Part I

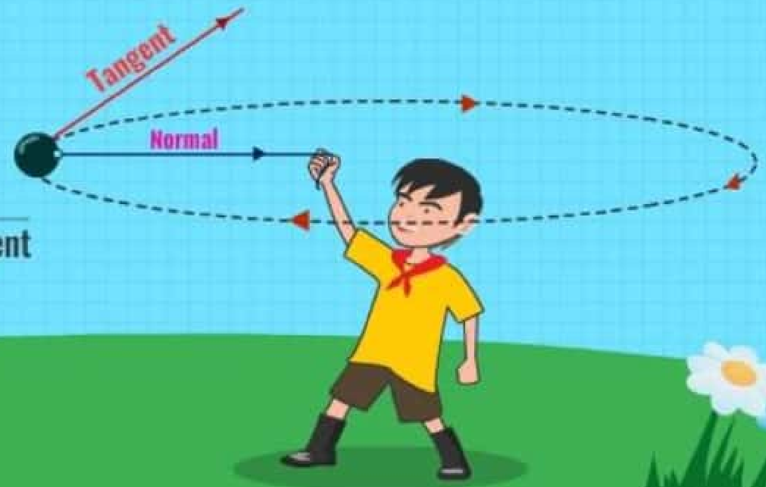
TANGENT & NORMAL

TANGENT

Tangent is a limiting case of a secant

NORMAL

A line that is perpendicular to a tangent line at the point of tangency.



CALCULATING TANGENT LINE & NORMAL LINE TO A CURVE

EQUATION OF TANGENT & ITS LENGTH

Equation: $y - y_1 = m_T\,(x - x_1)$

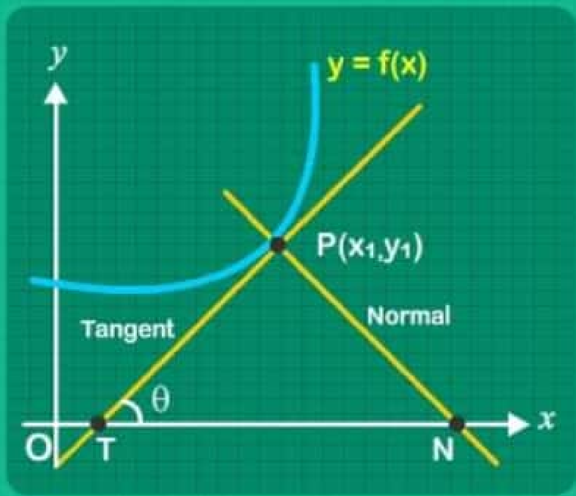
Length: $PT = \left|\dfrac{y_1\sqrt{1 + (m_T)^2}}{(m_T)}\right|$

EQUATION OF NORMAL & ITS LENGTH

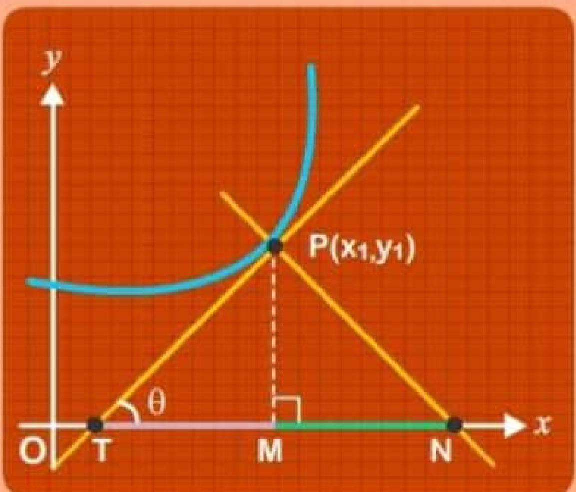
Equation: $y - y_1 = \dfrac{-1}{m_T}\,(x - x_1)$

