# PERMUTATIONS & COMBINATIONS

Part I

## COUNTING TECHNIQUES

### PERMUTATIONS

- Permutation is the number of ways to arrange things.

  **Eg : My safe code is 492.**

  **(order matters)**

- $P(n,r) = {}^nP_r = \frac{n!}{r!}$, Where $0 \leq r \leq n$

  n → the number of things to choose from

  r → the number of things we choose

  ! → factorial.

### COMBINATIONS

- Combination is the number of ways to choose things.

  **Eg : My Salad is a Combination of carrot, Onion, Tomato and Lemon.**

  **(order doesn't matter)**

- $C(n,r) = {}^nC_r = \frac{n!}{r!\,(n-r)!} = \binom{n}{r}$ ; Where $0 \leq r \leq n$

  n → the number of things to choose from

  r → the number of things we choose

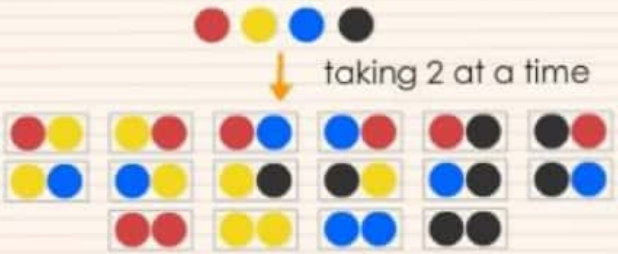
## TYPES OF PERMUTATIONS & COMBINATIONS

### When Repetition is Allowed.

**1. Permutations with Repetition**

Formula: $n^r$

(Repetition allowed, order matters)




**2. Combinations with Repetition**

Formula: ${}^{(n+r-1)}C_r$

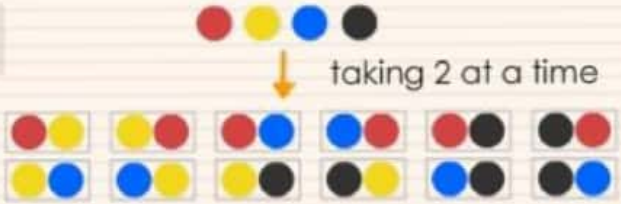
(Repetition allowed, order does not matter)


### When Repetition is not Allowed.

**1. Permutations without Repetition**

Formula: ${}^nP_r = \frac{n!}{(n-r)!}$

(No repetition, order matters)




**2. Combinations with Repetition**

Formula: ${}^nC_r = \frac{n!}{r!(n-r)!}$

(No repetition, order does not matter)

Part II

# PERMUTATION

If 'n' is the number of distinct things and 'r' things are chosen at a time.

## 1. Permutations of objects when all objects are not distinct.

Permutations = $\frac{n!}{P_1!\, P_2!\, \ldots\ldots\, P_r!}$ $P_r \rightarrow$ Number of things among 'n' are excatiy alike of $r^{th}$ type.

## 2. Permutations with Repetition

Number of Permutations = $n^r$

## 3. Circular Permutations

**Case 1 :** When clockwise and anticlock wise arrangements are different.

Number of Permutations : $(n-1)!$

**Case 2 :** When clockwise and anticlock wise arrangements are not different.

Number of Permutations : $\frac{1}{2}(n-1)!$

## 4. Permutation under Restrictions

**Case 1 :** When 's' particular things are always to be included.

Number of Permutations : $\frac{(n-s)!\, r!}{(n-r)!\, (r-s)!}$

**Case 2 :** When a particular thing is always to be included (s = 1).

Number of Permutations : $\frac{(n-1)!\, r!}{(n-r)!\, (r-1)!}$

**Case 3 :** When 's' particular things are never be included.

Number of Permutations : $\frac{(n-s)!}{(n-s-r)!}$

**Case 4 :** When a particular thing is never included (s = 1).

Number of Permutations : $\frac{(n-1)!}{(n-r-1)!}$

**Case 5 :** When 'm' particular things always come together.

Number of Permutations : $(n-m+1)! \times m!$

**Case 6 :** When 'm' particular things never come together.

Number of Permutations : $n! - (n-m+1)! \times m!$

Part III

# COMBINATION

If **'n'** is the number of distinct things and **'r'** things are chosen at a time.

## 1. Combinations with Repetition

Number of Combinations : $^{(n+r-1)}C_r$

## 2. Total Number of Combinations

**Case 1 :** Ways of selecting one or more than one things.

Number of Combinations : $^nC_1 + {}^nC_2 + \ldots\ldots + {}^nC_n = 2^n - 1$

**Case 2 :** When '$s_1$' alike objects of one kind, '$s_2$' alike objects of 2nd kind and so on ..... '$s_n$' alike objects of $n^{th}$ kind.

Number of Combinations : $(s_1 + 1)(s_2 + 1) \ldots\ldots (s_n + 1) - 1$

**Case 3 :** When '$s_1$' alike objects of one kind, '$s_2$' alike objects of 2nd kind and so on .... '$s_n$' alike objects of $n^{th}$ kind and rest '**p**' different objects.

Number of Combinations : $\left[(s_1 + 1)(s_2 + 1) \ldots\ldots (s_n + 1)\right] 2^p - 1$

## 3. Combinations Under Restrictions

**Case 1 :** When '**s**' particular things are always to be included.

Number of Combinations : $^{(n-s)}C_{(r-s)}$

**Case 2 :** When a particular thing is always to be included.

Number of Combinations : $^{(n-1)}C_{(r-1)}$

**Case 3 :** When '**s**' particular things are never included (s = 1).

Number of Combinations : $^{(n-s)}C_r$

**Case 4 :** When '**m**' particular things never come together.

Number of Combinations : $^nC_r - {}^{(n-m)}C_{(r-m)}$