DERIVATIVE

Part I

Derivative by First Principle

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = f'(x)$$

$\longrightarrow$ Instantaneous rate of change of y.w.r.t. x.

Fundamental Rules for Differentiation

1	PRODUCT RULE	$\frac{d}{dx}[f(x).g(x)] = f(x)\frac{d}{dx}\{g(x)\} + g(x)\frac{d}{dx}\{f(x)\}$
2	QUOTIENT RULE	$\frac{d}{dx}\left\{\frac{f(x)}{g(x)}\right\} = \frac{g(x).\frac{d}{dx}\{f(x)\} - f(x).\frac{d}{dx}\{g(x)\}}{\{g(x)\}^2}$
3	CHAIN RULE	if $y = f(u)$ & $u = g(x)$ then $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

Derivative of Standard Functions

For function $y = f(x)$, the derivative of the function is $\frac{dy}{dx}$

$\frac{d}{dx}(\sin x) = \cos x$	$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{\lvert x\rvert\sqrt{x^2-1}}, \lvert x\rvert>1$
$\frac{d}{dx}(\cos x) = -\sin x$	$\frac{d}{dx}(\text{cosec}^{-1} x) = \frac{-1}{\lvert x\rvert\sqrt{x^2-1}}, \lvert x\rvert>1$
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}, x \in R$
$\frac{d}{dx}(\sec x) = \sec x.\tan x$	$\frac{d}{dx}(x^n) = n.x^{n-1}; x \in R, n \in R, x>0$
$\frac{d}{dx}(\text{cosec} x) = -\text{cosec} x.\cot x$	$\frac{d}{dx}(a^x) = a^x.\ln a\,;\, a>0, a \neq 1$
$\frac{d}{dx}(\cot x) = -\text{cosec}^2 x$	$\frac{d}{dx}(e^{ax}) = ae^{ax}$
$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}, -1<x<1$	$\frac{d}{dx}(\log_a \lvert x\rvert) = \frac{1}{x}\log_a e$
$\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}, -1<x<1$	$\frac{d}{dx}(\ln \lvert x\rvert) = \frac{1}{x}$
$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}, x \in R$	$\frac{d}{dx}(\text{constant}) = 0$

METHOD OF DIFFERENTIATION Part II

L' Hopital Rule

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{0}{0} \text{ or } \frac{\infty}{\infty}$$

f(x) & g(x) are differentiable at x = a

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)} = \lim_{x \to a} \frac{f''(x)}{g''(x)} = \text{...till the indeterminate form vanishes.}$$

Guillaume de L' Hopital

Logarithmic Differentiation

$$\text{If } y = [f(x)]^{g(x)} \Rightarrow \ln y = g(x) \ln [f(x)]$$

$$\Rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = \frac{d}{dx}\left\{g(x).\ln[f(x)]\right\} \qquad \Rightarrow \frac{dy}{dx} = [f(x)]^{g(x)}.\left\{\frac{d}{dx}[g(x) \ln f(x)]\right\}$$

Parametric Differentiation

$$\text{If } x = f(t) \text{ \& } y = g(t) \text{ then } \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

Differentiation of Inverse Function

y = f(x) and x = g(y) are inverse function of each other $\dfrac{dx}{dy} = \dfrac{1}{dy/dx}$ or $g'(y) = \dfrac{1}{f'(x)}$

Derivative of a Determinant

$$\text{If } F(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ l(x) & m(x) & n(x) \\ u(x) & v(x) & w(x) \end{vmatrix}$$

where f,g,h,l,m,n,u,v,w are differentiable function of x then

$$F'(x) = \begin{vmatrix} f'(x) & g'(x) & h'(x) \\ l(x) & m(x) & n(x) \\ u(x) & v(x) & w(x) \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ l'(x) & m'(x) & n'(x) \\ u(x) & v(x) & w(x) \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ l(x) & m(x) & n(x) \\ u'(x) & v'(x) & w'(x) \end{vmatrix}$$