Elimination of variables from Linear equations

1. When we say eliminate so and so variables from so and so equations then it means that after elimination, the equation obtained is satisfied by left over variables for all values of eliminated variables.
   e.g. Eliminating f from x = r cos f and y = r sin f gives $x^2+y^2=r$ which is true in x and y for all values of f.

2. Thus eliminating x and y from the following:

   $$a_1x + b_1y + c_1 = 0$$
   $$a_2x + b_2y + c_2 = 0$$
   $$a_3x + b_3y + c_3 = 0$$

   is same as deducing the condition that these 3 equations in (x, y) have infinite solutions (i.e. these 3 lines are concurrent). Thus,

   $$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

3. To find the equation of a plane passing through 3 non-collinear points, we know:

   $$ax + by + cz + d = 0$$
   $$ax_1 + by_1 + cz_1 + d = 0$$
   $$ax_2 + by_2 + cz_2 + d = 0$$
   $$ax_3 + by_3 + cz_3 + d = 0$$

   These are equations in (a, b, c, d) to have infinite solutions in x,y,z. Thus:

   $$\begin{vmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix} = 0$$

4. Find condition that following 2 lines are coplaner:
   Line 1 : ax+by+cz+d=0, ex+fy+gz+h=0
   Line 2 : ix+jy+kz+l=0, mx+ny+oz+p=0

   $$\begin{vmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{vmatrix} = 0$$

   is the required condition.