Interpreting Cp and Cpk

About this text…

Focus on explanation by pictures, to develop an insight

Extensive, but NOT a formal and comprehensive information

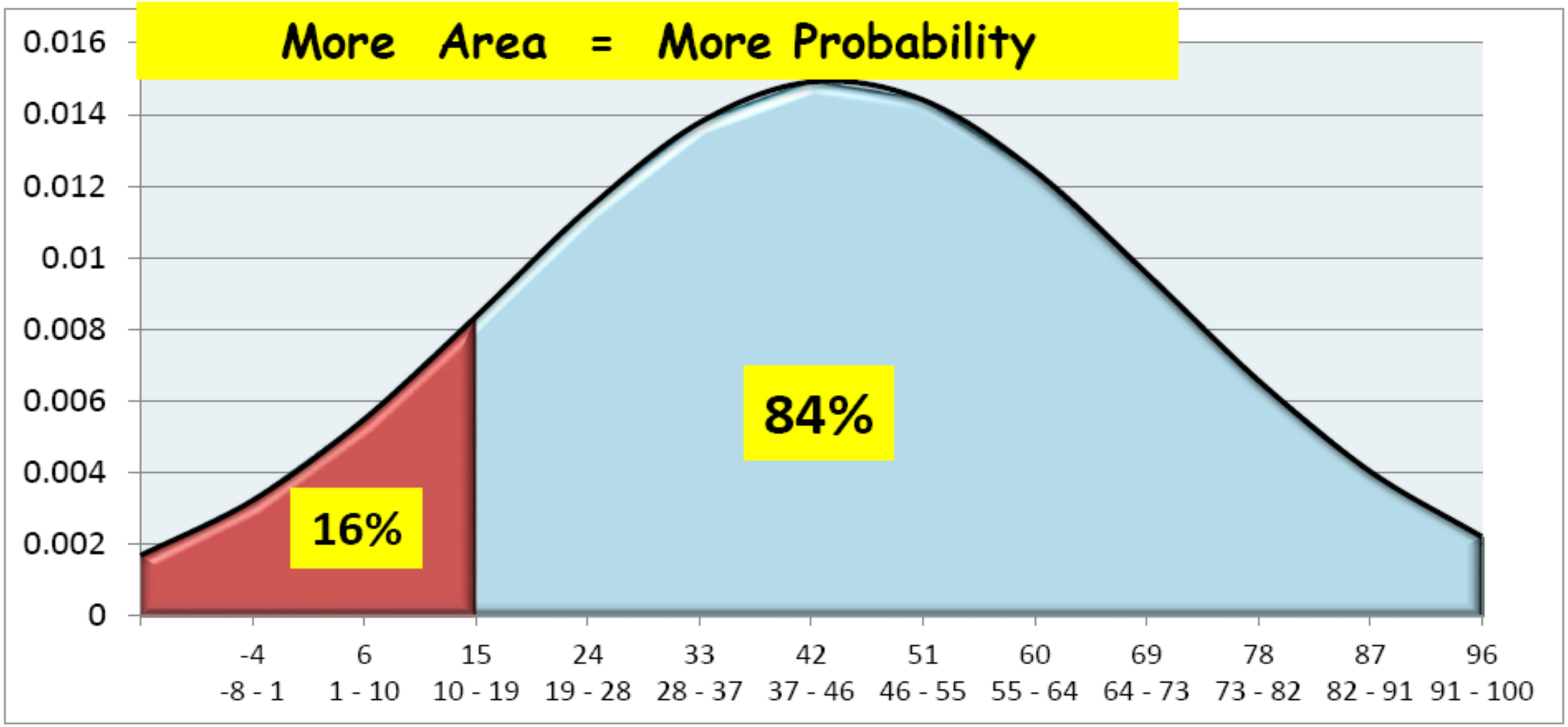
To be studied along with other (formal) text / literature

Assumes some previous knowledge of terms

Assumes some previous knowledge of Normal Distribution curve

Interpreting Cp and Cpk :

