UNDERSTANDING CONFIDENCE INTERVALS

(BY MEANS OF AN ANALOGY)

This is not a formal or complete text.

To be read for understanding purpose only in addition to a main formal text explaining the terms.

It just helps to understand the concept.

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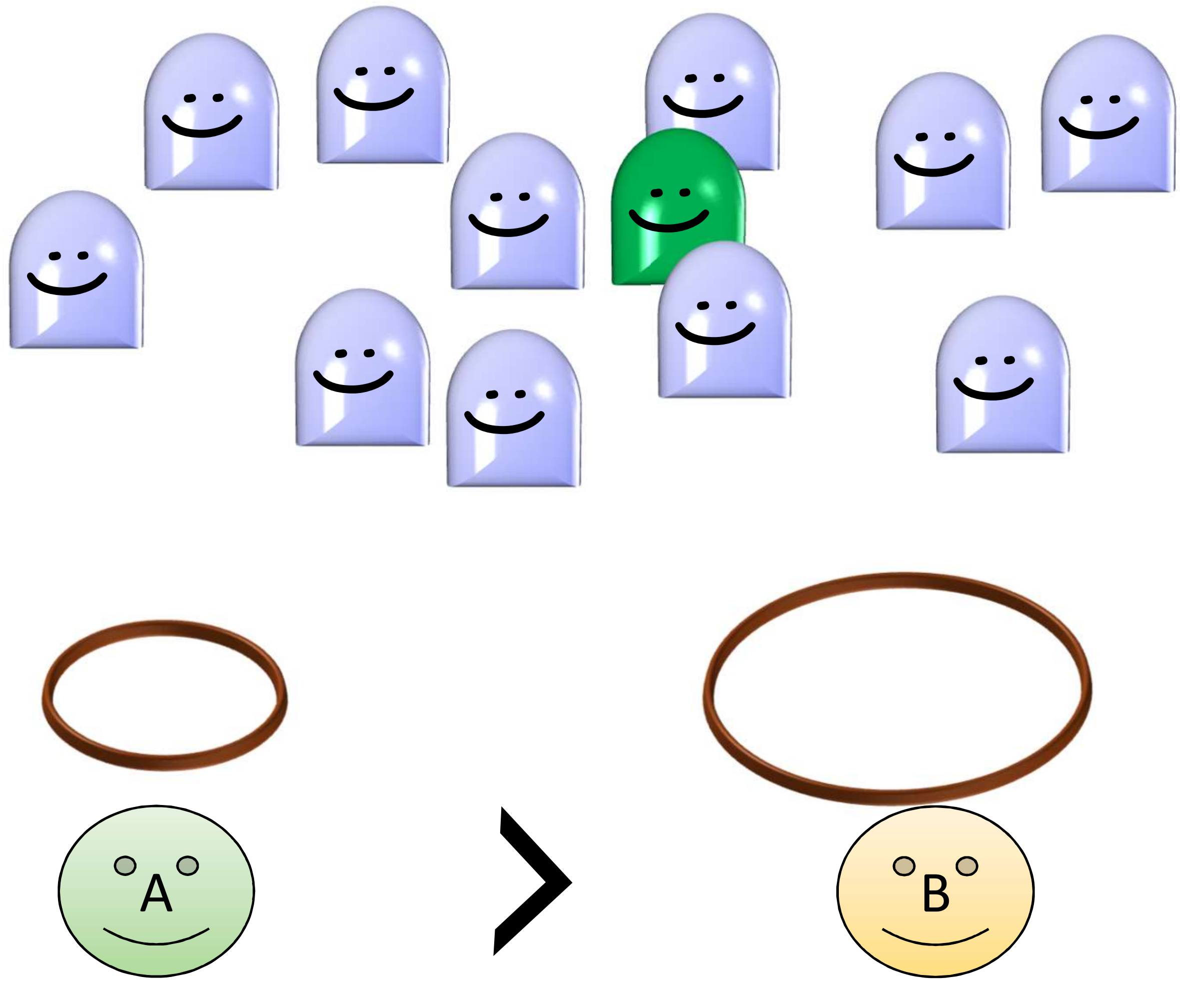
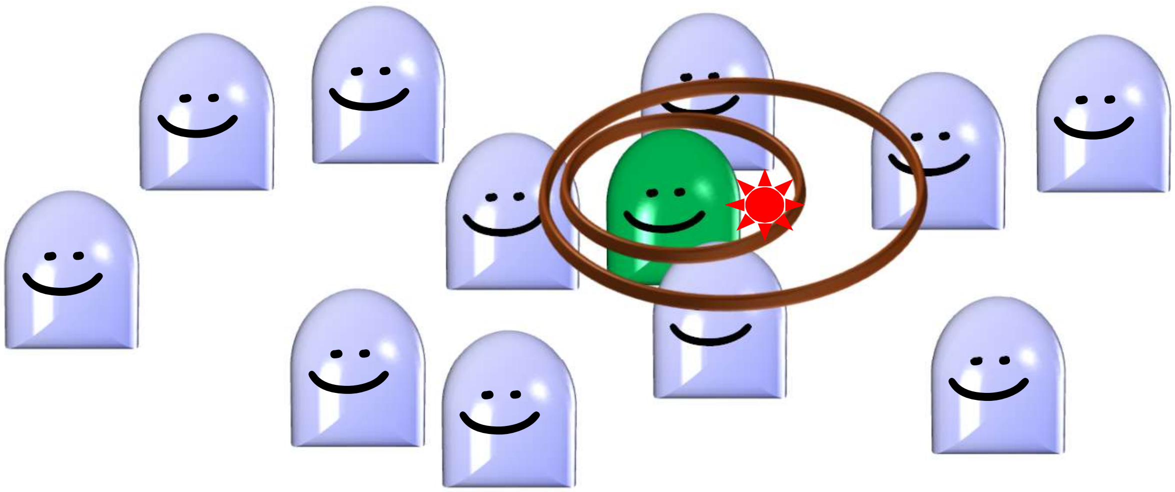
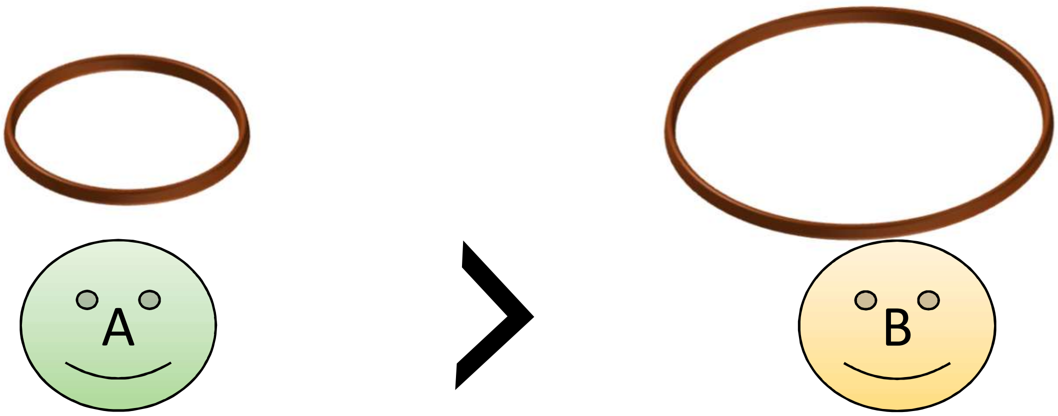


