

The total resistance coefficient  $C_T$  consists of four components:

$$C_T = C_F + C_A + C_{AA} + C_R \quad (2)$$

where  $C_F$  is the frictional resistance coefficient,  $C_A$  is the incremental resistance coefficient,  $C_{AA}$  is the air resistance coefficient and  $C_R$  is the residual resistance coefficient.

Once the ship's total resistance  $R_T$  is determined, the effective power of the ship,  $P_E$ , can be obtained according to the following equation:

$$P_E = R_T \cdot V_{sw} \quad (3)$$

Then, the required brake power of the ship,  $P_B$ , can be determined from  $P_E$  according to the following equations:

$$P_B = \frac{P_E}{\eta_T} \quad (4)$$

$$\eta_T = \eta_H \cdot \eta_O \cdot \eta_R \cdot \eta_S \quad (5)$$

where  $\eta_T$  is the total efficiency from  $P_B$  to  $P_E$ ,  $\eta_H$  is the hull efficiency,  $\eta_O$  is the propeller open water efficiency,  $\eta_R$  is the relative rotative efficiency and  $\eta_S$  is the shaft efficiency.

Finally, the ship's FCR,  $r$  (MT/h), can be determined as follows:

$$r = P_B \cdot SFOC \cdot 10^{-6} \quad (6)$$

where SFOC is the Specific Fuel Oil Consumption (SFOC) of the ship's main engine in g/kWh, which varies with the main engine load and can be obtained from the main engine performance documents.

Once the ship's main dimensions, payload  $L$  and SWS  $V_{sw}$  are known, the values of  $S$ ,  $C_F$ ,  $C_A$ ,  $C_{AA}$ ,  $C_R$ ,  $\eta_H$ ,  $\eta_O$ ,  $\eta_R$  and  $\eta_S$  can be calculated based on empirical formulae developed by Kristensen and Lützen [44]. Therefore, for a given ship carrying a certain amount of cargo, we can estimate the FCR of the ship at various SWSSs according to Equations (1)–(6).

### 3.2.2. The Speed Correction Model

To define the sailing time function, we need first to estimate the involuntary speed loss due to added resistance in wind and waves under the assumption that the ship's brake power remains constant. An approximate method developed by Kwon [45] can be used to achieve this goal. Kwon's method is easy and practical to use, as it depends on only a few parameters. Meanwhile, this method shows good accuracy in comparison to more extensive calculation methods [45]. According to Kwon [45], the percentage of speed loss can be expressed as:

$$\frac{\Delta V}{V_{sw}} 100\% = C_\beta C_U C_{Form} \quad (7)$$

from which, by using the relationship  $\Delta V = V_{sw} - V_w$ , it follows that the ship speed in the selected weather (wind and irregular waves) conditions may be expressed as:

$$V_w = V_{sw} - \left( \frac{\Delta V}{V_{sw}} 100\% \right) \frac{1}{100\%} V_{sw} = V_{sw} - \left( C_\beta C_U C_{Form} \right) \frac{1}{100\%} V_{sw} \quad (8)$$

where:

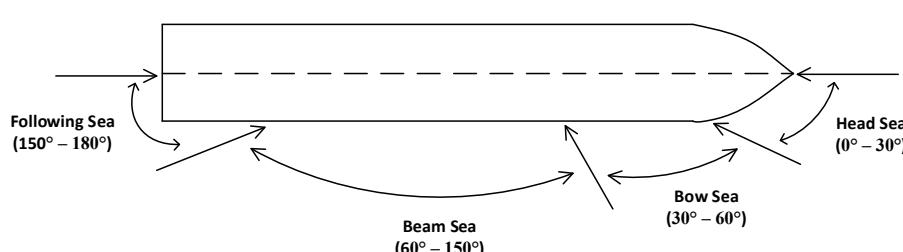
$V_w$ : The ship's STW in the selected weather (wind and irregular waves) conditions, given in m/s;

$\Delta V = V_{sw} - V_w$ : Absolute speed loss, given in m/s;

$C_\beta$ : Direction reduction coefficient, which is a non-dimensional number, dependent on the weather direction angle (with respect to the ship's bow)  $\theta$  and the Beaufort number  $BN$ , as shown in Figure 2 and Table 2;

$C_U$ : Speed reduction coefficient, which is a non-dimensional number, dependent on the ship's block coefficient,  $C_B$ , the loading conditions and the Froude number,  $Fn$ , as shown in Table 3;

$C_{Form}$ : Ship form coefficient, which is a non-dimensional number, dependent on the ship type, the Beaufort number,  $BN$ , and the ship displacement,  $\nabla$ , in  $m^3$ , as shown in Table 4.



