1 Lecture 0: Intro to Reaction Engineering

1. Reaction engineering

Understanding, modeling, designing, using, controlling, analyzing, improving anything in which chemical reactions happen.

- 1. Reaction engineering applications
 - (a) Traditional
 - i. Industrial chemical/petroleum processes
 - ii. Fine chemical/pharmaceutical processes
 - iii. Emerging, eg biorefinergy, shale gas, http://cistar.us
 - (b) Energy storage, batteries, fuel cells
 - (c) Environmental systems
 - i. Atmosphere, lake, bioreactor (water purification), catalytic convertor
 - (d) Biological systems
 - i. Cell, organ, body
 - (e) Laboratory reactors interrogate, quantify
 - (f) Research improved materials (catalysts), improved processes, understand limitations
 - i. Sabatier plot, https://doi.org/10.1038/nchem.121

2. Course structure

- (a) Quantifying chemical reactions
 - i. Stoichiometry
 - ii. Thermodynamics heat flow, direction, equilibrium
 - iii. Kinetics rates, mechanisms
- (b) Physical/chemical interactions
 - i. Transport, mixing, diffusion resistance, ...
- (c) Chemical reactors
 - i. Ideal 0 and 1-dimensional
 - ii. Non-ideal
 - iii. Non-isothermal
 - iv. Non-steady state
 - v. Multiphase
- (d) Chemical processes (beyond us)
- (e) Markets (beyond us)

2 Lecture 1: Stoichiometry and reactions

- 1. Substances
- 2. Amounts
 - (a) mass, moles, volumes
 - (b) flow rates
- 3. compositions
 - (a) amount/total amount
- 4. Reactions and stoichiometric coefficients
 - (a) Advancements $n_j = \sum_i \nu_{ij} \xi_i$
 - (b) Limiting reagents

3 Lecture 2: Chemical thermodynamics and equilibria

- 1. Chemical reactions $\sum_{j} \nu_{j} A_{j} = 0$
- 2. Thermodynamic potential differences
 - (a) Standard states
 - (b) Formation reactions
 - (c) Reaction enthalpy $\Delta H^{\circ}(T) = \sum_{j,j} H_{j}^{\circ}(T) = \sum_{j} \nu_{j} H_{f,j}^{\circ}(T)$
 - (d) Reaction entropy $\Delta S^{\circ}(T) = \sum_{j} {}_{j} S^{\circ}_{j}(T)$
- 3. Equilibrium-closed system
 - (a) Free energy vs reaction advancement, $G(\xi,T) = \sum_{j} n_{j} \mu_{j} = \sum_{j} (n_{j0} + \nu_{j} \xi) \left(\mu_{j}^{\circ}(T) + RT \ln a(\xi,T) \right)$
 - (b) Equilibrium $(\partial G/\partial \xi)_{T,P} = 0$
 - (c) Equilibrium constants and algebraic solutions
 - (d) Multiple reactions
- 4. Le'Chatlier principle system at equilibrium responds to oppose any perturbation
 - (a) Pressure, composition
 - (b) Temperature: Gibbs-Helmholtz and van't Hoff
- 5. Equilibrium-open system
 - (a) Reaction phase diagrams, see http://pubs.acs.org/doi/abs/10.1021/jacs.6b02651 for an example
 - (b) Electrochemical reactions
- 6. The molecular interpretation
- 7. Non-ideal activities
- 8. Surface adsorption
 - (a) Langmuir

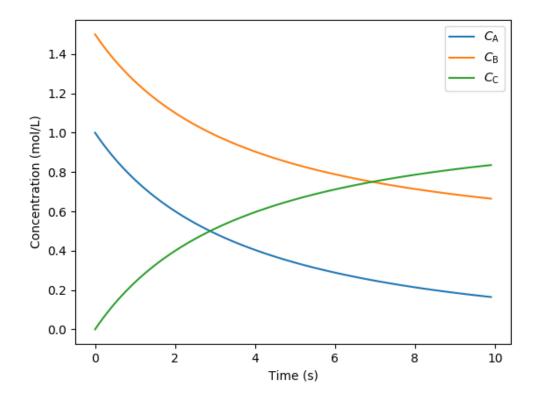
4 Lecture 3: Empirical kinetics

- 1. rates: number per unit time per unit something
- 2. reactor mass balance
- 3. rate expressions, Functions of T, P, composition C_i
- 4. rate orders
- 5. apparent orders
- 6. integrated rate expressions
- 7. temperature and Arrhenius expression, $k = Ae^{-E_a/k_BT}$
 - (a) Arrhenius plot, $\ln k$ vs 1/T

