

**Problem:** Person A has given loan of amount  $L$  to person B without record. After long time  $T$ , both person A and B cannot recall whether the loan has been paid back or not. There was no interest applicable. Describe the method to resolve this situation, assuming that neither A nor B wants to cheat other, nor wants to get cheated.

**Solution:** Both A and B are asked the probability  $P$  that the debt has been repaid. This question is asked by a neutral party in private. Alternatively, both A and B write this on slips in private without knowing each other's probability value. Let, the loan was taken at  $t=0$ . Then,

	Money with	
	A	B
At $t < 0$	$L$	$0$
At $t = 0$	$0$	$L$
At $t = T$	$?$	$?$

As per the recalled probabilities, person A will think that B has repaid the debt with probability  $P_a$ . Similarly, B recalls that the debt has been repaid with probability  $P_b$ . So,

As per A and B, the situation at  $t=T$  is :-

	Money with	
	A	B
At $t=T$ , as per A	$P_a * L$	$L * (1 - P_a)$
At $t=T$ , as per B	$P_b * L$	$L * (1 - P_b)$

Since, it is assumed that both A and B have no intent to cheat, so no one is given favour in determining the correct probability. Both are believed equally. This is *Equal Faith model*.

	Money with	
	A	B
At $t=T$ (probably)	$(P_a + P_b) / 2 * L$	$L * (1 - (P_a + P_b) / 2)$

So, A already has got amount  $(P_a + P_b) / 2 * L$ . The rest is payable by B.

Transaction =  $L * (1 - (P_a + P_b) / 2)$  from B to A.

1 **Example**,  $L=100$ ,  $P_a=0.5$ ,  $P_b=0.5$ , then,

A	B
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At t=T (money with)	50	50
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Amount payable by B to A =	50
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2 **Example**,  $L=100$ ,  $P_a=0.4$ ,  $P_b=0.6$ , then,

	A	B
At t=T (money with)	50	50

Amount payable by B to A =	50
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3 **Example**,  $L=100$ ,  $P_a=0.75$ ,  $P_b=0.75$ , then,

	A	B
At t=T (money with)	$(0.75+0.75)/2 \cdot 100$	
		$100 \cdot (1-(0.75+0.75)/2)$

Amount payable by B to A =	$100 \cdot (1-(0.75+0.75)/2)$
=	25

4 **Example**,  $L=100$ ,  $P_a=0.25$ ,  $P_b=0.25$ , then,

	A	B
At t=T (money with)	$(0.25+0.25)/2 \cdot 100$	
		$100 \cdot (1-(0.25+0.25)/2)$

Amount payable by B to A =	$100 \cdot (1-(0.25+0.25)/2)$
=	75

5 **Example**,  $L=100$ ,  $P_a=0$   $P_b=1$ , then,

	A	B
At t=T (money with)	$(0+1)/2 \cdot 100$	
		$100 \cdot (1-(0+1)/2)$

Amount payable by B to A =	$100 \cdot (1-(0+1)/2)$
=	50

6 **Example**,  $L=100$ ,  $P_a=1$   $P_b=0$ , then, *(Such a B exists?)*

	A	B
At t=T (money with)	$(1+0)/2 \cdot 100$	
		$100 \cdot (1-(1+0)/2)$

Amount payable by B to A =	$100 \cdot (1-(1+0)/2)$
=	50

7 Example,  $L=100$ ,  $P_a=1$   $P_b=1$ , then,

	A	B
At $t=T$ (money with)	$(1+1)/2 * 100$	
		$100 * (1 - (1+1)/2)$

Amount payable by B to A =	$100 * (1 - (1+1)/2)$
=	0

### Remarks

It is observed that the Equal Faith Model shown above, where both A and B are trusted equally, the transaction is always  $> 0$  (see examples 1 to 6). Only when both A and B are totally sure that the debt was paid back during T, does the transaction amount comes out to be 0. (see example 7).

The Equal Faith Model suggests that the onus to keep the debt repay transaction record is more with B.

