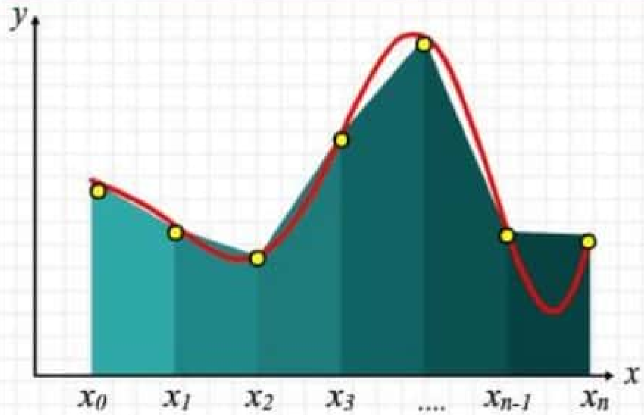
AREA UNDER THE CURVE

Trapezoidal Riemann Approximation



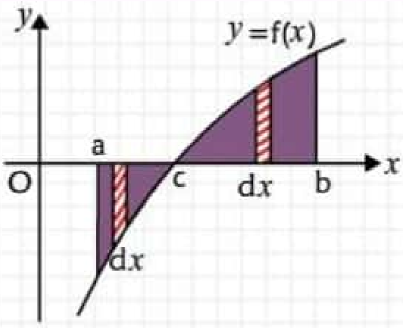
$$\text{Area} = \sum_{i=1}^{n} \frac{f(x_{i-1}) + f(x_i)}{2} \Delta x \quad ; \Delta x = \frac{b-a}{n}$$

Riemann Sum:

$$\text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{f(x_{i-1}) + f(x_i)}{2} \Delta x$$

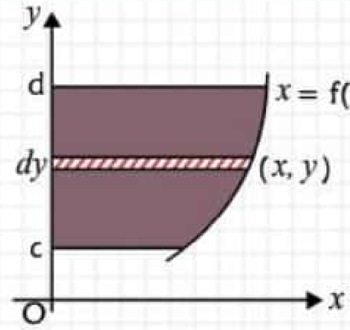
$$= \int_a^b f(x)dx \qquad dx - \text{infinitely small}$$

Area By Vertical Strips



$$A = \left|\int_a^c f(x)dx\right| + \int_c^b f(x)dx$$

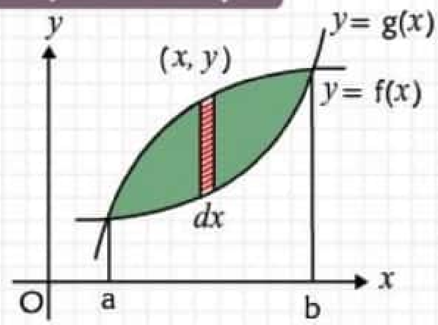
Area By Horizontal Strips



$$A = \int_c^d f(y)dy = \int_c^d x\,dy$$

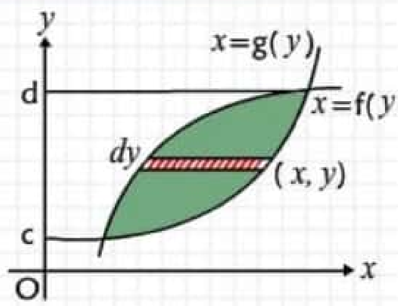
Area Enclosed Between Two Curves

Case – I : By vertical strips



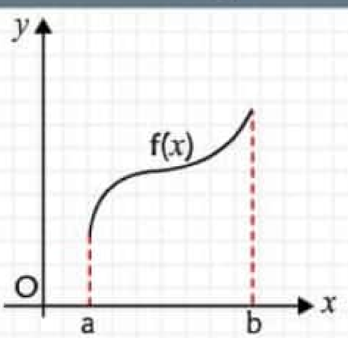
$$A = \int_a^b f(x)dx - \int_a^b g(x)dx = \int_a^b [f(x) - g(x)]dx$$

Case – II : By horizontal strips



$$A = \int_c^d f(y)dy - \int_c^d g(y)dy = \int_c^d [f(y) - g(y)]dy$$

Average Value of a Function



$y = f(x)$; $a \le x \le b$

$$y_{avg} = \frac{1}{b-a} \int_c^d f(x)dx$$

Useful Results

- Whole area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab (units)^2.
- Area enclosed between the parabola $y^2 = 4ax$ & $x^2 = 4by$ is $\frac{16\,ab}{3}$ (units)^2.
- Area enclosed between the parabola $y^2 = 4ax$ & $y = mx$ is $\frac{8\,a^2}{3m^3}$ (units)^2.

DIFFERENTIAL EQUATION

Differential equation is an equation that involves independent and dependent values and the derivatives of the dependent variables.

TYPES OF DIFFERENTIAL EQUATION

ORDINARY DIFFERENTIAL EQUATION

If the differential coefficients have reference to a single independent variable only.

$$\text{Eg}: \frac{d^2y}{dx^2} - \frac{2dy}{dx} + \cos x = 0$$

PARTIAL DIFFERENTIAL EQUATION

If there are two or more independent variables.

$$\text{Eg}: \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$$

ORDER AND DEGREE OF DIFFERENTIAL EQUATION

Order

Degree

$$f(x,y)\left[\frac{d^{m}y}{dx^m}\right]^{p} + \phi(x,y)\left[\frac{d^{m-1}y}{dx^{m-1}}\right]^{q} + \ldots\ldots = 0$$

Order : is the highest derivative.