Table 1: Basic kinetic rate laws

	differential rate	integrated rate	half-life
First order	$r = kC_A$	$C_A = C_{A0}e^{-k\tau}$	$\frac{1 \ln 2/k}{}$
Second order	$r = kC_A^2$	$1/C_A = 1/C_{A0} + k\tau$	$1/kC_{A0}$

```
import numpy as np
                                      #this lets up handles arrays of data
    import matplotlib.pyplot as plt
    from scipy.integrate import odeint, solve_ivp
4
5
    def dCdt(C,t,k):
       dC_Adt = -k*C[0]*C[1]
                                   \# A + B \rightarrow C; \quad r = k \ CA \ CB
6
        dC_Bdt = -k*C[0]*C[1]
7
        dC_Cdt = k*C[0]*C[1]
9
        dCdt = [dC_Adt,dC_Bdt,dC_Cdt]
        return dCdt
10
11
    # initialize concentrations
12
13
    C_0 = [1., 1.5, 0.]
14
    # initialize k's
15
    k = 0.2
16
17
18
    # Range of time to solve over
    t = np.arange(0,10,0.1)
19
20
    t_{span} = (0., 10.)
21
  p = (k,) # turn parameters into a tuple
    # Solve two ODEs with odeint
23
24
    #C = solve ivp(dCdt,t span,C 0,p,method='LSODA')
    C = odeint(dCdt,C_0,t,p)
^{25}
26
    C_A = C.transpose()[0] # Get C_A from C
   C_B = C.transpose()[1] # Get C_B from C
28
    C_C = C.transpose()[2]
29
    plt.figure()
    plt.plot(t,C_A,'-',label=r'$C_{\rm A}$')
32 plt.plot(t,C_B,'-',label=r'$C_{\rm B}$')
33 plt.plot(t,C_C,'-',label=r'$C_{\rm C}$')
    plt.xlabel('Time (s)')
    plt.ylabel('Concentration (mol/L)')
    plt.legend()
36
    plt.savefig('./conc.png')
```



5 Lecture 4: Analyzing reactor data

- 1. Differential methods
 - (a) Measuring rates
- 2. Integral methods
- 3. Half-lives

```
import numpy as np
                                      #this lets up handles arrays of data
    import matplotlib.pyplot as plt
    from scipy.optimize import curve_fit
3
    def differential(x, k, alpha):
        return k*x**alpha
7
    def integral(t, a, b):
8
        return (2*a/(2+a*b*t))**2
9
10
    t = np.array([0.00, 2.25, 4.50, 6.33, 8.00, 10.25, 12.00, 13.50, 15.60, 17.85, 19.60, 27.00, 30.00, 38.00, 41.00, 45.00, 47.00,
11
12
    C_Br2 = np.array([0.3335, 0.2965, 0.2660, 0.2450, 0.2255, 0.2050, 0.1910, 0.1794, 0.1632, 0.1500, 0.1429, 0.1160, 0.1053, 0.0830
13
14
    plt.figure()
15
    plt.plot(t,C_Br2,'o')
    plt.xlabel('Time (s)')
17
    plt.ylabel('Concentration (mol/L)')
    plt.legend()
19
    plt.savefig('./xylene-conc.png')
```

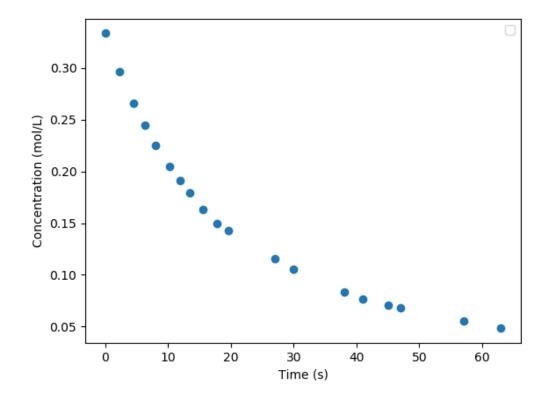
```
^{21}
    delta_t = np.ediff1d(t)
                                     # finite difference between adjacent points
22
23
    delta_C = np.ediff1d(C_Br2)
24
    grad_t = np.gradient(t)
grad_C = np.gradient(C_Br2)
                                         # second order approximation to gradient, allowing for unequal step size
25
26
    rate = -np.divide(grad_C,grad_t)
27
28
29
    plt.figure()
30
    plt.plot(C_Br2,rate,'o')
    plt.xlabel('Concentration (mol/L)')
31
    plt.ylabel('Rate (mol/L/x)')
32
    plt.legend()
34
35
    popt, pcov = curve_fit(differential, C_Br2, rate )
36
37
    print('k = {0:f}, alpha={1:f}'.format(popt[0],popt[1]))
38
    model = differential(C_Br2,popt[0],popt[1])
39
40
    plt.plot(C_Br2,model,'-')
41
    plt.savefig('./xylene-rate.png')
42
^{43}
    difference_array = np. subtract(rate, model)
44
45
    squared_array = np. square(difference_array)
    mse = squared_array. mean()
46
    print(mse)
47
48
    # Suggests order of 1.5
49
    popt1, pcov1 = curve_fit(integral, t, C_Br2)
50
    print('k = {0:f}'.format(popt[1]))
51
    model1 = integral(t, popt1[0], popt1[1])
53
54
    plt.figure()
55
    plt.plot(t,C_Br2,'o')
56
   plt.plot(t,model1,'-')
   plt.xlabel('Time (s)')
58
59
    plt.ylabel('Concentration (mol/L)')
60
    plt.legend()
    plt.savefig('./xylene-int-model.png')
61
```

