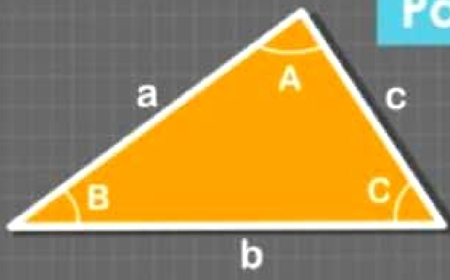
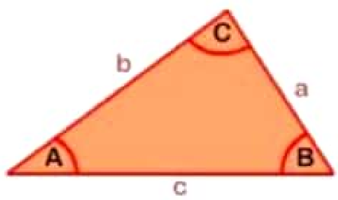
SOLUTION OF TRIANGLE

Part I



LAW OF SINES

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



LAW OF COSINES

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

PROJECTION FORMULA

$$a = b\cos C + c\cos B$$

$$b = c\cos A + a\cos C$$

$$c = a\cos B + b\cos A$$

AREA OF TRIANGLE

$$\text{Area} = \frac{1}{2}\, ab\sin C$$

$$\text{Area} = \frac{1}{2}\, bc\sin A$$

$$\text{Area} = \frac{1}{2}\, ca\sin B$$

NAIPER'S ANALOGY

$$\tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot\frac{A}{2}$$

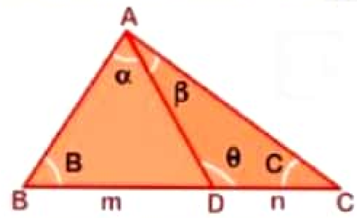
$$\tan\frac{C-A}{2} = \frac{c-a}{c+a}\cot\frac{B}{2}$$

$$\tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\frac{C}{2}$$

M–N THEOREM

$$(m+n)\cot\theta = m\cot\alpha - n\cot\beta$$

$$(m+n)\cot\theta = n\cot B - m\cot C$$



TRIGONOMETRIC HALF ANGLES

$$\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$$

$$\sin\frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{ca}}$$

$$\sin\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$$

$$\text{where, } s = \frac{a+b+c}{2}$$

$$\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$$

$$\cos\frac{B}{2} = \sqrt{\frac{s(s-b)}{ca}}$$

$$\cos\frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}$$

$$\text{where, } s = \frac{a+b+c}{2}$$

$$\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

$$\tan\frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{s(s-b)}}$$

$$\tan\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$$

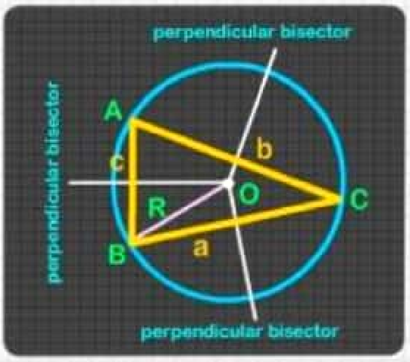
$$\text{where, } s = \frac{a+b+c}{2}$$

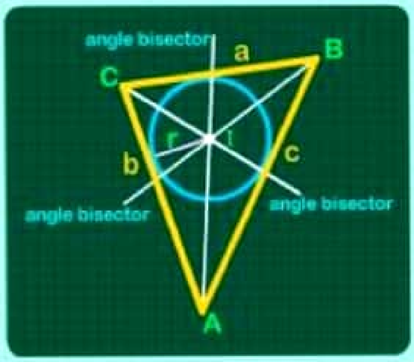
PROPERTIES OF TRIANGLES AND CIRCLES CONNECTED WITH THEM Part II

$R \rightarrow$ Circumradius of Δ ABC.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R \; ; \; \Delta = \frac{abc}{4R}$$

$$a \cos B \cos C + b \cos C \cos A + c \cos A \cos B = \frac{\Delta}{R}$$





$r \rightarrow$ Inradius of Δ ABC.

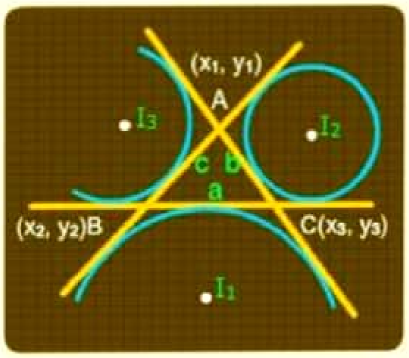
$$r = \frac{a \sin \frac{B}{2} \sin \frac{C}{2}}{\cos \frac{A}{2}} = \frac{b \sin \frac{C}{2} \sin \frac{A}{2}}{\cos \frac{B}{2}} = \frac{c \sin \frac{A}{2} \sin \frac{B}{2}}{\cos \frac{C}{2}}$$

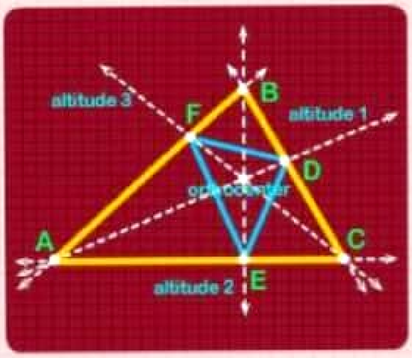
$$r = (s-a) \tan \frac{A}{2} = (s-b) \tan \frac{B}{2} = (s-c) \tan \frac{C}{2} \; ; \; r = \frac{\Delta}{s} \; ; \; s = \frac{a+b+c}{2}$$

$r_1, r_2, r_3 \rightarrow$ are radii of excircles of Δ ABC.

$$r_1 = \frac{\Delta}{s-a}; \quad r_2 = \frac{\Delta}{s-b}; \quad r_3 = \frac{\Delta}{s-c}; \quad s = \frac{a+b+c}{2}$$

$$r_1 = s \tan \frac{A}{2}; \quad r_2 = s \tan \frac{b}{2}; \quad r_3 = s \tan \frac{c}{2}$$





ORTHOCENTRE AND PEDAL TRIANGLE

ORTHOCENTRE - Point of intersection of 3 altitudes.

PEDAL - Pedal triangle is formed by joining the feet of altitudes.

Area of $\boxed{\Delta DEF = 4\Delta \cos A \cos B \cos C}$

where Δ is the area of triangle ABC.

LENGTH OF ANGLE BISECTOR MEDIAN & ALTITUDE

Length of an angle bisector from the angle A = $\boxed{\beta_a = \frac{2bc \cos A/2}{b+c}}$

Length of median from the angle A = $\boxed{m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}}$

Length of altitude from the angle A = $\boxed{A_a = \frac{2\Delta}{a}}$ $\Delta \rightarrow$ Area of triangle ABC.