Interpretation of area under Normal distribution curve :-

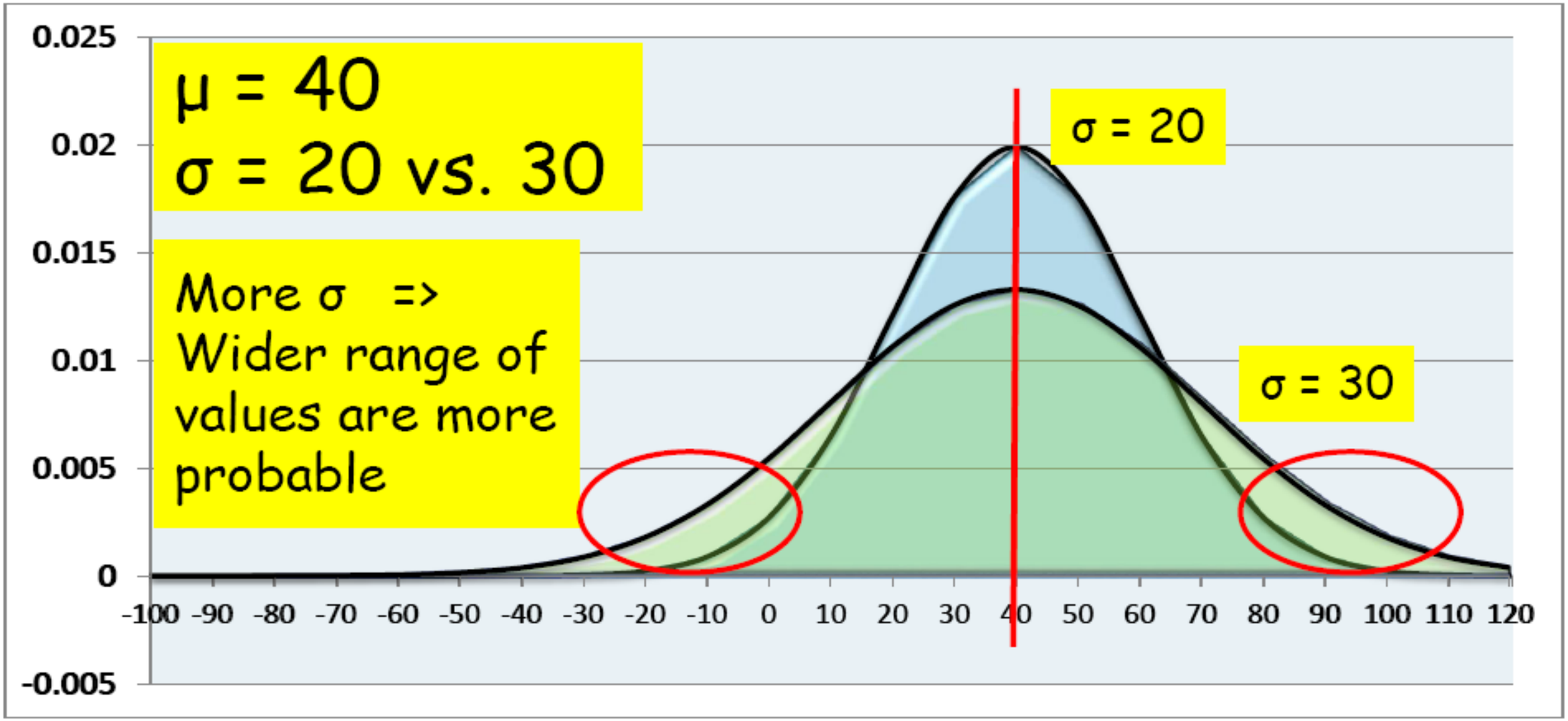


Probability of observations being < 15 = 16%

Probability of observations being > 15 = 84%

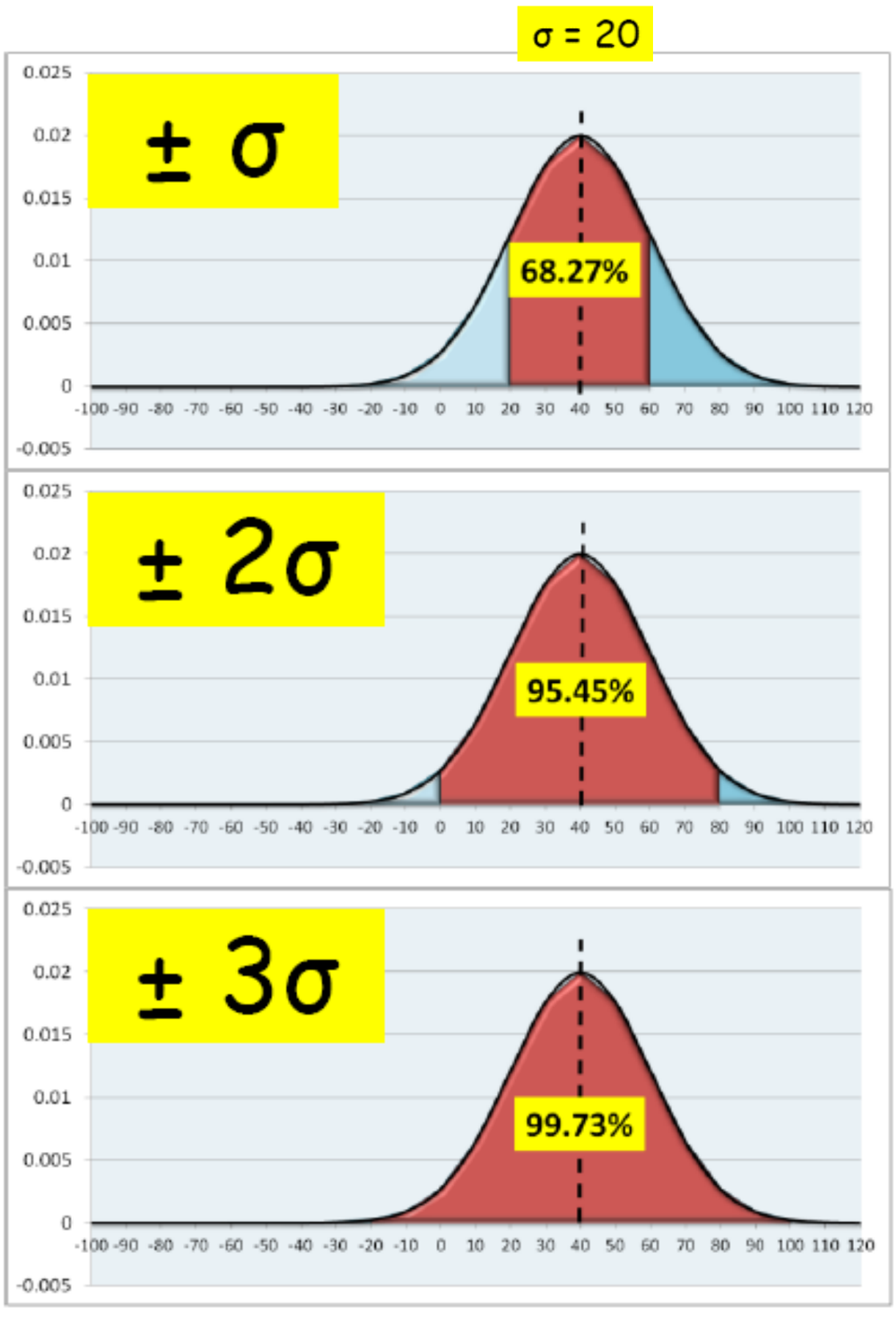
Interpreting Cp and Cpk :