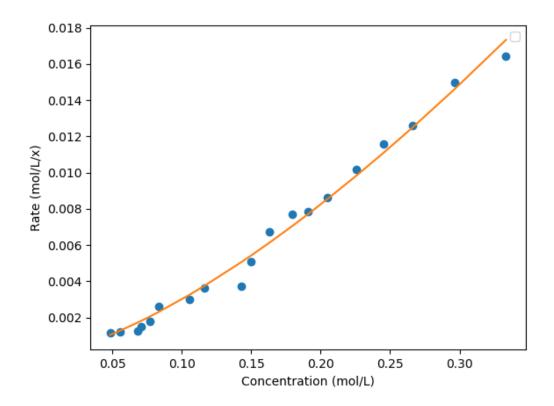
October 4, 2021

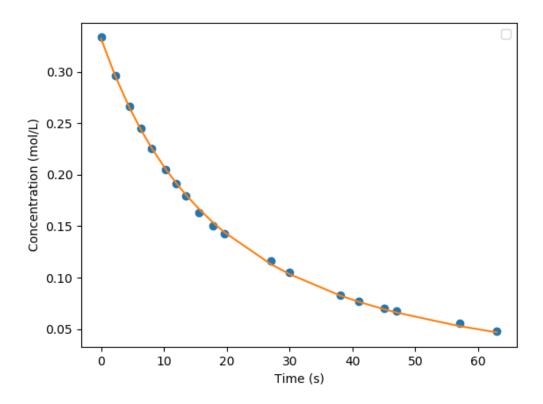
k = 0.085277, alpha=1.450860

2.4942019231742367e-07

k = 1.450860







6 Lecture 5: Molecular chemical kinetics

6.0.1 Reaction mechanisms

- 1. Elementary steps and molecularity
- 2. Ozone decomposition, rate second-order at high P_{O_2} , first-order at low P_{O_2}

$$\begin{array}{c} 2\mathrm{O}_3 \longrightarrow 3\mathrm{O}_2 \\ \mathrm{O}_3 \xrightarrow{k_1} \mathrm{O}_2 + \mathrm{O} \\ \mathrm{O}_2 + \mathrm{O} \xrightarrow{k_2} \mathrm{O}_3 \\ \mathrm{O} + \mathrm{O}_3 \xrightarrow{k_2} 2\mathrm{O}_2 \end{array}$$

- 3. Detailed balance and microscopic reversibility
- 4. Equilibrium requirement $K_{eq}(T) = k_f(T)/k_r(T)$
- 5. Reversibility $r_{\text{net}} = r_f(1 \beta), \ \beta = Q/K_c = \exp(-\Delta G(T, c_j)/RT)$
 - (a) $A \rightleftharpoons B$ example
- 6. Collision theory
 - (a) $A + B \rightarrow products$

(b) rate proportional to A/B collision frequency z_{AB} weighted by fraction of collisions with energy $> E_a$

$$r = kC_A C_B, k = \left(\frac{8k_B T}{\pi \mu}\right)^{1/2} \sigma_{AB} N_{av} e^{-E_a/k_B T}$$

(c) upper bound on real rates

6.0.2 Transition state theory (TST)

- 1. Assumptions
 - (a) Existence of reaction coordinate (PES)
 - (b) Existence of dividing surface
 - (c) Equilibrium between reactants and "transition state"
 - (d) Harmonic approximation for transition state
- 2. rate proportional to concentration of "activated complex" over reactants times crossing frequency

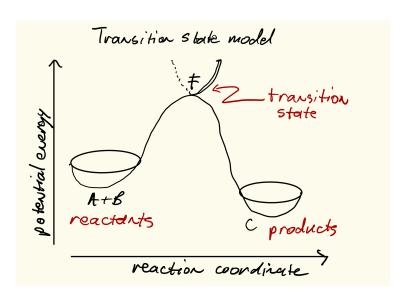
$$r = kC_A C_B$$

$$= k^{\ddagger} C_{AB}^{\dagger}$$

$$= \nu^{\ddagger} K^{\ddagger} C_A C_B$$

$$= \nu^{\ddagger} \frac{k_B T}{h \nu^{\ddagger}} \bar{K}^{\ddagger}(T) C_A C_B$$

