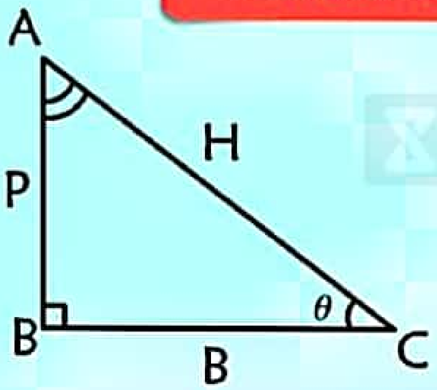
TRIGONOMETRY RATIO

" Pandit Badri Prasad Bole Hari Hari "

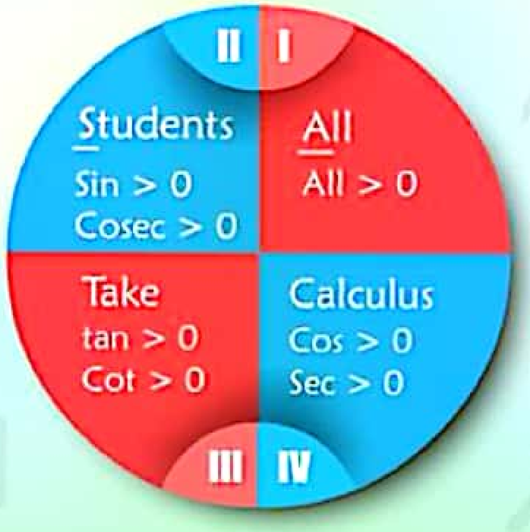


sin	cos	tan	cot	sec	cosec
P	B	P	B	H	H
H	H	B	P	B	P

Value

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin\theta$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	N.D
$\cot\theta$	N.D	$\sqrt{3}$	1	$1/\sqrt{3}$	0
$\sec\theta$	1	$2/\sqrt{3}$	$\sqrt{2}$	2	N.D
$\mathrm{cosec}\,\theta$	N.D	2	$\sqrt{2}$	$2/\sqrt{3}$	1

Quadrant



$(90^0 + \theta)$ Reduction

$\sin(90^0 + \theta) = \cos\theta$

$\cos(90^0 + \theta) = -\sin\theta$

$\tan(90^0 + \theta) = -\cot\theta$

$\cot(90^0 + \theta) = -\tan\theta$

$\sec(90^0 + \theta) = -\csc\theta$

$\csc(90^0 + \theta) = \sec\theta$

$(180^0 + \theta)$ Reduction

$\sin(180^0 + \theta) = -\sin\theta$

$\cos(180^0 + \theta) = -\cos\theta$

$\tan(180^0 + \theta) = \tan\theta$

$\cot(180^0 + \theta) = \cot\theta$

$\csc(180^0 + \theta) = -\csc\theta$

$\sec(180^0 + \theta) = -\sec\theta$

" Complementary angles are those whose sum is 90° "

$(360^0 - \theta)$ or $(2\pi - \theta)$ Reduction

$\sin(2\pi - \theta) = \sin(-\theta) = -\sin\theta$

$\cos(2\pi - \theta) = \cos(-\theta) = \cos\theta$

$\tan(2\pi - \theta) = \tan(-\theta) = -\tan\theta$

$\cot(-\theta) = -\cot\theta$

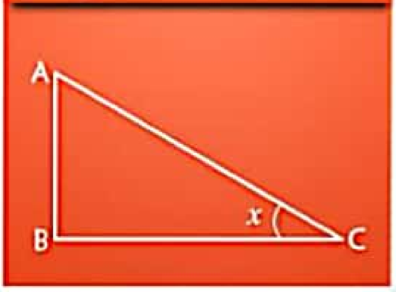
$\csc(-\theta) = -\csc\theta$

$\sec(-\theta) = \sec\theta$

TRIGONOMETRIC **IDENTITIES**

Part I

1 Quotient Identities



$$\tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\operatorname{cosec} x = \frac{1}{\sin x}$$

