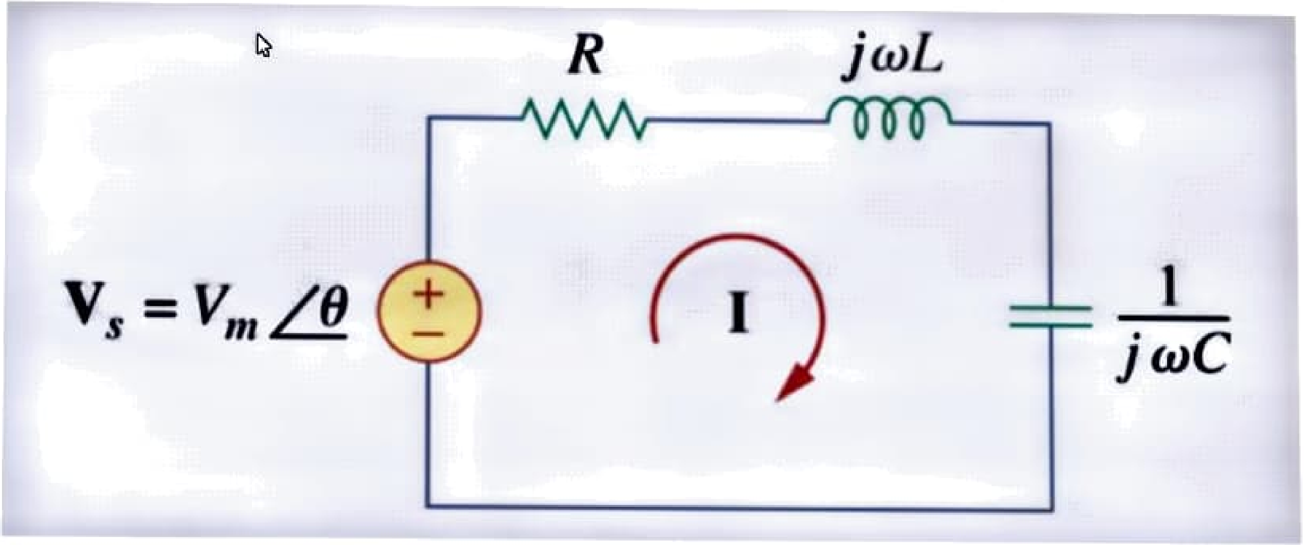
Introduction

- Resonance is a condition in an RLC circuit in which the capacitive and reactive reactance are equal in magnitude, thereby resulting in a purely resistive impedance.

$X_L = X_C$

- Resonance circuits are useful for constructing filters and used in many application.

Series Resonance Circuit



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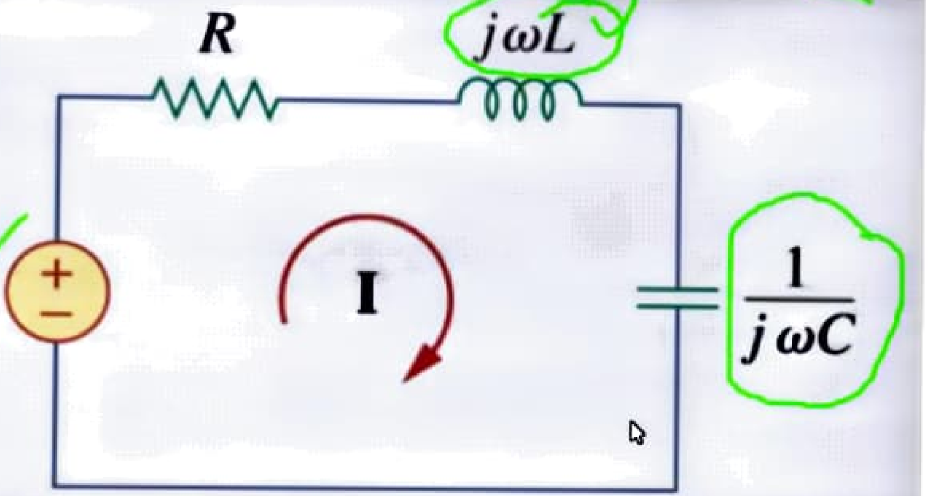