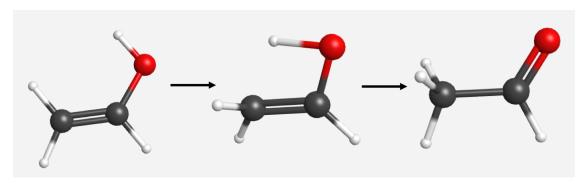
$$= \frac{k_B T}{h} \frac{q^{\ddagger}(T)}{q_A(T) q_B(T)} e^{-\Delta E(0)/k_B T} C_A C_B$$



6.0.3 Locating transition states computationally

- 1. Reactants/products are minima on potential energy surface
- 2. Transition state is first order saddle point. Unique point on pathway from reactant to product valley

- 3. vinyl alcohol to acetaldehyde example
- 4. https://www.webmo.net



6.0.4 Thermodynamic connection

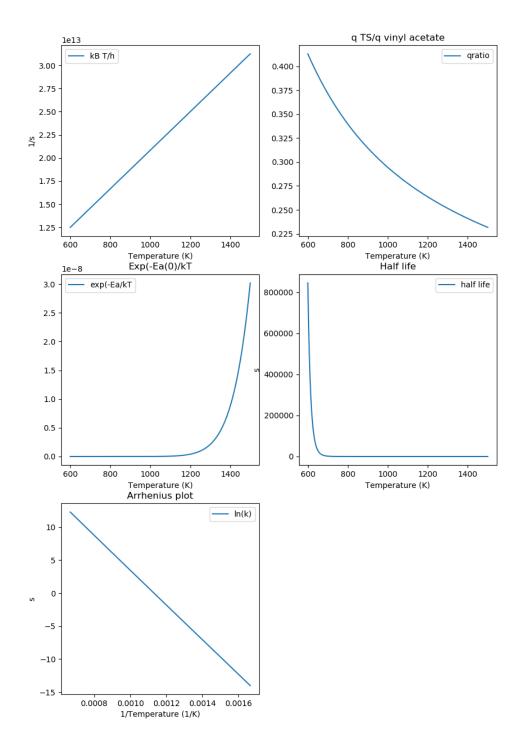
1. Relate activated complex equilibrium constant to activation free energy (isochoric standard state)

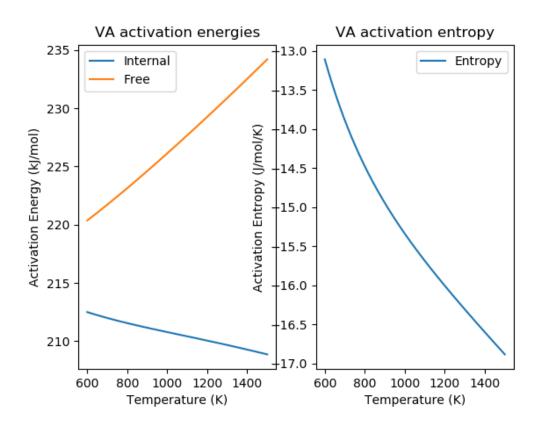
$$\bar{K}^{\ddagger}(T) = e^{-\Delta A^{\circ\ddagger}(T)/kT} = e^{-\Delta U^{\circ\ddagger}(T)/k_BT} e^{\Delta S^{\circ\ddagger}(T)/k_B}$$

2. Compare to Arrhenius expression

$$E_a = \Delta U^{\circ \ddagger}(T) + kT, A = \frac{k_B T}{h} e^1 e^{\Delta S^{\circ \ddagger}(T)/k_B}$$

Vinyl alcohol to TS 216 kJ/mol
Delta Uddagger (1000 K) = 211 kJ/mol
Delta Addagger (1000 K) = 226 kJ/mol
Delta Sddagger (1000 K) = -1480 J/mol K





6.0.5 Bimolecular reaction

1. Diels-Alder example

```
import numpy as np
1
2
                          # eV /K
    kB = 8.61733e-5
3
    h = 4.13566766e-15 # eV s
    eVtokJ = 96.485332
    Nav = 6.022e23
 6
                         # Avogadro's number
    RO = kB * eVtokJ * 1000.
                                    # gas constant in J/mol K
    A = 9.2e6 # liter/mole/second
    Ea = 99. \# kJ/mole
10
11
12
    T = 500.
                # K
13
14
    # 1 M standard state
15
    deltaUdd = Ea - R0 * T /1000
16
^{17}
    SS = 1.0 \# mol/liter
18
    deltaSdd = R0 * ( np.log(A/(1./SS)) - np.log(kB * T / h) - 1.)
19
20
21
    deltaAdd = deltaUdd - T * deltaSdd/1000.
22
    print('1 M standard state, 500 K:')
23
                                       Delta Sdd = {:4.0f} J/mol K Delta Add = {:4.0f} kJ/mol'.format(deltaUdd,deltaSdd,deltaAdd)
^{24}
    print('Delta Udd ={:4.0f} kJ/mol
25
26
    # 1 bar standard state
    PO = 1.0e5 # 1 bar = 10^5 Pa = 10^5 J/m^3
27
    PO = PO / 1e3 \# J/l
```

