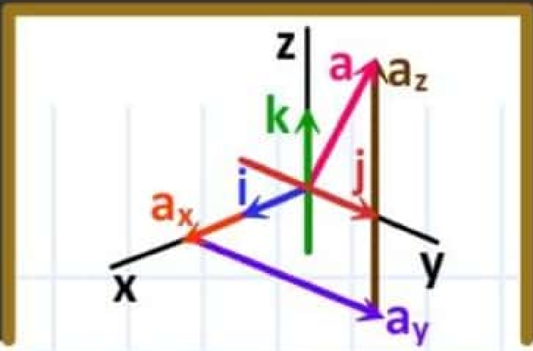


Part I

VECTOR

A quantity having both magnitude & direction

TYPES OF VECTORS

ZERO VECTOR

$\vec{a}$ is zero vector iff $|\vec{a}| = 0$

No Particular direction

UNIT VECTOR

$\vec{a}$ is unit vector iff $|\vec{a}| = 1$

Denote $\rightarrow \hat{a} = \dfrac{\vec{a}}{|\vec{a}|}$

EQUAL VECTORS

$\vec{a}$ & $\vec{b}$ are equal vectors iff $|\vec{a}| = |\vec{b}|$ & $\hat{a} = \hat{b}$

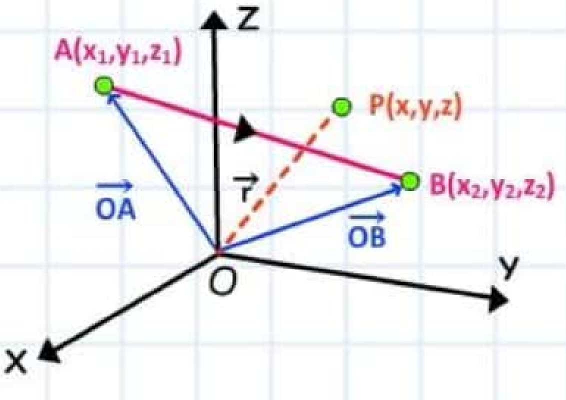
COLLINEAR VECTORS

$\vec{a}$ & $\vec{b}$ are collines vector iff $\vec{a} \parallel \vec{b}$

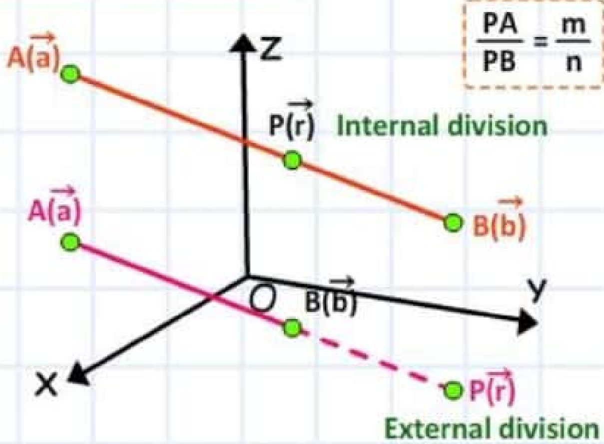
COPLANER VECTORS

vector parallel to or lying on the same plane.

POSITION VECTOR AND SECTION FORMULA



$$\frac{PA}{PB} = \frac{m}{n}$$



Position Vector

Postition Vector of pt. P is $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Vector joining two points A & B

$$\vec{AB} = \vec{OB} - \vec{OA}$$

$$\vec{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

Internal Section

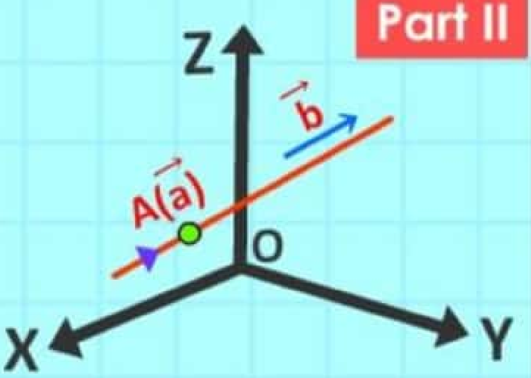
$$\vec{r} = \frac{(n\vec{a}) + (m\vec{b})}{m+n} \; ; m+n \neq 0$$