**Figure 2.** Weather direction.

**Table 2.** Direction reduction coefficient  $C_\beta$ .

Weather Direction	Weather Direction Angle (with Respect to the Ship's Bow) $\theta$	Direction Reduction Coefficient $C_\beta$
Head sea (irregular waves) and wind	0°–30°	$2C_\beta = 2$
Bow sea (irregular waves) and wind	30°–60°	$2C_\beta = 1.7 - 0.03(BN - 4)^2$
Beam sea (irregular waves) and wind	60°–150°	$2C_\beta = 0.9 - 0.06(BN - 6)^2$
Following sea (irregular waves) and wind	150°–180°	$2C_\beta = 0.4 - 0.03(BN - 8)^2$

**Table 3.** Speed reduction coefficient  $C_U$ .

Block Coefficient $C_B$	Ship Loading Conditions	Speed Reduction Coefficient $C_U$
0.55	Normal	$1.7 - 1.4Fn - 7.4(Fn)^2$
0.60	Normal	$2.2 - 2.5Fn - 9.7(Fn)^2$
0.65	Normal	$2.6 - 3.7Fn - 11.6(Fn)^2$
0.70	Normal	$3.1 - 5.3Fn - 12.4(Fn)^2$
0.75	Loaded or normal	$2.4 - 10.6Fn - 9.5(Fn)^2$
0.80	Loaded or normal	$2.6 - 13.1Fn - 15.1(Fn)^2$
0.85	Loaded or normal	$3.1 - 18.7Fn + 28.0(Fn)^2$
0.75	Ballast	$2.6 - 12.5Fn - 13.5(Fn)^2$
0.80	Ballast	$3.0 - 16.3Fn - 21.6(Fn)^2$
0.85	Ballast	$3.4 - 20.9Fn + 31.8(Fn)^2$

**Table 4.** Ship form coefficient  $C_{Form}$ .

Type of (Displacement) Ship	Ship form Coefficient $C_{Form}$
All ships (except container ships) in loaded loading conditions	$0.5BN + BN^{6.5} / (2.7\nabla^{2/3})$
All ships (except container ships) in ballast loading conditions	$0.7BN + BN^{6.5} / (2.7\nabla^{2/3})$
Container ships in normal loading conditions	$0.7BN + BN^{6.5} / (22.0\nabla^{2/3})$

The weather direction angle (with respect to the ship's bow)  $\theta$  can be calculated according to the following logical statement [46]:

$$\theta = \begin{cases} |\varphi - \alpha - 360^\circ|, & \text{IF } \varphi - \alpha > 180^\circ \\ |\varphi - \alpha + 360^\circ|, & \text{IF } \varphi - \alpha < -180^\circ \\ |\varphi - \alpha|, & \text{Otherwise} \end{cases} \quad (9)$$

where  $\varphi$  is the wind direction angle (with respect to the True North) and  $\alpha$  is the ship heading angle (with respect to the True North). Here, the wave direction angle is assumed to be the same as the wind direction angle, which is true in most instances of surface waves [47].

The Beaufort number,  $BN$ , is an empirical measure that relates wind speed to observed conditions at sea or on land. It is defined by a range of wind speeds at the standard height. Please refer to Townsin et al. [48] and Yang et al. [9] for more details.

The Froude number,  $Fn$ , can be determined by the following equation:

$$Fn = V_{sw} / \sqrt{L_{pp}g} \quad (10)$$

where  $L_{pp}$  is the ship length between perpendiculars in m and  $g$  is the acceleration of gravity in  $\text{m/s}^2$ .

The ship's block coefficient,  $C_B$ , and displacement,  $\nabla$ , are associated with the ship's main dimensions and payload,  $L$ . Detailed formulae for calculating  $C_B$  and  $\nabla$  can be seen in MAN Diesel & Turbo [49].

As can be seen from the above formulae and tables, for a given ship with a SWS  $V_{sw}$ , its corresponding STW  $V_w$  in selected weather (wind and irregular waves) conditions can be readily estimated once the following parameters are available: ship type; ship's main dimensions; ship's loading conditions; ship payload,  $L$ ; ship heading angle,  $\alpha$ ; wind direction angle,  $\varphi$ ; and Beaufort number,  $BN$ .

For the purpose of safety, the ship has a critical STW (maximum allowed STW) when sailing in wind and waves. When  $V_w$  is greater than the critical STW, the ship must be slowed down, which is known as the voluntary speed reduction. In this paper, the following equations are used to calculate the critical STW [50,51]:

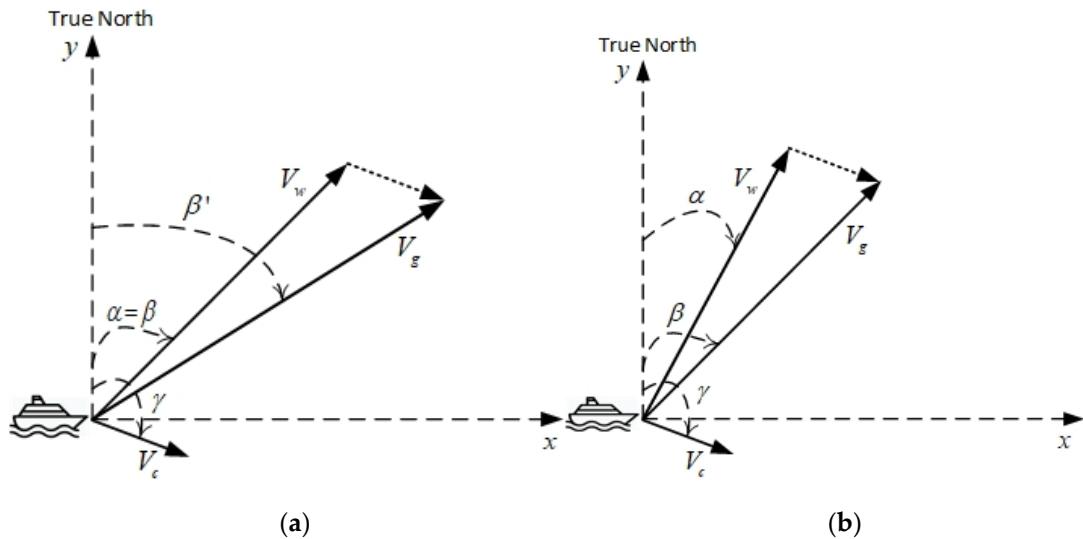
$$V_c = \exp \{0.13 \cdot [f(\theta) - h]^{1.6}\} + g(\theta) \quad (11)$$

