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A dynamic ship speed optimization method with time horizon segmentation

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ABSTRACT

The determination of the appropriate operational profile of a ship through the adjustment of its sailing speed can potentially present a fruitful prospect in the effort towards the maximization of energy resources exploitation in commercial ships. The dependency of fuel consumption on speed, always in inseparable conjunction with the ever-changing weather conditions prevailing over any specified route, leads to the formulation of a (from mathematical point of view) not well-defined problem, when practical considerations regarding the capabilities of currently available weather forecast services are taken into account. In the present work, a method for the dynamic determination of optimal ship speed, in a fixed route, is proposed. Within this method, the problem of deteriorating accuracy of weather predictions for relatively long time periods is addressed with the segmentation of the route's total time horizon in smaller time periods, in order for solvable and meaningful optimization problems to be formulated. The interdependency between the individual sub-problems is established and examined with the definition of appropriate method parameters, which are representative of the most important time scales of the engineering problem at hand. The effectiveness of the method is demonstrated with the application on an actual container ship route.

1. Introduction

Maritime transport has always been a major part of international commercial transportation, as it constitutes a highly cost-effective option of transferring large volumes of cargo between continents. However, in recent years, depressed markets, increasing fuel prices, low fares and new regulations regarding pollutants and greenhouse gases emissions from ships, have emphasized the need for optimizing the energy efficiency of ships, especially in the light of minimizing fuel consumption.

Important research work, in order to address this problem, is ongoing in the field of hydrodynamics and specifically on the optimization of hull shape design (Diez et al., 2018), with the goal of minimizing propulsion resistance, and thus fuel consumption. Furthermore, a large part of the research work nowadays is concerned with the design of propulsion machines with higher efficiency and consequently lower consumption. However, several alternative and interesting approaches to the problem also exist. One of them, which has been gaining increasingly more attention, concerns optimizing the cruising speed of the ship during voyage by taking into account the prevailing weather conditions.

Historically, the concept of optimizing the operational profile of a ship while taking into consideration the weather conditions initially appears in studies related to the broader topic of searching for the optimal route the ship must follow. In its early form, weather routing (Szlapczynska and Smierzchalski, 2008; Takashima et al., 2009; Vlachos, 2004) mainly tackles the problem of finding the shortest route between two ports while taking into account the weather conditions only in the light of the simultaneous avoidance of areas of adverse weather. The concept of ship speed optimization, i.e. the voluntary change of ship speed as a function of weather conditions during the voyage in order to minimize consumption, in the context of routing or weather routing, appears in the literature mainly in the last decade. A very comprehensive description of the aspects behind the idea of ship speed optimization is given in Psaraftis and Kontovas (2014), and the most important parameters of the problem are presented.

The problem of concurrently optimizing the speed for specific parts of a route, in order to minimize fuel consumption, is posed in Shao et al. (2011). Mathematically, the problem is stated as a Dynamic Programming problem and is solved using a suitable method (three dimensional forward dynamic programming method). Lin et al. (2013) formulate and solve a similar problem, in which, the simultaneous minimization of arrival time and fuel consumption is requested. The problem is formulated as a multistage problem and a specially designed three-dimensional modified isochrone method is used to solve it.

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Another case where this problem is formulated as a multi-stage problem can be found in the study of Zaccone et al. (2018). Weather is modeled through appropriate weather forecast maps and fuel consumption minimization is set as the objective function, while the problem is solved via dynamic programming methods. An interesting work in the same context can be found in Lee et al. (2018b), where the problem is solved using genetic algorithms. An important aspect of this last study is that the modeling of the ship and the weather conditions is done at a somewhat more detailed level.

Of course, ship speed optimization problems can also be found in studies beyond the general category of weather routing. In Wang and Meng (2012) the simultaneous optimization of the speed of multiple container vessels of a regular line network is studied in order to minimize the total operational cost. The problem is formulated as a mixed integer nonlinear programming problem. Wang and Hu (2015) in their study examine the optimization of a ship's speed for minimizing carbon emissions while different tax scenarios are considered. In their study, Kim et al. (2016) aim to the minimization of fuel consumption while considering variable arrival times at ports. The problem is posed as a mixed integer nonlinear programming problem and a special tree search algorithm is developed to solve it. A similar case of minimizing fuel consumption under the uncertainty regime of arrival and stay times in ports (use of stochastic variables) is considered in Aydin et al. (2017) where the problem is posed as a dynamic programming problem. All aforementioned publications present some very interesting approaches of the more general problem of ship speed optimization, but albeit, the consideration of the continuously changing weather conditions from route to route is not explicitly taken into account.

Research on the problem of speed optimization taking into account the weather conditions and defining the problem by excluding options of re-routing the ship's voyage can be found in relatively recent publications. Norlund and Gribkovsakaia (2017) formulate a ship speed optimization problem in order to simultaneously minimize fuel consumption and emissions on supply vessels. The effect of weather conditions on the ship speed is modeled through its effect in cruising time, while generic models for calculating resistance-propulsion (and therefore consumption) are used. The subject of uncertainty of weather conditions and its effect on the determination of optimal speed is also studied. In Wang et al. (2018) dynamic weather conditions are considered and the problem is formulated in the context of dynamic optimization. For the solution of the problem, a Model Predictive Control method is adopted and detailed non-linear models from the literature are used to model the ship performance. An appropriate ship performance indicator is selected as the minimization criterion. In Lee et al. (2018a) historical weather data for a given route are utilized and processed through data mining methods, in order to construct a model for correlating weather conditions and consumption. Then the problem of speed optimization is posed for simultaneous minimization of consumption and maximization of the SLA (service level agreement) and is solved with an appropriate particle swarm optimization algorithm.

In the work of Li et al. (2018), appropriate models are developed for correlating the weather conditions to the ship speed reduction, and speed optimization is performed in order to minimize consumption on a given route, while simultaneously maximizing profit. The use of big data analytics methods over a given route weather data in order to construct appropriate models for describing the energy efficiency of a ship is presented in Yan et al. (2018). Then, a suitable particle swarm optimization algorithm is used in order to find the optimal speed profile that minimizes consumption on a given trip.

The majority of these, relatively recent, publications demonstrates the importance of using real-time weather data combined with accurate models for the ship fuel consumption in order to predict the optimal ship speed operational profile. In addition, the optimization problem is properly grounded as a nonlinear programming problem and the benefits of speed optimization in reducing fuel consumption and emissions are demonstrated. However, the issue of uncertainty introduced in the prediction/forecast of future weather conditions, especially for long trips, is not discussed.

In the current work, the ship speed optimization problem for fuel consumption minimization is mathematically formulated as a dynamic optimization problem while the weather profile is input to the problem in the form of consecutive weather forecasts throughout a predefined route. Under this scope, a special method that attempts to tackle with the inherent uncertainty in the weather conditions forecasts is developed and applied.

In the next two Sections, the description and mathematical formulation of the problem, as a dynamic optimization problem are given. In Section 4, the specially developed method for tackling the uncertainty of the weather forecasts is presented. In Sections 5 and 6, the system modeling and general solution procedure are discussed. Finally, the method is applied on a real case scenario, and the results are presented and discussed in Section 7.

2. Description of the dynamic optimization problem

The engineering problem of ship speed optimization addressed in the present study can be stated by posing the following question:

«*What is the optimal propulsion power profile, or equivalently, what is the optimal speed profile that a ship must follow along a specified route and under variable weather conditions in order for the fuel consumption to be minimized, while satisfying certain constraints.*

An important additional consideration regarding any attempt to specify the optimal speed over a specified commercial ship route should, in any case, be the requirement that the ship must reach to the destination ports in predetermined and quite tight times of arrival. The practical applicability of any ship speed optimization method to be developed should be evaluated in conjunction with the ability of the last to ensure that these time limitations are satisfied.

Having these in mind, in order for the speed optimization problem to be mathematically well−posed, the following are required:

(i) The route that the ship will follow when traveling between two ports (e.g. from port A to port B), must be predetermined and known.
(ii) The Estimated Time of Departure (ETD) from port A, and the Estimated Time of Arrival (ETA) to port B, must be predetermined and known.
(iii) All ship characteristics (e.g. ship dimensions and characteristic coefficients, characteristics of ship engine(s), shaft and propeller characteristics etc.), that are required in order to develop the appropriate fuel consumption models, must be known.
(iv) The weather conditions along the route must be known a priori and used as inputs in the optimization procedure, since, in the current work the assumption is made that the weather can be predicted with sufficient accuracy, at least for a very short time horizon.