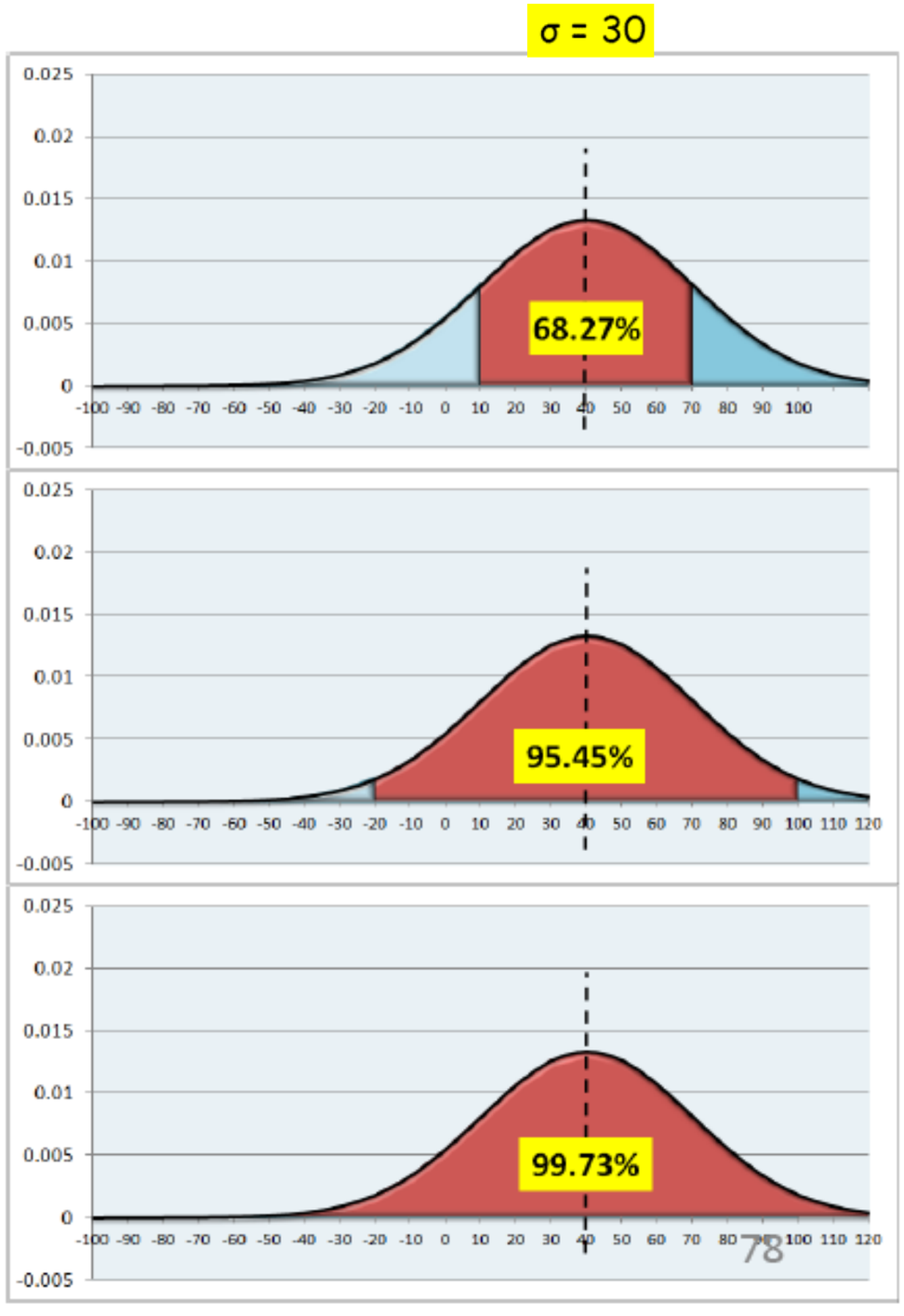
Interpretation of Standard Deviation



Interpreting Cp and Cpk :

In general, area b/w multiples of std. dev. about mean remains same irrespective of the value of mean or std. Dev.

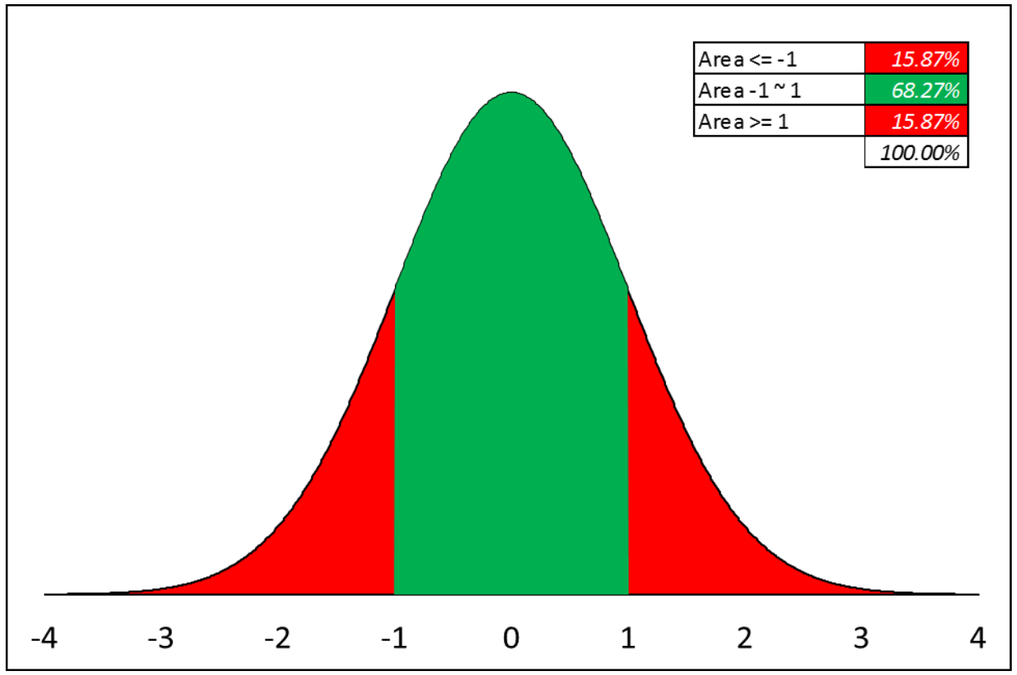




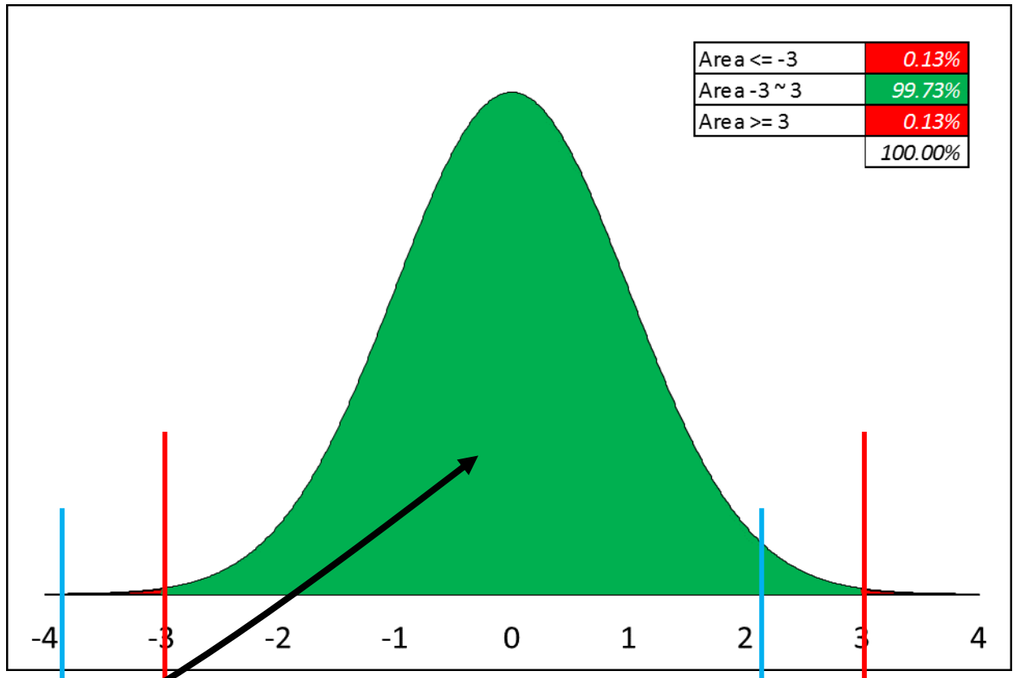
Interpreting Cp and Cpk :

In general, area b/w multiples of std. dev. about mean remains same irrespective of the value of mean or std. Dev.