Length: $PN = \left|\,y_1\sqrt{1 + (m_T)^2}\,\right|$

$$m_T = \left(\frac{dy}{dx}\right)_{P(x_1, y_1)} = \tan\theta$$



SUBTANGENT & SUBNORMAL



TM is the Subtangent and length of $TM = \left|\dfrac{y_1}{m_T}\right|$

MN is the Subnormal and length of $MN = \left|\,y_1\, m_T\right|$

ANGLE BETWEEN TWO INTERSECTING CURVES

Part II

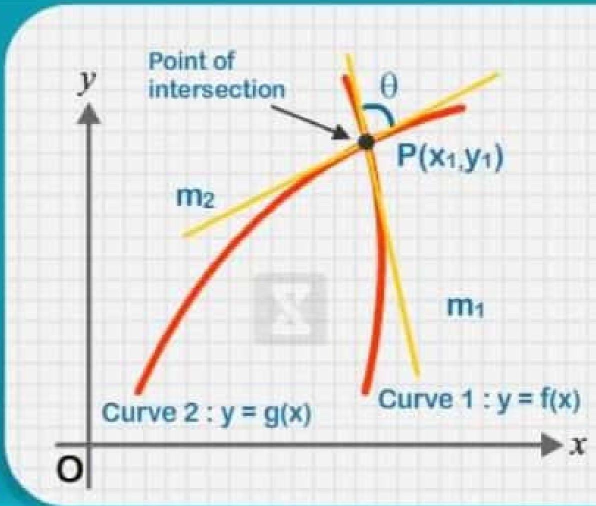
$m_1 \rightarrow$ slope to the curve 1 at point 'P'

$m_2 \rightarrow$ slope to the curve 2 at point 'P'

$$m_1 = \left.\frac{d\,f(x)}{dx}\right|_{(x_1,y_1)} ; \quad m_2 = \left.\frac{d\,g(x)}{dx}\right|_{(x_1,y_1)}$$

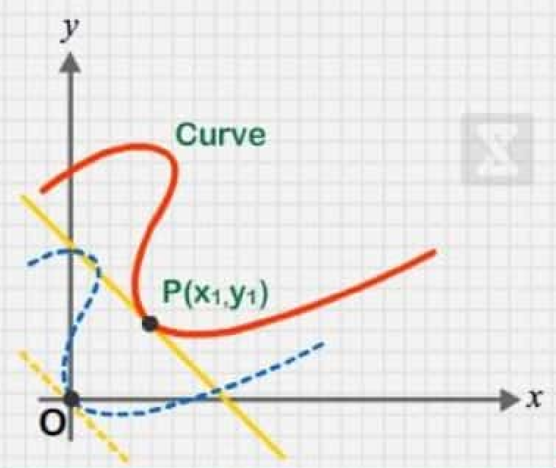
$$\theta = \tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

If $\theta = \frac{\pi}{2}$ then the curves are called Orthogonal Curves.



POINTS TO REMEMBER

1



Equation of Tangent at point $P(x_1, y_1)$ to any second degree general curve

$$ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$$

REPLACE:

$x^2 \rightarrow xx_1$; $y^2 \rightarrow yy_1$; $2x \rightarrow x + x_1$; $2y \rightarrow y + y_1$

$2xy \rightarrow xy_1 + x_1y$; $c \rightarrow c$

If curve passes through the 'O' then the equation of the tangent at 'O' may be directly written by comparing the lowest degree terms equal to 0.

$$2gx + 2fy = 0 \text{ or } gx + fy = 0$$

2

- If $\left(\frac{dy}{dx}\right)_{(x_1,y_1)} = 0 \Rightarrow$ Tangent is parallel to x-axis (Horizontal Tangent).
- If $\left(\frac{dy}{dx}\right)_{(x_1,y_1)} \rightarrow \infty$ or $\left(\frac{dx}{dy}\right)_{(x_1,y_1)} = 0 \Rightarrow$ Tangent is parallel to y-axis (Vertical Tangent).
- If $\left(\frac{dy}{dx}\right)_{(x_1,y_1)} = \pm 1 \Rightarrow$ Tangent at $P(x_1, y_1)$ is Equally inclined to the coordinate axis.