The problem described above is an inherently dynamic optimization problem, since the weather conditions that the ship encounters during her voyage are, in general, functions of time. Thus, time dependency is unavoidably inserted in all variables of the problem such as speed, ship resistance and required propulsive power from the engine(s). Of course, it must be clarified that time dependency alone is not sufficient to characterize an optimization problem as dynamic. The important characteristic that discriminates the specific problem at hand as a dynamic one is the issue of interdependency between time intervals. The fact that the route is specified and that the ship must conclude the trip in a prespecified time horizon, introduces such interdependency as it dictates that the optimal speed in each and every interval depends on the weather conditions of both the current interval and of every other interval.

It is true that, no matter how accurately the models utilized can calculate the ship resistance or no matter how efficiently the optimization method solves the problem, the reliability of the final optimal solution is dependent on the accuracy of the weather prediction. Ideally, if the weather is known with absolute accuracy during the whole time horizon of the trip, the underlying dynamic optimization problem can be solved with utmost precision, deriving the ultimately optimal solution. However, due to the fact that the time period during which the weather forecast can be considered accurate (up to an acceptable, for engineering purposes, degree) is several times shorter than the travel duration of a typical commercial trip, any attempt to solve the full time horizon problem will unavoidably yield questionable results. For this reason, a specialized method with the formulation of successive interdependent dynamic optimization problems, the combined solutions of which, approximate the solution of the full time horizon problem, has been developed and applied. The concept behind this time segmentation is the development of a formal way for determining the optimal ship navigation speed in each instance of time by solving each time an optimization problem of limited time horizon during which the important parameters related to the weather conditions can be considered as accurate, so the formulation is mathematically well posed.

The details of this method are presented in Section 4. The mathematical formulation of the general full time horizon dynamic optimization problem is given in the next Section.

3. Mathematical statement of the full time horizon dynamic optimization problem

The optimization problem at hand can be generally classified as a dynamic optimization problem (Bryson, 1999; Bryson and Ho, 1969; Kirk, 1998) and is mathematically stated as a minimization problem using a Differential – Algebraic Equation (DAE) formulation (Allgor and Barton, 1997; Biegler, 2010; Biegler and Grossman, 2004). The ship travels between ports A and B and one control variable is optimized, the speed of the ship, V. The final time, or time of arrival, t_f, is known a priori and the goal of optimization is the minimization of the fuel consumption, m_f: The objective function can be mathematically stated using a Boltza form by the following equation:

$$\min_V m_f = \int_{t_0}^{t_f} b_f \cdot \dot{W}_b(V) \cdot dt \tag{1}$$

where:

m_f Fuel consumption,
t_0 Initial time,
t_f Final travel time or travel duration,
b_f Specific Fuel Oil Consumption (SFOC) of the engine,
$\dot{W}_b$ Brake power of the engine,

The Specific Fuel Oil Consumption (SFOC) is in general (amongst other things) a function of Brake Power or equivalently of the engine load factor:

$$b_f = b_f\left(\dot{W}_b\right) \quad \text{or} \quad b_f = b_f(f_L) \tag{2}$$

while, the engine load factor, f_L, is defined as follows:

$$f_L = \frac{\dot{W}_b}{MCR} \tag{3}$$

with MCR the maximum continuous rating of the engine. More details regarding the Main Diesel Engine modeling are given in Section 5.

Since the ship will be sailing (through sea water) with a certain speed, it will experience resistance, which is, in general, a sum of several resistance components and dependent on several factors. In this text, the total resistance is denoted as R_T and will be, in general, a function of the ship speed, the weather conditions and the hydrodynamic characteristics and dimensions of the ship.

$$R_T = R_T(\mathbf{p}, V, \mathbf{WS}) \tag{4}$$

where:

$\mathbf{p}$ Vector that denotes time independent characteristics of the ship and the hull,
$\mathbf{WS}$ Weather State (weather conditions).

Considering the weather state (conditions), it is generally stated by the following equation:

$$\mathbf{WS} = (U_{wind}, \psi_{wind}, H_s, \theta_{waves}) \tag{5}$$

where:

U_{wind} Wind speed,
ψ_{wind} Wind direction,
H_s Significant wave height defined as the mean wave height (trough to crest) of the highest third of the waves,
θ_{waves} Wave direction.

These four variables, which essentially represent the weather prediction, are given as functions of time along the trip:

$$\begin{aligned} U_{wind} &= U_{wind}(t) \\ \psi_{wind} &= \psi_{wind}(t) \\ H_s &= H_s(t) \\ \theta_{waves} &= \theta_{waves}(t) \end{aligned} \tag{6}$$

The corresponding *Effective Power* (or towing power), $\dot{W}_e$, necessary to move the ship through the water, i.e. to tow the ship hull at the speed Vin absence of propulsive power is given as:

$$\dot{W}_e = R_T(V, \mathbf{WS}, \mathbf{p}) \cdot V \tag{7}$$

while, the resulting, required engine brake power is related to the ship resistance and speed using the equation:

$$\dot{W}_b = \frac{\dot{W}_e}{\eta_{prop}} = \frac{V \cdot R_T(V, \mathbf{WS}, \mathbf{p})}{\eta_{prop}(V, \mathbf{p}, \mathbf{q})} \tag{8}$$

where.

η_{prop} Total propulsive efficiency
$\mathbf{q}$ Vector that denotes time independent propeller and shaft characteristics.

The models applied in order to calculate ship resistance and propulsion (propulsive efficiency) as well as the performance of the Diesel engine(s) are presented in Section 5.

Furthermore, the distance travelled, d, the time elapsed and the ship speed are inter-connected by the equation:

$$d(t) = \int_{t_0}^{t} V \cdot dt \tag{9}$$

In order to conclude the mathematical statement of the problem, the necessary variable boundaries as well as the initial points and end points, if they exist, of the differential variables must be included. The ship speed must lie the interval:

$$V_{\min} \le V \le V_{\max} \tag{10}$$

where, $V_{\min}$ and $V_{\max}$ are the lower and upper bounds of the speed. Furthermore, it is important to ensure that the engine will operate inside the load limits specified by the manufacturer:

$$f_{L_{\min}} \leq f_L \leq f_{L_{\max}} \tag{11}$$

Also, for the two differential variables, distance travelled and fuel consumption, the initial points are known:

$$\begin{aligned} d(0) &= 0 \\ m_f(0) &= 0 \end{aligned} \tag{12}$$

while an endpoint constraint must be defined for the distance travelled, as follows:

$$d(t_f) = \int_{t_0}^{t_f} V \cdot dt = d_{A \to B} \tag{13}$$

with $d_{A \to B}$ is the total distance between ports A and B, which ensures that the ship has reached port B in the predefined time of arrival.

Finally, there may be need of additional constraints and variable bounds, but they are not included here, for brevity.

In the present section, it is seen that the variation of the optimal ship speed over a route can be mathematically uniquely determined in a strict manner, but this would be possible in practice only if the underlying weather conditions were defined with high accuracy over the whole time period spanning from the beginning of the navigation t_0 to the final time t_f. As this is not possible, in the present study a segmentation of the full time horizon is attempted for tackling the problem of the unavailability of valid weather predictions over the whole time span of the navigation.

4. Formulation of successive optimization problems by segmentation of the time horizon of the trip

The weather forecasts have nowadays attained sufficient prediction accuracy and resolution of predicted values for most of the quantities of interest for the purposes of the present work. However, the accuracy and the validity of the predictions naturally tend to degenerate as the prediction spans to longer time periods. In general, the predictions are considered to be accurate enough for a rather short time horizon or trust region (i.e. about two days). This poses a difficulty in the determination of the optimum speed profile in a trip between two ports that lasts for several days, as is the case in the majority of the merchant ships routes, because the weather conditions must be predetermined in order for the dynamic optimization problem to be solved. In other words, the full time horizon dynamic optimization problem cannot be practically solved with satisfying accuracy in the beginning of the route, since the more long-term a weather prediction is, the less accurate it will be.

To overcome this problem, a specialized *ad-hoc* procedure has been developed (Fig. 1). The key idea behind the procedure lies in the approximation of the initial full time horizon dynamic optimization problem by the solutions of several successive dynamic optimization problems, each spanning over a limited duration time horizon, and equal to the trust region of the weather prediction. The description of this procedure is given in the present Section.

The ship departs from Port A at time $t = t_0$ and needs to arrive at Port B according to the schedule, at time $t = t_f$. Two time discretization steps are used: i) Dt_V, which is the duration of the intervals that during each of them the ship sails with constant ship speed (speed, which is to be determined by the solution of the optimization problem), ii) Dt_w, which is the time resolution of the weather forecasts. It is noted that between the forecasts, the values of the predicted weather conditions are assumed to be varying linearly with time.

Two parameters, n_a and n_b are required for establishing the interrelation between the individual successive optimization problems to be solved. Both quantities are tunable parameters of the solution procedure.

The first parameter, denoted as n_a, is defined as the number of Dt_V intervals that are used for determining the time horizon for which the weather data provided are considered to be accurate up to an acceptable degree (weather trust region). For this time horizon an individual dynamic speed optimization sub-problem can be solved with satisfying accuracy. For example, with weather data assumed as accurate for the next 48 h, and Dt_V equal to 6 h, n_a is calculated as equal to 8, and the time horizon of each individual optimization problem will be: $n_a \cdot Dt_V = 48$ hours.