$$\mathbf{V}_s = V_m \angle\theta$$

At Resonance

$\boxed{Z = R}$

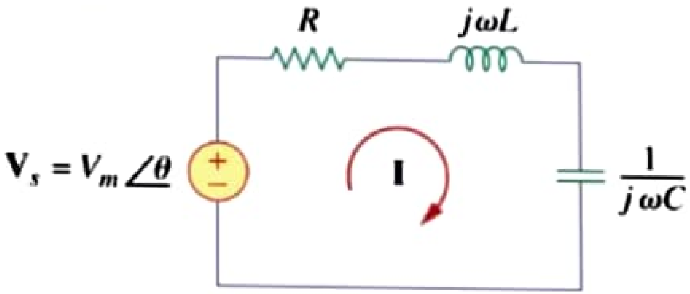
- At resonance, the impedance consists only resistive component R.
- The value of current will be maximum since the total impedance is minimum.
- The voltage and current are in phase.
- Maximum power occurs at resonance since the power factor is unity.

Series Resonance

Total impedance of series RLC Circuit is

$$Z_{Total} = R + jX_L - jX_C$$

$$Z_{Total} = R + j(X_L - X_C)$$



At resonance

$$X_L = X_C$$

The impedance now reduce to

$$Z_{Total} = R$$

The current at resonance

$$I_m = \frac{V_s}{Z_{Total}} = \frac{V_m}{R}$$

Resonance Frequency

(f_0)

Resonance frequency is the frequency where the condition of resonance occur.

Also known as center frequency.

Resonance frequency

$$\omega_o = \frac{1}{\sqrt{LC}} \text{ rad/s}$$

$$f_o = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

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Half-power Frequency

Half-power frequencies is the frequency when the magnitude of the output voltage or current is decrease by the factor of 1 / √2 from its maximum value.

Also known as cutoff frequencies.

$$\omega_1 = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)} \text{rad/s} \qquad \omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)} \text{rad/s}$$

At Resonance

$Z = R + j(X_L - X_C)$ 0

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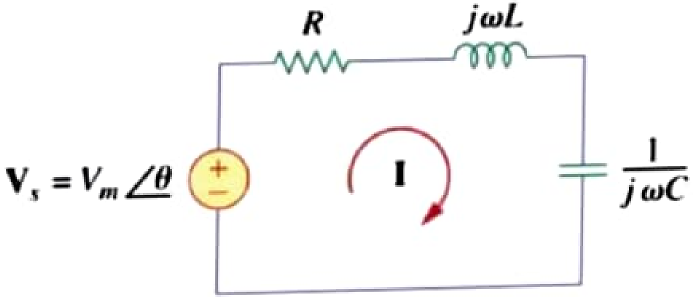

At resonance

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Resonance frequency

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$$\omega_0 = 2\pi f_0$$

$$f_0$$

$$\frac{1}{2\pi} \times (\omega_0) = f_0$$

$$\frac{1}{2\pi \times \sqrt{LC}} = f_0$$

The impedance of the circuit is

$$Z = R + j(X_L - X_C)$$

At resonance the imaginary part is zero

$$\begin{aligned}
X_L - X_C &= 0 \\
\omega_r L - \frac{1}{\omega_r C} &= 0 \\
\omega_r L &= \frac{1}{\omega_r C} \\
\omega_r^2 &= \frac{1}{LC} \\
\omega_r &= \frac{1}{\sqrt{LC}} \; radians/sec
\end{aligned}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}} \; Hz$$

The impedance of the circuit is

$$Z = R + j(X_L - X_C)$$

$2\pi f_r$

At resonance the imaginary part is zero

$$\begin{aligned}
\boxed{X_L - X_C} &= 0 \\
\omega_r L - \frac{1}{\omega_r C} &= 0 \\
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\omega_r^2 &= \frac{1}{LC} \\
\boxed{\omega_r = \frac{1}{\sqrt{LC}} \; radians/sec}
\end{aligned}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}} \; Hz$$

fr (or) fo

At resonance frequency f_r $\boxed{Z = R}$ and current is I_m
At half power frequencies f_1 and f_2 the current is $\frac{I_m}{\sqrt{2}}$

$$Z = \sqrt{2}R$$

$$Z = R + jX_L - JX_C = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\begin{aligned}
\sqrt{R^2 + (X_L - X_C)^2} &= \sqrt{2}R \\
R^2 + (X_L - X_C)^2 &= 2R^2 \\
(X_L - X_C)^2 &= R^2 \\
X_L - X_C &= R
\end{aligned}$$