External Section

$$\vec{r} = \frac{(m\vec{b}) - (n\vec{a})}{m-n} \; ; m-n \neq 0$$

Part II

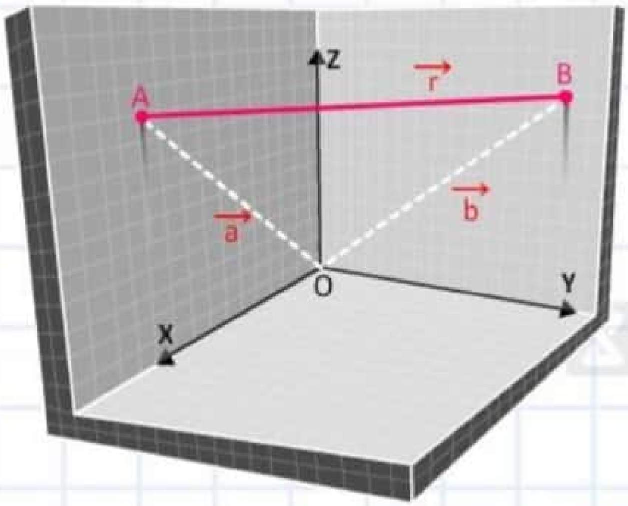
VECTOR LINES



Lines : For a Vector equation of a line one needs

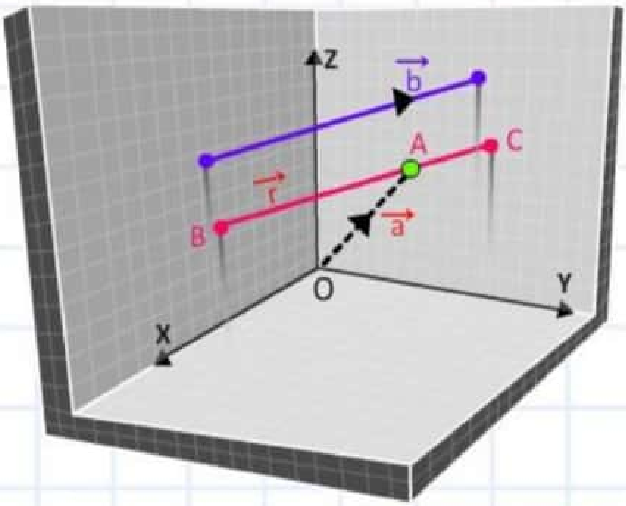
- A Point on line, let $\vec{a}$.
- A Parallel vector to the line, let $\vec{b}$.

When Two Points on Line are Known



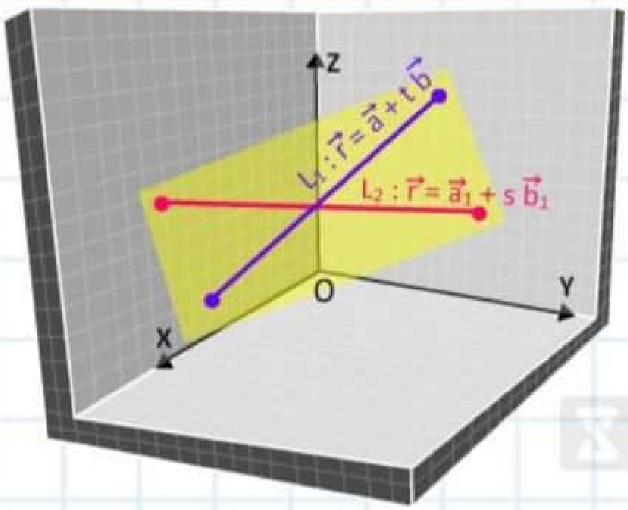
Equation of the Line AB $\quad \vec{r} = \vec{a} + t(\vec{b} - \vec{a})$

