2a

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$$\begin{bmatrix} q \\ \dot{q} \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ u/ml - bx_2/ml - g\sin(x_1)/l \end{bmatrix}$$

Write down the linearized dynamics

$$\dot{\tilde{x}} = \overbrace{D_x F\big|_{\substack{\bar{x} \\ \bar{u}}}}^{A} \cdot \tilde{x} + \overbrace{D_u F\big|_{\substack{\bar{x} \\ \bar{u}}}}^{B} \tilde{u}$$ → already linear

$$\frac{DF}{Dx}\Big|_{x^*} = \left[\frac{\partial f_i}{\partial x_j}\right] = \begin{bmatrix} \partial f_1/\partial x_1 \,(=0) & \partial f_1/\partial x_2 \,(=1) \\ \partial f_2/\partial x_1 & \partial f_2/\partial x_2 \end{bmatrix}$$

$$\frac{\partial f_2}{\partial x_1} = \frac{\partial}{\partial x_1}\left(u/ml - bx_2/ml - g\sin(x_1)/l\right) = -g\cos(x_1)/l \,\Big|_{x_1 = q_0}$$

$$\frac{\partial f_2}{\partial x_2} = \frac{\partial}{\partial x_2}\left(u/ml - bx_2/ml - g\sin(x_1)/l\right) = -b/ml$$

$$A(\tau) = \begin{bmatrix} 0 & 1 \\ -g\cos(q_0)/l & -b/ml \end{bmatrix}$$

$$D_u F\,\tilde{u} = \begin{bmatrix} \partial f_1/\partial u \,(=0) \\ \partial f_2/\partial u \end{bmatrix}$$

$$\partial f_2/\partial u = \frac{\partial}{\partial u}\left(u/ml - bx_2/ml - g\sin(x_1)/l\right) = \frac{1}{ml}$$

$$B(\tau) = \begin{bmatrix} 0 \\ \frac{1}{ml} \end{bmatrix}$$