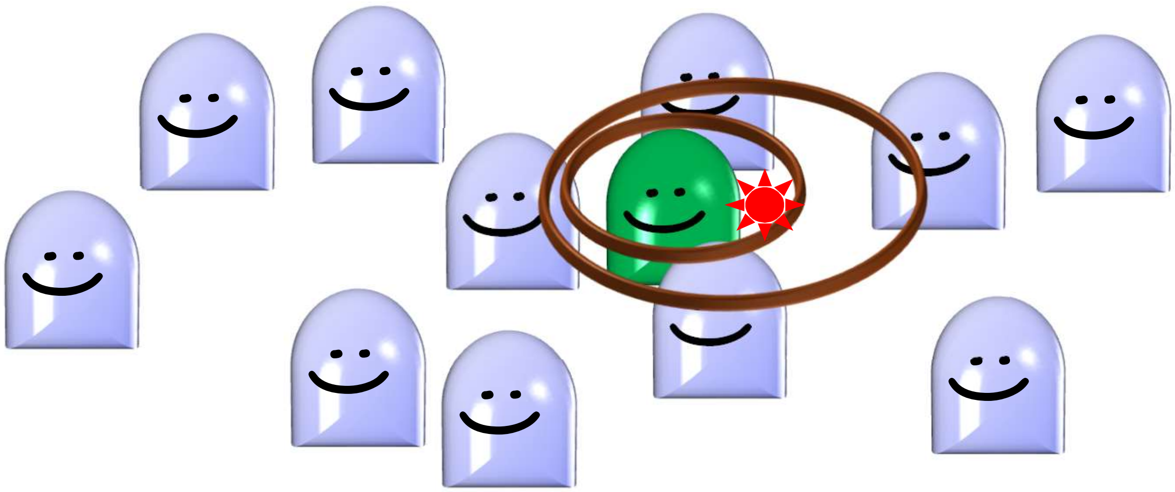
Suppose that A and B are throwing rings to catch the dark toy. Assume that the dark toy is hiding the 'mean' of population near it. And that if ring catches the dark toy, then it *by definition* signifies that 'mean' is somewhere within the ring, but we don't know exacty where in the ring. If the objective of throwing rings is to know the mean, then naturally we will want that a smaller ring is better, because it will locate the unknown mean with more accuracy.