Parallel to $\vec{b}$ & Passes Through Point A



Equation of the Line BC $\quad \vec{r} = \vec{a} + t\,\vec{b}$

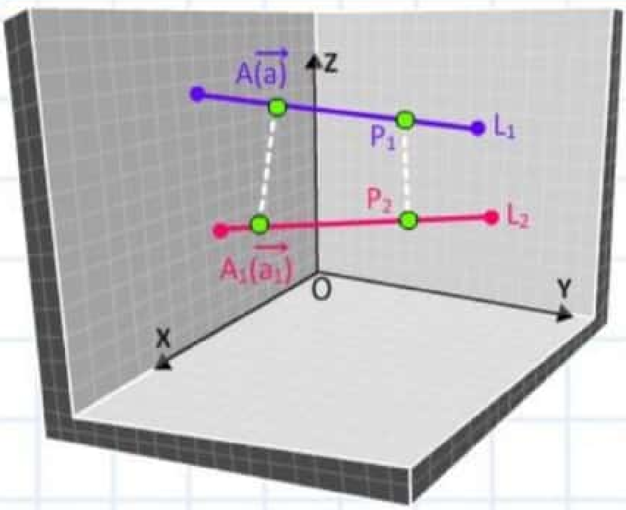
Condition for Intersection of Two Straight Lines



Condition of Intersection

- Line L_1 & L_2 will be coplaner
- Required Condition $[\vec{b}\,\vec{b}_1\,(\vec{a} - \vec{a}_1)] = 0$

Shortest distance between two Skew Lines

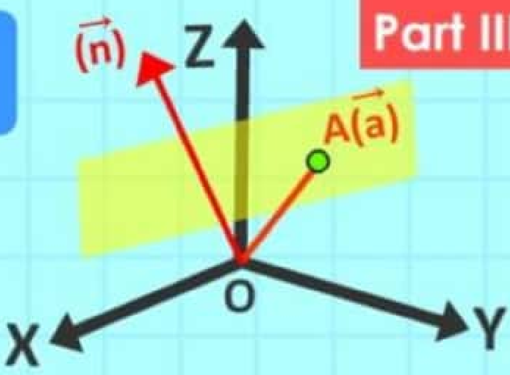


Shortest distance is measure along their common normal

$$\text{Shortest distance } P_1 P_2 = \frac{(\vec{b} \times \vec{b}_1) \cdot (\vec{a} - \vec{a}_1)}{|\vec{b} \times \vec{b}_1|}$$

PLANE OF VECTORS

Part III

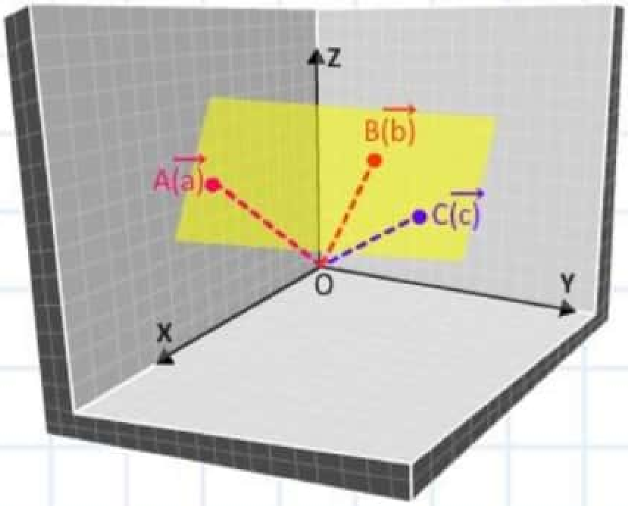


Planes : For Equation of a Plane, one needs

- A Point on Plane, let $\vec{a}$.
- A Perpendicular vector to the Plane, let $\vec{n}$.

Then Equation of Plane $(\vec{r}-\vec{a}).\vec{n}=0$

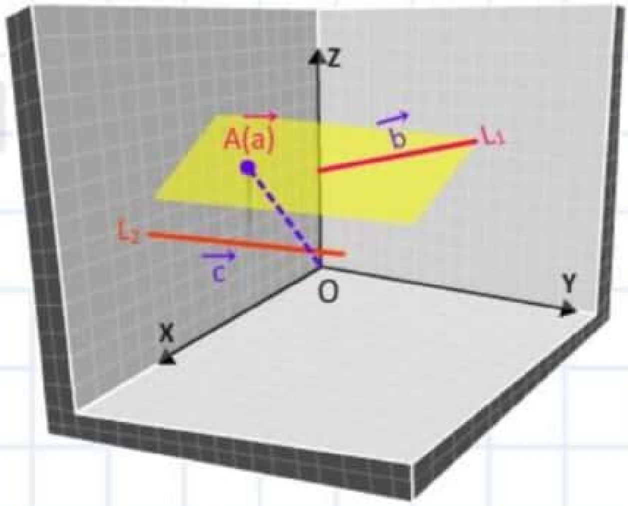
Plane Passing Through Three Points A,B,C