Once such an individual speed optimization problem is solved, the optimal solution found for the ship speed is not to be applied for the whole $n_a \cdot Dt_V$ time horizon, but instead is applied for $n_b \cdot Dt_V$ intervals, where n_b is the second parameter used for the interrelation of the successive optimization problems (the n_b parameter is to be lower than n_a). For example, if $n_b = 4$ in the previous example where $n_a = 8$, an optimization problem is solved for a 48 h horizon, but the optimal speed profile found is applied only in the first$n_b \cdot Dt_V = 24$ hours. At the end of the $n_b \cdot Dt_V$ period, the next dynamic optimization sub-problem of time horizon $n_a \cdot Dt_V$ is defined, with its initial time having been shifted by $n_b \cdot Dt_V$ hours and with the weather prediction (within the accuracy trust region) having also been updated and extended by $n_b \cdot Dt_V$ hours. The

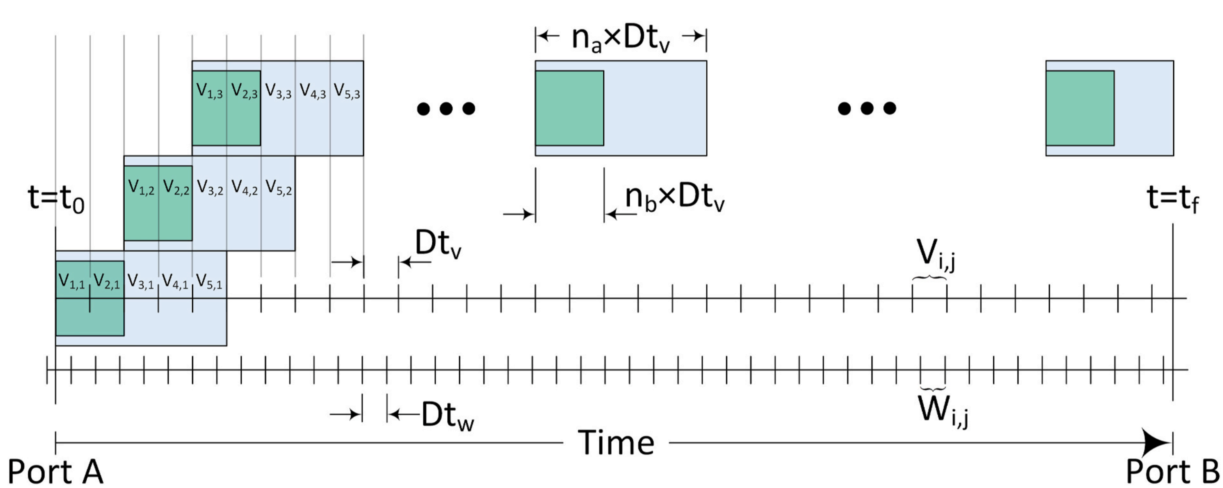

Fig. 1. Time horizon segmentation representation.
($V_{i,j}$: speed in interval i and problem j, $W_{i,j}$: weather variables in interval i and problem j, n_a: number of speed intervals in a sub-problem, n_b: number of weather intervals in a sub-problem, Dt_V: speed discretization interval, Dt_w: weather discretization interval).

same pattern, of formulating and solving successive optimization sub-problems is applied until the depletion of the time horizon of the full problem, $t = t_f$, and, consequently, the arrival of the ship at port B.

The number $j_{\max}$ of individual successive optimization problems to be solved is calculated as:

$$j_{max} = ceil\left(\frac{t_f - t_0}{n_b Dt_v} - \frac{n_a}{n_b} + 1\right) \tag{14}$$

where.

t_0 time of departure from port A
t_f required time of arrival at port B
ceil function representing rounding to the closer upper integer value

For each individual optimization problem *j*, the target distance $d_{f,j}$ that has to be reached (measuring from the initial port A) is calculated as:

$$d_{f,j} = d_{0,j} + \frac{d_f - d_{0,j}}{t_f - t_{0,j}} n_a \cdot Dt_v = d_{0,j} + V_{ship,mean,j} \cdot n_a \cdot Dt_v \tag{15}$$

where:

d_f total distance between ports A and B
$d_{0,j}$ distance already travelled at the beginning of optimization problem *j*
$t_{0,j}$ initial time at the beginning of optimization problem *j*

In Eq. (15), the fraction term:

$$V_{ship,mean,j} = \frac{(d_f - d_{0,j})}{(t_f - t_{0,j})} \tag{16}$$

is merely the mean speed that would be required for covering the total distance remaining to reach to the final destination at the period from $t_{0,i}$ to ETA ($t = t_f$). By defining a target total distance travelled to have been covered at the end of each of the successive optimization problems in the manner described, it is ensured that, in the overall procedure with the succession of individual sub-problems, the ship will reach the destination port at the predetermined total time. What is obtained by the solution of the sub problem, is, in effect, the deviation over $V_{ship,mean,j}$ at each instance of time, which is collectively compensated by deviations of the opposite sign and appropriate magnitude in other instances (lying within the $n_a * Dt_V$ time horizon of each sub-problem being solved) in order for the ship to have arrived at distance $d_{f,j}$, while also aiming at the minimization of the fuel consumption. As the time passes with the ship coming closer to the final destination and successive optimization problems are being solved, the value of $V_{ship,mean,j}$ is gradually being readjusted, and so it is assured the ship arrives at the predetermined ETA.

By applying this method, the initially difficult to solve (due to uncertainty in the long term prediction of the weather conditions) dynamic optimization problem for the total trip horizon is broken apart and approximated by a series of smaller successive, interdependent, dynamic optimization problems, that are easily solvable. Thus, the series of optimal solutions of the successive dynamic sub-problems will attempt to approximate the ideal optimal solution of the initial –full time horizon– dynamic optimization problem. Details regarding the overall solution procedure and the treatment of the successive dynamic sub-problems are given in Section 6. In the following Section, system modeling is discussed.

5. System modeling

The total resistance of a ship is composed of a number of different components (Politis, 2018; Politis and Skamnelis, 2007). In the present work, given the dynamic character of the problem due to the time varying weather conditions, the resistance components can be grouped in two main categories: the *calm water resistance*, R_{calm}, and the *added resistance*, R_{Added}:

$$R_T = R_{calm} + R_{Added} \tag{17}$$

The calm water resistance term (Holtrop, 1984; Holtrop and Mennen, 1982; Politis, 2018) is generally dependent on the speed of the ship, the trim of the ship as well as the characteristics and dimensions of the ship and the hull.

$$R_{calm} = R_{calm}(V, T, \mathbf{p}) \tag{18}$$

where:

T Trim of the ship.
p Vector that denotes time independent characteristics of the ship and the hull.

The calm water resistance term can be further decomposed to a sum of several terms (Holtrop and Mennen, 1982; Politis, 2018):

$$R_T = R_F(1 + k_1) + R_a + R_w + R_{APP} + R_B + R_{TR} + R_{AA} + R_{Aw} \tag{19}$$

where:

R_F Frictional resistance,
$1 + k_1$ Form factor of the hull,
R_a Model-ship correlation resistance
R_w Wave-making and wave-breaking resistance,
R_{APP} Appendage resistance due to the presence of bilge keels, rudders, bossings or open shafts and struts,
R_B Additional pressure resistance of bulbous bow near the water surface,
R_{TR} Additional pressure resistance of immersed transom stern,
R_{AA} Additional resistance due to the effects of wind (added wind resistance),
R_{Aw} Additional resistance due to the effects of waves (added wave resistance).

Considering the added resistance (ITTC, 2012; Politis, 2018), it can be broken up to two main terms: *the added wind resistance*, R_{AA}, and the *added wave resistance*, R_{Aw}:

$$R_{Added} = R_{AA} + R_{Aw} \tag{20}$$

Both those two terms are of great importance in the present study, since they essentially model the effect of the time varying weather conditions (wind and waves) on the total resistance of the ship, and consequently, in the required propulsive power from the engine(s). In contrast with the calm water resistance terms, which are generally dependent on the ship speed, trim and hull characteristics, those two terms are additionally dependent on the weather conditions, transforming the optimization problem to a dynamic one:

$$R_{Added} = R_{Added}(V, T, \mathbf{WS}, \mathbf{p}) \tag{21}$$

For the calculation of all the terms of the calm water resistance, models presented in the literature (Holtrop, 1984; Holtrop and Mennen, 1982; ITTC, 2012) are utilized. The calculation of the added wave resistance term is based on the calculation of mean resistance increase in regular waves induced by wave reflection (ITTC, 1987; 2012; Tsujimoto et al., 2008) and the calculation of the mean resistance increase in regular waves induced by ship motion (ITTC, 2012; Maruo, 1957, 1960). Finally, the calculation of the added wind resistance is based on a method (Fujiwara et al., 2005) developed by regression upon multiple data sets from model tests in wind tunnels.