3 The shortest distance between two non-intersecting curves is always along to the common normal of the curves.

MONOTONOCITY

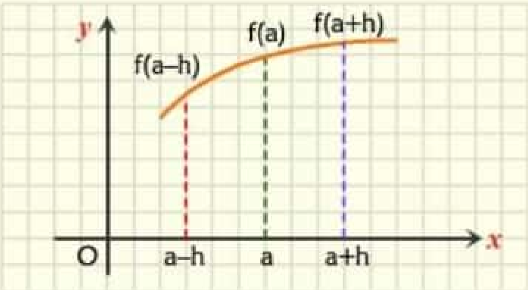
Part I

Definition

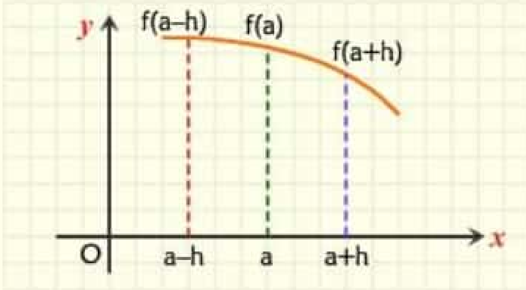
Functions are said to be monotonic if they are either increasing or decreasing in their entire domain, otherwise functions are called non–monotonic function.

Monotonocity of a Function at a Point

A function is said to be monotonically increasing at $x = a$ If $f(a + h) > f(a)$ & $f(a - h) < f(a)$ for small (+ve)h

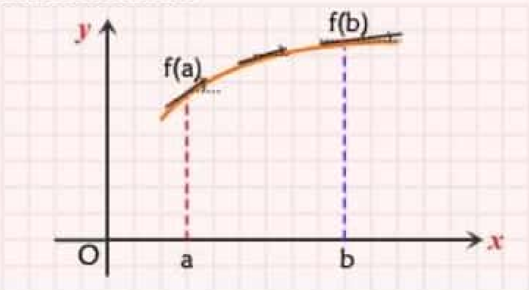


A function is said to be monotonically decreasing at $x = a$ If $f(a + h) < f(a)$ & $f(a - h) > f(a)$ for small (+ve)h

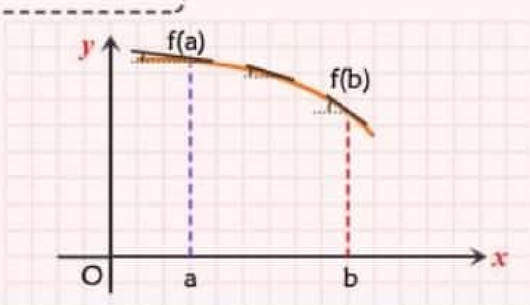


Monotonicity of a Function in an Interval (First Derivative Test)

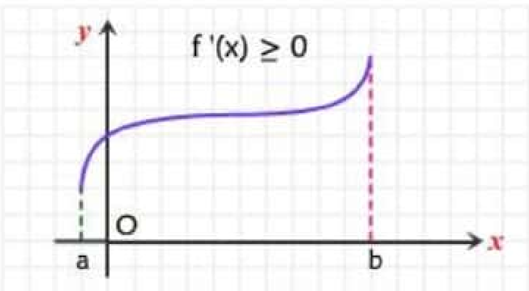
Function f(x) is said to be increasing in an interval (a, b) if $\boxed{\frac{dy}{dx} > 0 \text{ or } f'(x) > 0}$



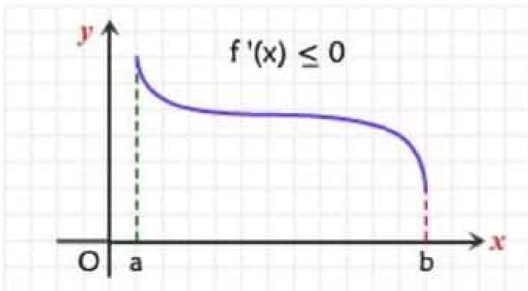
Function f(x) is said to be decreasing is an interval (a, b) if $\boxed{\frac{dy}{dx} < 0 \text{ or } f'(x) < 0}$