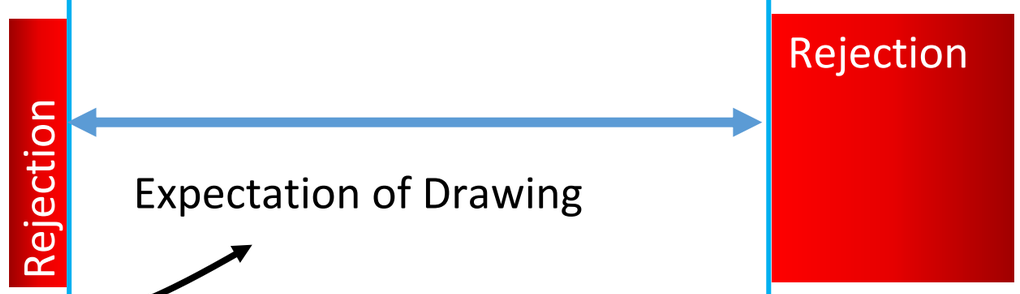
Example : Area between +/-1 sigma is = 69.27%



Example : Area between +/-3 sigma is = 99.73%



Study of Cp and Cpk aims to know what is the size and position of this 99.73% zone w.r.t. the zone defined by drawing (USL-LSL)



$$Cp = \frac{\text{USL} - \text{LSL}}{6\sigma} \quad \text{or} \quad \frac{2T}{6\sigma}.$$

$$Cpk = \text{lesser of } \frac{\text{USL} - \overline{\overline{X}}}{3\sigma} \quad \text{or} \quad \frac{\overline{\overline{X}} - \text{LSL}}{3\sigma}$$

Interpreting Cp and Cpk :

$$Cp = \frac{\text{USL} - \text{LSL}}{6\sigma} \quad \text{or} \quad \frac{2T}{6\sigma}.$$

Numerator = Drawing Band
Denominator = 99.73% Parts = say *'spread'*

=> Cp = How much is the drawing band
bigger than the *'spread' ?*

As drawing band is usually fixed,
Cp = Indicator of Size of the 'spread'
Bigger the Cp, => smaller is Spread wrt tolerance
The Better it is.

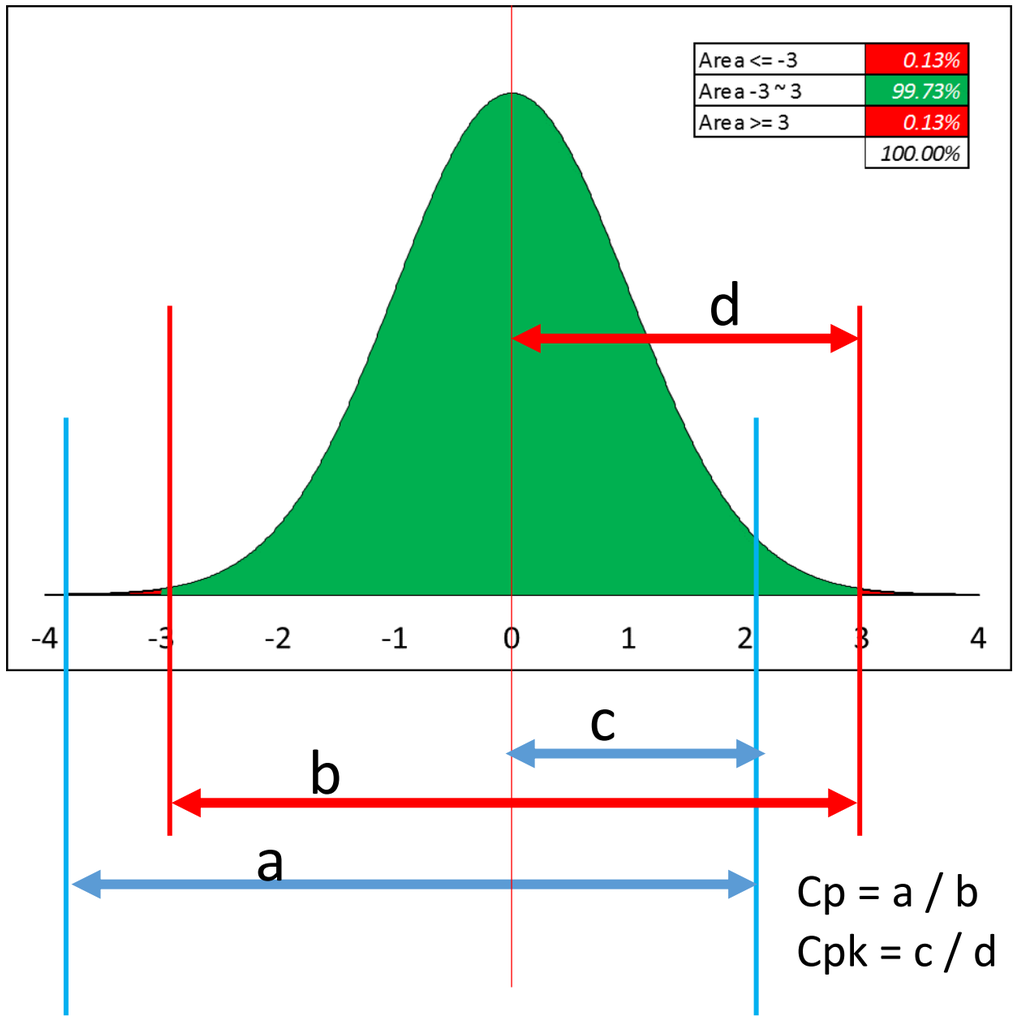
$$Cpk = \text{lesser of } \frac{\text{USL} - \overline{\overline{X}}}{3\sigma} \quad \text{or} \quad \frac{\overline{\overline{X}} - \text{LSL}}{3\sigma}$$

Numerator = Nearest distance from actual mean
where a drawing band. edge is located.
Denominator = 99.73% / 2 = half the 'spread'

=> Cpk = sitting on the actual mean,
distance of tolerance band edge upon
distance of the 'spread' edge

Yardstick for Cp = 6 sigma band about mean :-

Yardstick : Area between +/-3 sigma is = 99.73%



Cp = a / b
Cpk = c / d

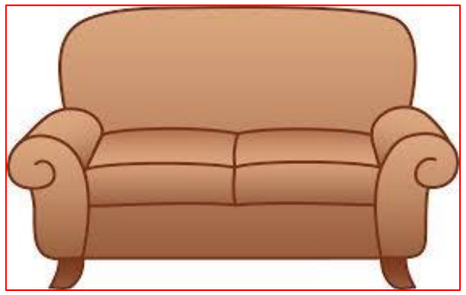
=> Cpk = Indicator of Position of the 'spread'
Bigger Cpk, => Nearer is the spread edge
wrt tol. Edge = > The Better it is.