After the calculation of the total resistance, its correlation with the required power from the engine in order for the ship to achieve speed *V*, is given by Eq. (8). The total propulsion efficiency, η_{prop}, is a term generally dependent on ship speed (Politis, 2018; Politis and Skamnelis, 2007), trim and characteristics (geometry, propeller characteristics, shaft efficiencies etc.):

$$\eta_{prop} = \eta_{prop}(V, T, \mathbf{p}, \mathbf{q}) \tag{22}$$

The calculation of the propulsive efficiency is performed based on a model from the literature (Holtrop, 1984; Holtrop and Mennen, 1982; ITTC, 2012) which uses as inputs the ship speed and characteristics (geometry, hull form) as well as other details considering the gear, transmission shaft and propeller (i.e. gearing, bearings, stern tube, propeller and open water efficiencies).

Further information regarding the models utilized for the calculation of total ship resistance and propulsion power can be found in Tzortzis (2018).

As seen in the preceding, for the development of the method, the weather conditions considered are mainly the wind and sea waves, in terms of their magnitudes and directions relative to the ship motion. The application of the proposed method in actual practice would require the additional consideration of the effects of certain other external weather conditions (such as currents, rain, fog, etc.) and other ship specific dynamic behavior quantities (e.g. ship motions) as well as the effect of additional operational constraints (e.g. constrained ship speed and squat effect on canal and waterways) on the total propulsion resistance. However, the purpose of the present study is not the exhaustive treatise of the several terms affecting the propulsion power and fuel consumption requirements. Any additional resistance term can be included in more advanced studies, without affecting the key concept of the time horizon segmentation designed to tackle the dynamic optimization problem.

In the current work, for the modeling of the Diesel Engine(s) (DE), the specific fuel oil consumption (SFOC) model is constructed based on regression upon data provided from the manufacturer and/or the operator of the ship. The model receives as inputs the load factor of the engine, the ambient conditions and the fuel characteristics and calculates the expected specific consumption:

$$b_f = b_f(f_L, T_{atm}, p_{atm}, T_{sea}, MCR, LHV) \tag{23}$$

where:

T_{atm} Ambient temperature (engine inlet),
p_{atm} Ambient pressure (engine inlet),
T_{sea} Sea temperature (coolant inlet).
MCR Maximum Continuous Rating of the Engine.
LHV Lower Heating Value of the fuel oil.

For different fuel LHV and other operating conditions, the SFOC is calculated (MAN, 2018) as:

$$b_f = b_{f,base}(f_L)\cdot\frac{42700}{LHV}\cdot[1 + 0.0002\cdot(25.0 - T_{inlet}) - 0.00002\cdot(1000.0 - P_{inlet}) \\ +0.00041\cdot(25.0 - T_{cool,in})] \tag{24}$$

with:

$$b_{f,base} = b_{f,base}(f_L, MCR) \tag{25}$$

for the regression of the base SFOC, $b_{f,base}$, a nonlinear model will be used, constructed from the available data.

6. Solution procedure

Based on the segmentation of the time horizon of the trip method and the related discussion presented in Section 4, the overall solution procedure of the total horizon dynamic optimization problem, can be outlined as follows:

a) Segmentation of the total time horizon according to all the time scale related problem parameters (n_a, n_b, Dt_V, Dt_W).
b) Formulation of the (initial) dynamic optimization sub-problem according to weather data input.
c) Solution of the dynamic optimization sub-problem:
   (i) Discretization of the dynamic optimization sub-problem and parameterization of the control variables (formulation of an equivalent Non Linear Programming (NLP) problem)
   (ii) Solution of the equivalent NLP sub-problem by an appropriate metaheuristic optimization algorithm.
d) Application of the current problem solution (according to parameter n_b).
f) Calculation of remaining total distance and remaining total time (needed for the formulation of the next sub-problem).
g) Update of weather data (forecast) and formulation of the next sub-problem to be solved.
h) Repetition of Steps c-g (and sub-steps) until the total time horizon and total distance have been covered.

Regarding Step c, the solution of the dynamic optimization sub-problems, modern solution approaches dictate that a level of discretization is to be applied on the original dynamic optimization problem and transform it into a static optimization problem (i.e. a large Non Linear Programming Problem), which can then be solved efficiently via static optimization methods. The method utilized for the transformation of each dynamic optimization problem to a NLP problem, Step c (i), is presented in Section 6.1. Next, in Section 6.2, the metaheuristic optimization algorithm which was used for the solution of the resulting NLP problems, Step c (ii), is discussed.

6.1. Treatment of the dynamic optimization sub-problems

The available solution approaches for dynamic optimization problems (Biegler, 2010; Logsdon and Biegler, 1989; Tzortzis and Frangopoulos, 2018) can be divided in *Indirect* and *Direct* (Fig. 2).

Indirect approach includes classical analytical methods such as *Calculus of Variations* (COV) (Pontryagin et al., 1962) and *Dynamic Programming* (DP) (Bellman, 2003), which may face serious issues when tackling with optimization problems that include inequality constraints and/or many variables (curse of dimensionality).

Direct approach constitutes of more modern techniques, where the optimization problem is solved by applying a certain level of discretization so that the original dynamic problem is transformed to a static NLP Problem which can be solved efficiently with evolutionary and/or gradient-based optimization algorithms. Depending on the level of discretization applied, they are distinguished between *Sequential* and *Simultaneous* (Logsdon and Biegler, 1989, 1992).

In the Sequential methods, such as the one used in this work, discretization is carried out only in the space of control variables (i.e. independent optimization variables), while the values of the state variables (i.e. dependent variables) are derived from the integration (in each iteration) of the system of Differential - Algebraic Equations (DAE) that describe the problem (Biegler, 2010; Chachuat, 2009; Edgar and Himmelblau, 1998; Feehery et al., 1997; Feehery and Barton, 1998; Logsdon and Biegler, 1989). Furthermore, in each iteration, the values of the objective function and the values of the constraints (and possibly the values of the required gradients, in the case of gradient based optimization) are evaluated. These methods are also known as feasible approach methods, since the underlying differential equations are solved once per each iteration. The control variables can be discretized (Fig. 3) using several schemas (Chachuat, 2009; Feehery and Barton, 1998; Feehery et al., 1997), depending on the requirements of the problem.

In general, sequential methods can be applied easily, with little

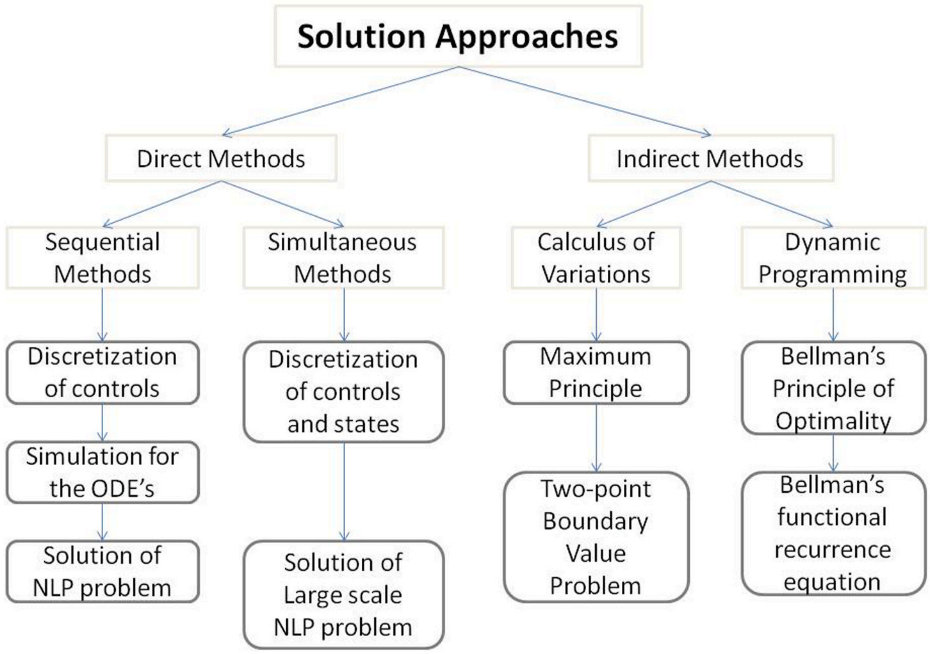

Fig. 2. Solution approaches for dynamic optimization problems (Tzortzis, 2018).