Equation of the Plane

$$(\vec{r}-\vec{a}).[(\vec{a}-\vec{b})\times(\vec{b}-\vec{c})]=0$$

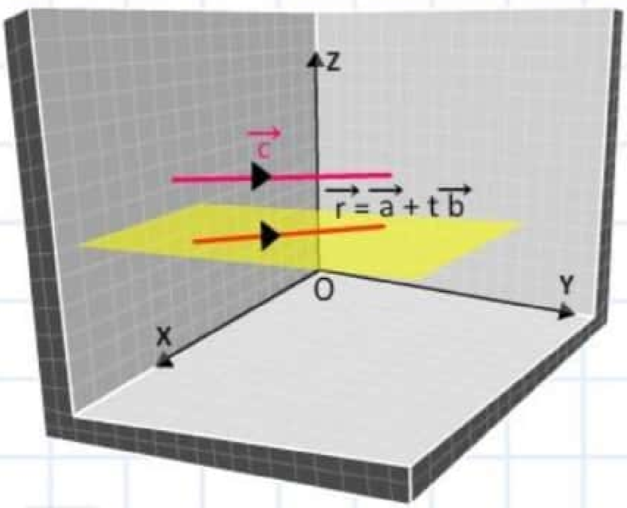
Plane Passing Through a Point and Parallel to two lines



Equation of the Plane

$$(\vec{r}-\vec{a}).(\vec{b}\times\vec{c})=0$$

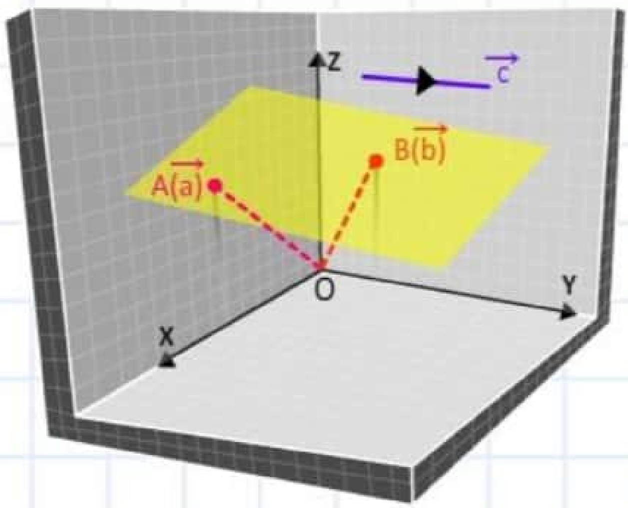
Plane Containing a Line and Parallel to another line



Equation of the Plane

$$\vec{r}.(\vec{b}\times\vec{c})=[\vec{a}\ \vec{b}\ \vec{c}]$$

Plane Passing Through Two Point and Parallel to a Lines



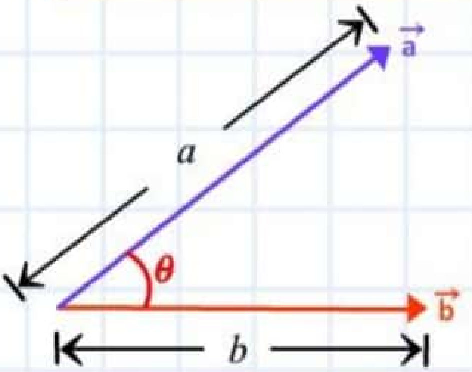
Equation of the Plane

$$\vec{r}.[(\vec{b}-\vec{a})\times\vec{c}]=[\vec{a}\ (\vec{b}-\vec{a})\ \vec{c}]=[\vec{a}\ \vec{b}\ \vec{c}]$$

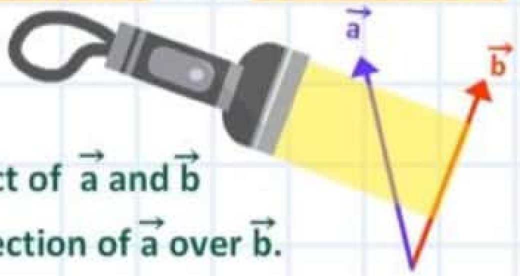
PRODUCT OF VECTORS

Part IV

Scalar or Dot Product



$\vec{a}\cdot\vec{b} = |\vec{a}|\;|\vec{b}|\cos\theta$ Or $\vec{a}\cdot\vec{b} = a\,b\cos\theta$



