UNSW Business School

School of Banking & Finance

FINS4781
SPECIAL TOPICS IN FINANCE: CONTINUOUS-TIME FINANCE

Course Outline
Semester 2, 2017

Course-Specific Information

The Business School expects that you are familiar with the contents of this course outline. You must also be familiar with the School's Course Outlines Policies webpage which contains key information on:

- Program Learning Goals and Outcomes
- Academic Integrity and Plagiarism
- Student Responsibilities and Conduct
- Special Consideration
- Student Support and Resources

This webpage can be found on the Business School website:
https://www.business.unsw.edu.au/degrees-courses/course-outlines/policies

Table of Contents

COURSE-SPECIFIC INFORMATION

1 STAFF CONTACT DETAILS

Lecturer-in-charge: Dr. David Colwell
Room BUS 367
Phone No: 9385 5851
Email: d.colwell@unsw.edu.au
Consultation Times – Mondays 12-2pm. (or by appointment)

To get to Dr. Colwell's office, please use the East wing elevator (closer to the bookstore), find the phone pad next to the glass door, hit the # key (i.e., "on") and dial my extension: 55851. I'll come open the door for you.

2 COURSE DETAILS

2.1 Teaching Times and Locations

Lectures start in Week 1 (to Week 12): The Time and Location are:
Tuesdays, 14:00 – 17:00, BUS 130.

2.2 Units of Credit

The course is worth 6 units of credit.
This course is taught in parallel to both undergraduate and postgraduate students.

2.3 Summary of Course

The theory of Option Pricing represents one of the success stories of modern Finance. Not only is the theory elegant and tractable mathematically, it has had enormous practical relevance to the development of complex financial instruments whose prices are contingent on underlying traded and non-traded assets. It paved the way for the analysis of financial problems from very different fields. There are many important time series in Finance that need to be modelled. Discrete-time models such as GARCH have been quite popular for describing stock prices, for example. These models give one a good picture of the distribution of stock prices for one period (e.g., daily prices). However, options require arbitrary maturities, and so the disadvantage to using discrete-time models for option pricing is that one often needs to resort to Monte Carlo methods, which are slow to converge in general. So, while GARCH type models are useful for estimating parameters based on time series, continuous-time methods are more effective for pricing options. I would also argue that going from a continuous-time to a discrete time model is much easier than going from a discrete time model to a continuous-time model; finding the continuous-time limit of a discrete-time model can be quite complicated. There have been a number of fiascos related to financial derivatives over the years. Hopefully, more careful mathematical modelling will help academics and practitioners better understand the riskiness of financial assets and their derivatives.

2.4 Course Aims and Relationship to Other Courses

My goal for this course is to explain and in a sense "de-mystify" some of the more sophisticated models in Continuous-Time Finance. This should enable the student to read current research papers at a high level.

In terms of modelling time series, we will discuss models with mean-reversion (such as the Ornstein-Uhlenbeck process and the square root process), stochastic volatility

models (Heston in particular), Lévy processes and other jump processes, and continuous-time regime switching models. These models describe the skewness and kurtosis of many financial time series more realistically than the standard models.

In terms of derivative securities we will discuss equity derivatives, interest rate derivatives (based on the Heath-Jarrow-Morton approach as well as the Libor Market Model approach) and if time allows, credit derivatives. We also discuss portfolio optimization in continuous-time. All other chapters are designed to lead us to these topics.

The earlier chapters in this course will emphasize the following topics: What is a filtration? Why is it useful to consider the conditional expected value of a random variable to be also a random variable? What are Markov Processes, Martingales, and Brownian Motions, and why are they so important to a dynamic Theory of Finance? Why do we need a new integration theory (the Itô integral) to determine properly the gains of dynamic portfolio strategies? Why do the rules of classical calculus no longer hold? Why do stochastic differential equations help us to characterize changes of asset prices over small time intervals? How do changes in probability measures (Girsanov's theorem) make option pricing easier? My approach is to include proofs to these results only if they're relatively easy. This means that, even if the most general results will not be proved, sometimes simpler results can be proved. My hope is that the more a student understands about these results, the easier it will be for him/her to apply them, or even extend them.