$$f(\theta) = 12.0 + 1.4 \times 10^{-4} \times \left( \frac{\pi \cdot \theta}{180} \right)^{2.3} \quad (12)$$

$$g(\theta) = 7.0 + 4.0 \times 10^{-4} \times \left( \frac{\pi \cdot \theta}{180} \right)^{2.3} \quad (13)$$

where  $V_c$  is the critical STW in knots,  $h$  is the significant wave height in m and  $\pi \cdot \theta / 180$  is the weather direction angle (with respect to the ship's bow) in radian.

To define the sailing time function more accurately, we need to correct the STW to the SOG by considering the influence of ocean currents. Actually, ocean currents can affect both the SOG and the ship course. Let  $\beta$  denote the expected ship course angle (with respect to the True North), which can be obtained based on the geographical coordinates of two adjacent waypoints along the sailing route, as explained in Section 3.1. During the voyage, the ship heading angle,  $\alpha$ , is a manipulated parameter. If  $\alpha$  is set equal to  $\beta$ , the ship will sail in an actual course of  $\beta'$  with an actual speed of  $V_g$  due to the existence of ocean currents, as shown in Figure 3a. Therefore, in practice, the crew needs to choose a ship heading angle which is different from the expected ship course angle to correct the yaw caused by ocean currents and guarantee that the ship can sail in the expected course, as shown in Figure 3b. In Figure 3,  $V_c$  is the current speed, given in m/s;  $\gamma$  is the current direction angle (with respect to the True North); and  $V_g$  is the actual sailing speed, namely the SOG, given in m/s.



**Figure 3.** Variations in speed and course due to ocean currents: (a)  $\alpha$  is set equal to  $\beta$  and (b)  $\alpha$  is set different from  $\beta$  to guarantee that the ship can sail in the expected course.

With the assistance of Figure 3b, we can develop formulae for calculating the SOG readily. According to the method of vector decomposition, the components of  $V_w$  in the  $x$  direction and the  $y$  direction are  $V_w \sin \alpha$  and  $V_w \cos \alpha$ , respectively. Similarly, the components of  $V_c$  in the  $x$  direction and the  $y$  direction are  $V_c \sin \gamma$  and  $V_c \cos \gamma$ , respectively. Therefore, the components of  $V_g$  in the  $x$  direction and the  $y$  direction can be expressed as Equations (14) and (15), respectively:

$$V_g^x = V_w \sin \alpha + V_c \sin \gamma \quad (14)$$

$$V_g^y = V_w \cos \alpha + V_c \cos \gamma \quad (15)$$

Hence, according to the method of vector synthesis, the SOG  $V_g$  can be determined by the following equation:

$$V_g = \sqrt{(V_g^x)^2 + (V_g^y)^2} \quad (16)$$

And the relationship among  $V_g^x$ ,  $V_g^y$  and  $\beta$  can be expressed as follows:

$$\beta = \arctan\left(\frac{V_g^x}{V_g^y}\right) \quad (17)$$

As can be seen from Equations (14)–(17), to calculate the SOG  $V_g$ , we need first to determine the STW  $V_w$  and the ship heading angle  $\alpha$ . As  $V_w$  is also dependent on  $\alpha$ , determining the value of  $\alpha$  becomes the prerequisite for calculating  $V_g$ . To address this problem, we present a heuristic algorithm, as shown in Section 4.1.

### 3.2.3. The Speed Optimization Model

In this section, a speed optimization model is developed for a fixed ship route between two ports to determine the SWS for each segment in the route, so that the fuel consumption of the ship over the whole voyage is minimized while the ETA is guaranteed. The notations of the speed optimization model are shown in Table 5.

**Table 5.** Notations of the speed optimization model.

Sets and indices	
$n$	Total number of sailing segments
$i$	Index of a segment, $i \in \{1, \dots, n\}$
<b>Parameters</b>	
$ETA$	The ETA at the destination port
$d_i$	Sailing distance in segment $i$ (nmi)
$V_{sw}^{\min}$	Minimum allowed value of the SWS (knots)
$V_{sw}^{\max}$	Maximum allowed value of the SWS (knots)
$V_c^i$	The critical STW in segment $i$ (knots)
<b>Derived variables</b>	
$r_i$	The FCR in segment $i$ (MT/h)
$V_w^i$	The STW in segment $i$ (knots)
$V_g^i$	The SOG in segment $i$ (knots)
<b>Decision variable</b>	
$V_{sw}^i$	The SWS in segment $i$ (knots)

Based on the notations in Table 5, the speed optimization model can be formulated as follows:

$$\min \sum_{i=1}^n (r_i \cdot \frac{d_i}{V_g^i}) \quad (18)$$

subject to

$$\sum_{i=1}^n \frac{d_i}{V_g^i} \leq ETA \quad (19)$$

$$V_{sw}^{\min} \leq V_{sw}^i \leq V_{sw}^{\max} \quad (20)$$

$$V_w^i \leq V_c^i \quad (21)$$

The objective function (18) minimizes the ship fuel consumption over the whole voyage. Constraint (19) ensures that the ship arrival time at the destination port is no later than the ETA. Constraints (20) ensure that the set SWS in each segment is within the ship's feasible SWS interval. Constraint (21) guarantees that the ship's STW does not exceed the critical STW in wind and waves.