SS = PO/(RO * T) # J/mol / J/l = mol/liter

```
Delta Udd = 95 \text{ kJ/mol} Delta Sdd = -124 \text{ J/mol} K Delta Add = 157 \text{ kJ/mol} 1 bar = 0.024 \text{ mol/liter} standard state, 500 K Delta Hdd = 91 \text{ kJ/mol} Delta Sdd = -164 \text{ J/mol} K Delta Gdd = 172 \text{ kJ/mol}
```

6.0.6 Correlations across reactions

- 1. early vs late transition states
- 2. Brønsted-Evans-Polyani relationship

$$E_a = \alpha \Delta H + \beta$$

3. linear free energy relationships between similar reactions (substituent effects)

$$\ln(k_1/k_1') \propto \ln(K_1/K_1')$$

4. compensation effect linear correlation across catalysts for the same reaction

$$\Delta H^{\circ \ddagger} \propto \Delta S^{\circ \ddagger}$$

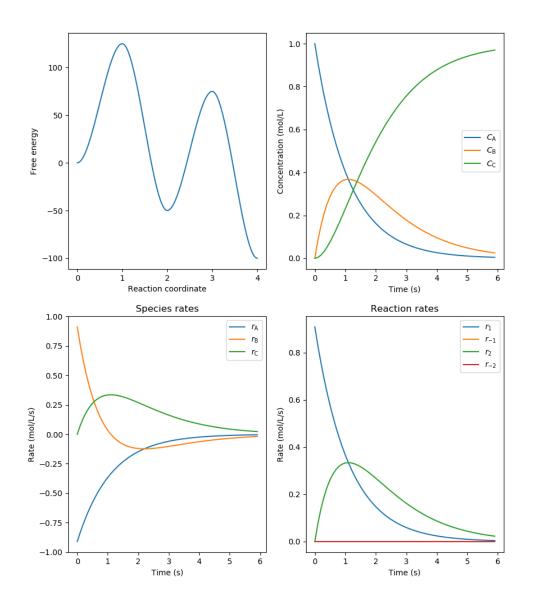
7 Lecture 6: Reaction networks

7.1 Simple and visible

- 1. Open vs closed (catalyzed) networks
- 2. $A \longleftrightarrow B \longleftrightarrow C$ reaction
- 3. free energy surface
- 4. Characterizations
 - (a) instantaneous selectivity
 - (b) overall selectivity
 - (c) yield
- 5. species rates vs reaction rates
 - (a) Vector representations of rates
- 6. Rate determining and degree of rate control
- 7. Equilibrium limited

```
import numpy as np
                                                                        #this lets up handles arrays of data
        import matplotlib.pyplot as plt
 2
         from scipy.integrate import odeint, solve_ivp
        from scipy.interpolate import BPoly
 4
        kB = 8.61733e-5
                                                 # eV /K
  6
        h = 4.13566766e-15 # eV s
 7
        def dCdt(C,t,k):
 9
                dC_Adt = -k[0]*C[0]+k[1]*C[1]
                                                                                  \# A \iff B \iff CA CB
10
11
                dC_Bdt = k[0]*C[0]-k[1]*C[1] -k[2]*C[1]+k[3]*C[2]
12
                dC_Cdt = k[2]*C[1]-k[3]*C[2]
13
                dCdt = [dC_Adt,dC_Bdt,dC_Cdt]
                return dCdt
14
15
        # initialize concentrations
16
17
        C_0 = [1., 0, 0.]
18
        R0 = 8.314
19
        T = 500.
20
21
        # initialize k's
22
        deltaGtot = -100.0; deltaG1 = -50.0; deltaG1d = 125. ; deltaG2d = 125.
23
24
        deltaG2 = deltaGtot - deltaG1
25
        deltaGn1d = deltaG1d-deltaG1
26
        deltaGn2d = deltaG2d-deltaG2
27
28
        p1 = BPoly.from_derivatives([0, 1, 2, 3, 4], [[0, 0], [deltaG1d, 0], [deltaG1, 0], [deltaG1+deltaG2d, 0], [deltaGtot, 0]])
29
30
        x=np.linspace(0,4,100)
31
32
        y=p1(x)
33
34
       plt.figure(figsize=(10,12))
35
       plt.subplot(2,2,1)
       plt.plot(x,y)
36
        plt.ylabel('Free energy')
37
        plt.xlabel('Reaction coordinate')
38
        k = (kB*T/h)*np.exp(-np.array([deltaG1d,deltaGn1d,deltaG2d,deltaGn2d])*1000./(R0*T))
40
        print('T = {:4.1f}'.format(T))
41