Interpreting Cp and Cpk :

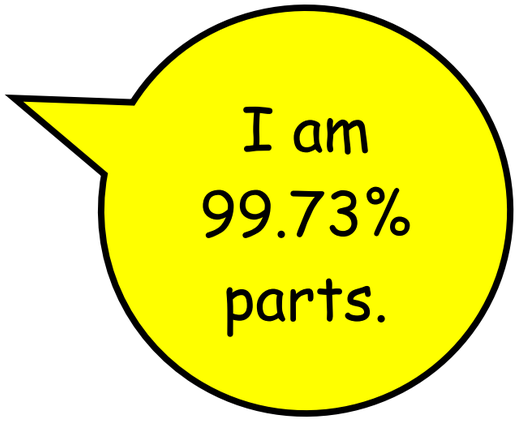
To understand further,

Let us introduce 2 characters :-

Sofa represents allowed/design range



Elephant represents the 99.73% of the parts

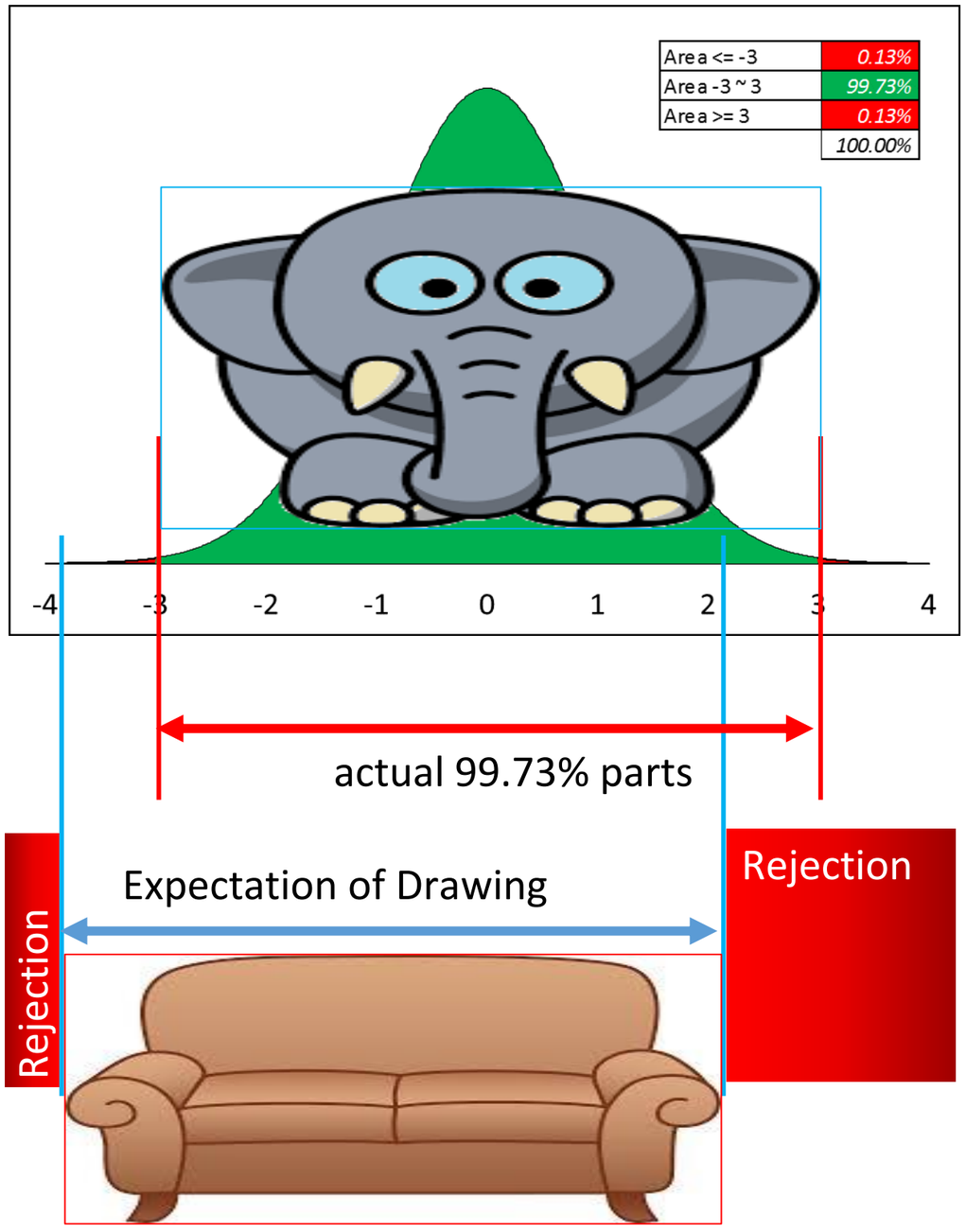


Then the study becomes :-

How well the Elephant sits on the Sofa ?

Yardstick for Cp = 6 sigma band about mean :-

Yardstick : Area between +/-3 sigma is = 99.73%



Interpreting Cp and Cpk :

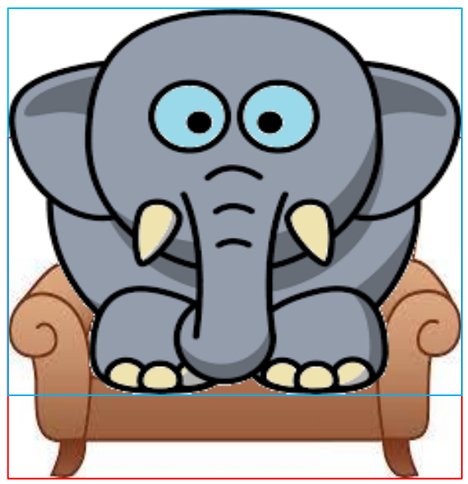
$$Cp = \frac{\text{USL} - \text{LSL}}{6\sigma} \quad \text{or} \quad \frac{2T}{6\sigma}.$$

Numerator = Drawing Band
Denominator = 99.73% Parts = say *'spread'*

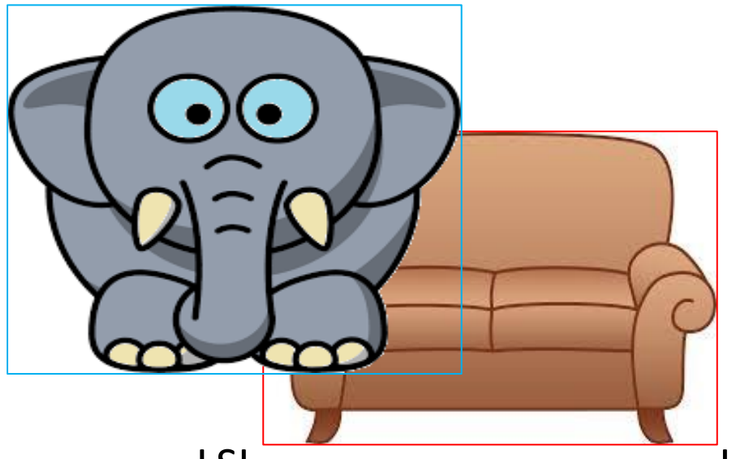
Cp = 1 means …

Numerator = Denominator

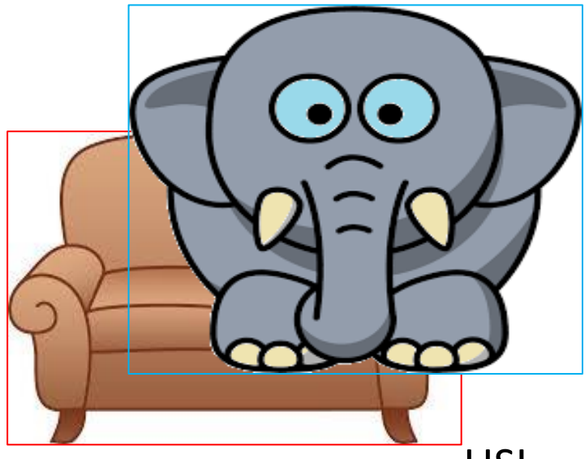
OR in other words…
99.73% parts band = Dwg. Band
in magnitude (width)



Cp = 1 does not necessarily mean …
that 99.73% parts are within dwg. Band :-



Width of sofa and width of elephant is same
99.73% parts band = Dwg. Band
=> Cp = 1
But some part of elephant (parts) is outside sofa (dwg.)