This course will not require memorization. No assessment will be closed book. The lecture notes are fairly thorough, and will hopefully be a useful reference after the course is finished.

In the spirit of full disclosure, let me also mention some topics that will not be discussed: (a) discrete-time models (such as binomial models or GARCH), (b) Monte Carlo methods, (c) programming, or (d) estimation. While all of these topics are important, students will have to learn these topics in other courses.

The course is a fourth year (Honours) and a Postgraduate (PhD) course. Participants should have a strong interest in asset pricing. The course FINS3635 (Options, Futures, and Risk Management) is a prerequisite. In addition, students would find it beneficial to have some background in probability theory.

2.5 Student Learning Outcomes

By the end of this course, you should be able to:

1. Understand the principles of stochastic calculus as far as they are needed in finance,
2. Understand the most important classical results from the finance literature that use the continuous-time approach,
3. Have a feeling about when this approach is useful,
4. Know about the advantages and disadvantages of modelling finance problems in continuous time,
5. Be able to read and critique a recently published paper that is related to continuous-time finance, and

6. Be able to transform a properly posed finance problem into a continuous-time model.

The Course Learning Outcomes are what you should be able to DO by the end of this course if you participate fully in learning activities and successfully complete the assessment items.

The Learning Outcomes in this course also help you to achieve some of the overall Program Learning Goals and Outcomes for all undergraduate / postgraduate coursework students in the Business School. Program Learning Goals are what we want you to BE or HAVE by the time you successfully complete your degree (e.g. 'be an effective team player'). You demonstrate this by achieving specific Program Learning Outcomes - what you are able to DO by the end of your degree (e.g. 'participate collaboratively and responsibly in teams').

For more information on Program Learning Goals and Outcomes, see the School's Course Outlines Policies webpage available at https://www.business.unsw.edu.au/degrees-courses/course-outlines/policies
The following table shows how your Course Learning Outcomes relate to the overall Program Learning Goals and Outcomes, and indicates where these are assessed (they may also be developed in tutorials and other activities):

Program Learning Goals and Outcomes		**Course Learning Outcomes**	**Course Assessment Item**
This course helps you to achieve the following learning goals for all Business undergraduate/ postgraduate coursework students:		*On successful completion of the course, you should be able to:*	*This learning outcome will be assessed in the following items:*
1	Knowledge	Use stochastic calculus to solve a variety of financial problems.	• Assignment problems
2	Critical thinking and problem solving	Use the mathematical methods discussed in this course to analyse a research paper and suggest ideas for future research.	• Paper review
3a	Written communication	Construct written work which is logically and professionally presented.	• Paper review
3b	Oral communication	Communicate ideas in a succinct and clear manner.	Not specifically assessed, although class participation is appreciated.
4	Teamwork	Work collaboratively to complete a task.	• Not specifically assessed.
5a.	Ethical, social and environmental responsibility	Gain some understanding of how derivatives played a role in a number of fiascos in finance, as well in the GFC.	• Not specifically assessed.
5b.	Social and cultural awareness	Not specifically addressed in this course.	

3 LEARNING AND TEACHING ACTIVITIES

3.1 Approach to Learning and Teaching in the Course

This course provides the basis to analyse and solve stochastic, dynamic problems in Finance at an advanced level. It is theoretically oriented with an enormous potential for practical application.

The course consists of weekly three-hour lectures. The lecture notes will be available before class, so that students can have an overview of the topics in advance. During the lecture, we discuss the details of some proofs, and omit the details of others, leaving them for the interested student. We discuss the intuition behind results and continually refer to the "big picture" issues, of how each topic relates to other topics. Questions and discussion in class are welcome. Some practice problems will be available for most chapters, and doing these should help students prepare for the take-home assignments. The paper to be reviewed (see below) will be chosen by the student according to the student's interests, but must be related to Continuous-Time Finance.

3.2 Learning Activities and Teaching Strategies

In order to obtain the full benefit from the course, students are expected to follow the following points below:

1. Read the relevant lecture notes before the lectures. This will make it easier for students to follow the lectures and to ask questions.
2. Attend class lectures.
3. Participate in the lectures, asking questions and answering the occasional questions posed by the lecturer.
4. Review the lectures after class.
5. Do the practice problems or take-home assignments when available.
6. Search the literature for a paper that interests you.