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At resonance frequency f_r $\boxed{Z = R}$ and current is I_m
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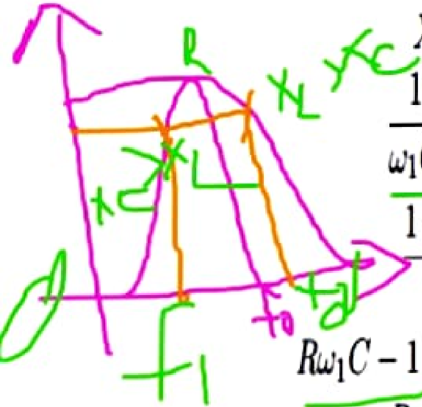
X_L - X_C &= R
\end{aligned}$$

At frequency ω_1 the circuit impedance $X_C > X_L$



$$
\begin{aligned}
X_C - X_L &= R\\
\frac{1}{\omega_1 C} - \omega_1 L &= R\\
\frac{1-\omega_1^2 LC}{\omega_1 C} &= R\\
R\omega_1 C - 1 + \omega_1^2 LC &= 0\\
\boxed{\omega_1^2 + \frac{R}{L}\omega_1 - \frac{1}{LC} = 0}
\end{aligned}
$$

$$a = 1, \quad b = \frac{R}{L}, \quad c = -\frac{1}{LC}$$

$$\omega_1 = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}}}{2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

Frequency is always positive

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$\omega_1 = 2\pi f_1$

In terms of frequency f_1

$$f_1 = \frac{1}{2\pi}\left[-\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right]$$

At frequency ω_2 the circuit impedance $X_L > X_C$

$$
\begin{aligned}
X_L - X_C &= R \\
\omega_2 L - \frac{1}{\omega_2 C} - &= R \\
\frac{\omega_2^2 LC - 1}{\omega_2 C} &= R \\
\omega_2^2 LC - R\omega_2 C - 1 &= 0 \\
\omega_2^2 - \frac{R}{L}\omega_2 - \frac{1}{LC} &= 0
\end{aligned}
$$

$$a = 1, \quad b = -\frac{R}{L}, \quad c = -\frac{1}{LC}$$

$$\omega_2 = \frac{\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}}}{2} = \frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

Half-power Frequency

Half-power frequencies is the frequency when the magnitude of the output voltage or current is decrease by the factor of 1 / √2 from its maximum value.

Also known as cutoff frequencies.

$$\omega_1 = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)} \text{rad/s} \qquad \omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)} \text{rad/s}$$

Half-power Frequency

Half-power frequencies is the frequency when the magnitude of the output voltage or current is decrease by the factor of $1 / \sqrt{2}$ from its maximum value.

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$$\omega_1 = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)} \text{rad/s} \qquad \omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)} \text{rad/s}$$

Relation between ω_r , ω_1 and ω_2

$\omega_1 \times \omega_2 =$

$$= \left[\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right] \times \left[\frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right]$$

$$= \frac{1}{LC}$$

$$\omega_r = \sqrt{\frac{1}{LC}}$$

$$\omega_r^2 = \frac{1}{LC} = \omega_1.\omega_2$$

$$\omega_r = \sqrt{\omega_1.\omega_2}$$

$$f_r = \sqrt{f_1 f_2}$$

Bandwidth, β

Bandwidth, β is define as the difference between the two half power frequencies.
The width of the response curve is determine by the bandwidth.

$$\beta = (\omega_{c2} - \omega_{c1})\text{rad/s}$$

$$\beta = \frac{R}{L}\text{rad/s}$$

Relation between ω_r , ω_1 and ω_2

$$
\begin{aligned}
\omega_1 \times \omega_2 &= \\
&= \left[\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right] \times \left[\frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right] \\
&= \frac{1}{LC}
\end{aligned}
$$

$$
\begin{aligned}
\omega_r &= \sqrt{\frac{1}{LC}} \\
\omega_r^2 &= \frac{1}{LC} = \omega_1 . \omega_2 \\
\omega_r &= \sqrt{\omega_1 . \omega_2}
\end{aligned}
$$

$$f_r = \sqrt{f_1 f_2}$$

Wa

Relation between ω_r , ω_1 and ω_2

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Wa

Relation between ω_r , ω_1 and ω_2

$$\omega_1 \times \omega_2 =$$

$$= \left[+\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right] \times \left[\frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right]$$