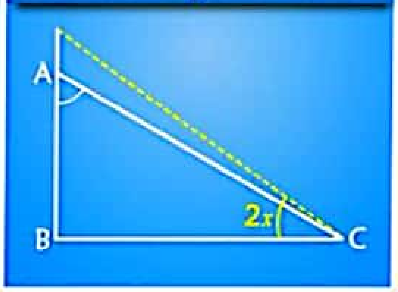
2 Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$\sec^2 x - \tan^2 x = 1$$

$$\operatorname{cosec}^2 x - \cot^2 x = 1$$

3 Double Angle Identities



$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

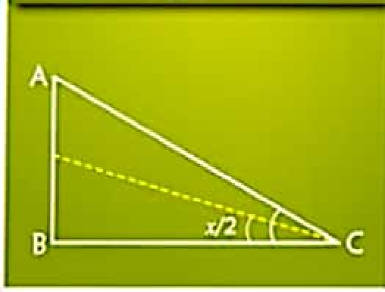
$$\cos 2x = 2 \cos^2 x - 1$$

$$\cos 2x = 1 - 2 \sin^2 x$$

$$\cos 2x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

4 Half Angle Identities



$$\sin \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}}$$

$$\cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}}$$

$$\tan \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}}$$

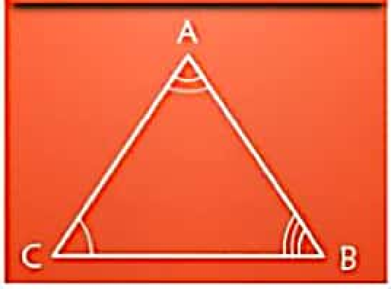
$$\tan \frac{x}{2} = \frac{1 - \cos x}{\sin x}$$

$$\tan \frac{x}{2} = \frac{\sin x}{1 + \cos x}$$

TRIGONOMETRIC IDENTITIES

Part II

5 Angle Sum & Difference Identities



$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\cot(A \pm B) = \frac{\cot A \,.\, \cot B \mp 1}{\cot B \pm \cot A}$$

6 Sum Identities

$$\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right).\cos\left(\frac{C-D}{2}\right)$$

$$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right).\sin\left(\frac{C-D}{2}\right)$$

$$\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right).\cos\left(\frac{C-D}{2}\right)$$

$$\cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right).\sin\left(\frac{C-D}{2}\right)$$

7 Product Identities

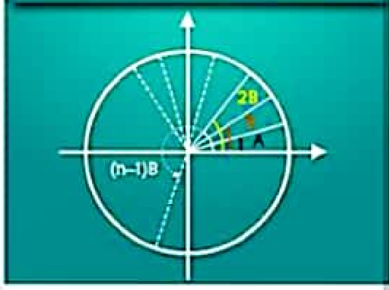
$$2\sin A \cos B = \left[\sin(A+B) + \sin(A-B)\right]$$

$$2\cos A \sin B = \left[\sin(A+B) - \sin(A-B)\right]$$

$$2\cos A \cos B = \left[\cos(A-B) + \cos(A+B)\right]$$

$$2\sin A \sin B = \left[\cos(A-B) - \cos(A+B)\right]$$

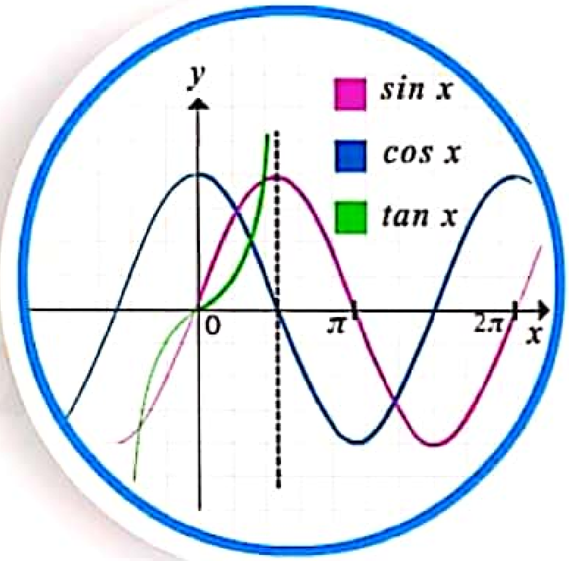
8 Summation of Trigonometric Series



$$\sin A + \sin(A+B) + \sin(A+2B) + \ldots + \sin(A+(n-1)B) = \frac{\sin nB/2}{\sin B/2}.\sin\left(A + \frac{(n-1)}{2}B\right)$$