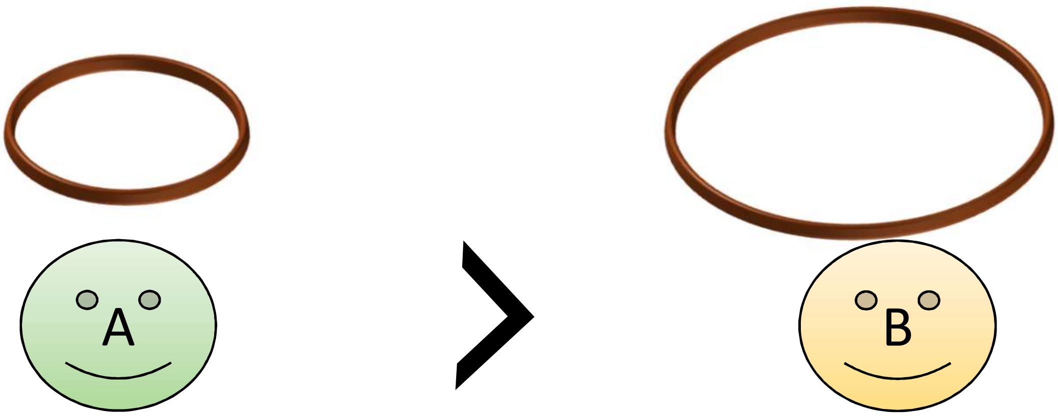




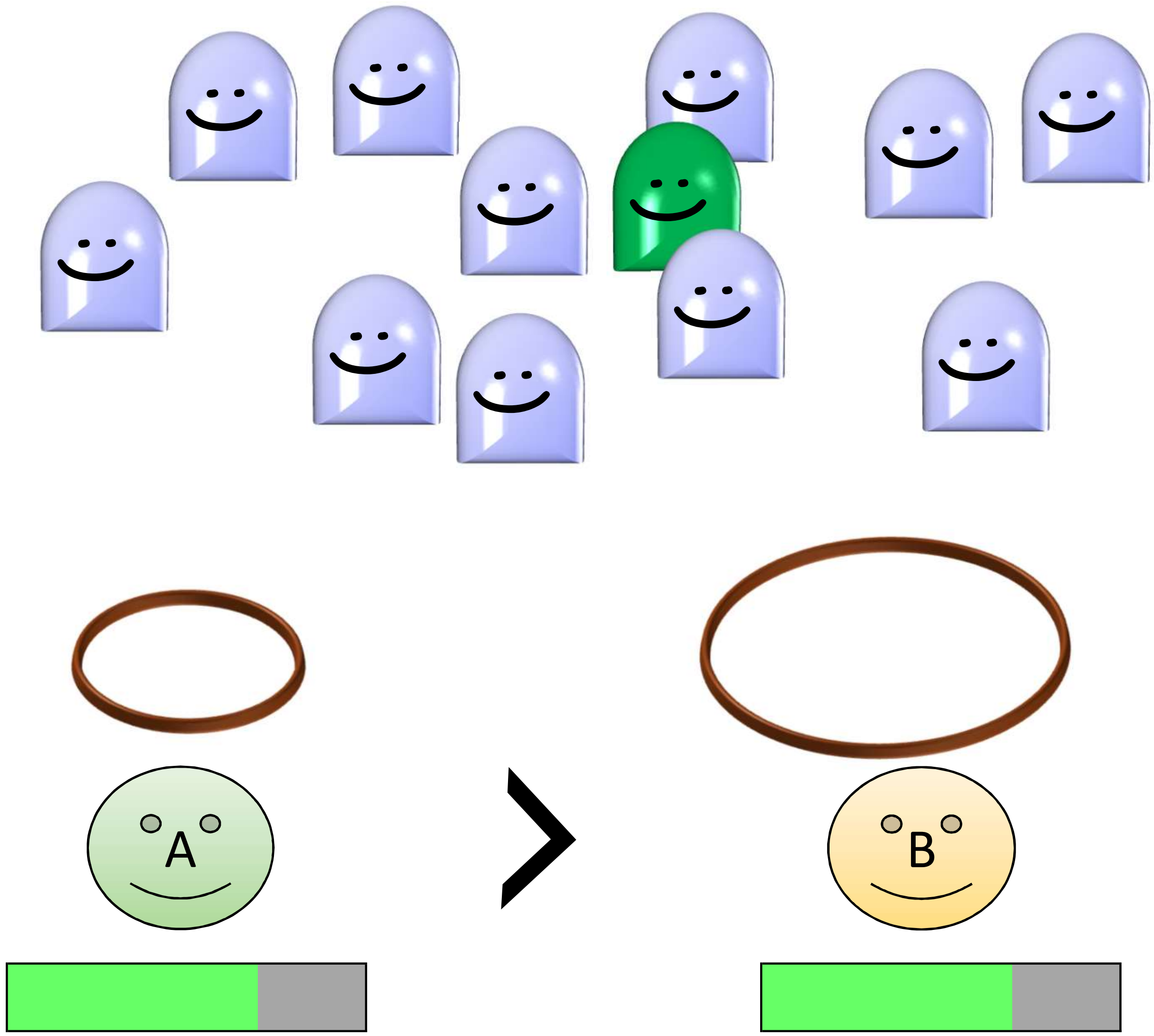
We can say that the center of the ring, after the throw is the most likely estimate of the 'mean' and that the radius of the ring is the plus minus bounds on this estimate.

The 'radius of the ring' in this situation is somewhat like the 'Margin of Error' of our estimated mean. The Margin of Error is less for A. Let it be known by theory that this radius is more if the standard deviation of population is more. And this radius is less if the sample size is more.