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$$\boxed{\omega_r = \sqrt{\frac{1}{LC}}}$$

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$$f_r = \sqrt{f_1 f_2}$$

$$\omega_r = \frac{1}{\sqrt{LC}}$$

$$2\pi f_r = \frac{1}{\sqrt{LC}}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

ω_0

Relation between ω_r, ω_1 and ω_2

6 marks

$$\omega_1 \times \omega_2 =$$
$$= \left[\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right] \times \left[\frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right]$$
$$= \frac{1}{LC}$$

$$\boxed{\omega_r = \sqrt{\frac{1}{LC}}}$$
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$$\omega_r = \frac{1}{\sqrt{LC}}$$
$$2\pi f_r = \frac{1}{\sqrt{LC}}$$
$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

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$$\beta = \frac{R}{L}\text{rad/s}$$

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$$\beta = (\omega_{c2} - \omega_{c1})\text{rad/s}$$

$$\beta = \frac{R}{L}\text{rad/s}$$

Relation between Bandwidth Quality factor

Bandwidth is $B = \omega_2 - \omega_1$

$$
\begin{aligned}
B &= \omega_2 - \omega_1 \\
&= \left[\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right] - \left[\frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right] + \\
&= \frac{R}{L} \ \textit{radians}
\end{aligned}
$$

$$B = \frac{1}{2\pi} \cdot \frac{R}{L} = \frac{R}{2\pi L} \ Hz$$

$$\omega_r = \sqrt{\frac{1}{LC}}$$

$$\omega_1 = -\frac{B}{2} + \sqrt{\left(\frac{B}{2}\right)^2 + \omega_r^2}$$

$$\omega_2 = \frac{B}{2} + \sqrt{\left(\frac{B}{2}\right)^2 + \omega_r^2}$$

Relation between Bandwidth Quality factor

Bandwidth is $B = \omega_2 - \omega_1$

$$B = \omega_2 - \omega_1$$

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$$B = \frac{R}{L} \text{ radians}$$

$$B = \frac{1}{2\pi} \cdot \frac{R}{L} = \frac{R}{2\pi L}\ Hz$$

$$\omega_r = \sqrt{\frac{1}{LC}}$$

$$\omega_1 = -\frac{B}{2} + \sqrt{\left(\frac{B}{2}\right)^2 + \omega_r^2}$$

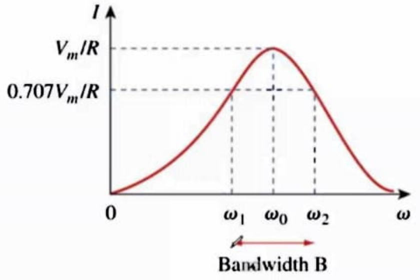
$$\omega_2 = \frac{B}{2} + \sqrt{\left(\frac{B}{2}\right)^2 + \omega_r^2}$$

$$+ \frac{R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\frac{R + R}{2L} = \frac{2R}{2L}$$

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

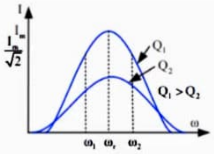
Current Response Curve



Quality factor Q

The quality factor Q is defined as the ratio of resonant frequency to the bandwidth

$$Q = \frac{\omega_r}{B} = \frac{\sqrt{\frac{1}{LC}}}{\frac{R}{L}} = \frac{1}{R}\sqrt{\frac{L}{C}}$$



