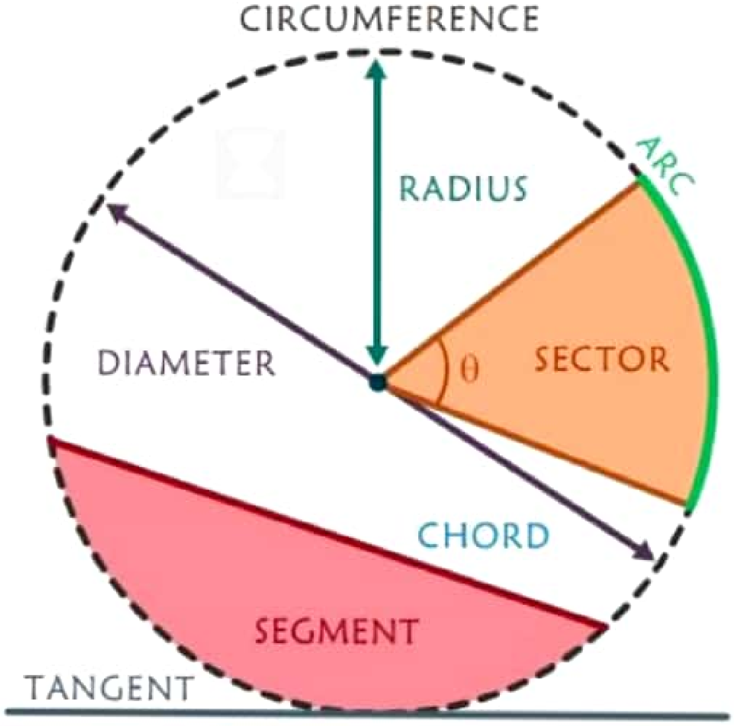
CIRCLE PROPERTIES
Part I

CIRCUMFERENCE
Length of the outer edge of a circle

DIAMETER
The distance from edge to edge passing through the center

CHORD
A straight line joining any two points on the circumference

SEGMENT
The area inside a circle enclosed by an arc and a chord



RADIUS
The distance from the center to the edge; half the diameter.

SECTOR
The area enclosed by an arc and two radii

ARC
A part of the circumference

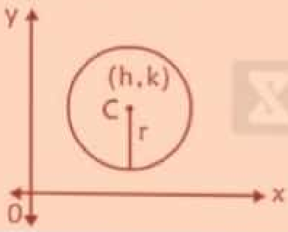
TANGENT
A straight line that touches a circle at two coincident points

Area $=\pi r^2$	Area of Sector $=\pi r^2 \cdot \frac{\theta}{360^\circ}$	Circumference $=2\pi r$	Length of Arc $=2\pi r \cdot \frac{\theta}{360^\circ}$

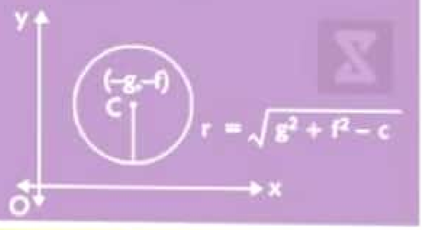
$\theta \rightarrow$ in degrees

STANDARD EQUATION OF THE CIRCLE

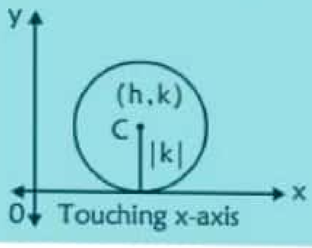
1. Central Form:
$(x-h)^2 + (y-k)^2 = r^2$



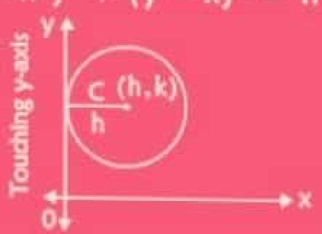
2. General equation of circle:
$x^2 + y^2 + 2gx + 2fy + c = 0$



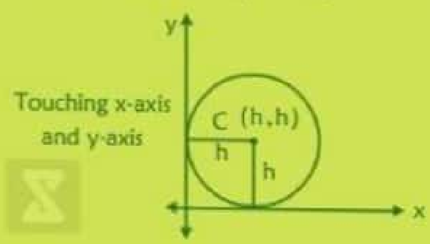
3. When circle touches x-axis
$(x-h)^2 + (y-k)^2 = k^2$



