INDEFINITE INTEGRAL

Part I

Indefinite Integral is the opposite of derivative

If $\frac{dy}{dx} = f(x)$ ➡ $y = \int f(x)\, dx$ — with respect to 'x'

Integral Symbol

Integrand: Function we want to integrate

METHODS OF INTEGRATION

1 SUBSTITUTION

$I = \int f(g(x))\, g'(x)\, dx$ ➡ Let $g(x) = u$ ➡ then $\frac{dg(x)}{dx} = g'(x) = \frac{du}{dx}$

$I = \int f(g(x))\, g'(x)\, dx$

$I = \int f(u)\, du$

Then we can integrate f(u), and finish by putting g(x) back as u.

2 BY PARTS (PRODUCT RULE)

$I = \int f(x).g(x)dx$ ➡ $I = f(x)\int g(x)dx - \int f'(x).\left(\int g(x)dx\right)dx$

A helpful rule of thumb is ILATE. Choose f(x) based on which of these comes first:

I L A T E

I	L	A	T	E
Inverse Trigonometric Functions	Logarithmic Functions	Algebraic Functions	Trigonometric Functions	Exponential Functions

3 MISCELLANEOUS

Euler's substitutions for integration $I = \int R(x, \sqrt{ax^2+bx+c})\, dx$

Substitutions

1. $\sqrt{ax^2+bx+c} = t \pm \sqrt{a}x$; $a > 0$
2. $\sqrt{ax^2+bx+c} = \sqrt{a(x-x_1)(x-x_2)} = t(x-x_1) = t(x-x_2)$,
3. $\sqrt{ax^2+bx+c} = tx \pm \sqrt{c}$; $c > 0$.

4 PARTIAL (FRACTION)

Expressing complicated algebraic fractions into 'Partial Fractions'.

Partial Fractions with 'Repeated Linear Factors' in the denominator.

Denominator containing	Expression	Form of Partial Fractions
Linear factor	$\frac{f(x)}{(x+a)(x+b)}$	$\frac{A}{x+a} + \frac{B}{x+b}$
Repeated linear factors	$\frac{f(x)}{(x+a)^3}$	$\frac{A}{x+a} + \frac{B}{(x+a)^2} + \frac{C}{(x+a)^3}$
Quadratic term (which cannot be factored)	$\frac{f(x)}{(ax^2+bx+c)(gx+h)}$	$\frac{Ax+B}{ax^2+bx+c} + \frac{C}{gx+h}$

BASIC INTEGRALS & TRICKS

ALGABRAIC

$$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$$

$$n \neq -1,\ n \in R$$

$$\int \frac{dx}{ax+b} = \frac{ln\,(ax+b)}{a} + C$$

$$\int a^{px+q}\,dx = \frac{a^{px+q}}{p.ln a}\ ;\ a > 0$$

TRIGONOMETRIC

$$\int \sin(ax+b)\,dx = -\frac{1}{a}\cos(ax+b) + C$$

$$\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b) + C$$

$$\int \sec^2(ax+b)\,dx = \frac{1}{a}\tan(ax+b) + C$$

$$\int \text{cosec}^2\,(ax+b)\,dx = -\frac{1}{a}\cot(ax+b) + C$$

$$\int \text{cosec}\,x \cot x\,dx = -\text{cosec}\,x + c$$

MISCELLANEOUS

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a}$$

$$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{x}{a}$$

$$\int \frac{dx}{(x-a)\sqrt{(x-a)(b-x)}}$$

Put : $x = a\cos^2\theta + b\sin^2\theta$

$$\int \frac{dx}{\sqrt{(x-a)(x-b)}}$$

Put : $x = a\sec^2\theta - b\tan^2\theta$

$$\int \frac{dx}{(ax+b)\sqrt{px+q}}$$

Put $px+q = t^2$

DEFINITE INTEGRALS

Part I

A Definite Integral represents the exact area under the curve between points 'a' and 'b'.

$$\int_a^b f(x)\,dx = [\,F(x)]_a^b = F(b) - F(a)$$ is called the definite integral of f(x) between the limits a and b. where $\frac{d}{dx}(F(x)) = f(x)$

DERIVATIVE OF ANTIDERIVATIVE (LEIBNITZ'S RULE)

If h(x) & g(x) are differentiable functions of x then,

$$\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)dt = f[h(x)].h'(x) - f[g(x)].\,g'(x)$$

DEFINITE INTEGRAL AS LIMIT OF A SUM

$$\int_a^b f(x)dx = \lim_{n\to\infty} h[f(a) + f(a+h) + f(a+2h) + \ldots + f(a+\overline{n-1}h)]$$

$$= \lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh) \text{ where } h = \frac{b-a}{n}$$

WALLI'S FORMULA & REDUCTION FORMULA

$$\int_0^{\pi/2} \sin^n x.\cos^m x dx = \frac{[(n-1)(n-3)\ldots 1 \text{ or } 2]\,[(m-1)(m-3)\ldots 1 \text{ or } 2]}{(m+n)(m+n-2)(m+n-4)\ldots.1 \text{ or } 2]}K$$

Where $K = \frac{\pi}{2}$ if both 'm' and 'n' are even ($m, n \in N$); otherwise K = 1

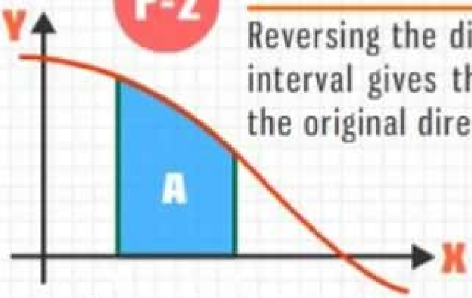
Part II

PROPERTIES OF DEFINITE INTEGRAL

P-1 **CHANGE OF VARIABLE:** $\int_a^b f(x)dx = \int_a^b f(t)dt = \int_a^b f(r)dr$

P-2 **REVERSING THE INTERVAL**

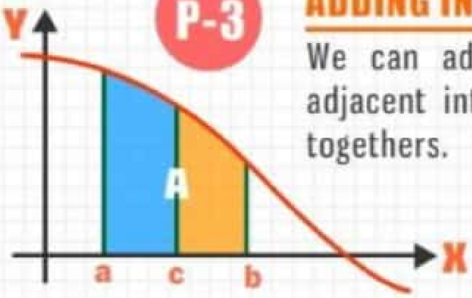
Reversing the direction of the interval gives the negative of the original direction.



$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

P-3 **ADDING INTERVAL**

We can add two adjacent intervals togethers.



$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

P-4

$$\int_{-a}^{a} f(x)dx = \int_0^a [f(x) + f(-x)]dx = \begin{cases} 0 & \text{if } f(x) \text{ is odd} \\ 2\int_0^a f(x)\,dx & \text{if } f(x) \text{ is even} \end{cases}$$

P-5

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx, \text{ in particular } \int_0^a f(x)dx = \int_0^a f(a-x)dx$$

P-6

$$\int_0^{2a} f(x)dx = \begin{cases} 2\int_0^a f(x)\,dx \;; & \text{if } f(2a-x) = f(x) \\ 0 \;; & \text{if } f(2a-x) = -f(x) \end{cases}$$

P-7

$$\int_0^{nT} f(x)dx = n\int_0^{T} f(x)dx$$

where 'T' is the period of the function i.e. $f(x+T) = f(x)$

P-8

$$\int_{a+nT}^{b+nT} f(x)dx = \int_a^b f(x)dx;$$ where $f(x)$ is periodic with period T & $n \in I$