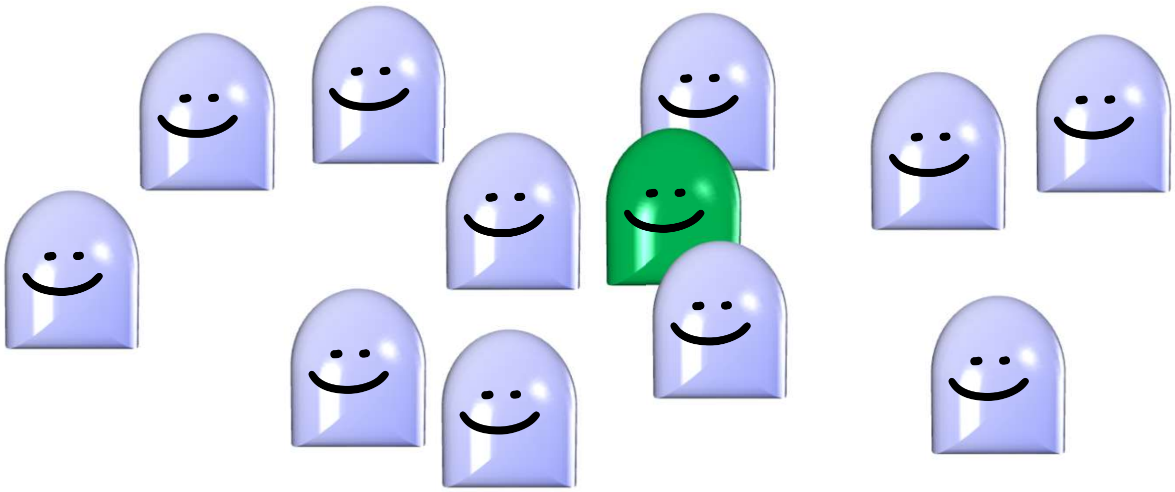


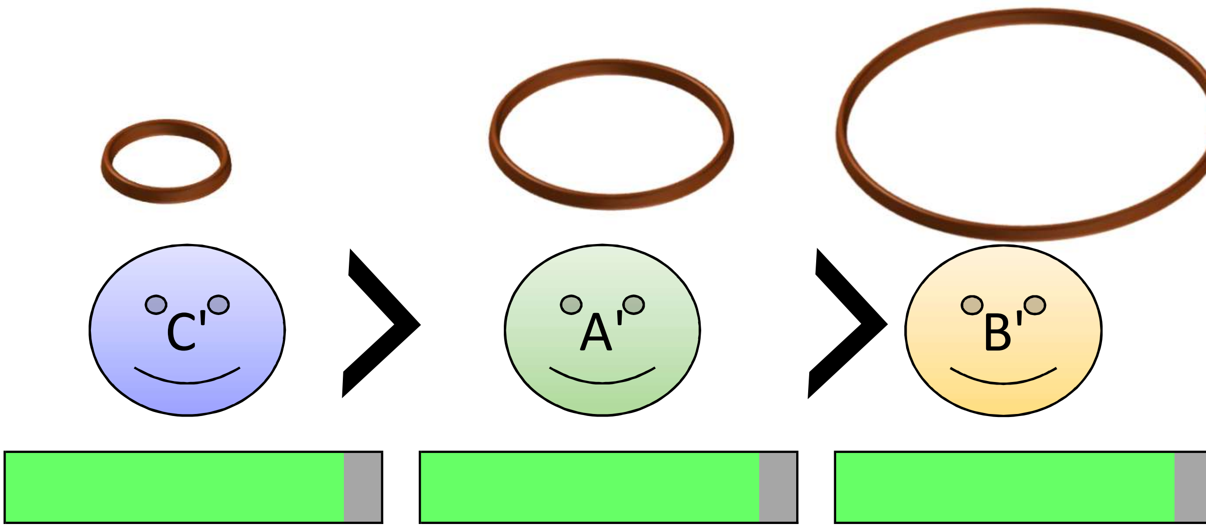


(If you think about it, this fact should not be hard to imagine, that more std. dev. means more variability which means more dispersed values, which means, that keeping all other things same, more likelyhood that mean is not where where we think it is likely to be. So more std. dev. means the ring to catch toy/mean is bigger and less accurate. And more sample size means that, we try out more values, in turn meaning that a bigger sample size reduces the radius of the ring.)