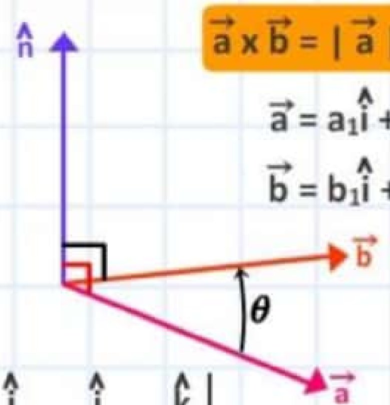
- Dot product of $\vec{a}$ and $\vec{b}$ is the projection of $\vec{a}$ over $\vec{b}$.

Vector or Cross Product

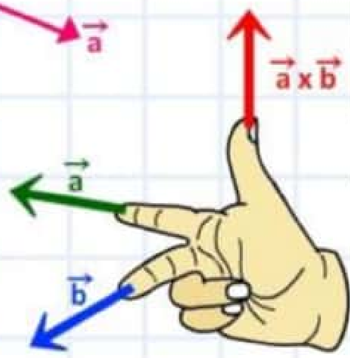
$\vec{a}\times\vec{b} = |\vec{a}|\;|\vec{b}|\;\sin\theta\,\hat{n}$

$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

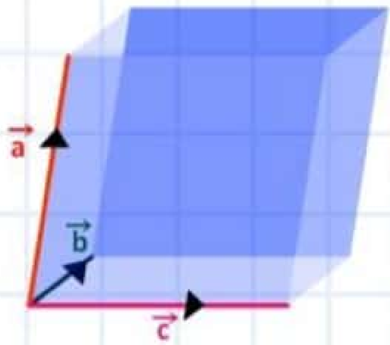


$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$



- Direction of resultant vector can be found using Right Hand Rule

Scalar Triple Product



⇐ Scalar triple product represents the volume of a parallelepiped

SCALAR TRIPLE PRODUCT

$\vec{a}\cdot(\vec{b}\times\vec{c}) = [\vec{a}\;\vec{b}\;\vec{c}]$ $\quad \vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

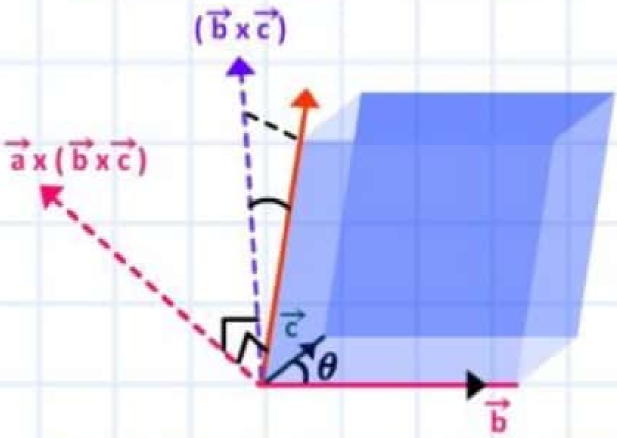
$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ $\quad \vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

$$\vec{a}\cdot(\vec{b}\times\vec{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = [\vec{a}\,\vec{b}\,\vec{c}]$$

NOTE

(i) $\vec{a}$, $\vec{b}$ & $\vec{c}$ are Coplanar iff $[\vec{a}\;\vec{b}\;\vec{c}] = 0$

Vector Triple Product



$\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}$

NOTE

(i) $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d}) = \vec{a}\cdot\vec{b}\times(\vec{c}\times\vec{d})$

$= \vec{a}\cdot\{(\vec{b}\cdot\vec{d})\vec{c} - (\vec{b}\cdot\vec{c})\vec{d}\}$

$= (\vec{a}\cdot\vec{c})(\vec{b}\cdot\vec{d}) - (\vec{a}\cdot\vec{d})(\vec{b}\cdot\vec{c})$

(ii) $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d}) = [\vec{a}\;\vec{c}\;\vec{d}]\vec{b} - [\vec{b}\;\vec{c}\;\vec{d}]\vec{a}$

Or $= [\vec{a}\;\vec{b}\;\vec{d}]\vec{c} - [\vec{a}\;\vec{b}\;\vec{c}]\vec{d}$