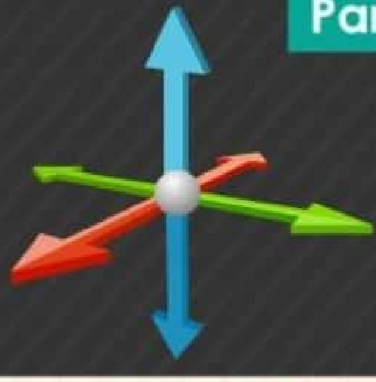
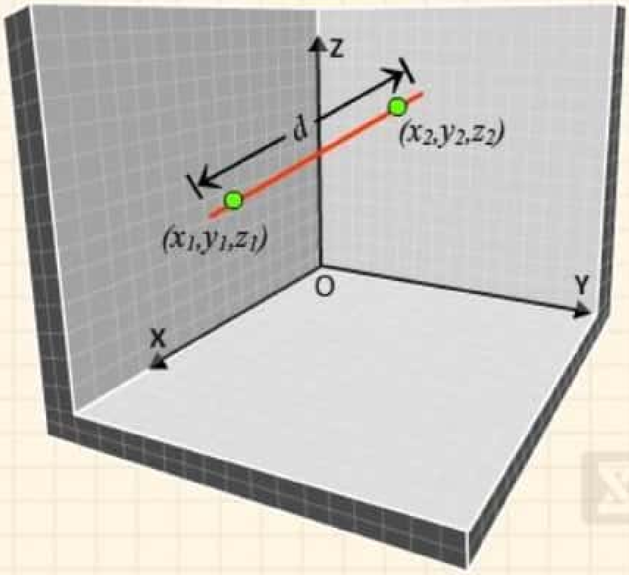
Part I

3D COORDINATE GEOMETRY

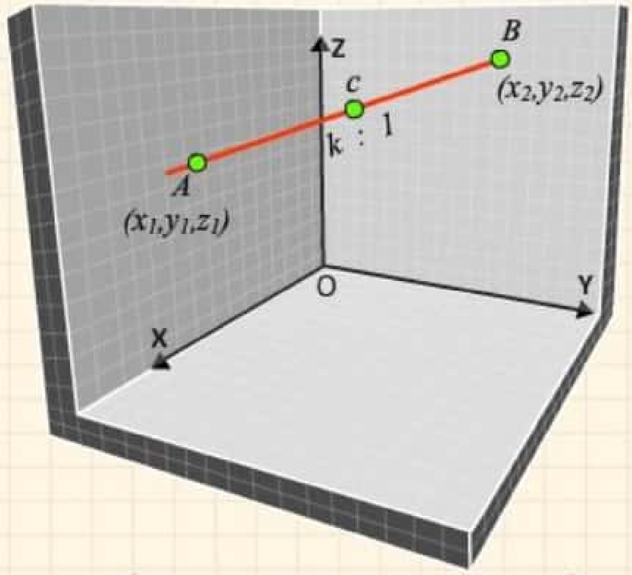


Distance between two points



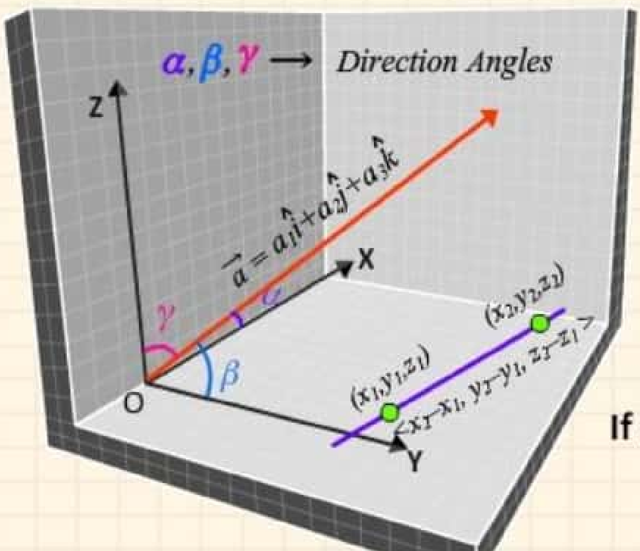
$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