Plot of frequency verses Q

IMPORTANT FORMULAE

Parameter	Formula
At resonance	$Z = R, \quad X_L = X_C \quad Current\ I_r = \frac{E}{R}$
Resonance	$\omega_r = \frac{1}{\sqrt{LC}} \qquad f_r = \frac{1}{2\pi\sqrt{LC}}$
Half power frequency	$\omega_1 = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \qquad f_1 = \frac{\omega_1}{2\pi}$ $\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \qquad f_2 = \frac{\omega_2}{2\pi}$
Half power frequency	$\omega_1 = \frac{-B}{2} + \sqrt{\left(\frac{B}{2}\right)^2 + \omega_r^2} \qquad f_1 = \frac{\omega_1}{2\pi}$ $\omega_2 = \frac{B}{2} + \sqrt{\left(\frac{B}{2}\right)^2 + \omega_r^2} \qquad f_2 = \frac{\omega_2}{2\pi}$
Bandwidth	$B = \omega_2 - \omega_1 = \frac{R}{L}$ Radians $B = f_2 - f_1 = \frac{R}{2\pi L}$ Hz
Quality factor	$Q = \frac{\omega_r L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$
$\omega_r\ \omega_1\ \omega_2$	$\omega_r = \sqrt{\omega_1\omega_2} \quad OR\ f_r = \sqrt{f_1 f_2}$
Voltage across capacitor/inductor	$V_{Lr} = V_{Cr} = IX_{Lr}$
Value of inductor at f_1, f_2	$L_1 = \frac{1}{\omega_1^2 C} - \frac{R}{\omega_1}, \ L_2 = \frac{1}{\omega_2^2 C} + \frac{R}{\omega_2}$
Value of capacitor at f_1, f_2	$C_1 = \frac{1}{\omega_1^2 L} - \frac{R}{\omega_1}, \ C_2 = \frac{1}{\omega_2^2 L} + \frac{R}{\omega_2}$

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At resonance	$Z = R,\ X_L = X_C$ $Current\ I_r = \frac{V}{R}$
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$\omega_r\ \omega_1\ \omega_2$	$\omega_r = \sqrt{\omega_1 \omega_2}$ $OR\ f_r = \sqrt{f_1 f_2}$
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Value of inductor at f_1, f_2	$L_1 = \frac{1}{\omega^2 C} - \frac{R}{\omega},\ L_2 = \frac{1}{\omega^2 C} + \frac{R}{\omega}$
Value of capacitor at f_1, f_2	$C_1 = \frac{1}{\omega^2 L} - \frac{R}{\omega},\ C_2 = \frac{1}{\omega^2 L} + \frac{R}{\omega}$

IMPORTANT FORMULAE

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At resonance	$Z = R,\ X_L = X_C\ \ Current\ I_r = \frac{E}{R}$
Resonance	$\omega_r = \frac{1}{\sqrt{LC}}\quad f_r = \frac{1}{2\pi\sqrt{LC}}$
Half power frequency	$\omega_1 = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\quad f_1 = \frac{\omega_1}{2\pi}$ $\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\quad f_2 = \frac{\omega_2}{2\pi}$
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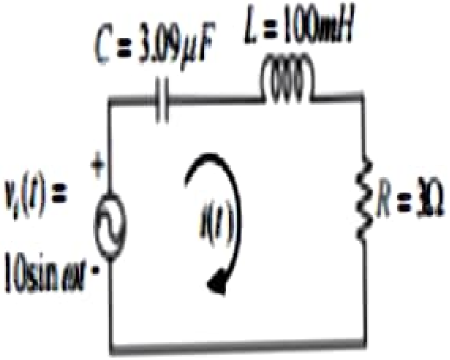
Quality factor	$Q = \frac{\omega_r L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$
$\omega_r\ \omega_1\ \omega_2$	$\omega_r = \sqrt{\omega_1\omega_2}\quad OR\ f_r = \sqrt{f_1 f_2}$
Voltage across capacitor/inductor	$V_{Lr} = V_{Cr} = IX_{Lr}$
Value of inductor at f_1, f_2	$L_1 = \frac{1}{\omega^2 C} - \frac{R}{\omega},\ L_2 = \frac{1}{\omega^2 C} + \frac{R}{\omega}$
Value of capacitor at f_1, f_2	$C_1 = \frac{1}{\omega^2 L} - \frac{R}{\omega},\ C_2 = \frac{1}{\omega^2 L} + \frac{R}{\omega}$

Selectivity: is property of circuit in which the circuit is allowed to select a band of frequencies between f_1 and f_2.

