Cp Cpk - Graphing Quickly from Values

Amit Bhola

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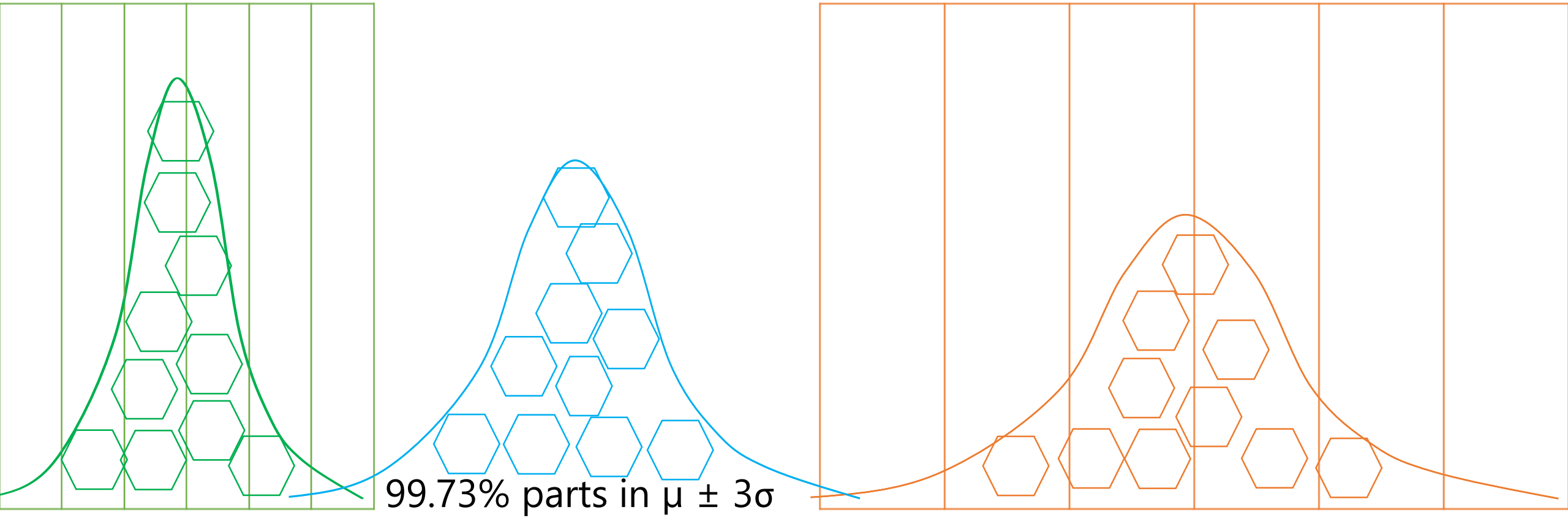
Recap