Section Formula



$$C\left(\frac{kx_2+x_1}{k+1}, \frac{ky_2+y_1}{k+1}, \frac{kz_2+z_1}{k+1}\right)$$

Direction Cosines & Ratios



Direction Cosines

$$\cos\alpha = \frac{\vec{a_1}}{|\vec{a}|}, \quad \cos\beta = \frac{\vec{a_2}}{|\vec{a}|}, \quad \cos\gamma = \frac{\vec{a_3}}{|\vec{a}|}$$

Note : $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\cos\alpha \equiv \ell$; $\cos\beta \equiv m$; $\cos\gamma \equiv n$

So $\ell^2 + m^2 + n^2 = 1$

Direction Ratios

If a,b and c are direction ratios then $a \propto \ell$; $b \propto m$; $c \propto n$

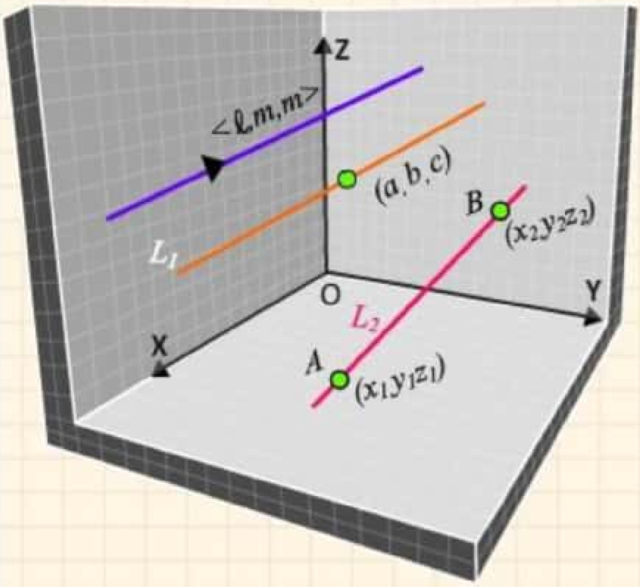
$$\frac{\ell}{a} = \frac{m}{b} = \frac{n}{c} = \lambda \text{ (say)}$$

$$\therefore (a^2+b^2+C^2)\,\lambda^2 = \ell_2 + m_2 + n_2 = 1$$

Therefore, $\ell = \pm \dfrac{a}{\sqrt{(a^2+b^2+c^2)}}$; $m = \pm \dfrac{b}{\sqrt{(a^2+b^2+c^2)}}$; $n = \pm \dfrac{c}{\sqrt{(a^2+b^2+c^2)}}$

THREE DIMENSIONAL LINES

Part II



Line passing through point (a, b, c) parallel to line having direction cosines ℓ,m,n is

$$L_1 : \frac{x-a}{\ell} = \frac{y-b}{m} = \frac{z-c}{n}$$

Equation of a line passing through two points A & B

$$L_2 : \frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

Angle between two lines

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\ \sqrt{a_2^2+b_2^2+c_2^2}}$$

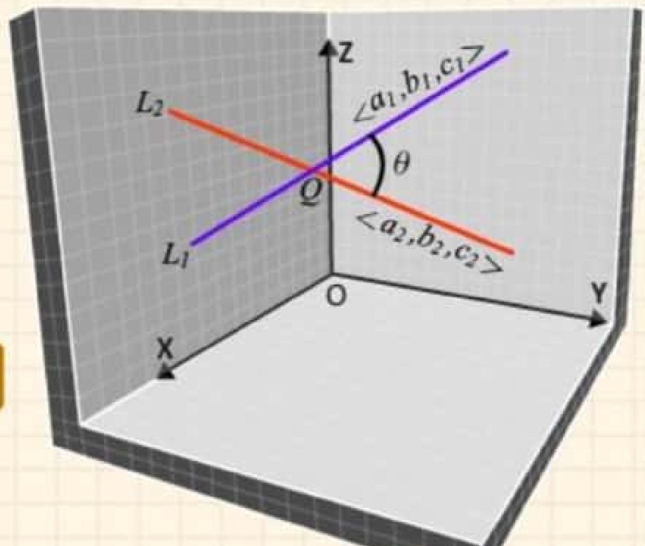
$$\cos\theta = \ell_1\ell_2 + m_1m_2 + n_1n_2$$