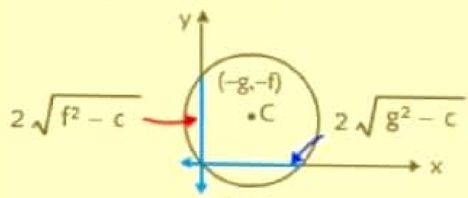
4. When circle touches y-axis
$(x-h^2) + (y-k)^2 = h^2$



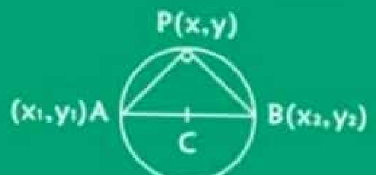
5. When circle touches both the axis
$(x-h^2) + (y-h)^2 = h^2$



6. Intercepts cut by the circle on axes:
$x^2 + y^2 + 2gx + 2fy + c = 0$



7. Diametrical form of circle:
$(x-x_1)(x-x_2) + (y-y_1)(y-y_2) = 0$



8. The parametric forms of the circle:

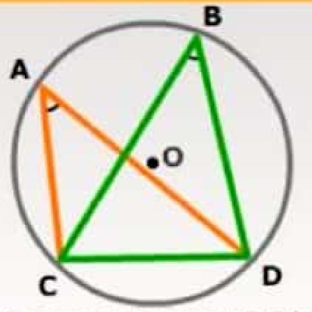
Circle	Parametric forms
$x^2 + y^2 = r^2$	$x = r\cos\theta,\ y = r\sin\theta$
$(x-h)^2 + (y-k)^2 = r^2$	$x = h + r\cos\theta,\ y = k + r\sin\theta$

where θ is the parameter $\theta \in [0, 2\pi)$

CIRCLE THEOREMS

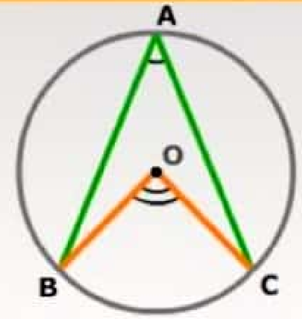
Part III

1 $\angle CAD = \angle CBD$



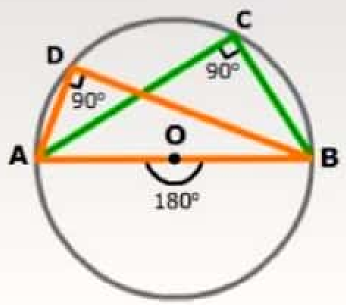
Angle in the same segment and standing on the same chord are always equal.

2 $\angle BOC = 2\angle BAC$



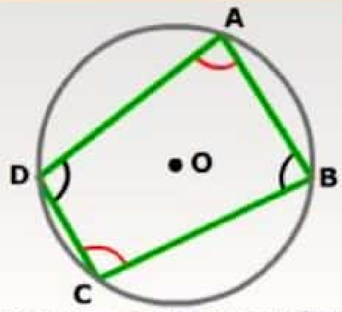
The angle at the centre of a circle is twice the angle at the circumference.

3 $\angle BCA = \angle BDA = 90^\circ$



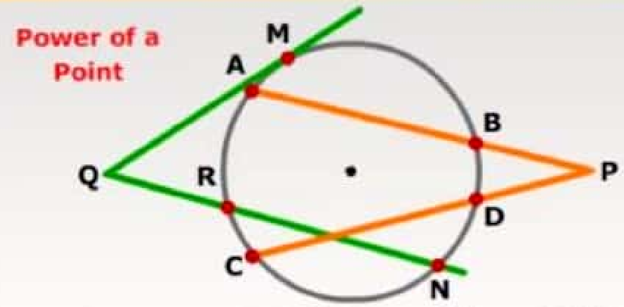
The angle in a semi-circle is always 90°.

4 $\angle B + \angle D = 180^\circ$, $\angle A + \angle C = 180^\circ$



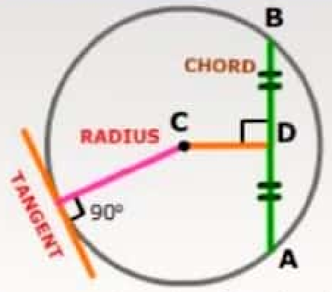
ABCD is a cyclic Quadrilateral,
Diagonally opposite angles add up to 180°.