Width of sofa and width of elephant is same
99.73% parts band = Dwg. Band
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Interpreting Cp and Cpk :

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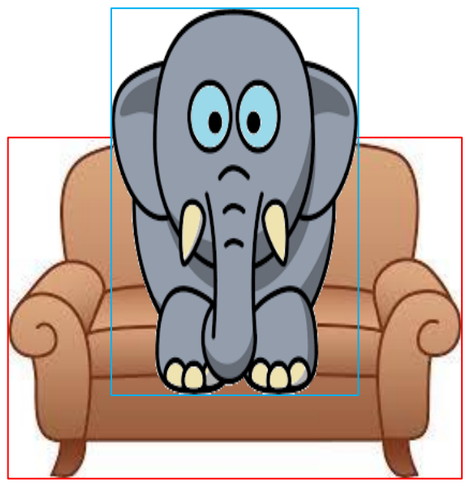
Numerator = Drawing Band
Denominator = 99.73% Parts = say *'spread'*

Cp > 1 means …

Numerator > Denominator

OR in other words…
99.73% parts band < Dwg. Band
in magnitude (width)



Cp > 1 does not necessarily mean …
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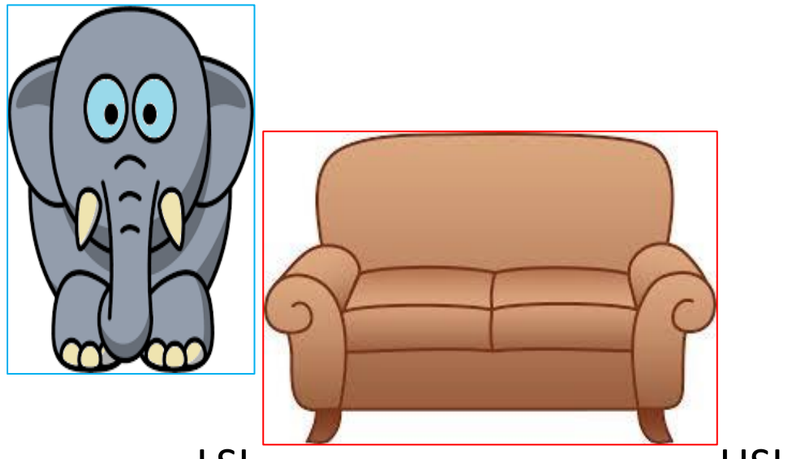
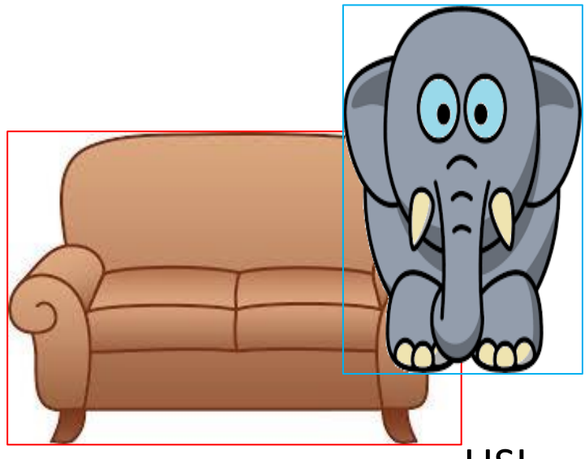


Width of sofa > width of elephant
99.73% parts band < Dwg. Band
=> Cp > 1
But whole of elephant (parts) is outside sofa (dwg.)



Width of sofa > width of elephant
99.73% parts band < Dwg. Band
=> Cp > 1
But some part of elephant (parts) is outside sofa (dwg.)

Interpreting Cp and Cpk :

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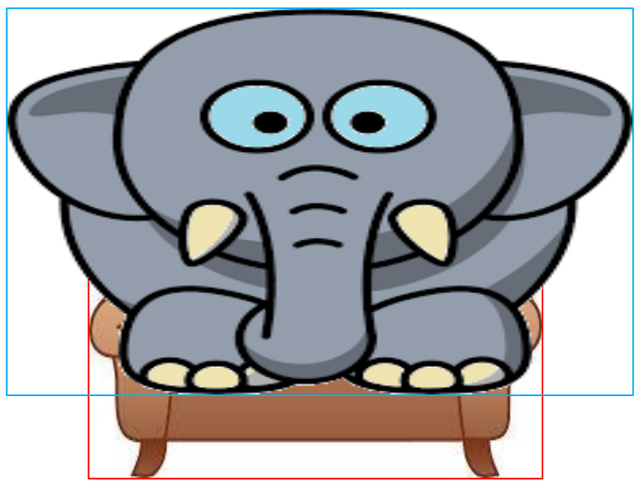
Numerator = Drawing Band
Denominator = 99.73% Parts = say *'spread'*

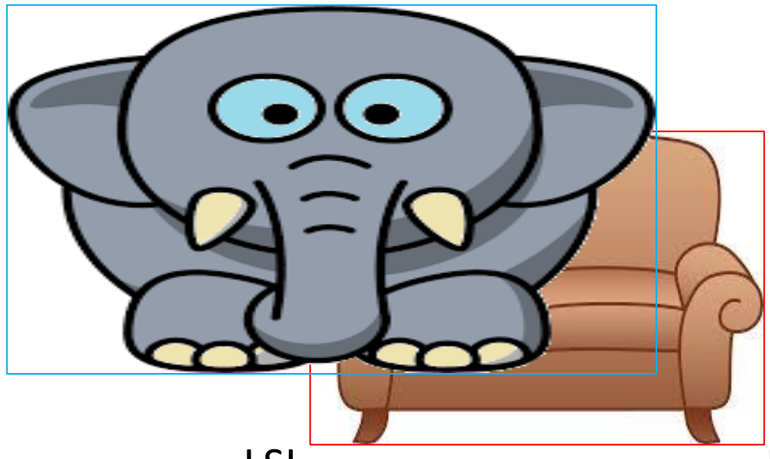
Cp < 1 means …

Numerator < Denominator

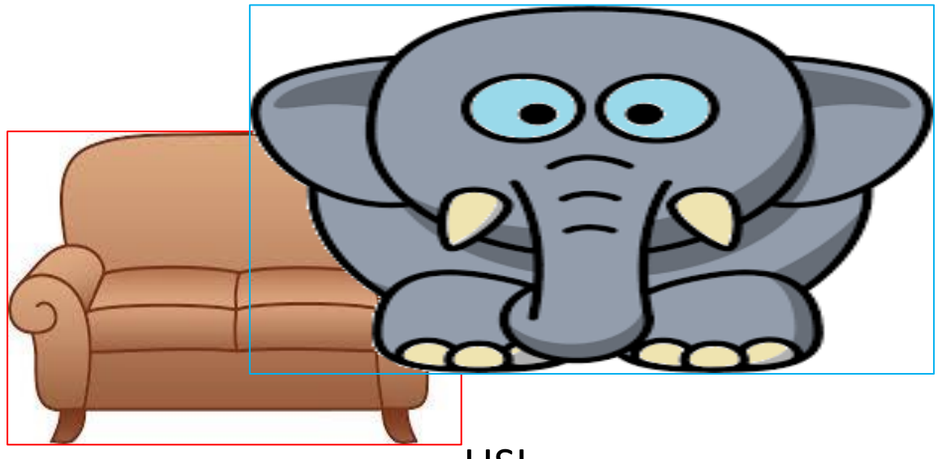
OR in other words…
99.73% parts band > Dwg. Band
in magnitude (width)