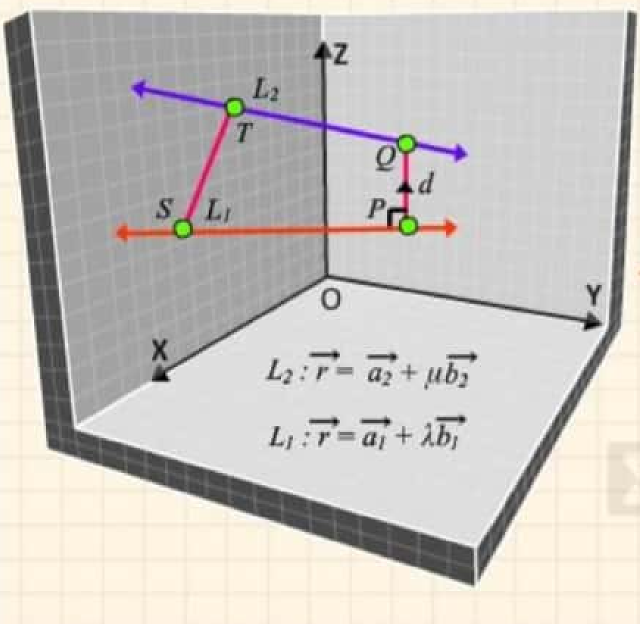
If L_1 and L_2 are Perpendicular ($\theta = 90^\circ$)

$\ell_1\ell_2 + m_1m_2 + n_1n_2 = 0$ Or $a_1a_2 + b_1b_2 + c_1c_2 = 0$

If L_1 and L_2 are Parallel

$\frac{\ell_1}{\ell_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$ Or $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$





Distance between two skew lines

$$d = \left| \frac{(\vec{b}_1 \times \vec{b}_2)\ (\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

Cartesian form

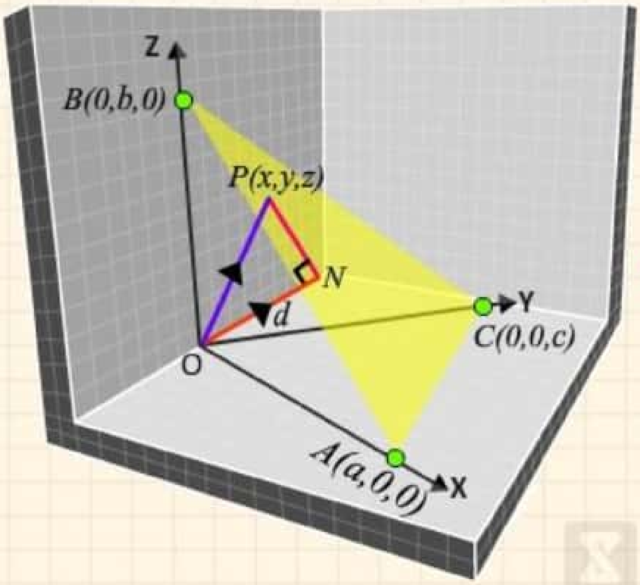
Line $L_1 : \frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$

Line $L_2 : \frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$

$$d = \left| \frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2+(a_1b_2-a_2b_1)^2} \right|$$

Part III

THREE DIMENSIONAL PLANES



Equation of a plane in Normal form

Equation : $\vec{r}\cdot\hat{n} = d$

$\hat{n}$: unit normal vector along $\overrightarrow{ON}$

d: Perpendicular distance of plane from 'O'

Cartesian form

Equation : $\ell x + my + nz = d$

Here ℓ, m, n are the direction cosines of $\hat{n}$

Intercept form of the equation of a plane ABC

Equation : $\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{z}{c} = 1$

a, b and c are the direction ratios.