Degree : is the exponent of the highest derivative.

SOLVING DIFFERENTIAL EQUATION

1 ELEMENTARY TYPE OF 1ST ORDER AND 1ST DEGREE

SEPERATION OF VARIABLES METHOD :

Step 1 - Move all the y terms (including dy) to one side and all the x terms (inculding dx) to the other side.

Step 2 - Intergrate one side with respect to 'y' and the other side with respect to 'x'.

Step 3 - Simplify it.

SEPERATION OF VARIABLES TYPES :

Type-1 : Equation of the form $f(x)dx = -g(y)dy \Longrightarrow \int f(x)dx + \int g(y)dy = C$

Type-2 : Equation of the form $\frac{dy}{dx} = f(ax + by + c);\ b \neq 0 \Longrightarrow$ put $ax + by + c = t$ & reduce to Type 1

Type-3 : Equation of the form $\frac{dy}{dx} = \frac{a_1x + b_1y + c_1}{a_2x + b_2y + c_2}$ {if $a_2 + b_1 = 0$} Cross multiply and note the perfect differential $d(xy)$.

Type-4 : Transformation to polar coordinates

(a) $x = r\cos\theta$; $y = r\sin\theta$; $x^2 + y^2 = r^2$; $\frac{y}{x} = \tan\theta$; $xdx + ydy = rdr$; $xdy - ydx = r^2d\theta$

(b) $x = r\sec\theta$; $y = r\tan\theta$; $x^2 - y^2 = r^2$; $\frac{y}{x} = \sin\theta$; $xdx - ydy = rdr$; $xdy - ydx = r^2\sec\theta\, d\theta$

Part II

2 HOMOGENEOUS DIFFERENTIAL EQUATION

T-(1) An equation of the form $\frac{dy}{dx} = \frac{f(x, y)}{g(x, y)}$ (where f(x, y) and g(x, y) are homogeneous functions of x and y and of same degree) To solve put $y = tx$ or $x = ty$

T-(2) Equation reducible to homogeneous differential $\frac{dy}{dx} = \frac{a_1x + b_1y + c_1}{a_2x + b_2y + c_2}$ When $a_1b_2 - a_2b_1 \neq 0$ or $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$. Substitution : $x = u + h$ and $y = v + k$ then reduce differential equation to Homogeneous Differential Equation T-1

3 LINEAR DIFFERENTIAL EQUATION

A differential equation is said to be linear if the dependent variable and its differential coefficients occur in the first degree only and are not multiplied together.

$$a_0(x)\frac{d^ny}{dx^n} + a_1(x)\frac{d^{n-1}y}{dx^{n-1}} + a_2(x)\frac{d^{n-2}y}{dx^{n-2}} + \ldots\ldots + a_n(x)\, y = \phi(x)$$

where $a_0(x), a_1(x), a_2(x) \ldots\ldots\ldots, a_n(x)$ are called coefficient of Differential Equation.

Linear differential equation of first order

$$\frac{dy}{dx} + P(x)y = Q(x)$$

To solve

Calculate : Integrating factor (I.F.) $= e^{\int P(x)dx}$

Solution : $y\,(\text{I.F.}) = \int Q(x)(\text{I.F.})dx + C$

Bernoulli's Equation

Equation reducible to linear differential equation

Equation of the form $\frac{dy}{dx} + P(x)y = Q(x)y^n$

To solve divide by y^n and substitute $y^{1-n} = t$

$$\frac{dt}{dx} + (1 - n)P(x)t = Q(x)(1 - n)$$

Now solve as 1st order Linear Differential Equation

SOME IMPORTANT EXACT DIFFERENTIALS

- $xdy + ydx = d(xy)$
- $\frac{xdy - ydx}{xy} = d\left(\ln\left(\frac{y}{x}\right)\right)$
- $\frac{xdy - ydx}{x^2 + y^2} = d\left(\tan^{-1}\frac{y}{x}\right)$
- $\frac{xdy - ydx}{x^2} = d\left(\frac{y}{x}\right)$
- $\frac{ydx - xdy}{xy} = d\left(\ln\frac{x}{y}\right)$
- $d\left(\frac{e^y}{x}\right) = \left(\frac{xe^ydy - e^ydx}{x^2}\right)$
- $\frac{xdy - ydx}{y^2} = d\left(-\frac{x}{y}\right)$
- $\frac{xdy + ydx}{x^2y^2} = d\left(-\frac{1}{xy}\right)$
- $d\left(\frac{e^x}{y}\right) = \left(\frac{ye^xdy - e^xdx}{y^2}\right)$
- $\frac{xdy + ydx}{xy} = \frac{d(xy)}{xy} = d(\ln xy)$
- $\frac{2(xdx + ydy)}{x^2 + y^2} = [\ln(x^2 + y^2)]$
- $\frac{dx + dy}{x + y} = d(\ln(x + y))$