Cp < 1 does however necessarily mean …
that some of 99.73% parts are outside dwg. Band :-



Width of sofa < width of elephant
99.73% parts band > Dwg. Band
=> Cp < 1
Necessarily some part of elephant (parts) is outside sofa (dwg.)



Width of sofa < width of elephant
99.73% parts band > Dwg. Band
=> Cp < 1
Necessarily some part of elephant (parts) is outside sofa (dwg.)

Interpreting Cp and Cpk :

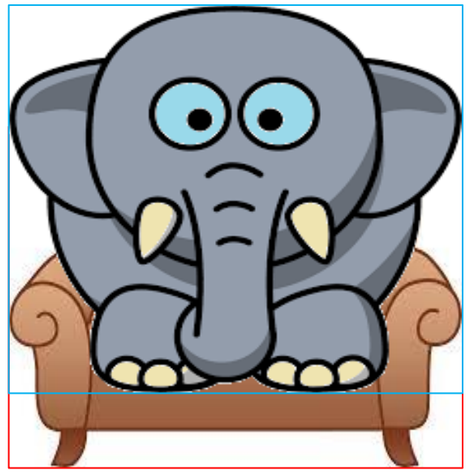
$$Cpk = \text{lesser of } \frac{\text{USL} - \overline{\overline{X}}}{3\sigma} \quad \text{or} \quad \frac{\overline{\overline{X}} - \text{LSL}}{3\sigma}$$

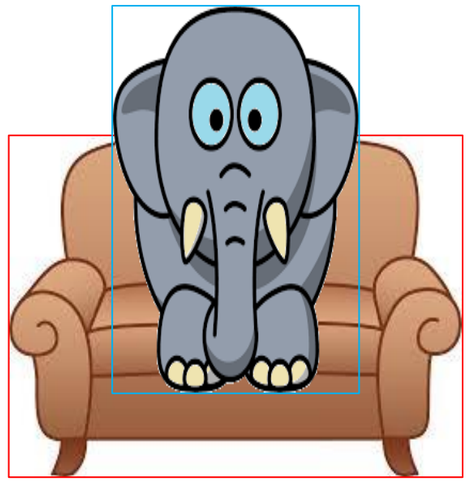
Numerator = Nearest distance from actual mean
where a drawing band. edge is located.
Denominator = 99.73% / 2 = half the 'spread'

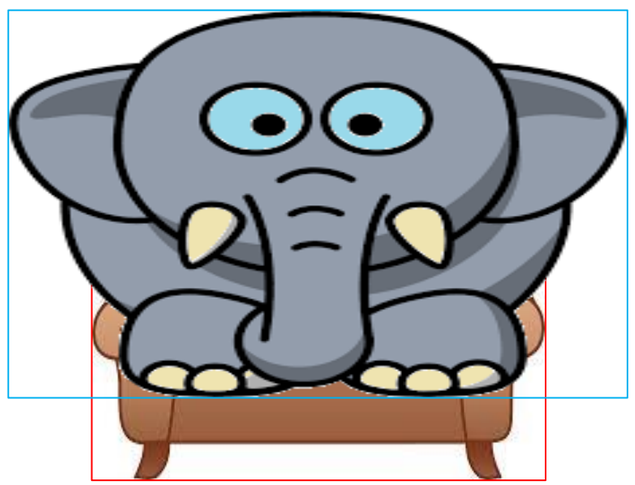
=> Cpk = sitting on the actual mean,
distance of tolerance band edge upon
distance of the 'spread' edge

Cp = Cpk means…

… Parts spread symmetric about Dwg. Mean







Max. possible +ve value of Cpk is Cp

"Criteria for Centering of process is : Cp = Cpk"

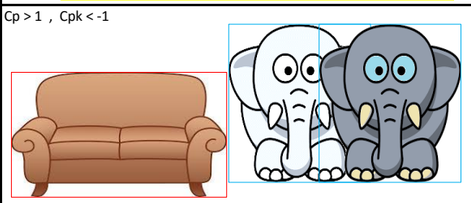
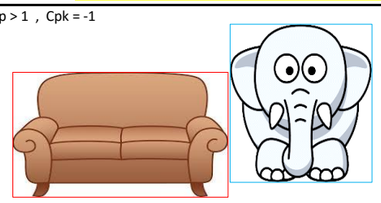
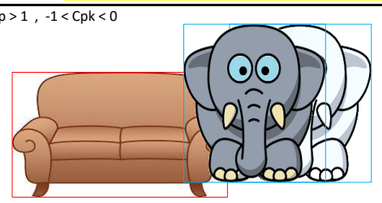
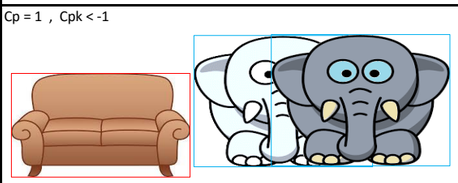
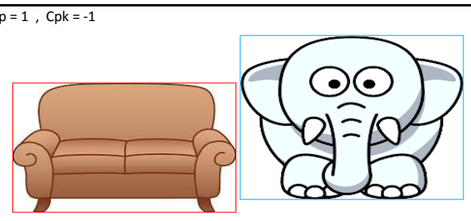
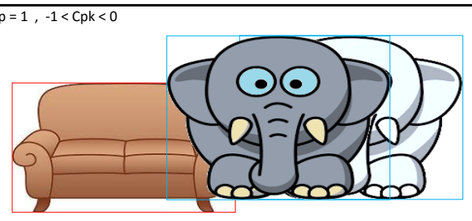
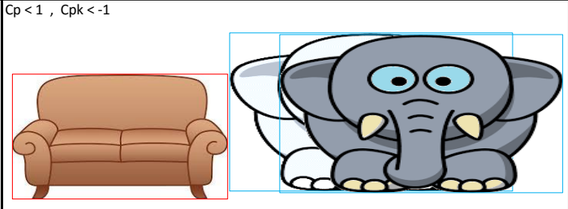
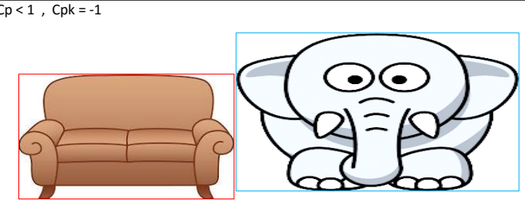
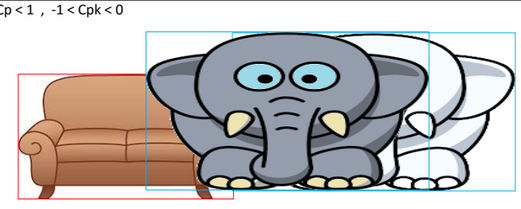
What does Cpk < 0 , Cpk = 0 , Cpk = -1 , Cpk = 1 etc. mean…?