Non-Decreasing Function

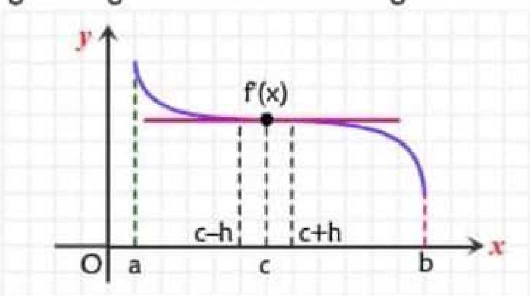


Non-Increasing Function



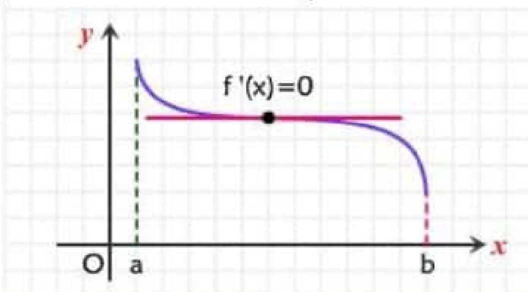
Point of Inflection

Point in the domain of f(x) where f '(x) does not changes its sign as x increases through c.



Critical Point

Point in the domain of f(x) where f '(x) is equal to zero or f '(x) fails to exist. Due to any reason.

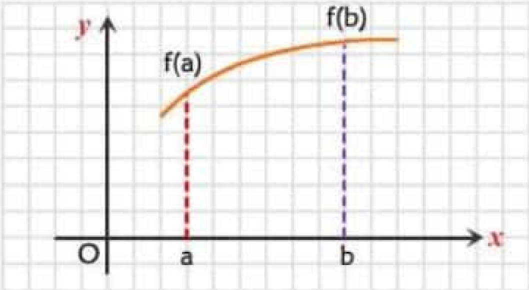


MONOTONOCITY

Part II

Greatest & Lowest Value of a Function

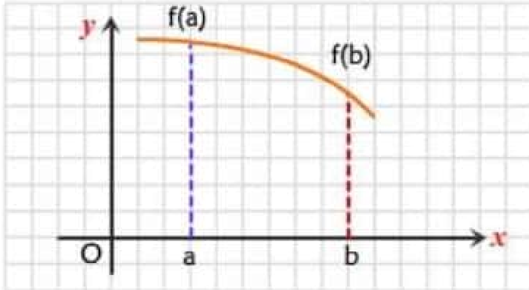
If a continuous function $y = f(x)$ is strictly increasing in $[a, b]$ then



Lowest value $= f(a)$

Greatest value $= f(b)$

If a continuous function $y = f(x)$ is strictly decreasing in $[a, b]$ then



Lowest value $= f(b)$

Greatest value $= f(a)$

Rolle's Theorem

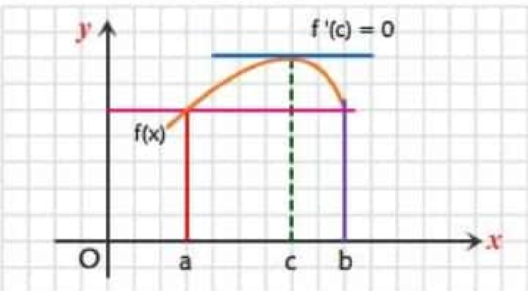
Let $f(x)$ be a function of x satisfying following conditions:

(1) $f(x)$ is continuous in $[a, b]$

(2) $f(x)$ is differentiable in (a, b)

(3) $f(a) = f(b)$

Then there exists atleast one point $x = c$, which belongs to (a, b) such that $\boxed{f'(c) = 0}$