If any issues are still not clear, ask me, send me an e-mail, or come to my office during my consultation hours, or by appointment.

4 ASSESSMENT

4.1 Formal Requirements

In order to pass this course, you must:

- achieve a composite mark of at least 50; and
- make a satisfactory attempt at all assessment tasks (see below).

4.2 Assessment Details

Assessment Task	Weighting	Length	Due Date
Take-home assignment	20%		Week 5
Take-home assignment	20%		Week 9

Take-home assignment	20%		Week 13
Paper review/research proposal	40%	5 to 10 pages	University Exam Period
Total	100%		

Details of each assessment task:

The take-home assignments will involve applying the material learned in class to derive extensions of the results in class. For example, we may extend a calculation done in class to the case where the underlying asset exhibits mean-reversion. There will be roughly three or four questions per assignment. Your solutions can be hand-written, if your writing is clear. The assignment will be due at the beginning of class, in the week specified. I should be able to give you at least one week to finish each assignment.

For the paper review, I would like each student first to find a fairly current, published paper that has something to do with continuous-time finance. Find a paper that interests you. Then I would like you to summarize the paper, somewhat briefly. Show me that you understand the paper and understand its contribution. Finally, I would like you to make some suggestions for future research. Your suggestion(s) may involve empirically testing the paper in some way, applying it to a different problem, or extending it in some way. I imagine this paper review/research proposal (or "report") could be of about 5 to 10 typed pages in length. You are not expected to carry out original research, but I would like you to make a proposal that seems attainable. This paper review will be due, say, three days before the end of the University Exam Period. You may discuss your choice of paper with me before you start writing.

4.3 Assignment Submission Procedure

Ordinarily assessment items are to be submitted electronically, but for a small class such as this, I will allow you to hand me hard copies of the assignments in class. For the paper review, please email me a copy of your review together with a copy of the paper that you are reviewing.

4.4 Late Submission and Penalties

Assessments submitted late will have marks deducted (10% of the total marks per day).

Quality Assurance
The Business School is actively monitoring student learning and quality of the student experience in all its programs. A random selection of completed assessment tasks may be used for quality assurance, such as to determine the extent to which program learning goals are being achieved. The information is required for accreditation purposes, and aggregated findings will be used to inform changes aimed at improving the quality of Business School programs. All material used for such processes will be treated as confidential.

5 COURSE RESOURCES

The website for this course is on Moodle at:
http://moodle.telt.unsw.edu.au

The only materials that are required for this course are the lecture notes, which will be made available on Moodle in a timely manner.

The lecture notes are influenced by a wide variety of sources. The following is a list of some of those sources, and could be considered extra reading if the student needs more detail on a particular subject.

Bates, D.S., 1996, "Jumps and Stochastic Volatility: Exchange Rate Processes Implicit in Deutsche Mark Options," *Review of Financial Studies*, (Spring) Vol. 9, No. 1, 69-107.

Baxter, M. and Rennie, A., 1996, *Financial Calculus: An Introduction to Derivative Pricing*, Cambridge University Press.

Bielecki, T.R. and Rutkowski, M., 2002, *Credit Risk: Modeling, Valuation and Hedging,* Springer Finance.

Black, F. and Cox, J.C., 1976, "Valuing Corporate Securities: Some Effects of Bond Indenture Provisions," Journal of Finance, 31 (May), 351-368.

Black, F., and Scholes, M., 1973, "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy*, 81 (May-June), 637-654.

Brennan, M. and Schwartz, E., 1985, "Evaluating Natural Resource Investments," *Journal of Business*, 58, 135-157.

Brigo, D. and Mercurio, F. 2006, *Interest Rate Models—Theory and Practice: With Smile, Inflation and Credit*, 2nd Ed., Springer Finance.

Buchen, P.W., 2001, "Image Options and the Road to Barriers," *Risk Magazine*, 14 (9), 127-130.

Buchen, P.W., 2012, *An Introduction to Exotic Option Pricing*, Chapman & Hall.

Caouette, Altman, and Narayanan, 1998, *Managing Credit Risk: The Next Great Financial Challenge*, Wiley.