#### 4. Solution Approach

It is challenging to use derivative-based methods to solve the speed optimization model presented in Section 3.2.3, since there is a very complicated relationship between the decision variable (i.e.,  $V_{sw}^i$ ) and each of the derived variables (i.e.,  $r_i$ ,  $V_w^i$  and  $V_g^i$ ). Hence, in this paper, a direct search method, namely a genetic algorithm (GA), is used for optimization. Before describing the GA in Section 4.2, a heuristic algorithm for determining the ship heading angle is presented in Section 4.1.

##### 4.1. A Heuristic Algorithm for Determining the Ship Heading Angle

In this paper, as explained in Section 3.2.2, determining the ship heading angle,  $\alpha$ , is the prerequisite for ship speed optimization and needs to be addressed first. In order to address this problem, we present a heuristic algorithm, as shown in Algorithm 1.

**Algorithm 1.** A heuristic algorithm for determining the ship heading angle.

- 
- Input:** Ship type, ship's main dimensions, ship's loading conditions,  $L$ ,  $V_{sw}$ ,  $\beta$ ,  $BN$ ,  $\varphi$ ,  $V_c$  and  $\gamma$ .
- Output:**  $\alpha$  and  $V_w$ .
- Basis:** Normally,  $V_w/V_c > 10$ . Hence, the difference between  $\alpha$  and  $\beta$  will not be too great.
- Step 1.** Replace  $\alpha$  in Equation (9) with  $\beta$  to obtain the relative angle between  $\varphi$  and  $\beta$ . This relative angle is denoted as  $\theta'$ .
- Step 2.** Replace  $\theta$  in Table 2 with  $\theta'$  to determine the weather direction. This weather direction is denoted as  $WD$ .
- Step 3.** Assume that  $\theta$  also belongs to  $WD$ , as the difference between  $\alpha$  and  $\beta$  is small.
- Step 4.** Calculate the value of  $V_w$  according to the equations and tables in Section 3.2.2. In this step, the above  $WD$  is used as the weather direction.
- Step 5.** Once the value of  $V_w$  is determined, we can calculate the value of  $\alpha$  according to Equations (14), (15) and (17).
- Step 6.** Calculate the value of  $\theta$  based on the obtained  $\alpha$  (refer to Equation (9)).
- Step 7.** Determine the weather direction to which  $\theta$  belongs. This weather direction is denoted as  $WD'$ .
- Step 8.** If  $WD$  equals to  $WD'$ , output the obtained  $V_w$  in Step 4 and the obtained  $\alpha$  in Step 5;  
Otherwise, re-execute Step 4 and Step 5 using  $WD'$  and output the obtained  $V_w$  and  $\alpha$ .
- 

As can be seen from Algorithm 1, the proposed heuristic algorithm can be used to determine the values of  $\alpha$  and  $V_w$  simultaneously. Once  $\alpha$  and  $V_w$  are determined, the SOG  $V_g$  can be readily calculated according to Equations (14)–(16).

#### 4.2. The GA for Speed Optimization

A real-coded GA is employed to solve the speed optimization problem. The procedures of the GA are described as follows:

Step 1: Population initialization. For a sailing route with  $n$  segments, the individual is represented as a vector of length  $n$ . The following equation represents the  $j^{\text{th}}$  individual of the population:

$$I_j = [V_{sw}^{1j}, V_{sw}^{2j}, \dots, V_{sw}^{ij}, \dots, V_{sw}^{nj}]^T \quad (22)$$

Then, a population of  $N$  individuals is represented as a  $n \times N$  matrix:

$$Pop = [I_1, I_2, \dots, I_j, \dots, I_N] \quad (23)$$

Each individual of the population represents a solution to the speed optimization.  $V_{sw}^{ij}$  is the  $V_{sw}$  in  $i^{\text{th}}$  segment of the  $j^{\text{th}}$  solution. The first step of the GA is to initialize the  $Pop$ . To this end, the genes of each chromosome (individual) are randomly generated within the range of  $V_{sw}^{\min}$  and  $V_{sw}^{\max}$ .

Step 2: Fitness evaluation. Evaluating the fitness value of each individual according to the following equation:

$$F(I_j) = -[\sum_{i=1}^n (r_{ij} \cdot \frac{d_i}{V_g^{ij}}) + p_1(I_j) + p_2(I_j)] \quad (24)$$

where  $r_{ij}$  is the FCR corresponding to  $V_{sw}^{ij}$ ,  $V_g^{ij}$  is the SOG corresponding to  $V_{sw}^{ij}$ ,  $p_1(I_j)$  is the penalty function that is related to Constraint (19) and  $p_2(I_j)$  is the penalty function that is related to Constraint (21).