Equation of a plane perpendicular to a given vector and passing through a given point

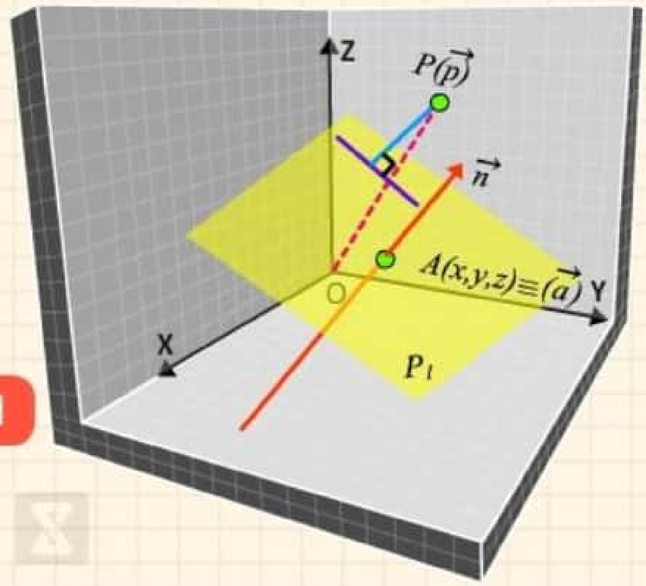
Equation : Cartesian Form

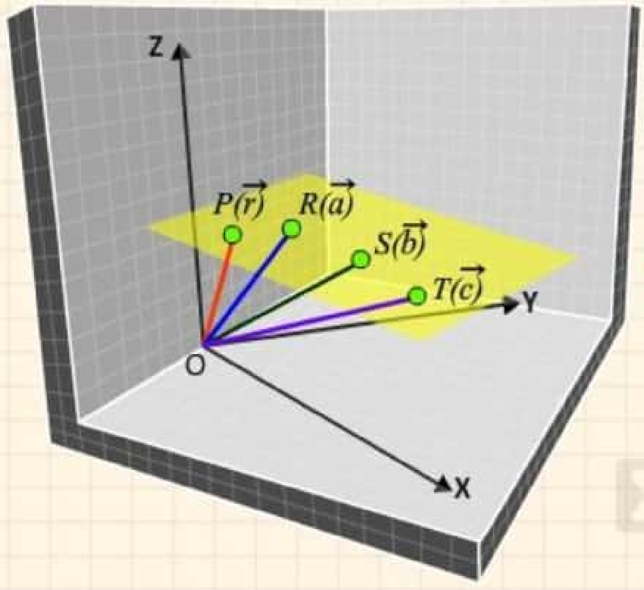
$(\vec{r} - \vec{a})\cdot\vec{n} = 0$ if $\vec{n} = a\hat{i} + b\hat{j} + c\hat{k}$

Plane : $a(x-x_1)+b(y-y_1)+c(z-z_1) = 0$

The Distance of a Point P From Plane $P_1 : \vec{r}\cdot\hat{n} = d$

$$\text{Perpendicular distance} = \left| \frac{\vec{p}\cdot\hat{n} - d}{|\hat{n}|} \right|$$





Equation of a plane passing through three non - collinear points

Equation : $(\vec{r} - \vec{a})\cdot[(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a})] = 0$

Cartesian Form

If $R = (x_1, y_1, z_1)$, $S = (x_2, y_2, z_2)$ & $T = (x_3, y_3, z_3)$

$$\text{Equation : } \begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

Plane P passing through the Intersection of two given planes P_1 & P_2

$P_1 : \vec{r}\cdot\hat{n}_1 = d_1$ Or $P_2 : \vec{r}\cdot\hat{n}_2 = d_2$

Equation of Plane P

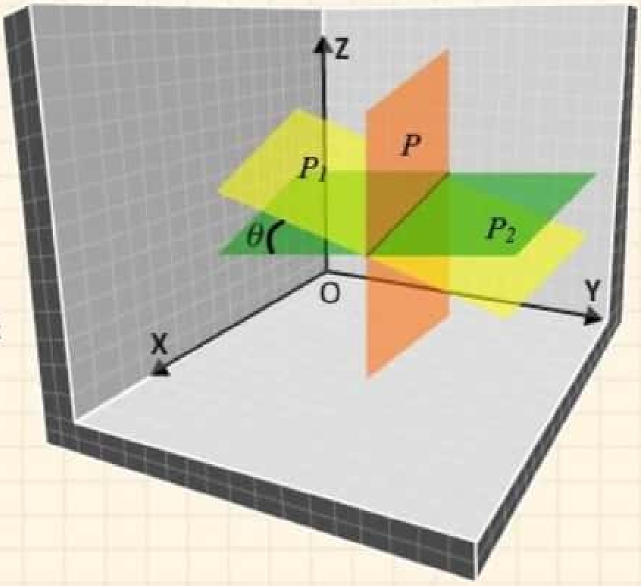
$$\vec{r}\cdot(\vec{n}_1 + \lambda\vec{n}_2) = d_1 + \lambda d_2 \; ; \quad \lambda : \text{Constant}$$