5 $QR.QN = (QM)^2$, $PA.PB = PC.PD$



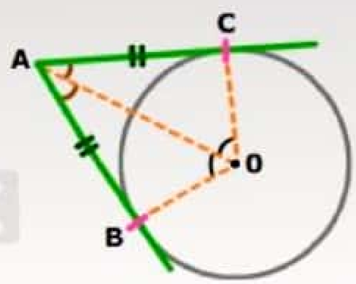
If QRN is a secant & QM is tangent then $QR.QN=(QM)^2$
If PBA & PDC are secant to circle then $PA.PB=PC.PD$

6 $CD \perp AB$, $BD = DA$, $\angle BDC = 90^\circ$



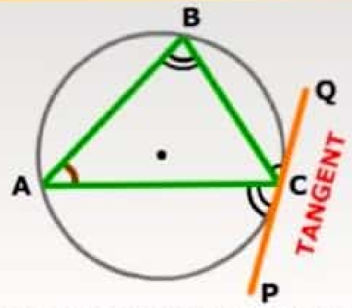
Angle between tangent and radius is always 90°.
Perpendicular bisector of any chord, pass through center.

7 $AB = AC$, $\angle AOB = \angle AOC$



Tangents from a common point (A) to a circle are always equal.

8 $\angle PCA = \angle ABC$, $\angle QCB = \angle BAC$



The angle between the tangent and the side of the triangle is equal to the interior opposite angle.

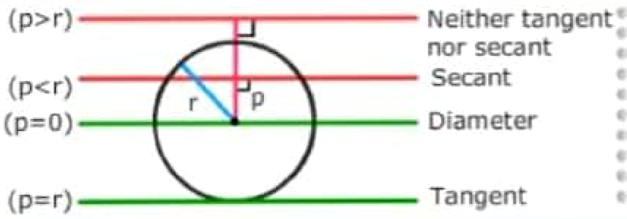
CIRCLE'S TANGENTS

Part II

Condition of Tangency :

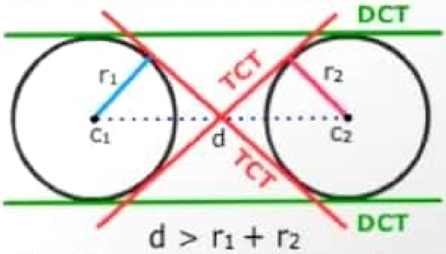
p = perpendicular distance from center to line

r = radius to circle



Common tangents of two circles

1 If two circle are separated

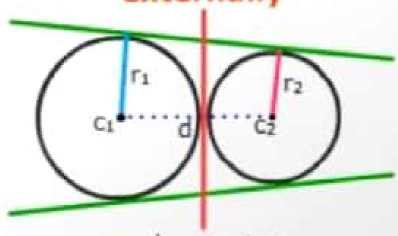


$d > r_1 + r_2$

d=distance between centers

$\Rightarrow$ 4 common tangents (2DCT,2TCT)

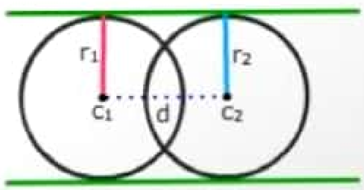
2 If two circles touch externally



$d = r_1 + r_2$

$\Rightarrow$ 3 common tangents (2DCT,1TCT)

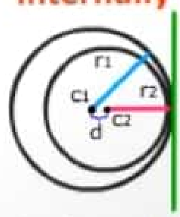
3 If two circles intersect each other



$|r_1 - r_2| < d < r_1 + r_2$

$\Rightarrow$ 2 common tangents (2DCT)

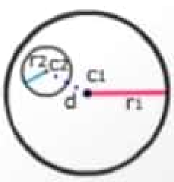
4 If two circles touch internally



$d = |r_1 - r_2|$

$\Rightarrow$ 1 common tangent (1DCT)

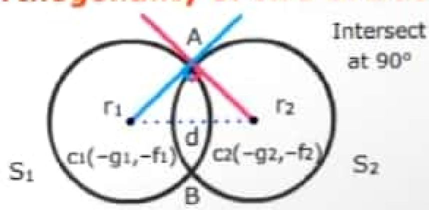
5 If one circles is completely contained in another circle



$d < |r_1 - r_2|$

$\Rightarrow$ No common tangent

6 Orthogonality of two circles



$x^2+y^2+2g_1x+2f_1y+d_1=0$

$x^2+y^2+2g_2x+2f_2y+d_2=0$

Condition : $2g_1g_2 + 2f_1f_2 = d_1+d_2$

*DCT = Direct Common Tangent, TCT = Transverse Common Tangent

Position of a point $P(x_1,y_1)$ w.r.t. circle :

$x^2+y^2+2gx+2fy+c = 0$

deponds on

$S_1 = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$