The penalty functions  $p_1(I_j)$  and  $p_2(I_j)$  are defined as follows:

$$p_1(I_j) = \begin{cases} M, & \text{If } \sum_{i=1}^n \frac{d_i}{V_g^{ij}} > ETA \\ 0, & \text{Otherwise} \end{cases} \quad (25)$$

$$p_2(I_j) = \begin{cases} M, & \exists V_w^{ij} > V_c^i \\ 0, & \text{Otherwise} \end{cases} \quad (26)$$

where  $M$  is a large enough number,  $V_w^{ij}$  is the STW corresponding to  $V_{sw}^{ij}$  and  $V_c^i$  is the critical STW in segment  $i$ .

Step 3: Selection. Selecting  $N$  parent individuals to build a mating pool. The selection strategy used in this paper is the roulette wheel.

Step 4: Reproduction. Repeat  $N/2$  times:

- (a) Crossover. Picking up two parent individuals randomly from the mating pool and creating offspring by using a crossover operator. The crossover operator used in this paper is the BLX- $\alpha$  [52]. The crossover probability is  $p_c$ .
- (b) Mutation. The above newly generated offspring are reprocessed with a mutation operator. The mutation operator used in this paper is the uniform random mutation. The mutation probability is  $p_m$ .
- (c) Adding the children individuals to a new population.

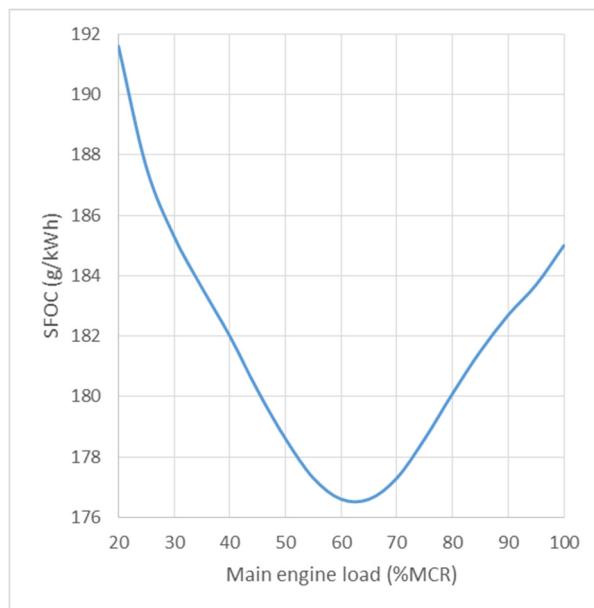
Step 5: Termination. Stopping the GA when it has reached a predefined number of generations  $G$ .

## 5. Case Study

In this section, a voyage of an oil products tanker is selected to perform the case study. The main dimensions of the selected ship are shown in Table 6, the SFOC curve of the ship's main engine is shown in Figure 4, and the voyage plan is shown in Table 7.

**Table 6.** Main dimensions of the selected ship.

Name	Unit	Value
Ship type	-	Oil products tanker
Length overall	m	244.6
Length between perpendiculars ( $L_{pp}$ )	m	233.0
Beam molded	m	42.0
Depth molded	m	22.2
Summer deadweight	MT	109,672
Summer draft	m	15.5
Design speed	knots	15.7
Minimum allowed value of the SWS ( $V_{sw}^{\min}$ )	knots	8.0
Maximum allowed value of the SWS ( $V_{sw}^{\max}$ )	knots	15.7
Rated power of main engine	kW	15,260



**Figure 4.** The SFOC curve of the ship's main engine.

**Table 7.** Voyage plan.

Departure Port	Destination Port	Ship Payload $L$ (MT)	Total Distance (nmi)	ETA (h)
Port A	Port B	87,689	3393.24	280.00

The sailing route between port A and port B is divided into 12 segments according to the weather and sea conditions on this route. These segments are connected by 13 consecutive waypoints (incl. port A and port B). The information of these waypoints and segments is presented in Table 8. Parts of these data are obtained directly from the ship's noon reports, and others are calculated from the available data. In Table 8, please note that (a) waypoint 1 represents port A and waypoint 13 represents port B; and (b) a segment is defined as the route between the current waypoint and the previous waypoint, e.g., segment 3 is the sailing route from waypoint 3 to waypoint 4.

### 5.1. Model Verification

The reliability of the speed optimization model depends mainly on two factors: (a) the accuracy of the fuel consumption function in Section 3.2.1 and (b) the accuracy of the speed correction model in Section 3.2.2. Therefore, before optimizing the ship's speed, we validate the above two models based on the measured data.