1: For the circuit shown in Figure Find (a) The resonant and half power frequencies (b) Calculate the quality factor and bandwidth (c) Determine the amplitude of the current at

$\omega_0, \omega_1, \omega_2$



$LC = 100 \times 10^{-3} \times 3.09 \times 10^{-6} = 3.09 \times 10^{-7}$

The resonant frequency ω_o is

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{3.09 \times 10^{-7}}} = 1800\ rad/s$$

The half power frequency ω_1, ω_2 is

$$\frac{R}{2L} = \frac{3}{2 \times 100 \times 10^{-3}} = 15$$

$$\begin{aligned}
\omega_1 &= -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \\
&= -15 + \sqrt{225 + \frac{1}{3.09 \times 10^{-7}}} \\
&= -15 + \sqrt{225 + 3.236 \times 10^6} \\
&= -15 + 1798.96 = 1784\ rad/s \\
\omega_2 &= \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \\
&= 15 + 1798.96 = 1814\ rad/s
\end{aligned}$$

Frequency in Hz is

$$\begin{aligned}
f_1 &= \frac{\omega_1}{2\pi} = \frac{1784}{2\pi} = 284\ Hz \\
f_2 &= \frac{\omega_2}{2\pi} = \frac{1814}{2\pi} = 289\ Hz
\end{aligned}$$

$$LC = 100 \times 10^{-3} \times 3.09 \times 10^{-6} = 3.09 \times 10^{-7}$$

The resonant frequency ω_0 is

W_r (or) ω_0

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{3.09 \times 10^{-7}}} = 1800\ rad/s$$

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

The half power frequency ω_1, ω_2 is

$$\frac{R}{2L} = \frac{3}{2 \times 100 \times 10^{-3}} = 15$$

$$\begin{aligned}
\omega_1 &= -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \\
&= -15 + \sqrt{225 + \frac{1}{3.09 \times 10^{-7}}} \\
&= -15 + \sqrt{225 + 3.236 \times 10^6} \\
&= -15 + 1798.96 = 1784\ rad/s \\
\omega_2 &= \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \\
&= 15 + 1798.96 = 1814\ rad/s
\end{aligned}$$

Frequency in Hz is

$$\begin{aligned}
f_1 &= \frac{\omega_1}{2\pi} = \frac{1784}{2\pi} = 284\ Hz \\
f_2 &= \frac{\omega_2}{2\pi} = \frac{1814}{2\pi} = 289\ Hz
\end{aligned}$$

Bandwidth B is

$$B = \omega_2 - \omega_1 = 1814 - 1784 = 30\ rad/s$$

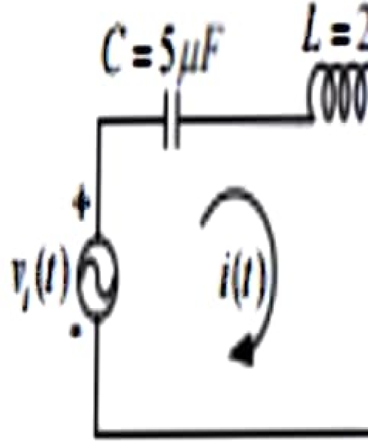
Also B is

$$B = \frac{R}{L} = \frac{3}{100 \times 10^{-3}} = 30\ rad/s$$

Quality factor Q is

$$Q = \frac{\omega_0}{B} = \frac{1800}{30} = 60$$

2: For the circuit shown in Figure find the resonant frequency, quality factor and bandwidth for the circuit. Determine the change in Q and the bandwidth if R is changed from $R = 2\ \Omega$ to $R = 0.4\ \Omega$



Homework Problem

A series resonant RLC circuit has resonant frequency of 80 K rad/sec and a quality factor of 8. Find the bandwidth, the upper cutoff frequency and lower cutoff frequency

A coil of resistance R=20 Ω and inductance L=0.2 H is connected in series with a capacitance across 230 V supply Find (a) the value of the capacitance for which resonance occurs at 100 Hz frequency (b) the current through and voltage across the capacitor (c) Q factor of the coil

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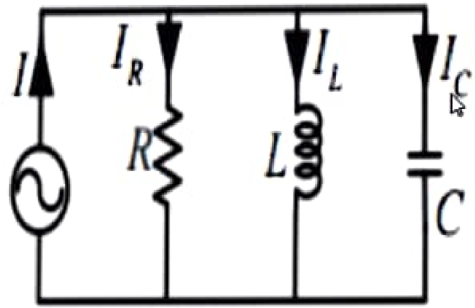
PARALLEL RESONANCE