        print('Delta G1 = {:4.1f} deltaG1dagger = {:4.1f} deltaG2 = {:4.1f} deltaG2d = {:4.1f} kJ/mol'.format(deltaG1,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,deltaG1d,d
42
        print('k1 = {:6.3e}
                                                   k-1 = \{:6.3e\}
                                                                                                k2 = \{:6.3e\}
                                                                                                                                  k-2 = \{:6.3e\} /s'.format(k[0],k[1],k[2],k[3]))
43
44
        # Range of time to solve over
45
        t = np.arange(0,6,0.1)
46
47
        p = (k,) # turn parameters into a tuple
48
        C = odeint(dCdt,C_0,t,p)
        C_A = C.transpose()[0] # Get C_A from C
50
51
        C_B = C.transpose()[1] # Get C_B from C
        C_C = C.transpose()[2]
52
53
      plt.subplot(2,2,2)
54
       plt.plot(t,C_A,'-',label=r'$C_{\rm A}$')
55
        plt.plot(t,C_B,'-',label=r'$C_{\rm B}$')
        plt.plot(t,C_C,'-',label=r'$C_{\rm C}$')
57
       plt.xlabel('Time (s)')
58
       plt.ylabel('Concentration (mol/L)')
59
       plt.legend()
60
61
       plt.subplot(2,2,3)
62
       plt.plot(t,-k[0]*C_A+k[1]*C_B,label=r'$r_{\mbox{\em A}}$')
       plt.plot(t,k[0]*C_A-k[1]*C_B-k[2]*C_B + k[3]*C_C,label=r'$r_{\rm B}$')
64
       plt.plot(t,k[2]*C_B-k[3]*C_C,label=r'$r_{\rm C}$')
65
66
        plt.xlabel('Time (s)')
        plt.ylabel('Rate (mol/L/s)')
67
```

```
plt.title('Species rates')
   plt.legend()
69
70
   plt.subplot(2,2,4)
71
   plt.plot(t,k[0]*C_A,label=r'$r_1$')
72
   plt.plot(t,k[1]*C_B,label=r'$r_{-1}$')
   plt.plot(t,k[2]*C_B,label=r'$r_2$')
74
  plt.plot(t,k[3]*C_C,label=r'$r_{-2}$')
76 plt.xlabel('Time (s)')
77
   plt.ylabel('Rate (mol/L/s)')
   plt.title('Reaction rates')
   plt.legend()
79
   plt.savefig('./Images/ABC-rxn.png')
81
   T = 500.0
   Delta G1 = -50.0 delta G2 = -50.0
                                                                         deltaG2d = 125.0 kJ/mol
   k1 = 9.092e-01
                          k-1 = 5.433e-06
                                                     k2 = 9.092e-01
                                                                             k-2 = 5.433e-06 /s
```



7.2 Simply and invisible

- 1. Lindemann-Hinshelwood model for first order reactions
- 2. Irreversible steps
- 3. Pseudo-steady state approximation
 - (a) A \longrightarrow B \longrightarrow C in limit $k_2 > k_1$

 $r_{\rm B} \approx 0$

$$C_{\rm B,max} \approx 0$$

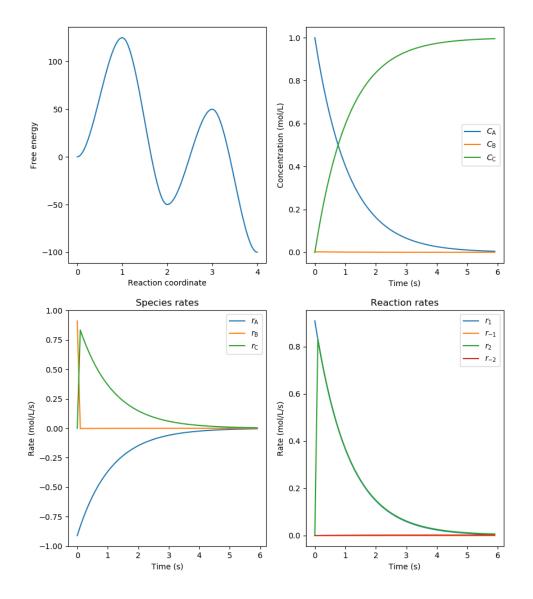
$$C_A = C_{A0}e^{-k_1t}$$

$$C_B = C_{A0}\frac{k_1}{k_2 - k_1} \left(e^{-k_1t} - e^{-k_2t} \right)$$

(a) B intermediate \rightarrow reactive intermediate

T = 500.0

Delta G1 = -50.0 deltaG1dagger = 125.0 delta G2 = -50.0 deltaG2d = 100.0 kJ/mol k1 = 9.092e-01 k-1 = 5.433e-06 k2 = 3.720e+02 k-2 = 2.222e-03 /s