#### 5.1.1. Verification of the Fuel Consumption Function

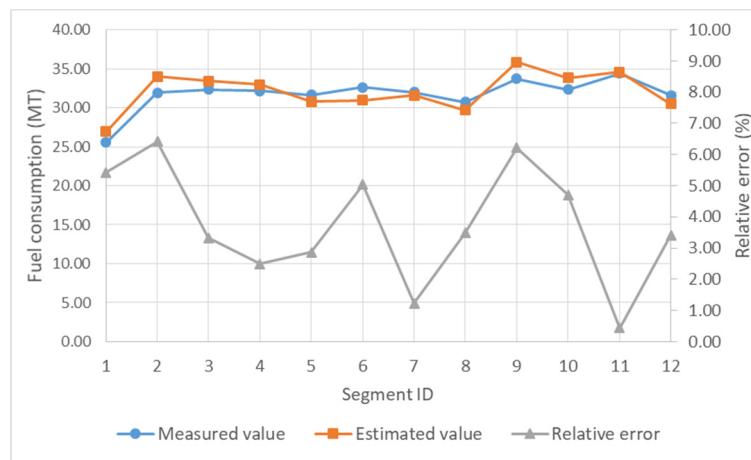
For segment  $i$ , the FCR  $r_i$  can be estimated using the fuel consumption function in Section 3.2.1 and the value of  $V_{sw}^i$  in Table 8. Then, we can estimate the fuel consumption in segment  $i$  by multiplying  $r_i$  by the corresponding sailing time in Table 8. For each segment, the measured and the estimated fuel consumption are compared. Meanwhile, the relative error between them is calculated, as shown in Table 9. The following two figures can be extracted from Table 9: (a) among the 12 segments of the target route, the maximum relative error is less than 6.50%; and (b) from an overall perspective, the average relative error of these 12 segments is 3.75%. These figures indicate that the fuel consumption function we proposed in Section 3.2.1 has a high accuracy. Intuitive verification results of the fuel consumption function are shown in Figure 5.

**Table 8.** Information of waypoints and segments of the selected route.

Waypoint	Latitude (°)	Longitude (°)	Segment ID $i$	$\beta_i$ (°)	$d_i$ (nmi)	$V_{sw}^i$ (knots)	Sailing Time (h)	Fuel Consumption (MT)	$\varphi_i$ (°)	BN	$h_i$ (m)	$\gamma_i$ (°)	$V_c^i$ (knots)
1	24.75	52.83	-	-	-	-	-	-	-	-	-	-	-
2	26.55	56.45	1	61.25	223.86	12.7	18.70	25.54	139	3	1.0	245	0.30
3	24.08	60.88	2	121.53	282.54	12.6	24.10	31.93	207	3	1.0	248	0.72
4	21.73	65.73	3	117.61	303.18	12.7	23.20	32.33	9	4	1.5	158	0.73
5	17.96	69.19	4	139.03	298.44	12.5	23.90	32.18	201	4	1.5	178	0.21
6	14.18	72.07	5	143.63	280.51	12.3	23.30	31.66	88	5	2.5	135	0.49
7	10.45	75.16	6	140.84	287.34	12.2	24.00	32.60	86	4	1.5	113	0.22
8	7.00	78.46	7	136.42	284.40	12.2	24.50	32.00	353	3	1.0	338	0.54
9	5.64	82.12	8	110.37	233.25	12.2	23.00	30.74	35	5	2.5	290	1.25
10	4.54	87.04	9	102.57	301.80	12.8	24.20	33.72	269	4	1.5	270	0.28
11	5.20	92.27	10	82.83	315.70	12.6	24.00	32.32	174	3	1.0	93	0.72
12	5.64	97.16	11	84.87	293.80	12.7	24.00	34.41	60	1	0.1	185	0.62
13	1.81	100.10	12	142.39	288.42	12.3	23.10	31.57	315	3	1.0	90	0.30

**Table 9.** Verification results of the fuel consumption function.

Segment ID $i$	Measured Fuel Consumption (MT)	Estimated FCR (MT/h)	Estimated Fuel Consumption (MT)	Relative Error (%)
1	25.54	1.44	26.93	5.43
2	31.93	1.41	33.98	6.42
3	32.33	1.44	33.41	3.33
4	32.18	1.38	32.98	2.49
5	31.66	1.32	30.76	2.86
6	32.60	1.29	30.96	5.03
7	32.00	1.29	31.61	1.23
8	30.74	1.29	29.67	3.48
9	33.72	1.48	35.82	6.22
10	32.32	1.41	33.84	4.70
11	34.41	1.44	34.56	0.44
12	31.57	1.32	30.49	3.41

**Figure 5.** Intuitive verification results of the fuel consumption function.

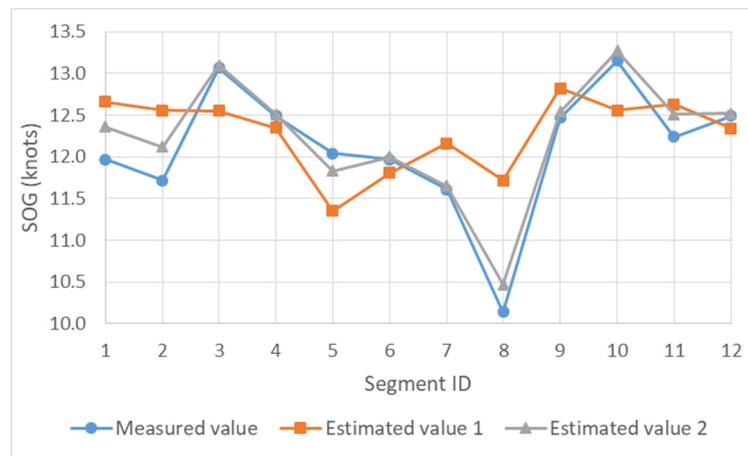
### 5.1.2. Verification of the Speed Correction Model