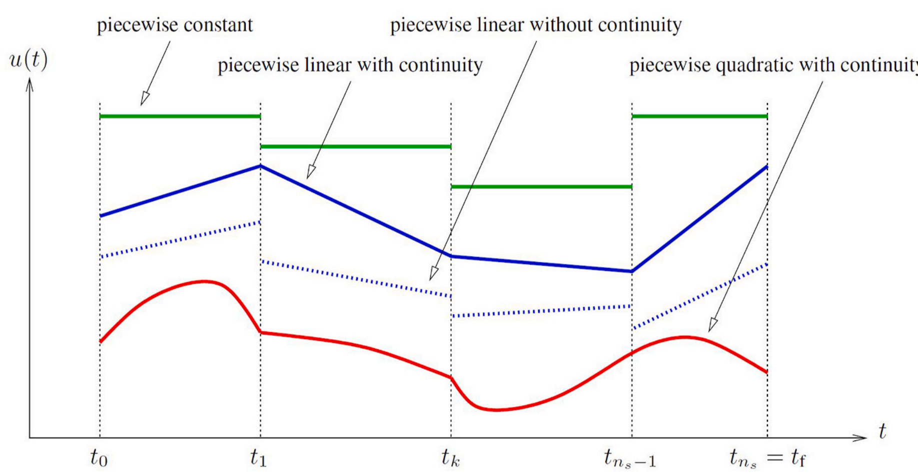

Fig. 3. Piecewise constant, linear and polynomial representation of variables (Chachuat, 2009).

programming effort and have been proven to be very effective considering accuracy and required computational time (Feehery and Barton, 1998; Feehery et al., 1997; Logsdon and Biegler, 1989, 1992).

More specifically, in the current study, for the solution of the successive DO sub-problems a hybrid sequential method (*Control Vector Parameterization*) has been developed and applied, where the time horizon is divided in equally spaced intervals. For the control variable (e.g. ship speed) a piecewise constant representation (Fig. 3) is used (step Dt_V). For the weather conditions, a parameter of the problem, a piecewise linear (with continuity) discretization scheme (Fig. 3) is applied (step Dt_w) which essentially models when the weather conditions are changing; something dictated by the weather prediction/provider forecast resolution and it may be a rather small time period (e.g. every 1 h). The speed discretization interval, Dt_V, is used for defining the time period during which the ship speed is kept at a constant value and in practice is defined by the time intervals at the end of which the captain of the ship will re-adjust the sailing speed. Due to the practical limitations imposed by the ship operational requirements and safety, the Dt_V step may be chosen to be much larger than the Dt_w interval. The mathematical statement of the resulting discrete optimization sub-problems, after application of the sequential method, is given next.

Each sub-problem j can be stated as follows:

$$\underset{V=\{V_{i,j}\}}{\text{minimise}} m_{f,j} = \sum_{i=1}^{n_a} \int_{t_{0_{i,j}}}^{t_{f_{i,j}}} b_f\left(f_{L_{i,j}}\right)\cdot \dot{W}_{b,i}\left(V_{i,j}, \mathbf{WS}_{i,j}\right)\cdot dt \tag{26}$$

where:

$t_{0_{i,j}}$ Initial time of i-th interval in j-th problem
$t_{f_{i,j}}$ Final time of i-th interval in j-th problem

Subject to:

$$d_{f,j} = \sum_{i=1}^{n_a} V_{i,j}\cdot Dt_V \tag{27}$$

$$\dot{W}_b = \frac{V_{i,j} \cdot R_T\left(V_{i,j}, \mathbf{WS}_{i,j}, \mathbf{p}\right)}{\eta_{prop}\left(V_{i,j}, \mathbf{p}, \mathbf{q}\right)} \tag{28}$$

$$\mathbf{WS}_{i,j} = \left(U_{wind i,j}, \psi_{wind \psi_{wind i,j}}, H_{s H_{s i,j}}, \theta_{waves \theta_{waves i,j}}\right) \tag{29}$$

and bounds:

$$V_{i,j,\min} \leq V_{i,j} \leq V_{i,j,\max} \tag{30}$$

$$f_{L_{i,j,\min}} \leq f_{L f_{L i,j}} \leq f_{L_{i,j,\max}} \tag{31}$$

Of course, additional, necessary, constraints and bounds exist and are included in the system modeling, but they are not written here, for brevity.

For the solution of this problem, Step c (ii) of the overall solution procedure, a specialized metaheuristic optimization algorithm (Kennedy and Eberhart, 1995) has been applied. Details are given in the next Section.

6.2. Particle swarm optimization algorithm

The resulting NLP sub-problems are solved with the use of an algorithm based on Particle Swarm Optimization (PSO) (Eberhart et al., 1996; Parsopoulos and Vrahatis, 2002). For n time intervals (each one having duration equal to Dt_V), the independent optimization variables are the ship speed on each interval, V_k, with their number being equal to $n-1$ since the ship speed in the last interval will be a dependent variable due to the requirement of predetermined total travel time (k = [1, $n-1$]). A candidate solution contained in particle p among a population of $p_{\max}$ particles is represented by the vector$\widehat{x}$:

$$\widehat{x}_p = \left(V_{1,p}, V_{2,p}, ..., V_{n-1,p}\right) \quad , \quad p = 1, p_{max} \tag{32}$$

Prior to the application of the optimization algorithm, the boundary values for each of the independent optimization variables must be set. The lower bound $b_{l,Vk}$ for each optimization variable V_k is set to the constant value of 3 m/s (approximately 6 knots). The upper bound $b_{u,Vk}$ for $k = [1, n-1]$ is set equal to the ship speed for which the prime mover will attain a load factor equal to 1 in any moment during the Dt_V time interval, and is calculated according to the weather predictions provided. This was required for not allowing the optimal solution search procedure to move in regions which cannot be applied in practice due to overloading of the prime mover and to facilitate the convergence of the optimization algorithm.

The initialization of the population is another important factor for the successful application of the optimization algorithm. Instead of allowing an entirely randomly generated population, the mean speed $V_{ship,mean}$ is used, which is merely defined as the total distance divided by the total time. Each independent variable $V_{k,p}$ for $p = 1$,$p_{\max}$ is initialized as:

$$V_{k,p} = \begin{cases} V_{ship,mean} + \left(b_{u,Vk} - V_{ship,mean}\right)\dfrac{rand() - 0.5}{0.5} & , \quad V_{ship,mean} < b_{u,Vk} \\ b_{l,Vk} + rand()\left(b_{u,Vk} - b_{l,Vk}\right) & , \quad V_{ship,mean} \geq b_{u,Vk} \end{cases} \quad , \quad \begin{array}{l} k = 1, n-1 \\ p = 1, p_{max} \end{array} \tag{33}$$

with $rand()$ being a function representing a uniform random distribution from 0 to 1.

In the case where $V_{ship,mean} < b_{u,Vk}$, the initial values for variable $V_{k,p}$ among the population of $p_{\max}$ particles are distributed uniformly around the $V_{ship,mean}$, otherwise a typical randomization between the lower and upper bounds is used. It was found that this intervention had a very beneficial effect on the convergence of the optimization algorithm and the repeatability of its results.

Table 1
Tuning parameters of the PSO algorithm.

Description	Symbol	Value
Maximum number of particles	$p_{\max}$	100
Maximum iterations	$m_{\max}$	200
Tuning parameter (Eq. (34))	α	1
Tuning parameter (Eq. (34))	β	1.49
Maximum inertia parameter	θ_1	0.7

According the typical particle swarm optimization algorithm, after the initialization, the positions of the particles are updated as follows:

$$\widehat{x}_p^{m+1} = \widehat{x}_p^m + \widehat{u}^{m+1}\widehat{u}^{m+1} = \theta\widehat{u}^m + \alpha\widehat{r} \odot \left[g^* - \widehat{x}_p^m\right] + \beta\widehat{r} \odot \left[\widehat{x}_p^* - \widehat{x}_p^m\right] \tag{34}$$

where:

$\widehat{x}_p^m$: Vector containing the values of the independent variables of particle p (position) at iteration m
$\widehat{u}_p^m$: Vector containing the updates of the independent variables of particle p (velocity) at iteration m
g^*: Vector containing the globally best solution found among the population up to iteration m
$\widehat{x}_p^*$: Vector containing the best solution found for particle p iteration m
$\widehat{r}$: Vector of random values between 0 and 1 (updated for each iteration m)
$\odot$: Hadamard product representation
θ, α, β: Tuning parameters of the PSO algorithm.

The parameter, referred to as the inertia tuning parameter, is not constant during the succession of the m iterations, but is adjusted from a high value θ_1 towards a lower value θ_2 among the m iterations as

$$\theta = (\theta_1 - \theta_2)\frac{m_{max} - m}{m_{max}} + \theta_2 \tag{35}$$

with $m_{\max}$ being the maximum permitted number of iterations.

The tuning of the algorithms parameters was executed with several trial and error procedures, in order for the final solutions obtained to present a virtually absolute repeatability and fast convergence. Their values are presented in Table 1.