Refer next pages and observe carefully…

Interpreting Cp and Cpk :

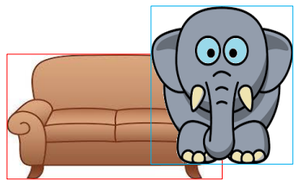
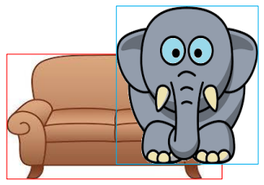
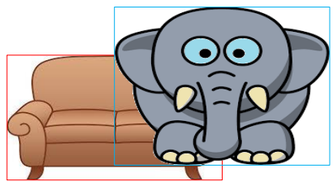
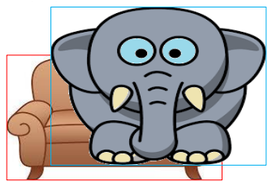
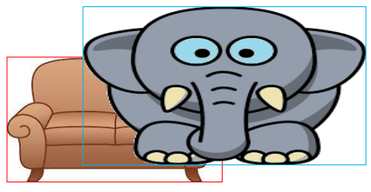
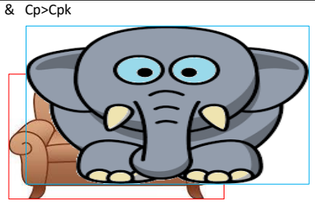
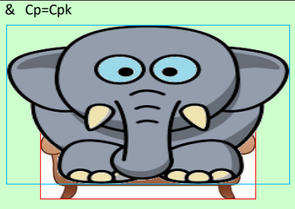
Sofa : design range
Elephant : 99.73% parts

Meaning of Cpk < 0

		Cpk < -1	Cpk = -1	-1 < Cpk < 0
	Cp > 1			
	Cp = 1	Elephant is more than *just* outside Sofa, its edge is away from Sofa edge	Whole of the Elephant is *just* outside Sofa, its edge same as Sofa edge	Less than half of Elephant is on the Sofa, and most part of Elephant is outside Sofa
	Cp < 1	100% spread is way beyond dwg. band	100% spread is just beyond dwg. band	< 50% spread OK
Elephant smaller than Sofa	Cp > 1			
Elephant size = Sofa	Cp = 1			
Elephant bigger than Sofa	Cp < 1			
		Cpk < -1	Cpk = -1	-1 < Cpk < 0

Interpreting Cp and Cpk :

Sofa : design range
Elephant : 99.73% parts

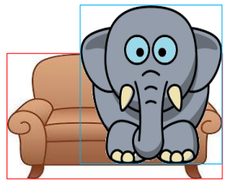
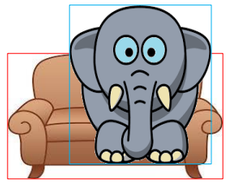
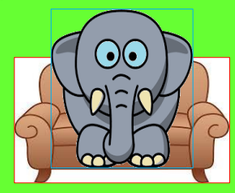
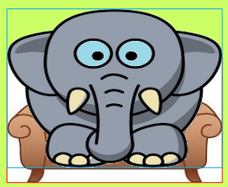
		Meaning of 0 <= Cpk < 1		
		Cpk = 0	**0 < Cpk < 1 & Cp>Cpk**	**0 < Cpk < 1 & Cp=Cpk**
	Cp > 1			- - -
	Cp = 1	**Exactly half of the Elephant is on the Sofa, rest half is outside Sofa**	**More than half of Elephant is on the Sofa, and some (non zero and <half) part of Elephant is outside Sofa**	**Still there are rejections at both ends of tol. band**
	Cp < 1	**50% spread OK**	**> 50% spread OK < 50% spread Rejected**	**Elephant at Sofa center (as Cp = Cpk)**
Elephant smaller than Sofa	**Cp > 1**	Cp > 1 , Cpk = 0 	Cp > 1 , 0 < Cpk < 1 & Cp>Cpk 	Cp > 1 , 0 < Cpk < 1 & Cp=Cpk
Elephant size = Sofa	**Cp = 1**	Cp = 1 , Cpk = 0 	Cp = 1 , 0 < Cpk < 1 & Cp>Cpk 	Cp = 1 , 0 < Cpk < 1 & Cp=Cpk
Elephant bigger than Sofa	**Cp < 1**	Cp < 1 , Cpk = 0 	Cp < 1 , 0 < Cpk < 1 & Cp>Cpk 	Cp < 1 , 0 < Cpk < 1 & Cp=Cpk 
		Cpk = 0	**0 < Cpk < 1 & Cp>Cpk**	**0 < Cpk < 1 & Cp=Cpk**

Made by : amit bhola, amit_aromatic@yahoo.com, Feb'18

Interpreting Cp and Cpk :

Sofa : design range
Elephant : 99.73% parts

Meaning of Cpk >= 1

		Cpk = 1	Cpk > 1 & Cp > Cpk	Cpk > 1 & Cp = Cpk
	Cp > 1	**Elephant fully on the Sofa, and at the Edge of Sofa**	**Elephant fully on the Sofa, but offset from center**	**Elephant at Sofa center (as Cp = Cpk)**
	Cp = 1	**" ditto "** *(NOTE : Since Elephant size = Sofa, so it also means that Elephant at Sofa center as Cp = Cpk)*	- - -	- - -
	Cp < 1	**100% spread is just within tol. band**	**100% spread is within tol. band, some safety margin**	**100% spread is within tol. band, Best safety margin**
Elephant smaller than Sofa	**Cp > 1**	Cp > 1 , Cpk = 1 	Cp > 1 , Cpk > 1 & Cp > Cpk 	Cp > 1 , Cpk > 1 & Cp = Cpk 
Elephant size = Sofa	**Cp = 1**	Cp = 1 , Cpk = 1 	Cp = 1 , Cpk > 1 & Cp > Cpk	Cp = 1 , Cpk > 1 & Cp = Cpk
Elephant bigger than Sofa	**Cp < 1**	Cp < 1 , Cpk = 1	Cp < 1 , Cpk > 1 & Cp > Cpk	Cp < 1 , Cpk > 1 & Cp = Cpk
		Cpk = 1	**Cpk > 1 & Cp > Cpk**	**Cpk > 1 & Cp = Cpk**