Part I

BINOMIAL THEOREM

BINOMIAL

A binomial is a polynomial with two terms. e.g

$5y^3 - 3$

2 terms

THE BINOMIAL THEOREM

Helps us expand binomials to any given power without direct multiplication.

General formula for $(x+y)^n$

$$(x + y)^n = {}^nC_0 x^n y^0 + {}^nC_1 x^{n-1} y + {}^nC_2 x^{n-2} y^2 + \ldots + {}^nC_r x^{n-r} y^r + \ldots + {}^nC_n x^0 y^n = \sum_{r=0}^{n} {}^nC_r x^{n-r} y^r$$

PROPERTIES OF THE BINOMIAL EXPANSION $(x + y)^n$

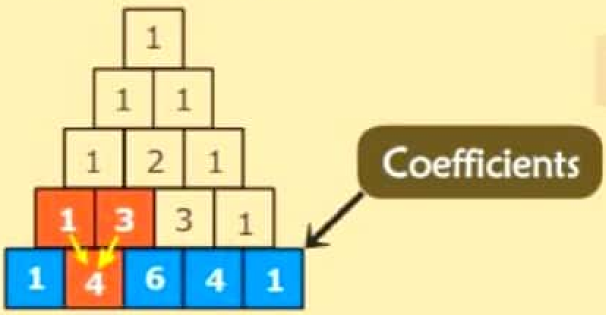
- There are $n + 1$ terms.
- The first term is x^n and the final term is y^n
- The exponent of x decreases by 1 while the exponent of y increases by 1.

BINOMIAL COEFFICIENTS

Expanding $(x+y)^n$, the binomial coefficients are simply the number of ways of choosing x from a number of brackets and y from the rest and are found using pascal's triangle.

PASCAL'S TRIANGLE

Assuming $n = 4$, We have $(a+b)^4$ and pascal's triangle would look like



$(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

Formula for the coefficient from pascal's Triangle.

$$\binom{n}{k} = {}^nC_k = \frac{n!}{k!(n-k)!}$$

It is commonly called "n choose k".

1 IMPORTANT TERMS IN THE BINOMIAL EXPANSION

Part II

IN THE EXPANSION OF $(x+y)^n$

- **GENERAL TERM :** $T_{r+1} = {}^nC_r\, x^{n-r}.\, y^r$
- **MIDDLE TERM :** $T_{(n+2)/2} = {}^nC_{n/2}.\, x^{n/2}.\, y^{n/2}$; when n is even

  $T_{(n+1)/2}$ & $T_{[(n+1)/2]+1}$; when n is odd
- **NUMERICALLY GREATEST TERM :** $T_{r+1} \geq T_r \Rightarrow \dfrac{T_{r+1}}{T_r} \geq 1 \Rightarrow \dfrac{n-r+1}{r}\left|\dfrac{a}{x}\right| \geq 1 \Rightarrow r \leq \dfrac{(n+1)}{\left(\left|\dfrac{x}{a}\right|+1\right)}$
- **TERM INDEPENDENT OF x :** Term independent of x contains no x; Hence find the value of r for which the exponent of x is zero.

2 SOME RESULTS ON BINOMIAL COEFFICIENTS

- $C_0 + C_1 + C_2 + \ldots\ldots.. + C_n = 2^n$
- $C_0 + C_2 + C_4 + \ldots. = C_1 + C_3 + C_5 + \ldots. = 2^{n-1}$
- $C_0^2 + C_1^2 + \ldots + C_n^2 = {}^{2n}C_n = \dfrac{(2n)!}{n!n!}$
- $C_0.\, C_r + C_1.\, C_{r+1} + \ldots\ldots.. + C_{n-r}\, C_n = \dfrac{(2n)!}{(n+r)!(n-r)!}$

3 SOME IMPORTANT EXPANSIONS

EXPONENTIAL SERIES

- $e^x = 1 + \dfrac{x}{1!} + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \ldots..\infty$; where x may be any real or complex number.
- $a^x = 1 + \dfrac{x}{1!}\ln a + \dfrac{x^2}{2!}\ln^2 a + \dfrac{x^3}{3!}\ln^3 a + \ldots\ldots..\infty$ where $a > 0$.
- $e = 1 + \dfrac{1}{1!} + \dfrac{1}{2!} + \dfrac{1}{3!} + \ldots\ldots\ldots\infty$

LOGARITHMIC SERIES

- $\ln(1+x) = x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \dfrac{x^4}{4} + \ldots\ldots\infty$ where $-1 < x \leq 1$
- $\ln(1-x) = -x - \dfrac{x^2}{2} - \dfrac{x^3}{3} - \dfrac{x^4}{4} - \ldots\ldots\infty$ where $-1 \leq x < 1$

APPROXIMATIONS

- $(1+x)^n = 1 + nx + \dfrac{n(n-1)}{1.2}x^2 + \dfrac{n(n-1)(n-2)}{1.2.3}x^3 + \ldots\ldots.$

  if x be very small, then $(1+x)^n = 1 + nx$, approximately.

FOLLOWING EXPANSION SHOULD BE REMEMBERED ($|x| < 1$)

- $(1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 - \ldots.\infty$
- $(1-x)^{-1} = 1 + x + x^2 + x^3 + x^4 + \ldots.\infty$
- $(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 \ldots.\infty$
- $(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \ldots.\infty$