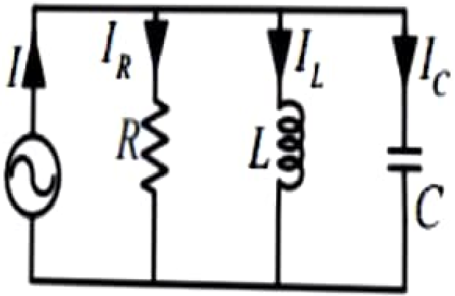


The total admittance of the circuit is

$$Y = G + j\left(\omega C - \frac{1}{\omega L}\right)$$

When the circuit is at resonance the imaginary part is zero

PARALLEL RESONANCE



The total admittance of the circuit is

$$Y = G + j\left(\omega C - \frac{1}{\omega L}\right)$$

When the circuit is at resonance the imaginary part is zero

$$\left(\omega_r C - \frac{1}{\omega_r L}\right) = 0$$

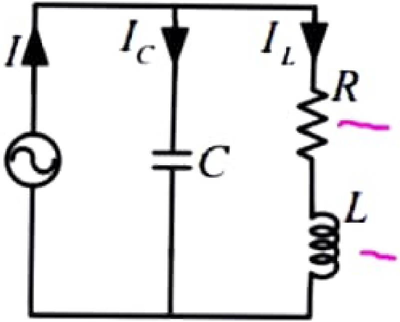
$$\omega_r^2 = \frac{1}{LC}$$

$$\omega_r = \sqrt{\frac{1}{LC}}$$

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

PRACTICAL PARALLEL RESONANT CIRCUIT

Consider a parallel circuit consists of resistor and inductor in one branch and capacitor in another branch which is as shown in Figure



The impedance of the inductor branch is

$$Z_L = R + j\omega L$$

The admittance of the inductor branch is

$$Y_L = \frac{1}{Z_L} = \frac{1}{R + j\omega L}$$

$$Y_L = \frac{1}{Z_L} = \frac{1}{R + j\omega L}$$

$$= \frac{1}{R + j\omega L} \times \frac{R - j\omega L}{R - j\omega L} = \frac{R - j\omega L}{R^2 + \omega^2 L^2}$$

Similarly the impedance of the capacitor branch is

$$Z_C = \frac{1}{j\omega C}$$

The admittance of the capacitor branch is

$$Y_C = \frac{1}{Z_C} = j\omega C$$

Total admittance of the circuit is

$$Y = Y_L + Y_C = \frac{R - j\omega L}{R^2 + \omega^2 L^2} + j\omega C$$

Separating real and imaginary parts

$$Y = \frac{R}{R^2 + \omega^2 L^2} + j\left[\omega C - \frac{\omega L}{R^2 + \omega^2 L^2}\right]$$

$$\omega_r C - \frac{\omega_r L}{R^2 + \omega_r^2 L^2} = 0$$

$$\omega_r C = \frac{\omega_r L}{R^2 + \omega_r^2 L^2}$$

$$R^2 + \omega_r^2 L^2 = \frac{\omega_r L}{\omega_r C} = \frac{L}{C}$$

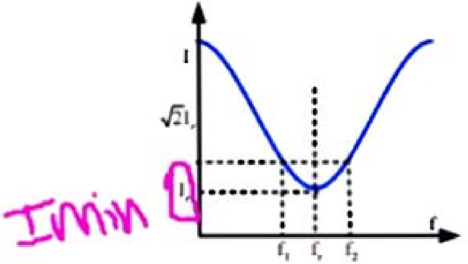
$$\omega_r^2 L^2 = \frac{L}{C} - R^2$$

$$\omega_r^2 = \frac{\frac{L}{C} - R^2}{L^2} = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\omega_r = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

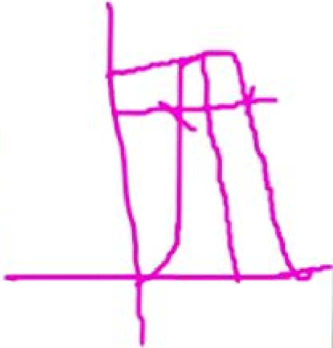
$$f_r = \frac{1}{\;}\sqrt{\frac{1}{\;} - \frac{R^2}{\;}}$$

The Frequency Response of Parallel Graph is as shown in Figure

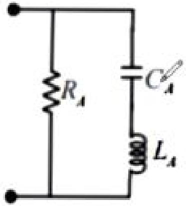


plot of parallel resonant circuit

From the figure it is observed that
the current is minimum at resonance. The parallel circuit is called as a rejector circuit. The circuit impedance is maximum at the resonance. The half power frequencies are at $\sqrt{2}I_r$.



