SQUENCE AND SERIES

Part I

WHAT IS A PROGRESSION ?

A progression is a list of things (usually numbers) that are in order.

Example : 2 (1st term) 4 (2nd term) 8......... (3rd term) ← Dots Denote Infinite Progression

TYPE OF PROGRESSION

- Arithmetic Progression
- Geometric progression
- Harmonic Progression
- Arithmetico Geometric progression
- Miscellaneous Progression

Arithmetic Progression

Definition

A pattern of numbers that increases or decreases by a constant number.
E.g. 4, 7, 10, 13............

General Progression

General form of an arithmetic progression is given as $a, a+d, a+2d..., a+(n-1)d$

Where: a – First term | d – Common difference

n^{th} term

General term of an arithmetic progression is given as $\boxed{T_n = a + (n-1)d}$

Sum of 'n' terms

If 'n' terms $a, a+d, a+2d..., a+(n-1)d$ are in arithmetic progression Then the sum of 'n' terms: $\boxed{S_n = \frac{n}{2}\left[2a + (n-1)d\right]}$

Airthmetic Mean

If $a_1, a_2, ..., a_n$ are in arithmetic progression then the Arithmetic Mean (AM) is: $\boxed{A_m = \frac{a_1 + a_2 + ... + a_n}{n} \text{ or } = \frac{S_n}{n}}$

If $A_1, A_2 .... A_n$ are 'n' arithmetic means between two numbers 'a' and 'b' then $a, A_1, A_2 ..... A_n, b$ are in AP.

Where common diffrence $\boxed{d = \frac{b-a}{n+1}}$ and arithmetic means are $\boxed{A_i = a + i\frac{b-a}{n+1}}$

Geometric Progression

Definition

The progression, where the ratio of successive terms of a progression is constant
E.g. 4, 8, 16, 32, 64, ... here the common ratio is 2.

General Progression

General form of a geometric progression is given as $a, ar, ar^2, \ldots, ar^{n-1}$
Where: a – First term | r – Common ratio

n^{th} term

General term of a geometric progression is given as $T_n = a \cdot r^{(n-1)}$

Sum of 'n' terms

If 'n' terms $a, ar, ar^2, \ldots, ar^{n-1}$ are in geometric progression then the sum of 'n' terms:

$$S_n = \frac{a(r^n-1)}{r-1};\ r \neq 1$$

Geometric Mean

If $a_1, a_2, \ldots, a_n$ are in geometric progression then the geometric mean (GM) is:

$$G_m = (a_1 \cdot a_2 \cdot a_3 \ldots\ldots a_n)^{1/n}$$

If $G_1, G_2 \ldots G_n$ are 'n' geometric means between two numbers 'a' and 'b' then $a, G_1, G_2 \ldots G_n, b$ are in G.P.

Where common ratio $r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$ and geometric means are $G_i = a\left(\frac{b}{a}\right)^{\frac{i}{n+1}}$

Arithmetico Geometric Progression

Definition

The result of the multiplication of a geometric progression with the corresponding terms of an arithmetic progression

General Progression

$a, (a+d)r, (a+2d)r^2, (a+3d)r^3 \ldots$ Where:

a – First term | r – Common ratio of GP | d – Common difference of AP

n^{th} Term

General term of a arithmetico geometric progression is $T_n = [a+(n-1)d]r^{(n-1)}$

Sum of 'n' Terms

$a, (a+d)r, (a+2d)r^2, \ldots\ldots$ are in AGP then sum of the terms is:

$$S_n = \frac{a}{1-r} + \frac{rd(1-r^{n-1})}{(1-r)^2} - \frac{[a+(n-1)d]r^n}{1-r}$$

If $|r|<1$ and 'n' tends to infinity then sum of infinite terms is:

$$\lim_{n\to\infty} S_n = \frac{a}{(1-r)} + \frac{rd}{(1-r)^2}$$

HARMONIC PROGRESSION & MISC.

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HARMONIC PROGRESSION

- It is a sequence in which the reciprocal of the terms are in Arithmetic Progression.
- If $a, a+d, a+2d, \ldots\ldots\ldots$ in Arithmetic Progression.

  then $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \ldots\ldots$ in Harmonic Progression.
- n^{th} term of the Harmonic term is $T_n = \frac{1}{a+(n-1)d}$
- Sum of 'n' terms ⇒ No direct way but can be found with the help of A.P.
- If $a_1, a_2, \ldots\ldots a_n$ in Arithmatic Progression the Harmonic mean H_m is

  $$\frac{n}{H_m} = \frac{1}{a_1} + \frac{1}{a_2} + \ldots\ldots + \frac{1}{a_n}$$
- R.M.S $\geq$ AM $\geq$ GM $\geq$ HM
- $GM^2 = AM \times HM$ ⟶ AM, GM, and HM are in Geometric Progression.

MISCELLANEONS PROGRESSION

Sequences which sometimes follow a particular pattern and sometimes not.

POWER SERIESES

- Sum of the first 'n' natural number $1+2+3+\ldots\ldots n = \sum n = \frac{n(n+1)}{2}$
- Sum of Squares of the first 'n' natural numbers $1^2+2^2+3^2+\ldots\ldots n^2 = \sum n^2 = \frac{n(n+1)(2n+1)}{6}$
- Sum of cubes of the first 'n' natural numbers $1^3+2^3+\ldots+n^3 = \sum n^3 = \frac{[n(n+1)]^2}{4} = (1+2+\ldots\ldots n)^2$