7.3 Simplifying reaction networks

- 1. Simplifications helpful when parameters of full model are unknown or to fit to experimental observations
- 2. Ozone decomposition, $2O_3 \longrightarrow 3O_2$

$$O_3 \rightleftharpoons O_2 + O$$

$$O + O_3 \longrightarrow 2O_2$$

(a) PSSA

$$-r_{O3} = \frac{2k_1k_2C_{\mathrm{O}_3}^2}{k_{-1}C_{\mathrm{O}_2} + k_2C_{\mathrm{O}_3}}$$

- (b) Limiting behaviors
- 3. CO chlorination (closed)

$$\begin{array}{c} \operatorname{Cl}_2 \xrightarrow{h\nu} \operatorname{2Cl} \cdot \\ \operatorname{CO} + \operatorname{Cl} \cdot & \longrightarrow \operatorname{COCl} \cdot \end{array}$$

$$\operatorname{COCl} \cdot + \operatorname{Cl}_2 & \longrightarrow \operatorname{COCl}_2 + \operatorname{Cl} \cdot \end{array}$$

4. Ethane pyrolysis $C_2H_6 \longrightarrow C_2H_4 + H_2$ (open)

	$A (s^{-1})$	$E_a (\mathrm{kJ} \mathrm{mol}^{-1})$
$C_2H_6 \longrightarrow 2CH_3$	1.0×10^{17}	356
$CH_3 + C_2H_6 \longrightarrow CH_4 + C_2H_5$	2.0×10^{11}	44
$C_2H_5 \longrightarrow C_2H_4 + H$	3.0×10^{14}	165
$H + C_2H_6 \longrightarrow H_2 + C_2H_5$	3.4×10^{12}	28
$H + C_2H_5 \longrightarrow C_2H_6$	1.6×10^{13}	0

- 1. Chain reactions
 - (a) Polymerization

7.4 Complex reaction networks

1. Ethane pyrolysis redux, doi:10.1021/jp206503d

7.5 Stochastic solutions

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import odeint, solve_ivp

Nav = 6.022e23
    l_to_nm3 = 1e21

def dCdt(C,t,k):
    dC_Adt = -k[0]*C[0]  # A -> B -> C; r = k CA CB

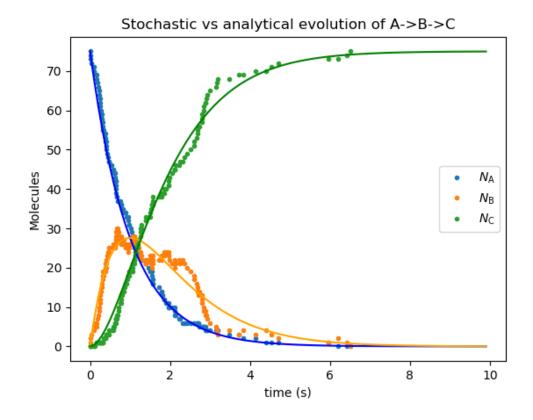
dC_Bdt = k[0]*C[0]-k[1]*C[1]

dC_Cdt = k[1]*C[1]

dC_ddt = [dC_Adt,dC_Bdt,dC_Cdt]

