Part I

WHAT IS A MATRIX ?

A Matrix

$$\begin{bmatrix} -1 & 3 & 0 \\ 6 & -3 & -1 \\ 2 & 0 & 1 \end{bmatrix}$$

Matrix is an array/arrangement of numbers

Order of a matrix = Number of Rows X Number of Columns

$$\begin{array}{cc} \text{Column} & \\ \begin{bmatrix} -1 & 3 & 0 \\ 6 & -3 & -1 \\ 2 & 0 & 1 \end{bmatrix} & \begin{array}{l} \text{Row} \\ \\ \\ \end{array} \end{array}$$

TYPE OF MATRICES

1 Row Matrix

Matrix having only one row.

$A_{1\times3} = [1\ 2\ 3]$

2 Column Matrix

Matrix having only one column.

$A_{2\times1} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

3 Square Matrix

Matrix having same number of rows and columns.

$A_{3\times3} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 6 \\ 7 & 1 & 9 \end{bmatrix}$

4 Zero/Null Matrix

Matrix having all elements equal to zero.

$A_{3\times3} = 0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

5 Upper Triangular Matrix

All entries below the main diagonal are zero.

$A_{3\times3} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 6 \\ 0 & 0 & 9 \end{bmatrix}$

6 Lower Triangular Matrix

All entries above the main diagonal are zero.

$A_{3\times3} = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 5 & 0 \\ 7 & 8 & 9 \end{bmatrix}$

7 Diagonal Matrix

All entries above and below the principal diagonal are zero.

$A_{3\times3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 9 \end{bmatrix}$

8 Identity/Unit Matrix

Diagonal matrices in which all diagonal elements are unity/one.

$A_{3\times3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

OPERATIONS ON MATRICES

Addition Matrix

Matrices must have same order.

$3 + 4 = 7$

$A+B = \begin{bmatrix} 3 & 8 \\ 4 & 6 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix} = \begin{bmatrix} 7 & 8 \\ 5 & -3 \end{bmatrix}$

Subtraction Matrix

Matrices must have same order.

$3 - 4 = -1$

$A-B = \begin{bmatrix} 3 & 8 \\ 4 & 6 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix} = \begin{bmatrix} -1 & 8 \\ 3 & 15 \end{bmatrix}$

Equality Matrix

Matrices having same order with all the corresponding elements being equal.

$A_{3\times3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 9 \end{bmatrix}$; $B_{3\times3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 9 \end{bmatrix}$; $A = B$

Transpose of a Matrix

A matrix formed by turning all the rows into columms and vice-versa. Symbol ⇨ A^T.

$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$

Part II

MATRIX MULTIPLICATION

Multiplication of Matrix with a Scalar

Each element of the Matrix is multiplied by the scalar

$2 \times 4 = 8$

$$2 \times \begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 2 & -18 \end{bmatrix}$$

Multiplication of a Matrix with another Matrix

If a matrix A and another matrix B then A x B Possible if

Post factor = Pre factor

$$A_{m \times n} \qquad B_{p \times q}$$

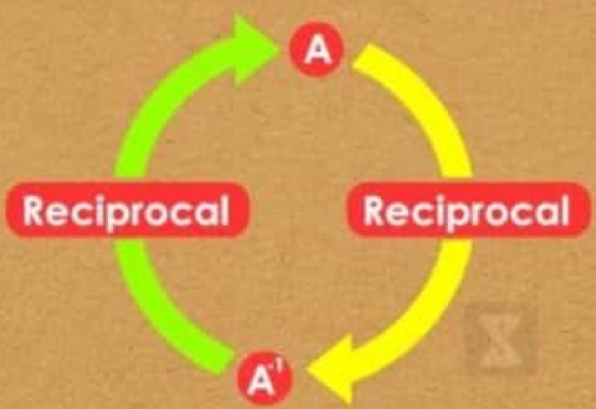
$$\begin{bmatrix} 1 & 3 & 4 \\ 7 & 5 & 6 \\ 2 & 9 & 8 \end{bmatrix}_{3 \times 3} \qquad \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}_{3 \times 1}$$

These must match.