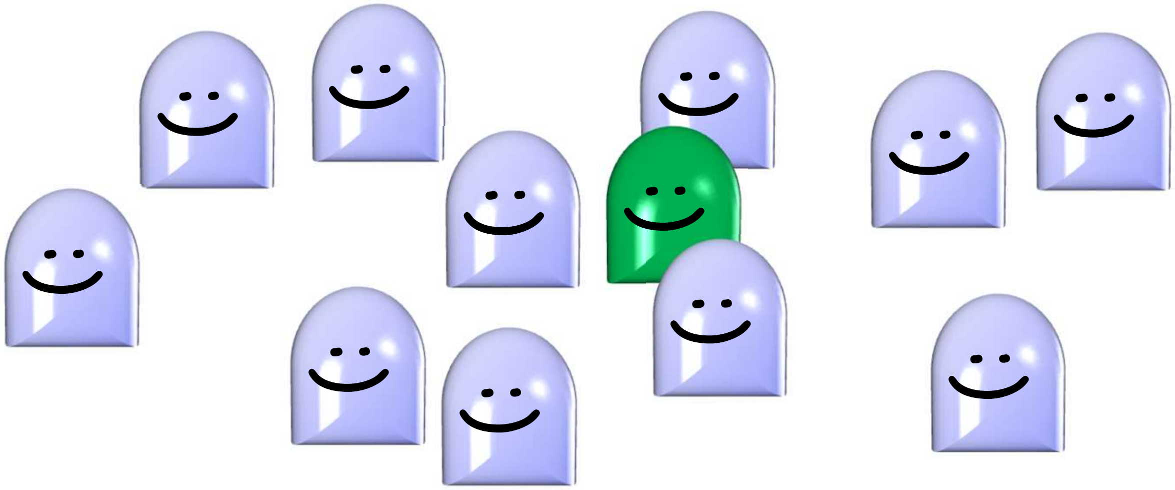
Next imagine that the accuracy of throw to catch the toy, (and hence to catch the mean) of A and B is both 70% each. 30% times, they *miss* the toy. We say that they both catch mean at 70% confidence. The *diameter* of ring is called *confidence interval*. So we say that they are *estimating the mean in a confidence interval at 70% confidence level*. Clearly A with less confidence interval is better.

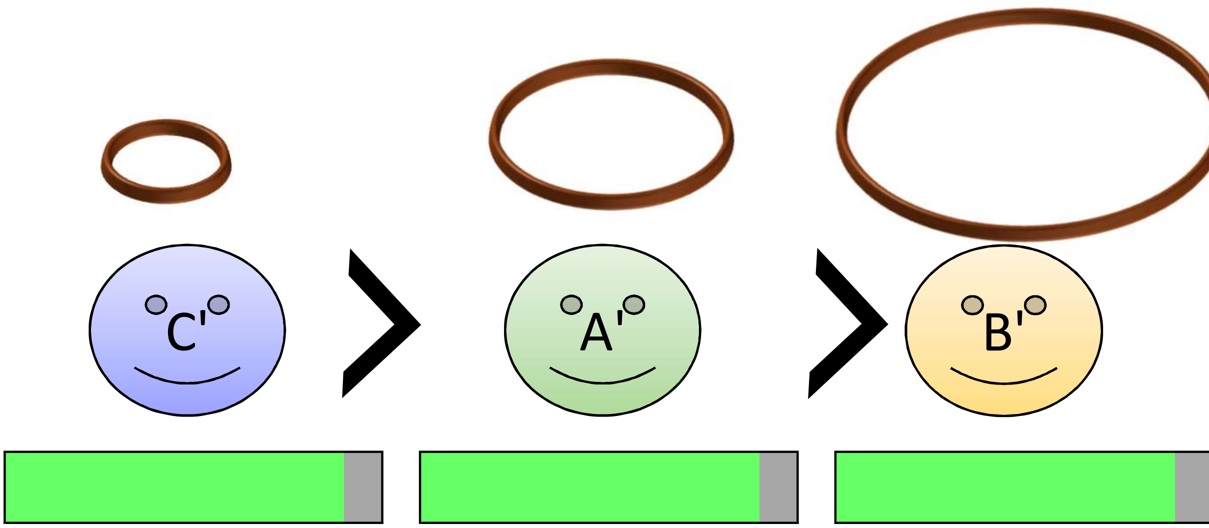




Shown are the ones with 90% confidence intervals each. Suppose that a C' has confidence interval (ring diameter) also low because of lower standard deviation of population & larger sample size; and that A,B,C all have accuracy of 90% each. Definitely we will prefer C' > A' > B' > A > B

So, we prefer...