Calculate the resonant frequency of the circuit shown in Figure



General parallel resonant circuit

$$Z_{LC} = jX_L + jX_C = j\omega L + \frac{1}{j\omega C} = \frac{1-\omega^2 LC}{j\omega C}$$

$$Z_{LC} = \frac{j(\omega^2 LC - 1)}{\omega C}$$

$$\omega^2 LC = 1$$

$$\omega^2 = \frac{1}{\sqrt{LC}}$$

At Resonance

- At resonance, the impedance consists only conductance G.
- The value of current will be minimum since the total admittance is minimum.
- The voltage and current are in phase.

Parameters in Parallel Circuit

Parallel resonant circuit has same parameters as the series resonant circuit.

Resonance frequency

$$\omega_o = \frac{1}{\sqrt{LC}} \text{ rad/s}$$

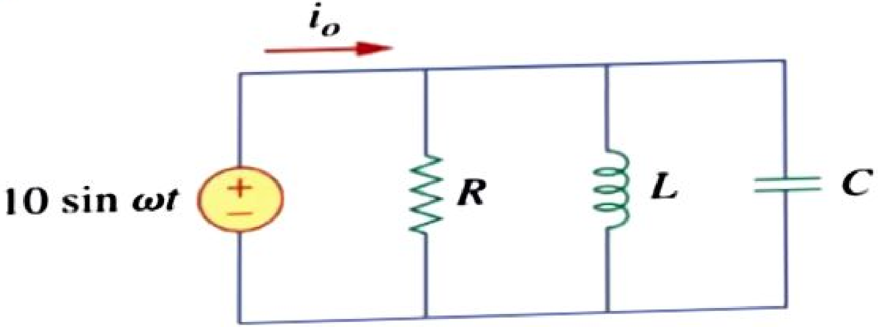
Half-power frequencies

$$\omega_1 = \frac{-1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \left(\frac{1}{LC}\right)} \text{ rad/s} \qquad \omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \left(\frac{1}{LC}\right)} \text{ rad/s}$$

Example

If R=8kΩ, L=0.2mH and C=8μF, calculate

- ω_o
- Q and β
- ω_1 and ω_2
- Power dissipated at ω_o, ω_1 and ω_2.



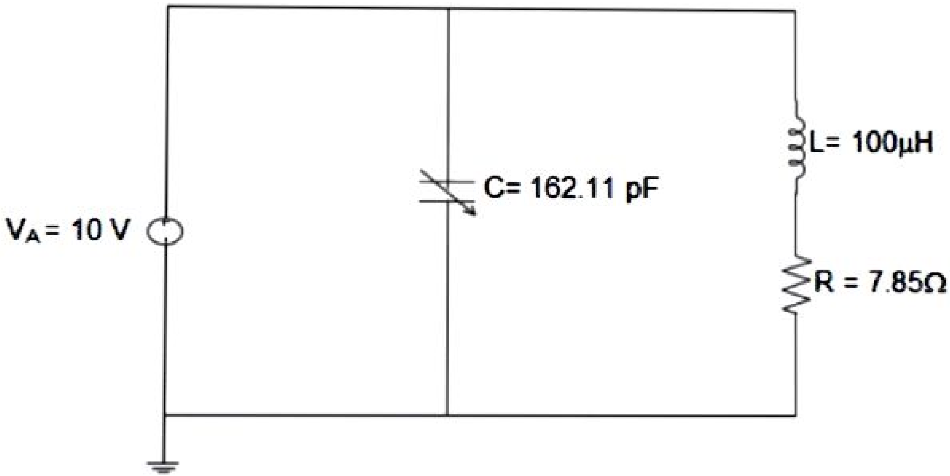
Problem

2

- A parallel resonant circuit has $R=100k\Omega$, $L=25mH$ and $C=5nF$. Calculate
  - F_o
  - F_1 and F_2
  - Q
  - BW

3

Find F0,F1,F2,Xl,Xc,BW,current through L and C.



3

Find F0,F1,F2,Xl,Xc,BW,current through L and C.

Q