How to Multiply a Matrix by Another Matrix ?

$$\begin{bmatrix} 1 & 3 & 4 \\ 7 & 5 & 6 \\ 2 & 9 & 8 \end{bmatrix}_{3 \times 3} \times \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}_{3 \times 1} = \begin{bmatrix} 1 \times 3 + 3 \times 5 + 4 \times 2 \\ 7 \times 3 + 5 \times 5 + 6 \times 2 \\ 2 \times 3 + 9 \times 5 + 8 \times 2 \end{bmatrix}_{3 \times 1} = \begin{bmatrix} 26 \\ 58 \\ 67 \end{bmatrix}_{3 \times 1}$$

INVERSE OF A MATRIX



Reciprocal of a Matrix.

- For a matrix A inverse of this.

  i.e. $A^{-1} \neq \frac{1}{A}$ Why ?

- For a matrix A

  (matrix) × (Inverse of matrix) = I

  i.e. $A \times A^{-1} = I$ or $A^{-1} \times A = I$

  But $A \times A^{-1} \neq A^{-1} \times A$

How to find Inverse of a Matrix ?

Step - I

Check whether Matrix A is singular or non-singular i.e.

$|A| = 0 \Rightarrow$ singular

$|A| \neq 0 \Rightarrow$ Non-singular

Step - II

If Matrix A is Non-singular, then find the value of determinant and also find one adjoint matrix A.

Step - III

Follow the formula

$$A^{-1} = \frac{1}{|A|} \text{adj } A.$$

- $(A^{-1})^{-1} = A$, if A is non-singular.
- $(A^{-1})^{T} = (A^{T})^{-1}$
- If $A = \text{diag}(a_{11}, a_{22}, \ldots\ldots a_{nn})$ Then, $A^{-1} = \text{diag}\left(\frac{1}{a_{11}}, \frac{1}{a_{22}}, \ldots\ldots, \frac{1}{a_{nn}}\right)$

TYPE OF SQUARE MATRICES

Nilpotent Matrix

If $B^P = 0$ where 'P' is the least +ve integer. Then, 'B' is a Nilpotent matrix.

$$B = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$$

Idempotent Matrix

If $B^2 = B$. Then, 'B' is an Idempotent matrix.

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Involutory Matrix

If $B^2 = I$. Then, 'B' is a Involutory matrix

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Symmetric Matrix

If $B^T = B$. Then, 'B' is a Symmetric matrix.

$$B = \begin{bmatrix} 1 & 4 & 5 \\ 4 & 2 & 6 \\ 5 & 6 & 3 \end{bmatrix}$$

Skew Symmetric Matrix

If $B^T = -B$ and all Principal diagonal elements are zero. Then 'B' is a skew symmetric matrix.

$$B = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 3 \\ -2 & -3 & 0 \end{bmatrix}$$

Unitary Matrix

If $B' (B')^T = I$ where B' is the complex conjugate of B. Then, B is a unitary matrix.

$$B = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$

Orthogonal Matrix

A square matrix 'B' if $B^T B = I = B\ B^T$ or $B^T = B^{-1}$. Then, 'B' is an Orthogonal matrix.

Points To Remember

- In a skew-symmetric matrix all the Principal diagonal elements are zero.
- For any square matrix A, $A + A^T$ is symmetric & $A - A^T$ is skew-symmetric.
- Every square matrix can be uniquely expressed as a sum of two square matrices of which one is symmetric and the other is skew-symmetric
- $A = B + C$, where $B = \frac{1}{2}(A + A^T)$ & $C = \frac{1}{2}(A - A^T)$
- For any matrix A ⇨ $(A^T)^T = A$
- Let λ be a scalar & A be a matrix. Then $(\lambda A)^T = \lambda A^T$
- $(A_1 \pm A_2 \pm \dots \pm A_n)^T = A_1^T \pm A_2^T \pm \dots \pm A_n^T$ where A_j are comparable.
- $(A_1 . A_2 \dots A_n)^T = A_n^T . A_{n-1}^T \dots A_2^T . A_1^T$ provided the product is defined.
- $A + B = B + A$
- $(A + B) + C = A + (B + C)$
- $0 = [0]_{m \times n}$ is the additive identity.
- $\lambda(A + B) = \lambda A + \lambda B$