Chen, R. R., Cheng, X., Fabozzi, F. J., & Liu, B. (2008). An explicit, multi-factor credit default swap pricing model with correlated factors. *Journal of Financial and Quantitative Analysis*, *43*(1), 123.

Colwell, D.B., Feldman, D. and Hu, W., 2015, "Non-Transferable Non-Hedgeable Executive Stock Option Pricing," *Journal of Economic Dynamics and Control*, Vol. 53, April, pp. 161-191.

Colwell, D.B., Henker, T., Fong, K., and Ho, J., 2003, "Real Options Valuation of Australian Gold Mines and Mining Companies." *The Journal of Alternative Investments*, 23-38.

Chung, K.L., 1974, *A Course in Probability Theory*, 2nd Ed., Academic Press.

Cochrane, J.H., 2001. *Asset Pricing*, Princeton University Press.

Cont, R. and Tankov, P., 2008, *Financial Modelling with Jump Processes*, 2nd Ed., Chapman and Hall.

Cox, J. C., & Huang, C. F., 1989, "Optimal consumption and portfolio policies when asset prices follow a diffusion process," *Journal of economic theory*, *49*(1), 33-83.

Cox, J.C., Ingersoll, J.E., and Ross, S.A., 1985, "A Theory of the Term Structure of Interest Rates," *Econometrica*, 53, 385-407.

Cvitanić, J., and Karatzas, I., 1992, "Convex Duality in Constrained Portfolio Optimization," *Annals of Applied Probability*, 2(4), 767-818.

Duffie, D., Pan J., and Singleton, K., 2000, "Transform Analysis and Asset Pricing for Affine Jump-Diffusions," *Econometrica*, 1343-1376.

Elliott, R.J., 1982, *Stochastic Calculus and Applications*, Springer.

Elliott, R.J., Aggoun, L., and Moore, J.B., 1995, *Hidden Markov Models: Estimation and Control*, Springer.

Harrison, J.M., and Pliska, S., 1981, "Martingales and Stochastic Integrals in the Theory of Continuous Trading," *Stochastic Processes and their Applications*, 11, 215-260.

Heath, D., Jarrow, R.A. and Morton, A., 1992, "Bond Pricing and the Term Structure of Interest Rates: A New Methodology for Contingent Claims Valuation," *Econometrica*, 60, 77-105.

Heston, S.L., 1993, "A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options", *Review of Financial Studies*, 6, 327-343.

Ho, T.S.Y., and Lee, S.-B., 1986, "Term Structure Movements and the Pricing of Interest Rate Contingent Claims," *The Journal of Finance*, 41, 1011-1029.

Hull, J.C., 2012, *Options, Futures, and Other Derivatives*, 8th Ed., Pearson.

Hull, J. and White, A., 1990, "Pricing Interest Rate Derivative Securities," *The Review of Financial Studies*, 3, 573-592.

Jacod, J. and Shiryaev, A.N., 1987, *Limit Theorems for Stochastic Processes*, Springer.

Karatzas, I., Lehoczky, J.P., Shreve, S.E., 1987. Optimal portfolio and consumption decisions for a "small investor" on a finite horizon. SIAM Journal on Control and Optimization 25 (6) 1557-1586. Doi: 10.1137/0325086

Karatzas, I. and Shreve, S.E., (I) 1991, *Brownian Motion and Stochastic Calculus*, 2nd Ed., Springer.

Karatzas, I. and Shreve, S.E., (II) 1998, *Methods of Mathematical Finance*, Springer.

Konstandatos, O., 2008, *Pricing Path Dependent Exotic Options: A Comprehensive Mathematical Framework*, VDM Verlag Dr. Mueller e.K.

Leland, H.E., 1994, "Corporate Debt Value, Bond Covenants, and Optimal Capital Structure," *Journal of Finance*, 49, No. 4, (Sept), 1213-1252.

Liptser, R.S. and Shiryaev, A.N 1977, *Statistics of Random Processes I: General Theory*, Springer-Verlag.

Merton, R. C., 1973, "An intertemporal capital asset pricing model." *Econometrica: Journal of the Econometric Society*, 867-887.

Merton, R.C., 1974, "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates," *Journal of Finance*, 29, 449-470.

Merton, R.C., 1990, *Continuous-Time Finance*, Blackwell.