$$\cos A + \cos(A+B) + \cos(A+2B) + \ldots + \cos(A+(n-1)B) = \frac{\sin nB/2}{\sin B/2}.\cos\left(A + \frac{(n-1)}{2}B\right)$$

TRIGNOMETRIC EQUATION



Principal Solution

The solutions of a trigonometric equation which lie in the interval $[0, 2\pi]$ are called principal solutions.

Eg: $\sin x = \dfrac{1}{2} \Rightarrow x = \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{9\pi}{6} \ldots\ldots\ldots$

But, principal solution of

$\sin x = \dfrac{1}{2}$ are $\dfrac{\pi}{6}, \dfrac{5\pi}{6} \in [0, 2\pi]$

General Solution

$\sin\theta = \sin\alpha \Rightarrow \theta = n\pi + (-1)^n\alpha,\ \alpha \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right], n \in I$

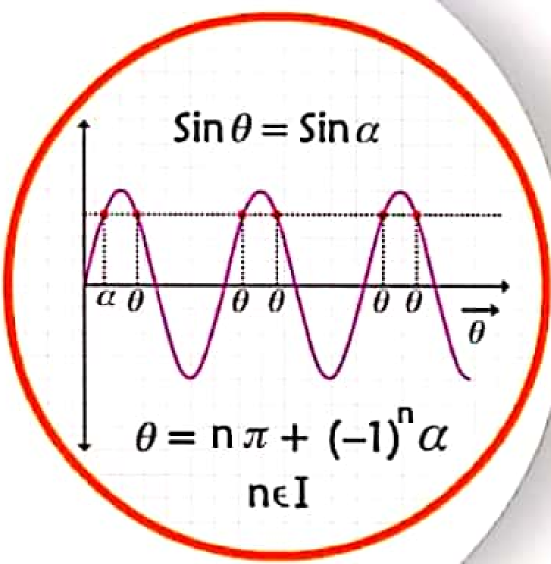
$\cos\theta = \cos\alpha \Rightarrow \theta = 2n\pi \pm \alpha, \alpha \in [0, \pi], n \in I$

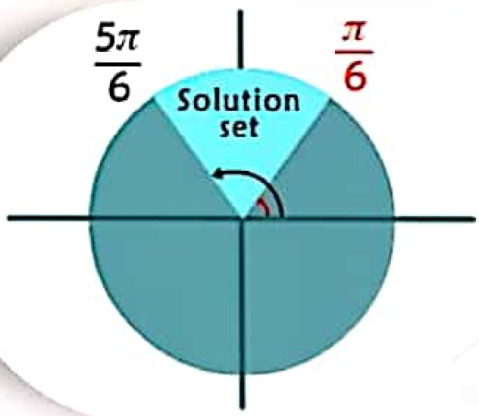
$\tan\theta = \tan\alpha \Rightarrow \theta = n\pi + \alpha, \alpha \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right), n \in I$

$\sin^2\theta = \sin^2\alpha \Rightarrow \theta = n\pi \pm \alpha, n \in I$

$\cos^2\theta = \cos^2\alpha \Rightarrow \theta = n\pi \pm \alpha, n \in I$

$\tan^2\theta = \tan^2\alpha \Rightarrow \theta = n\pi \pm \alpha, n \in I$; α is called one principal angle.





Trigonometric Inequalities

Eg: $\sin x > \dfrac{1}{2} \Rightarrow \dfrac{\pi}{6} < x < \dfrac{5\pi}{6}$