The maximum number of iterations is not practically reached, as the procedure stops when an improvement on the global best objective function is not larger than a very low limit for more than 10 iterations. Due to the probabilistic nature of the algorithm, the convergence is achieved at a different time of iterations on each execution, and the value of $m_{\max} = 200$ is set to a relatively large value for safety.

In the development of the PSO optimization algorithm, one intervention was carried out in order to improve its performance. The standard PSO algorithm dictates that in each iteration the particles should be attracted towards the global best found g^*. Instead of this, in the initial iterations, the particles at iteration $m+1$ are attracted towards the g^m, which is the best candidate solution found among the population at iteration *m,* instead towards g^*. The value of g^m is, in general, suboptimal compared to g^*, but it was observed that this practice largely improves the performance of the optimization algorithm, due to the fact that helps the population to retain its divergence, which is needed, especially in the initial iterations of the algorithm execution in order for avoiding convergence to local optima.

The appropriate handling of the values of independent variables in cases that they exceed the permissible bounds after the application of the positions update step was found to be another important aspect of the successful application of the optimization algorithm. In the case that an independent variable is temporarily given a value exceeding the limits posed by either the lower or upper bound, this unacceptable value is overridden, and is replaced by another value derived from a stochastic distribution, the codomain of which is adapted in each iteration in a manner that the final outcome remains in the permissible range.

7. Case study

For the case study, an actual trip of an existing 6900 TEU containership was considered. The ship departed from Port A (port of Ensenada, Baja California) and arrived to Port B (Negishi Bay, Japan) in the span of 295 h (12.3 days). The total sailing distance between the two ports was $d_{A \rightarrow B} = 5136.5$ nautical miles (9512.9 km).

The ship has one propeller and one main engine (two stroke, Diesel engine). For the modeling of the system, the procedures presented in Section 5 were applied. All necessary information was taken from the GA (general arrangement) of the ship. Furthermore, data from multiple sea trials as well as real time stored data from onboard sensors were used. Related data include power, mass flow and SFOC measurements of the main engine as well as ambient conditions (pressure and temperature). Also, ship itineraries and measurements of ship drafts, trim, propeller power, latitude and longitude of the ship, speed over ground, speed through water, ship course, apparent wind speed and direction, in 5–10 min intervals. Some of the most essential characteristics of the container ship are given in Table 2.

It is clarified that, since no actual (measured) data were available for the wave height and direction, the following steps were taken: the wave height was correlated with the wind speed utilizing the Beaufort scale, while the wave direction was set roughly equal to the wind direction. Thus, in order to describe the weather state two parameters were required as inputs: the wind speed and direction.

In Fig. 4 and Fig. 5, characteristic curves of the required brake power vs ship speed are given in relation to actual (measured) data, for two different sets of conditions of draft, trim, wind speed and direction. It is observed that the resulting resistance-propulsion model for calculating the required propulsion power from the main engine presents satisfying accuracy for the requirements of the current work.

The main engine, a two-stroke Diesel engine, was modeled using data from the manufacturer as well as data from sea trials and shop tests. The resulting model and related polynomial coefficients are given in Appendix A, Table A1. Some basic characteristics of the main engine can be found in Table 3, while a plot of the resulting SFOC calculation vs load percentage, for specific ambient conditions, can be found in Fig. 6.

In Fig. 7, the actual (measured) weather conditions (wind speed and direction) encountered by the ship during her trip are given. Additional weather data, measured from sensors onboard the ship during the route, and input in the models, include the air pressure and temperature and

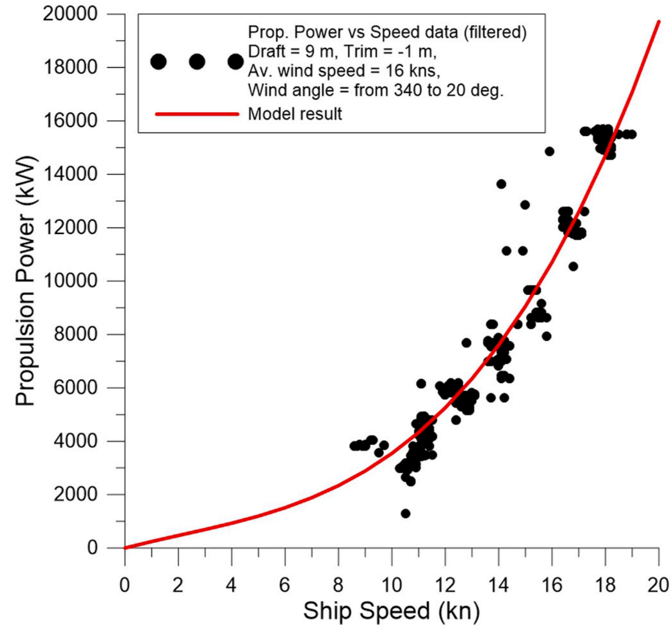

Fig. 4. Propulsion power vs Ship Speed and comparison with data (case 1).

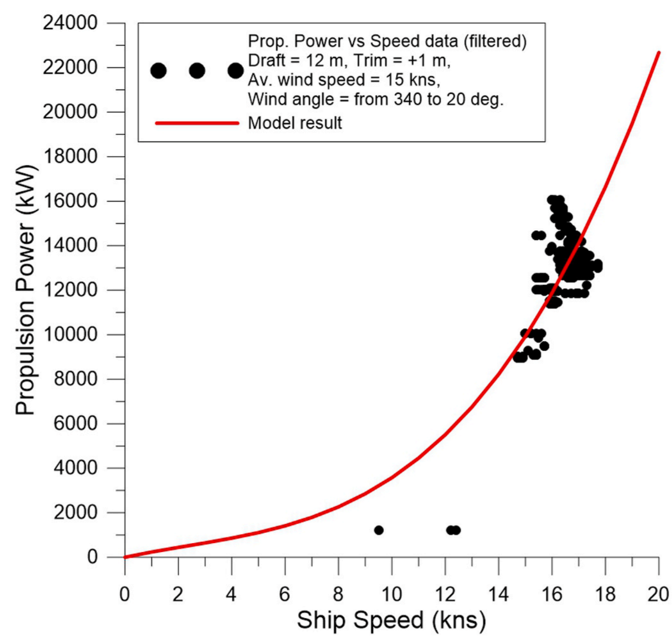

Fig. 5. Propulsion power vs Ship Speed and comparison with data (case 2).

Table 2
Basic characteristics and dimensions of the container ship.

Parameter	Symbol	Value
Length (Overall)	L_{OA}	270.9 m
Length (Between perpendiculars)	L_{BP}	258.4 m
Breadth	B	42.8 m
Draft (scantling)	T_L	14.55 m
Draft (design)	T_B	13 m
Block coefficient*	C_B	0.6234
Midship coefficient*	C_M	0.975
Service Speed at design Draft	V_s	22.5 kn
Propeller diameter	d_p	9.1 m
Deadweight	DWT	72294 tons

Table 3
Main engine characteristics.

Parameter	Symbol	Value
Engine Type	–	Two Stroke
Nominal Power	MCR	32.9 MW
Nominal Speed	RPM	72 rpm
Fuel type	–	HFO

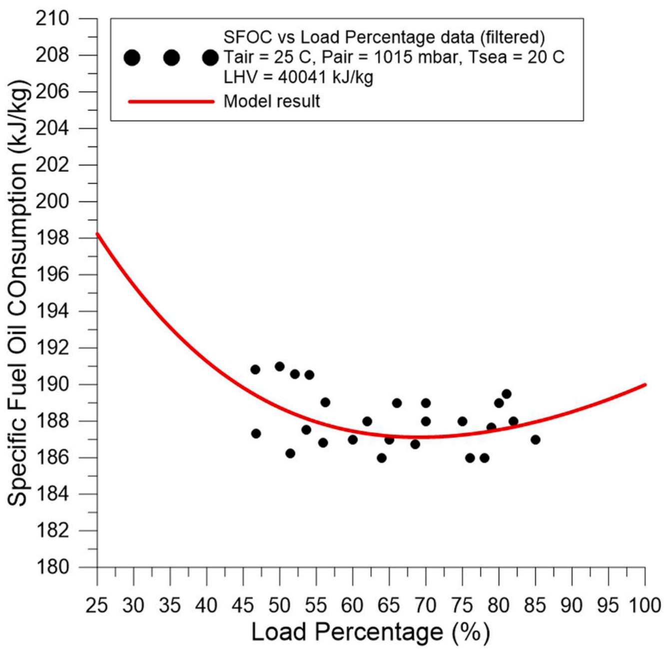

Fig. 6. SFOC vs load percentage of the main 2-X Diesel engine.