Musiela, M. and Rutkowski, M., 1997, *Martingale Methods in Financial Modelling*, Springer.

Protter, P.E., 2005, *Stochastic Integration and Differential Equations*, Springer.

Revuz, D., and Yor, M., 1999, *Continuous Martingales and Brownian Motion*, 3rd Ed., Springer.

Rogers, L.C.G. and Williams D., 1994, *Diffusions, Markov Processes and Martingales, Vol. 2: Ito Calculus*, Cambridge University Press.)

Ross, S.M., 1980, *Introduction to Probability Models*, 2nd Ed., Academic Press.

Royden, H.L., 1968, *Real Analysis*, 2nd Ed., MacMillan.

Schönbucher, 2003, *Credit Derivatives Pricing Models: Models, Pricing and Implementation*, Wiley Finance.

Schwartz, E., and Smith, J. E., 2000, "Short-term variations and long-term dynamics in Commodity Prices," *Management Science*, 46(7), 893-911.

Shreve, S.E., 2004, *Stochastic Calculus for Finance II*, Springer.

Stein, E.M., and Stein, J.C., 1991, "Stock Price Distributions with Stochastic Volatility: An Analytic Approach," *Review of Financial Studies*, Vol. 4, No. 4, 727-752.

Vasicek, O.A., 1977, "An Equilibrium Characterization of the Term Structure," *Journal of Finance*, 5, 177-188.

Wong, B., and Heyde, C.C., 2006, "On Changes of Measure in Stochastic Volatility Models," *Journal of Applied Mathematics and Stochastic Analysis*, 1-13.

6 COURSE EVALUATION AND DEVELOPMENT

Each year feedback is sought from students and other stakeholders about the courses offered in the School and continual improvements are made based on this feedback. UNSW's myExperience survey is one of the ways in which student evaluative feedback

is gathered. In this course, we will seek your feedback through, end of semester myExperience responses. Feedback from previous students indicated that I sometimes speak too quickly. As a result of this feedback, the lecturer is careful about speaking slowly and clearly.

7 COURSE SCHEDULE

NOTE: The timing of these lectures is only approximate. If I do not finish a topic one week, I will continue with that chapter the following week. Also, if I finish a topic early, I will go right to the next topic. Usually, you will need to bring two sets of lecture notes to a given lecture. Each week I'll try to let you know what you'll need to bring the following week.

I am also flexible about what topics we cover. Please let me know if there are some topics you particularly want to cover, or if there are some topics related to continuous-time finance that you are interested in but are not on this list. Suggestions are welcome.

COURSE SCHEDULE			
Week	Topic	References	Other activities / assessment
Week 1 24 July	Ch. 1 Measure Theory	Royden, Chung	
Week 2 31 July	Ch. 2 Stochastic Processes	Elliott, Jacod and Shiryaev, Karatzas and Shreve (I) Revuz and Yor	
Week 3 7 August	Ch. 3 Stochastic Integration	As above	
Week 4 14 August	Ch. 4 Girsanov's Theorem	As above	First assignment due
Week 5 21 August	Ch. 5 Black-Scholes	Hull, Harrison, M., and Pliska	
Week 6 28 August	Ch. 6 Applications of the Black-Scholes partial differential equation	Merton, Leland, Barrier options, Black and Cox, Brennan and Schwartz	
Week 7 4 September	Ch. 7 Portfolio Optimization	Karatzas and Shreve (II), Merton.	
Week 8 11 September	Ch. 8 Mean-reversion	Vasicek, Cox-Ingersoll-Ross	
Week 9 18 September	Ch. 9 Stochastic Volatility	Heston	Second assignment due
Mid-semester break: 23 September – 2 October inclusive (2 Oct = Labour Day Public Holiday)			

Week 10 3 October	Ch. 10 Heath-Jarrow-Morton	Baxter and Rennie	
Week 11 9 October	Ch. 12 Lévy Processes	Jacod and Shiryaev, Cont and Tankov	
Week 12 16 October	Ch. 13 Regime-Switching or Ch. 14 Credit Default Swaps	Elliott, Aggoun and Moore or Brigo and Mercurio	
Week 13 23 October			Third assignment due (Paper review is due during the examination period.)