Cartesian form

$P_1 : a_1x+b_1y+c_1z = d_1$ Or $P_2 : a_2x+b_2y+c_2z = d_2$

Equation of plane P :

$$(a_1x+b_1y+c_1z-d_1) + \lambda(a_2x+b_2y+c_2z-d_2) = 0$$



If Angle between two planes P_1 & P_2 is θ

$$\cos\theta = \left|\frac{\vec{n_1}\cdot\vec{n_2}}{|\vec{n_1}|.|\vec{n_2}|}\right|$$

Cartesian form

If $P_1 : a_1x+b_1y+c_1z+d_1 = 0$, $P_2 : a_2x+b_2y+c_2z+d_2 = 0$

$$\cos\theta = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\,\sqrt{a_2^2+b_2^2+c_2^2}}\right|$$

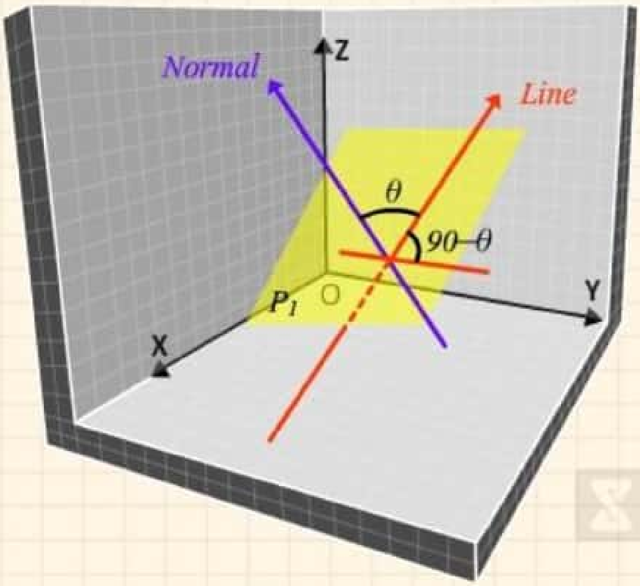
- If $P_1 \perp P_2 \Rightarrow \theta = 90^\circ$

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

- If $P_1 || P_2 \Rightarrow \theta = 0^\circ$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

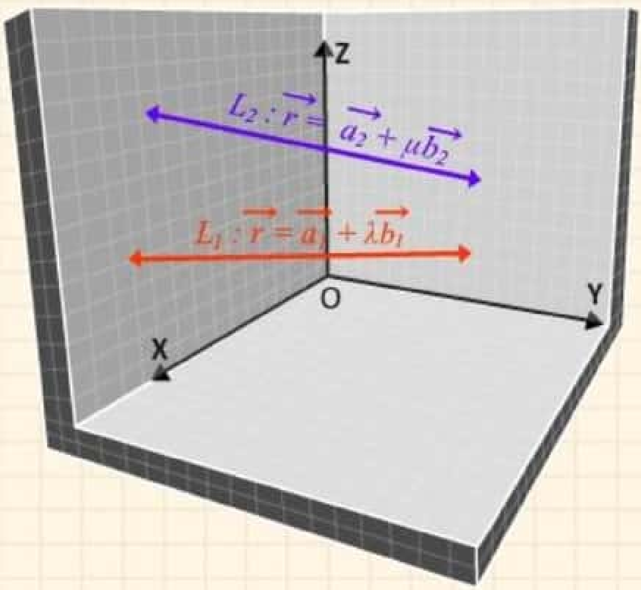
Angle Between a Line and a Plane



Line : $\vec{r} = \vec{a} + \lambda\vec{b}$ Plane : $\vec{r}\cdot\vec{n} = d$

Therefore $$\cos\theta = \left|\frac{\vec{b}\cdot\vec{n}}{|\vec{b}|.|\vec{n}|}\right|$$

Coplanarity of Two Lines



If L_1 & L_2 are coplaner, then

$$(\vec{a_2} - \vec{a_1})\cdot(\vec{b_1} \times \vec{b_2}) = 0$$