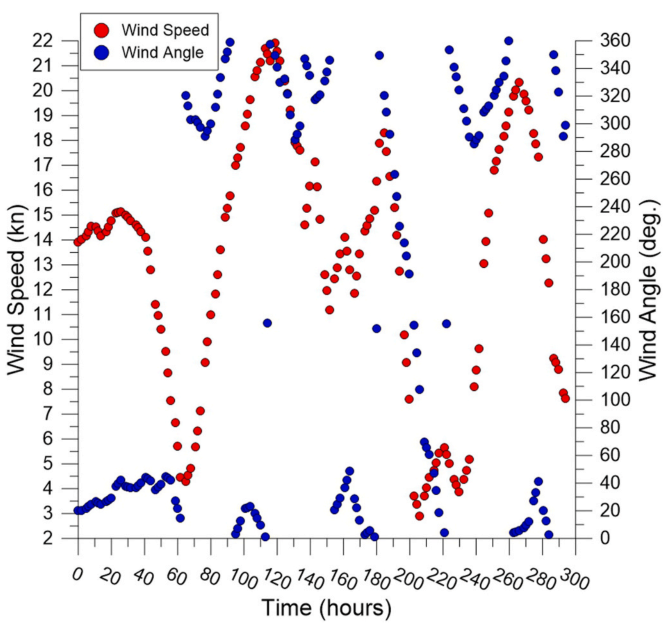

Fig. 7. Wind speed and direction vs Time.

the sea temperature. However, their variations during the trip and consequently their effects are minor, thus their profiles are not included here for brevity.

Fig. 8 depicts the actual ship speed profile that was followed by the ship during the specific trip in conjunction with the wind speed encountered.

In order for the effectiveness of the methodology developed to be assessed, it is applied at the case of the specific ship and the specified trip characteristics and weather conditions. Using the wind speed and direction readings from the ship, and due to lack of actual, successive, weather forecast data from the actual trip, we suppose that, *within the trust region of the weather forecast* (a parameter in the method), the predictions of the forecast coincide with the ship measurements. Of course, such an assumption may not always be completely accurate.

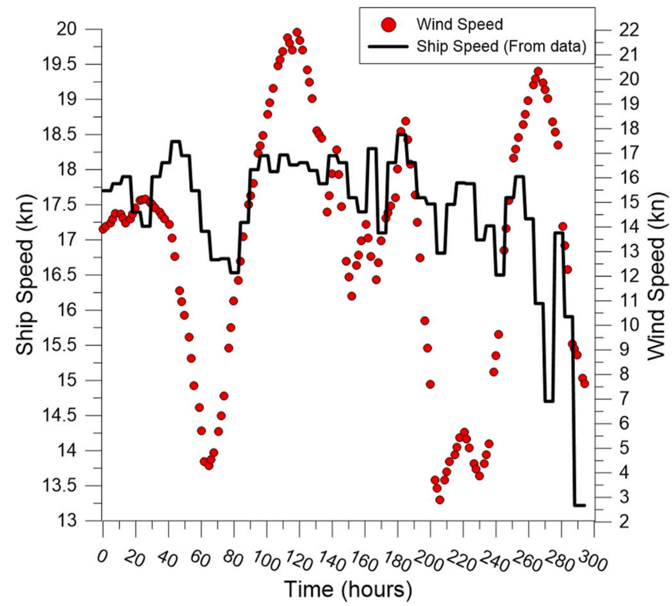

Fig. 8. Actual ship speed and wind speed vs Time.

Table 4
Problem parameters.

Parameter	Symbol	Value
Fuel LHV	LHV	40041.8 kJ/kg
Total distance	$d_{A \to B}$	5136.5 nm
Total travel duration	t_f	295 h
Discretization of weather inputs	Dt_w	3 h
Discretization of ship speed	Dt_V	6 h

However, it is adequate under the scope of testing the effectiveness of the method in successfully approximating the theoretical optimal solution of the full time horizon.

Some of the most important parameters of the problem are summarized in Table 4.

By setting the parameter Dt_w equal to 3 h, we are assuming that the time discretization of the weather forecast is equal to 3 h. Equivalently, by setting $Dt_V = 6$ h, we accept that the ship will effectively readjust its speed every 6 h. Effectively, any combination of Dt_V and Dt_w can be used as input; even the case of $Dt_V = Dt_w$, which states that the ship speed changes concurrently with the weather profile.

First, the optimization problem is solved for the total 295 h horizon, as if the exact weather profile is known a priori with accuracy. For its solution, the same procedure (presented in Section 6) as for each successive optimization problem is applied, but having omitted the total horizon segmentation step (Step a). Of course, this solution is only theoretical, but it will serve as a baseline for the assessment of the method. Next, the problem is solved for several combinations of n_a and n_b. By manipulating n_a we essentially change the *trust region* of the weather prediction; in other words, given we are at time t, we admit that the weather forecast will, from then on, accurately predict the weather conditions (Fig. 8) that the ship will encounter for the next $n_a{\cdot}Dt_V$ hours. Correspondingly, by manipulating n_b, we change the *solution application region*; or in other words, we change the frequency of when the next weather prediction will be received in order to formulate and solve the next sub-problem.

In Table 5 the list of parameter combinations, along with their corresponding weather forecast trust and solution application regions, is given. The total fuel consumption resulting from the solution of each parameter combination is given in Table 6, along with the calculated gain in comparison with the actual trip. In Fig. 9, the optimal ship speed

Table 5
List of parameter combinations and corresponding time regions for $Dt_V = 6$ h.

Description	n_a	n_b	Weather Trust Region	Solution application region	Number of successive (sub) problems
Full horizon	–	–	*295 h*	*295 h*	*1*
Combination 1	4	1	24 h	6 h	47
Combination 2	4	2	24 h	12 h	24
Combination 3	4	3	24 h	18 h	17
Combination 4	8	1	48 h	6 h	43
Combination 5	8	4	48 h	24 h	12
Combination 6	8	7	48 h	42 h	7
Combination 7	12	1	72 h	6 h	39
Combination 8	12	6	72 h	36 h	8
Combination 9	12	11	72 h	66 h	5

Table 6
Results of the optimizations in terms of total consumption (objective function), computational times and difference from theoretical optimum.

Description	Computational Time (sec)	Total Consumption (kg)	Difference (%) from Theoretical Optimum	Gain in Fuel Consumption (%)
Actual Speed	–	1080185	2.064	–
Full horizon (Theoretical Optimum)	*1195*	*1058339*	–	*2.022*
Combination 1	1470	1059911	0.149	1.877
Combination 2	735	1060019	0.159	1.867
Combination 3	525	1060092	0.166	1.860
Combination 4	2660	1059196	0.0810	1.943
Combination 5	805	1059412	0.101	1.923
Combination 6	420	1059533	0.113	1.912
Combination 7	4025	1058975	0.061	1.964
Combination 8	840	1059015	0.064	1.959
Combination 9	455	1059042	0.066	1.957

profiles derived from several selected parameter combinations, along with the actual ship speed, are presented.

The results indicate that for a fixed value of the parameter n_a, the best value for parameter n_b is 1, that is, as small as possible. This result is logical and expected. With a low value of n_b, the individual optimization sub-problems succeed each other with the smallest possible step. In this way, the weather (forecast) data input in each of them is updated frequently and manage to efficiently predict the actual weather conditions. Thus, the individual successive problem solutions are able to better approximate the optimal theoretical solution of the full time horizon problem.

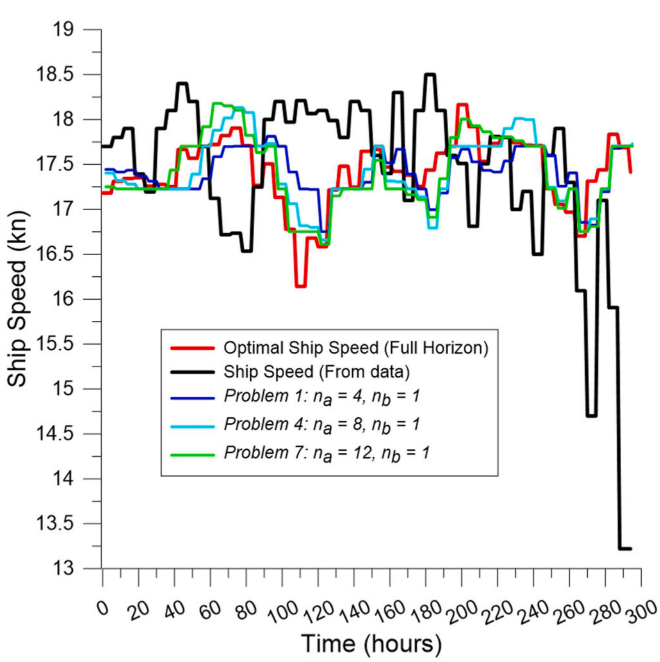

Fig. 9. Optimal ship speed profiles for $Dt_V = 6$ h.