Lagrange's Mean Value Theorem

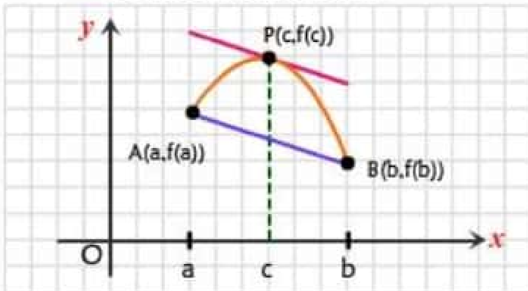
Let $f(x)$ be a function at x satisfying the following:

(1) $f(x)$ is continuous in $[a, b]$

(2) $f(x)$ is differentiable in (a, b)

(3) There exist atleast one $c \in (a, b)$ such that

$$\boxed{f'(c) = \frac{f(b) - f(a)}{b - a}}$$

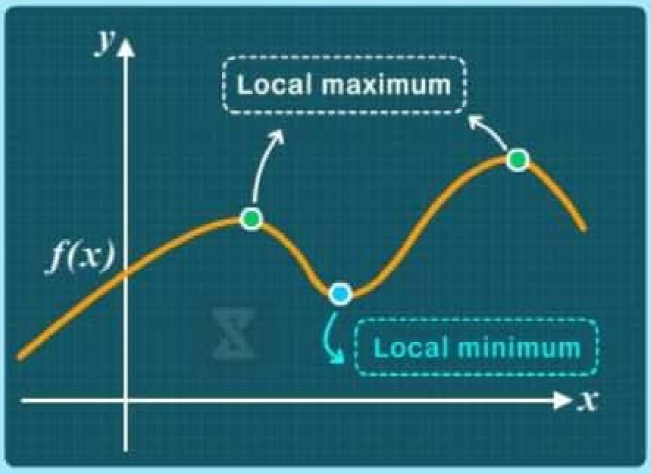


Points to Remember

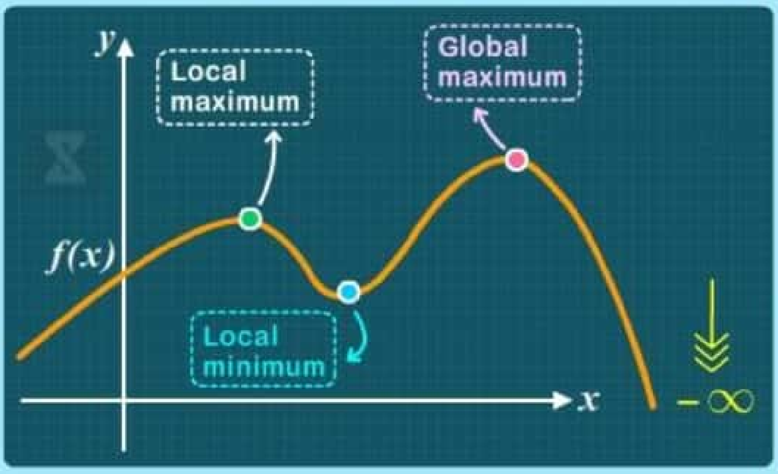
- If a function is invertible, it has to be either increasing or decreasing.
- If f is an increasing function then its negative i.e. $h = -f$ is a decreasing function.
- Reciprocal of an increasing function is a decreasing function.
- If f is an increasing function and g is also an increasing function their sum $h = f + g$ is an increasing function.
- If f and g both are increasing function then $h = f \times g$ is also an increasing function.
- If a function f is increasing (I) and takes negative values and another function g is decreasing (D) and takes positive values, then their product is an increasing function.
- Monotonocity of the difference of two function can be predicted as if I – I = can't say, I – D = increasing, D – I = decreasing, D – D = can't say.