$$\overline{x} \pm z\frac{s}{\sqrt{n}}$$

x bar = estimated mean = center

z = confidence level = skill

s = std dev

n = sample size

z*s/root(n) = margin of error = radius

2* margin of error = confidence interval = diameter

C' > A' > B' > A > B

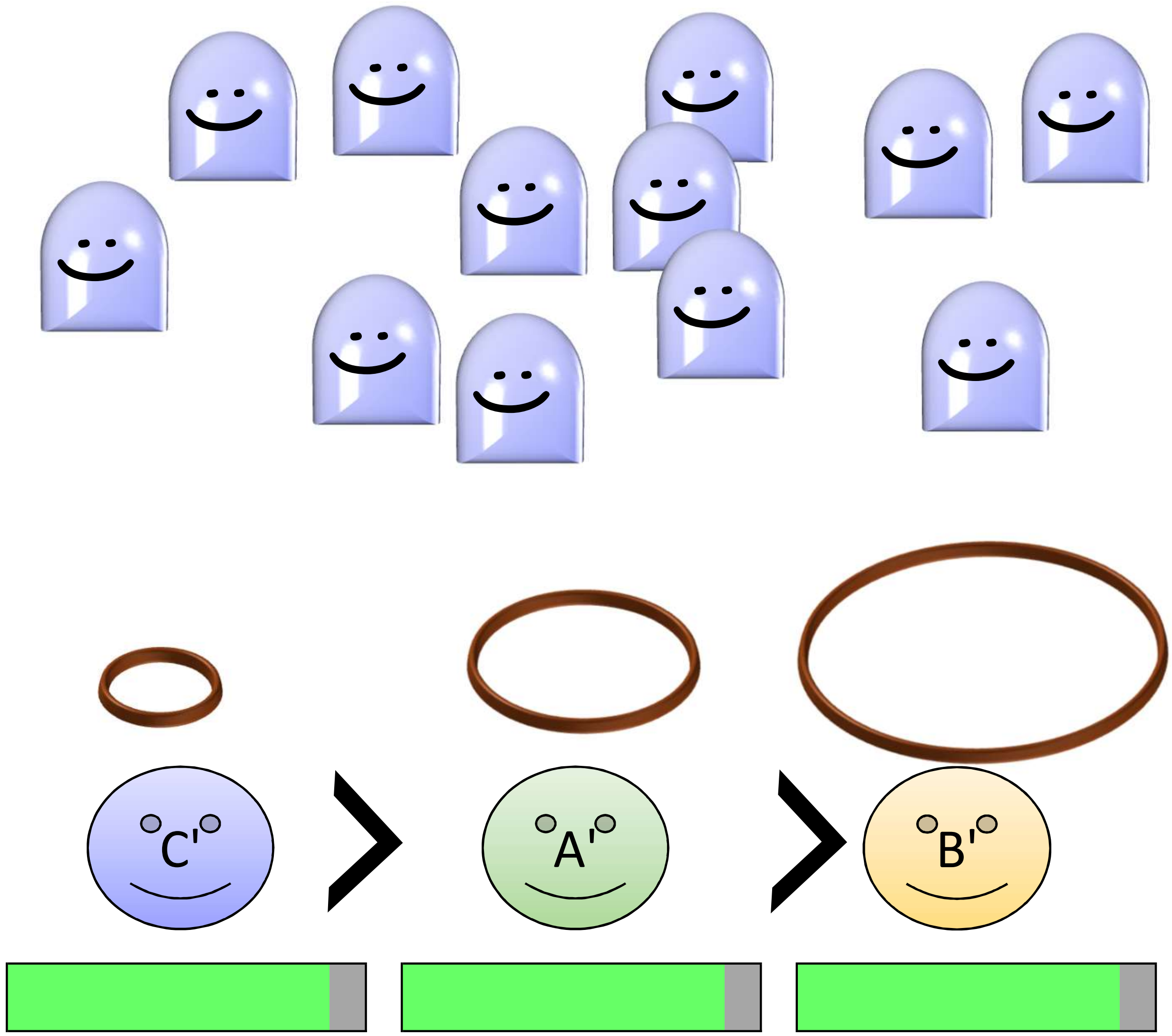
So, we prefer...

Std. deviation : Less (for less dia.)

Sample size : More (for less dia.)

Confidence Level : High (for higher accuracy of throw) OR

Alpha value : Less



The concept of dark toy, or even any toy is actually unnecessary. It was introduced just to demonstrate that we are targetting to find something, and if it is in the ring (our limit of confidence interval), then we declare the search a success. As the mean is actually unknown, so in fact we do not know the dark toy beforehand. It can be any toy. Rest all analogies remain same : Dia - confidence interval, radius - margin of error determined by sample size and std dev., skill - confidence level. 100%-skill = alpha.