return dCdt
```

```
14
    Ntot = 75 # set total number of particles
15
16
    {\tt CAO} = 1.0 # set initial concentrations, mol/l
17
    CBO = 0.0
18
    CCO = 0.0
19
20
^{21}
    k = np.array([1.0,1.0])
                               # /second
22
23
    Ctot = CAO + CBO + CCO
24
    Volume = Ntot/Ctot # number/mol/liter
25
26
    side = Volume**(1/3)
27
28
    na = Ntot *(CAO/Ctot); nb = Ntot *(CBO/Ctot); nc = Ntot *(CCO/Ctot); tnow = 0.
29
    NA = np.array([na])
30
31
    NB = np.array([nb])
    NC = np.array([nc])
32
    t = np.array([tnow])
33
34
    while na > 0 or nb > 0: # While there is still A and B left, there will be reaction occurring.
35
36
        p1 = np.random.random_sample() # Get random number from 0 to 1.
        p2 = np.random.random_sample()
37
38
        r1 = k[0] * na # Definition for the reaction mechanism.
39
        r2 = k[1] * nb
        if p2 < (r1/(r1+r2)): # Monte Carlo step
40
41
            na-=1; nb+=1
42
        else:
43
            nb-=1; nc+=1
44
45
        dt = -np.log(p1) / (r1 + r2) # This is the time interval until the next reaction to occur.
        tnow+= dt # To refresh the time for next circulation.
46
47
        NA = np.append(NA,na) # inelegant and wasteful
48
        NB = np.append(NB,nb)
49
        NC = np.append(NC,nc)
50
        t = np.append(t,tnow)
51
52
         open('KMC\_data.txt','w+').write('\n'.join('\%f, \%i, \%i, \%i' \%x for x in data))
53
         # Record data of this circulation to the .txt file.
54
55
    # analytical solution
56
    C_0 = [CA0, CB0, CC0]
57
    ta = np.arange(0,10,0.1)
58
    p = (k,) # turn parameters into a tuple
59
60
    C = odeint(dCdt,C_0,ta,p)
61
    C_A = C.transpose()[0] # Get C_A from C
62
    C_B = C.transpose()[1] # Get C B from C
63
    C_C = C.transpose()[2]
64
65
    # Plot with the data acquired above.
66
    plt.figure()
67
    plt.plot(t, NA, 'o', markersize = '3', label=r'$N_{\rm A}$') # Configure for the first line.
68
    plt.plot(t, NB, 'o', markersize = '3', label=r'$N_{\rm B}$')
    plt.plot(t, NC, 'o', markersize = '3', label=r'$N_{\rm C}$')
70
    plt.plot(ta,C_A*Volume,'-',color='blue')
71
    plt.plot(ta,C_B*Volume,'-',color='orange')
72
    plt.plot(ta,C_C*Volume,'-',color='green')
73
    plt.legend()
    plt.title('Stochastic vs analytical evolution of A->B->C')
75
76
    plt.xlabel('time (s)') # Labeling the axis.
77
    plt.ylabel('Molecules')
    plt.savefig('./Images/kMC.png')
```



8 Heterogeneous reactions

- 1. adsorption, L-H
- 2. TPD
- 3. catalysis
- 4. Sabatier analysis

8.0.1 Heterogeneous reactions and catalysis

- 1. molecule-surface collisions
- 2. surface reactions
- 3. Ammonia oxidation, $\mathrm{NH_3} + \mathrm{O_2} \longrightarrow \mathrm{NO} + \mathrm{N_2},$ doi:10.1021/acscatal.8b04251

9 Liquid-phase reactions

9.0.1 Diffusion-controlled reactions

- 1. Intermediate complex
- 2. Steady-state approximation

Table 2: Equilibrium and Rate Constants

Equilibrium Constants $a A + b B \rightleftharpoons c C + d D$

$$\begin{split} K_{eq}(T) &= e^{\Delta S^{\circ}(T)/k_{B}} e^{-\Delta H^{\circ}(T)/k_{B}T} \\ K_{c}(T) &= \left(\frac{1}{c^{\circ}}\right)^{\nu_{c}+\nu_{d}-\nu_{a}-\nu_{b}} \frac{(q_{c}/V)^{\nu_{c}}(q_{d}/V)^{\nu_{d}}}{(q_{a}/V)^{\nu_{a}}(q_{b}/V)^{\nu_{b}}} e^{-\Delta E(0)\beta} \\ K_{p}(T) &= \left(\frac{k_{B}T}{P^{\circ}}\right)^{\nu_{c}+\nu_{d}-\nu_{a}-\nu_{b}} \frac{(q_{c}/V)^{\nu_{c}}(q_{d}/V)^{\nu_{d}}}{(q_{a}/V)^{\nu_{a}}(q_{b}/V)^{\nu_{b}}} e^{-\Delta E(0)\beta} \end{split}$$

Unimolecular Reaction $[A] \rightleftharpoons [A]^{\ddagger} \rightarrow C$

$$k(T) = \nu^{\ddagger} \bar{K}^{\ddagger} = \frac{k_B T}{h} \frac{\bar{q}_{\ddagger}(T)/V}{q_A(T)/V} e^{-\Delta E^{\ddagger}(0)\beta}$$

$$E_a = \Delta H^{\circ \ddagger} + k_B T$$
 $A = e^1 \frac{k_B T}{h} e^{\Delta S^{\circ \ddagger}}$

Bimolecular Reaction $A + B \rightleftharpoons [AB]^{\ddagger} \rightarrow C$

$$k(T) = \nu^{\ddagger} \bar{K}^{\ddagger} = \frac{k_B T}{h} \frac{q_{\ddagger}(T)/V}{(q_A(T)/V)(q_B(T)/V)} \left(\frac{1}{c^{\circ}}\right)^{-1} e^{-\Delta E^{\ddagger}(0)\beta}$$
$$E_a = \Delta H^{\circ \ddagger} + 2k_B T \quad A = e^2 \frac{k_B T}{h} e^{\Delta S^{\circ \ddagger}}$$

- 3. Diffusion-controlled limit $(k_D = 4\pi(r_A + r_B)D_{AB})$
- 4. Reaction-controlled limit $(k_{app} = (k_D/k_{-D})k_r)$