MAXIMA AND MINIMA

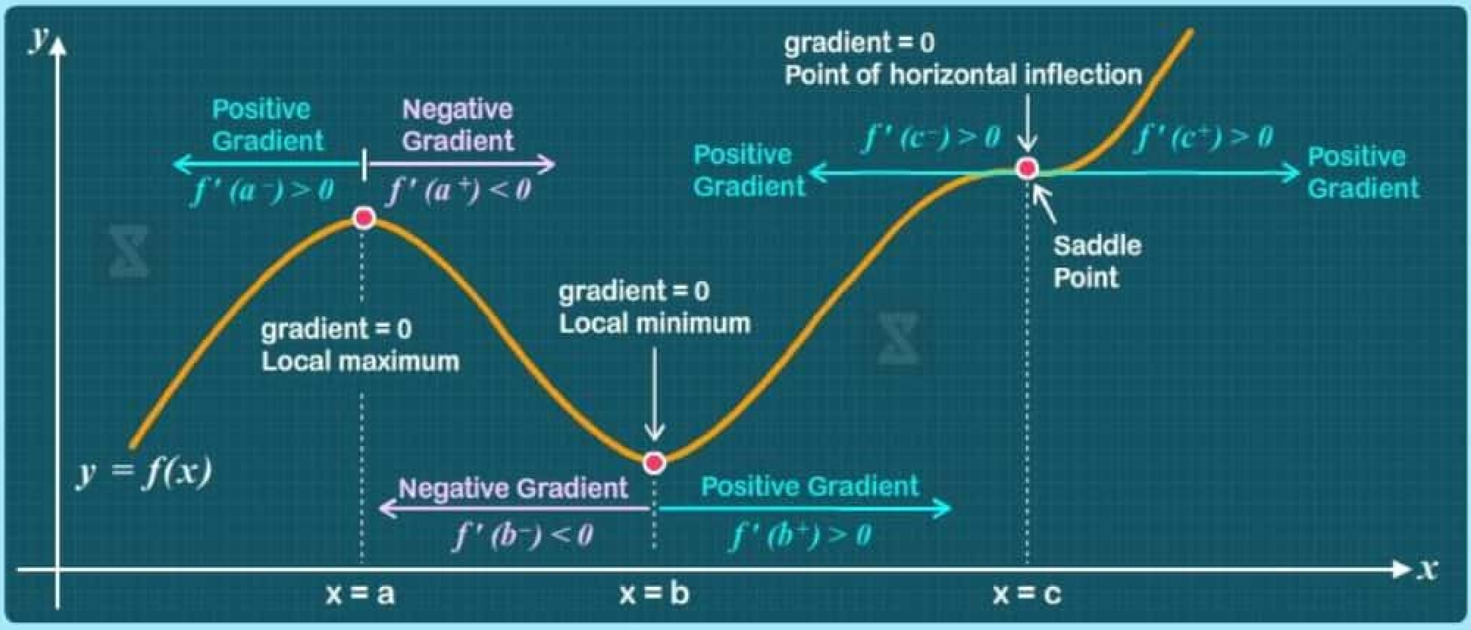
LOCAL MAXIMUM & MINIMUM



ABSOLUTE MAXIMUM & MINIMUM



FINDING MAXIMUM & MINIMUM



FIRST DERIVATIVE TEST

LOCAL MAXIMUM	$f'(a) = 0$	$f'(a^-) > 0$	$f'(a^+) < 0$
LOCAL MINIMUM	$f'(b) = 0$	$f'(b^-) < 0$	$f'(b^+) > 0$
SADDLE POINT	$f'(c) = 0$	$f'(c^-) > 0$	$f'(c^+) > 0$

- In general at saddle point (let x = c) $f'(c^+)$ and $f'(c^-)$ both are either positive or negative.

SECOND DERIVATIVE TEST

LOCAL MAXIMUM	$f'(a) = 0$	$f''(a) < 0$
LOCAL MINIMUM	$f'(b) = 0$	$f''(b) > 0$
SADDLE POINT	$f'(c) = 0$	$f''(c) = 0$

- In general at saddle point (let x = c) $f'(c) = f''(c) = \ldots\ldots = f^n(c) = 0$.