For segment  $i$ , the measured value of  $V_g^i$  can be calculated based on  $d_i$  and the corresponding sailing time in Table 8. Meanwhile, the estimated value of  $V_g^i$  can be obtained based on the value of  $V_{sw}^i$  in Table 8 and the speed correction model in Section 3.2.2. Here, two estimated values of  $V_g^i$  are obtained. The first one is obtained under the condition that the influence of ocean currents is not considered, while the second one is calculated under the condition that the influence of ocean currents is taken into account. In the first case, the estimated value of  $V_g^i$  is actually the estimated value of  $V_w^i$ . For each segment, the measured  $V_g^i$  and the estimated  $V_g^i$  are compared, and the relative error between them is calculated, as shown in Table 10. From this table, we can see that: (a) when the influence of ocean currents is not considered, the average relative error of the speed correction model is 4.75%; and (b) when the influence of ocean currents is taken into account, this value becomes 1.36%. Intuitive verification results of the speed correction model are shown in Figures 6 and 7. Based on these results, we can draw the conclusion that the error between the estimated  $V_g^i$  and the measured  $V_g^i$  can be effectively reduced if the influence of ocean currents is taken into account.

**Table 10.** Verification results of the speed correction model.

Segment ID $i$	Measured $V_g^i$ (knots)	Estimated $V_g^i$ 1 <sup>a</sup> (knots)	Relative Error 1 <sup>a</sup> (%)	Estimated $V_g^i$ 2 <sup>b</sup> (knots)	Relative Error 2 <sup>b</sup> (%)
1	11.97	12.66	5.74	12.36	3.25
2	11.72	12.56	7.12	12.12	3.38
3	13.07	12.55	3.97	13.10	0.24
4	12.49	12.35	1.11	12.51	0.18
5	12.04	11.35	5.74	11.83	1.74
6	11.97	11.81	1.38	12.00	0.23
7	11.61	12.16	4.73	11.65	0.36
8	10.14	11.72	15.56	10.47	3.24
9	12.47	12.82	2.78	12.54	0.55
10	13.15	12.56	4.53	13.27	0.88
11	12.24	12.63	3.19	12.51	2.19
12	12.49	12.34	1.16	12.52	0.27

<sup>a</sup> The influence of ocean currents is not considered. <sup>b</sup> The influence of ocean currents is taken into account.

**Figure 6.** Comparison of measured speed over ground (SOG) and estimated SOG.**Figure 7.** Comparison of relative error 1 and relative error 2.

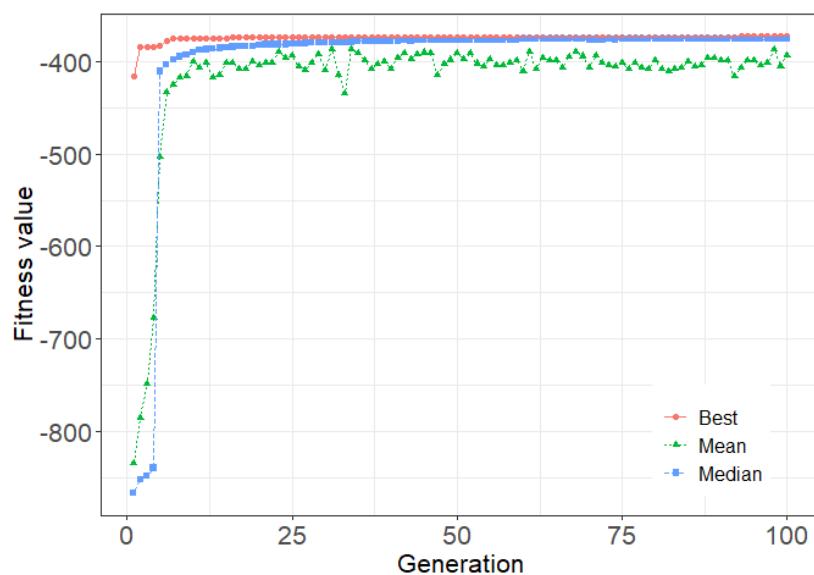
## 5.2. Speed Optimization Results and Analyses

The solution approach in Section 4 is programmed with R programming language. The parameter settings of the GA are presented in Table 11. We ran the program 10 times. The average running time of the program was 14.86 s. The results of the 10 runs of the program were consistent, indicating that the algorithm had a good stability. We randomly selected one of the above 10 running results

for further analysis. The convergence curve of this run is presented in Figure 8. Based on this figure, we can draw the conclusion that the designed GA has good convergence and convergence speed on the target problem.