Furthermore, another important observation is that for greater values of the parameter n_a, the optimal solutions derived from the method are getting better and better and closer to the optimal theoretical solution of the full time horizon problem. Again, this result is logical, since by increasing the parameter n_a, we essentially increase the trust region of the weather. This means that, for each successive sub problem, the weather data input is considered to be accurate for a longer horizon.

Considering the computational times, a trade-off is observed. For a fixed n_b (i.e. $n_b = 1$), as n_a becomes greater, less successive sub problems have to be solved. However, these sub problems become more and more complicated and harder to converge, thus requiring more computations. Parameter combination 4, when compared with combination 1, nearly doubles the accuracy of the approximation, but takes twice as much in terms of computational time to converge. The same can be observed when comparing combinations 7 and 4. On the other hand, for a fixed n_a, by increasing n_b, the number of successive sub problems, and thus computational times are decreased, but approximation accuracy is sacrificed.

Of course, by observing the results of Table 6, one could argue that even in the worst case scenarios (parameter combinations 1–3) the difference of total consumption of the approximate optimal solution from the optimal theoretical solution of the full time horizon problem, is less than 0.2%. These quite low differences are positive indications for the practical applicability of the method, since the cases presented in parameter combinations 1–3 are essentially expected to be closer to what the application of this method in in real life scenarios would resemble, as the weather forecast trust region is expected to be very short.

Finally, the most important observation is that with the application of the speed optimization method presented, the ship speed profile can be optimized according to the upcoming weather data provided and result to improvements of the overall fuel consumption in the order of

Table 7
Sensitivity analysis for the effect of Dt_V on the optimal solution.

Description	Dt_V *(hours)*	n_a	n_b	Total Consumption (kg)	Difference from baseline (Combination 4) (kg)
Combination 10	3	16	1	1058977	−219 (−0.02%)
Combination 4	*6*	*8*	*1*	*1059196*	–
Combination 11	12	4	1	1059602	406 (+0.04%)
Combination 12	24	2	1	1060353	1157 (+0.11%)

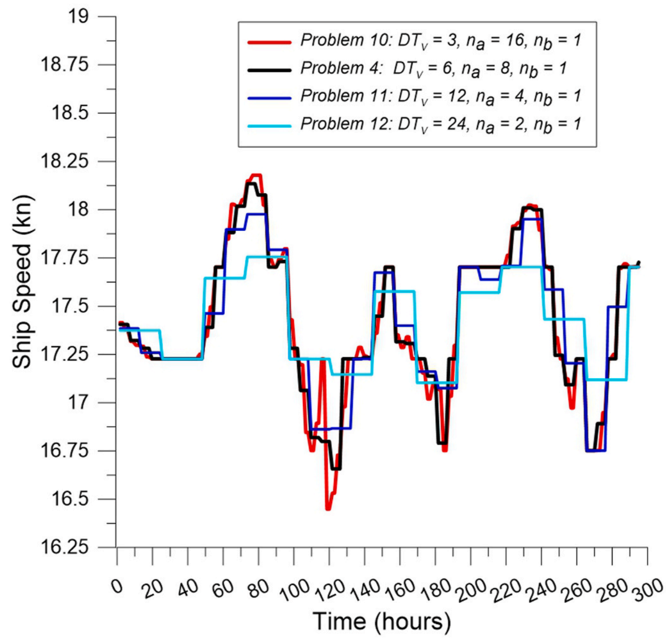

Fig. 10. Optimal ship speed profiles for parameter combinations 10, 11, 12 and 4.

1.86- 2.0%. The gain in fuel consumption could potentially be considered small, but it is certainly not negligible. It is worth pursuing this optimization and more so, if we take into consideration that the gain is achieved with a nearly zero investment. Also, it has to be noted, that, the expected improvements will be highly dependent on the form of weather profile and the duration of the trip. Longer trips and more aggressive weather profiles (steep and rapidly changing) will lead to increase of the gains achieved. In any case, another beneficial aspect of the application of the method, is that the optimal speed can be determined in a formal mathematical manner, without relying upon the subjective estimations of the navigation crew.

Next, in order to investigate the effect of the ship speed readjustment interval (parameter Dt_V) on the optimal solution of the problem, three additional cases, for $Dt_V = 3$, 12 and 24 h, are formulated and solved. In all three cases, the n_a parameter is adjusted so that the weather trust region ($n_a \cdot Dt_V$) is equal to 48 h, while the n_b parameter is set to 1. Details and optimization results are given in Table 7. The optimal ship speed profiles are given in Fig. 10.

It is observed that setting the speed discretization equal to the weather conditions discretization (3 h) does not greatly affect the optimal solution, although it slightly improves it, as expected. The same can be said in the case of increasing the Dt_V to 12 h; only a slight deterioration of the optimal total consumption value is observed. The deterioration is more apparent in the case of $Dt_V = 24$ h. However, it is very interesting that even in the case of using 24 h intervals for the ship speed (i.e. change speed every two days) and a weather forecast trust region of 48 h, the proposed optimal speed profile still produces significant gains when compared with the initial, actual consumption.

Summarizing, the sensitivity analysis performed in the current Section reveals that the best performance, in terms of accurately finding the optimal ship speed profile that minimizes fuel consumption, is achieved by setting parameter n_a as large as possible, parameter n_b as small as possible and parameter Dt_V (how often the ship readjusts the speed) equal to Dt_w (the weather forecast interval). However, the differences in the way the method performs when different values are selected for these parameters, are not significantly large, as the overall optimal solution is satisfactorily approached in most of the cases presented (except, perhaps, in the case where the Dt_V interval becomes much larger than 12 h). In any case, the aforementioned parameters are to be considered as tunable parameters in order to enhance the practicality and range of applicability of the method. Thus, the final choices for their values are to be made upon each application at hand and while having taken under consideration all the details and practical constraints of the problem as well as the operator preferences. For example, the navigation crew may consider that a change of sailing speed every time the weather changes ($Dt_V = Dt_w$) is not practically realizable in a route and thus set the Dt_V parameter to be higher. Also, the weather prediction time step (parameter Dt_w), which was set equal to 3 h for the presentation of the method in the current work, can be set to arbitrarily smaller values as long as smaller time resolutions of weather data are supported by the weather provider; the method can effectively incorporate any given weather forecast interval length. Furthermore, the weather forecast trusted region, which was assumed to be 48 h in the present example, may actually be much higher or much lower and this will affect the selection of the value of parameter n_a, while the parameter n_b can be determined according to the navigation plan rescheduling frequency as determined by the crew. In a final comment, it is noted that the current method is proposed under the assumption that the weather prediction data are satisfactorily accurate in the short term prediction region (denoted as trust region in the text). However, further developments can be carried out on the method, based in stochastic and robust optimization formulations of the problem, for incorporating the effects of the probabilistic nature of the weather data in the final optimal solution.

8. Closure

In the present work, a method for the determination of the optimal operational ship speed is proposed, aiming at the minimization of the overall total fuel consumption over specified routes. This method can be thought of as being included in the more general context of weather based ship operational profile optimization. From a mathematical point of view, the problem at hand is posed as a dynamic optimization problem. The distinctive characteristic of the method is that the full time horizon is segmented in smaller time regions, and a limited time horizon sub-problem is solved for each. This temporal segmentation and solution of successive sub-problems was employed for addressing the problem of the deteriorating accuracy of weather forecasts for longer prediction periods. The interdependence between the individual sub-problems is controlled by two parameters of important practical significance, which are representative of a) the frequency that a ship may readjust its sailing speed, and b) the time resolution of weather forecasts. The method is applied for the case of an actual container ship route, and the effect of control parameters is assessed. It is demonstrated that fuel consumption savings of the order of 2% in comparison with usual current practice may be achieved.

CRediT authorship contribution statement

George Tzortzis: Conceptualization, Methodology, Supervision, Writing – original draft, Writing - review & editing, Visualization, Investigation. **George Sakalis:** Conceptualization, Software, Methodology, Writing – original draft, Validation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A

For the calculation of the Main engine specific fuel Oil Consumption, Equations (23) and (24) are used. The base SFOC (Eq. (24)) has been modeled with a 4th order polynomial, Eq. (A1), utilizing data from the manufacturer as well as data from sea trials and shop tests:

$$b_{f,base} = \sum_{n=0}^{4} a_n \cdot f_L^n \tag{A.1}$$

with the a_n coefficients of the polynomial given in Table A1.

Table A1
Coefficients for the base SFOC polynomial.

Coefficient	Value
a_0	+208.0024724478
a_1	- 1.278690684100
a_2	+0.017639081300
a_3	- 0.000106074100
a_4	+0.000000273400

* It is noted that the values of the block and midship coefficients given here are the reference values at design draft. In reality, these parameters are functions of ship draft, and respective models have been created (from real data) and are used in the optimization procedure.

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