**Table 11.** The parameter settings of the genetic algorithm (GA).

Parameter	Symbol	Value
Population size	$N$	200
Chromosome size	$n$	12
Crossover probability	$p_c$	0.8
Mutation probability	$p_m$	0.1
Generations	$G$	100
A big enough number	$M$	500

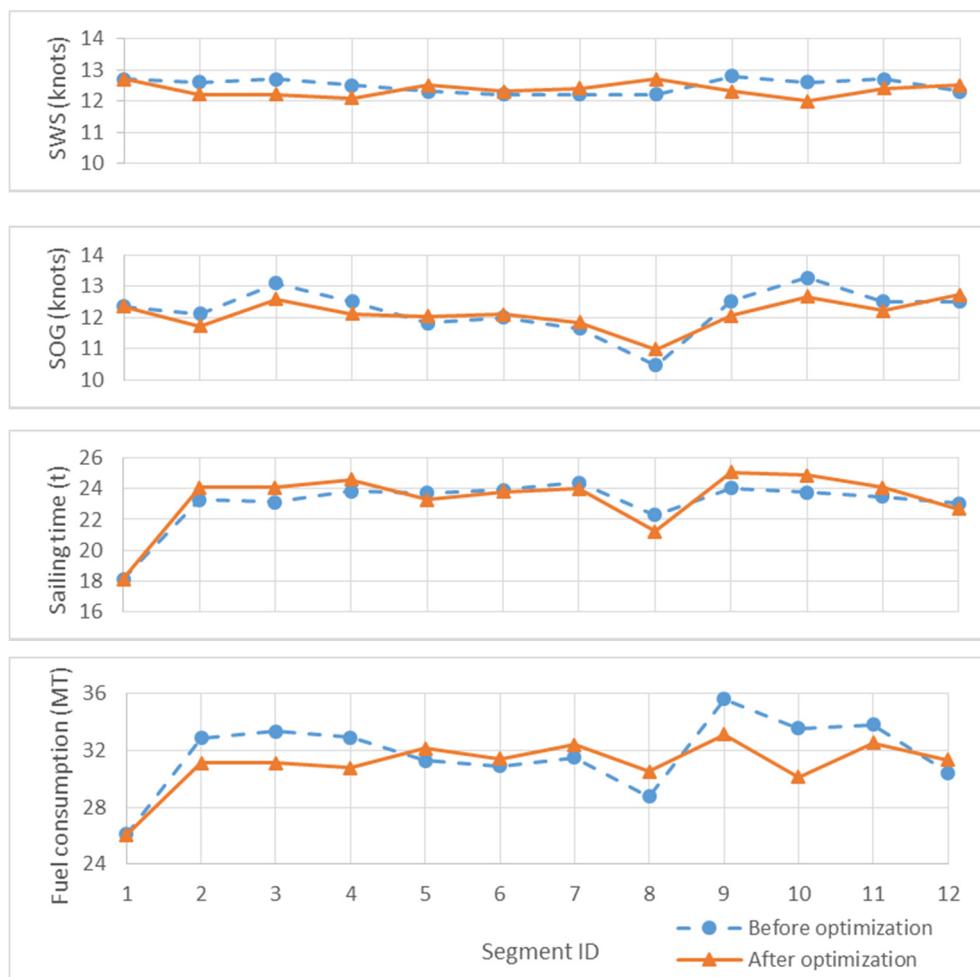


**Figure 8.** Convergence curve of the GA.

After optimization, the SWS, SOG, sailing time, FCR and fuel consumption of each segment are shown in Table 12. From this table, we can see that: (a) the total sailing time over the whole voyage is 280.00 h, which remains unchanged; and (b) the total fuel consumption over the whole voyage is 372.62 MT, which is 8.38 MT less than the actual value (i.e., 381.00 MT). Considering that there is a little deviation between the actual and the estimated fuel consumption, the latter should be chosen as the benchmark for comparison. Before optimization, the estimated total fuel consumption over the whole voyage is 381.01 MT. Therefore, after optimization, the total fuel consumption over the whole voyage is actually reduced by 8.39 MT, accounting for 2.20% of the estimated fuel consumption before optimization. We can thus conclude that the proposed speed optimization model can help the selected oil products tanker save about 2.20% of bunker fuel to complete a 280-h voyage. The SWS, SOG, sailing time and fuel consumption of each segment before and after optimization are compared in Figure 9.

**Table 12.** Results of ship speed optimization.

Segment ID $i$	SWS $V_{sw}^i$ (knots)	SOG $V_g^i$ (knots)	Sailing Time (h)	FCR $r_i$ (MT/h)	Fuel Consumption (MT)
1	12.7	12.36	18.10	1.44	26.06
2	12.2	11.72	24.10	1.29	31.09
3	12.2	12.59	24.10	1.29	31.09
4	12.1	12.11	24.60	1.25	30.75
5	12.5	12.04	23.30	1.38	32.15
6	12.3	12.10	23.80	1.32	31.42
7	12.4	11.85	24.00	1.35	32.40
8	12.7	10.98	21.20	1.44	30.53
9	12.3	12.05	25.10	1.32	33.13
10	12.0	12.67	24.90	1.21	30.13
11	12.4	12.21	24.10	1.35	32.54
12	12.5	12.72	22.70	1.38	31.33

**Figure 9.** Comparison of still water speed (SWS), SOG, sailing time and fuel consumption of each segment before and after optimization.

### 5.3. Analysis of GHG Emissions

The impact of the speed optimization model on GHG emissions from ships is also a major concern of this paper. According to the IMO [6], CO<sub>2</sub> is the dominant GHG produced by shipping. Hence, we mainly emphasize emissions of CO<sub>2</sub> in our analysis. As mentioned in Section 1, GHGs emitted by