Height adjusts as per σ to keep μ± zσ area constant

Has σ < 1

Has σ = 1

Has σ > 1



99.73% parts in μ ± 3σ

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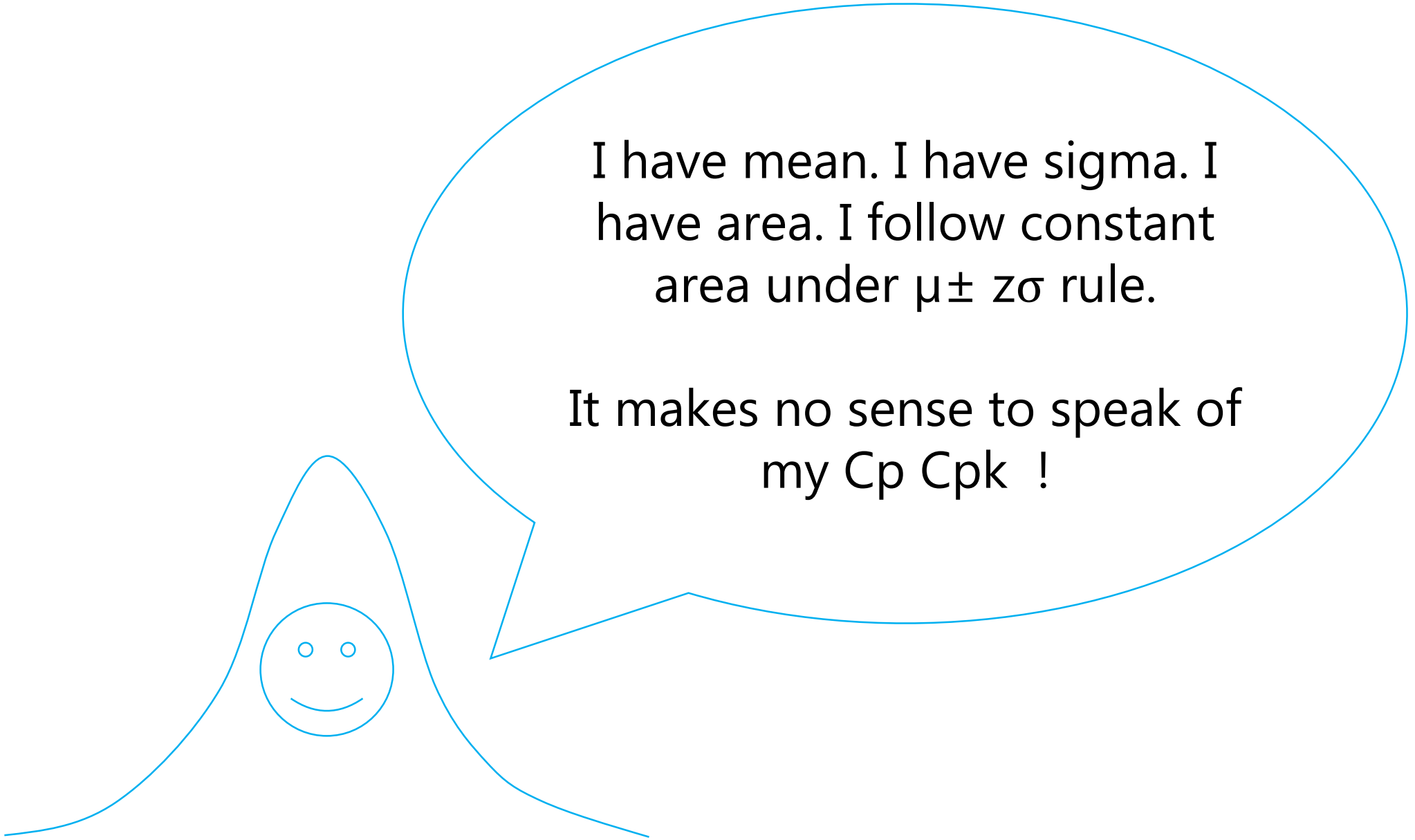
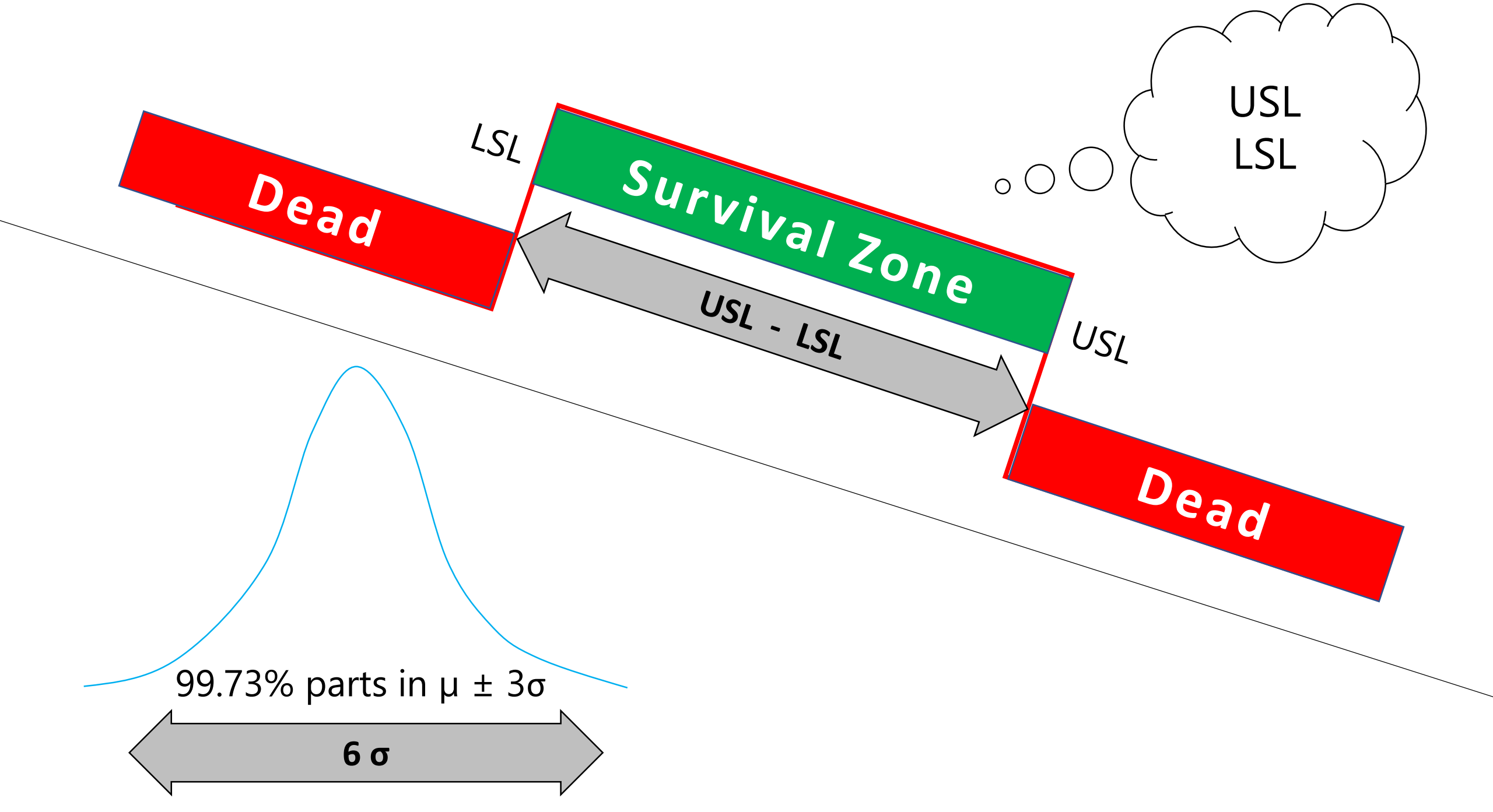
99.73% parts in μ ± 3σ

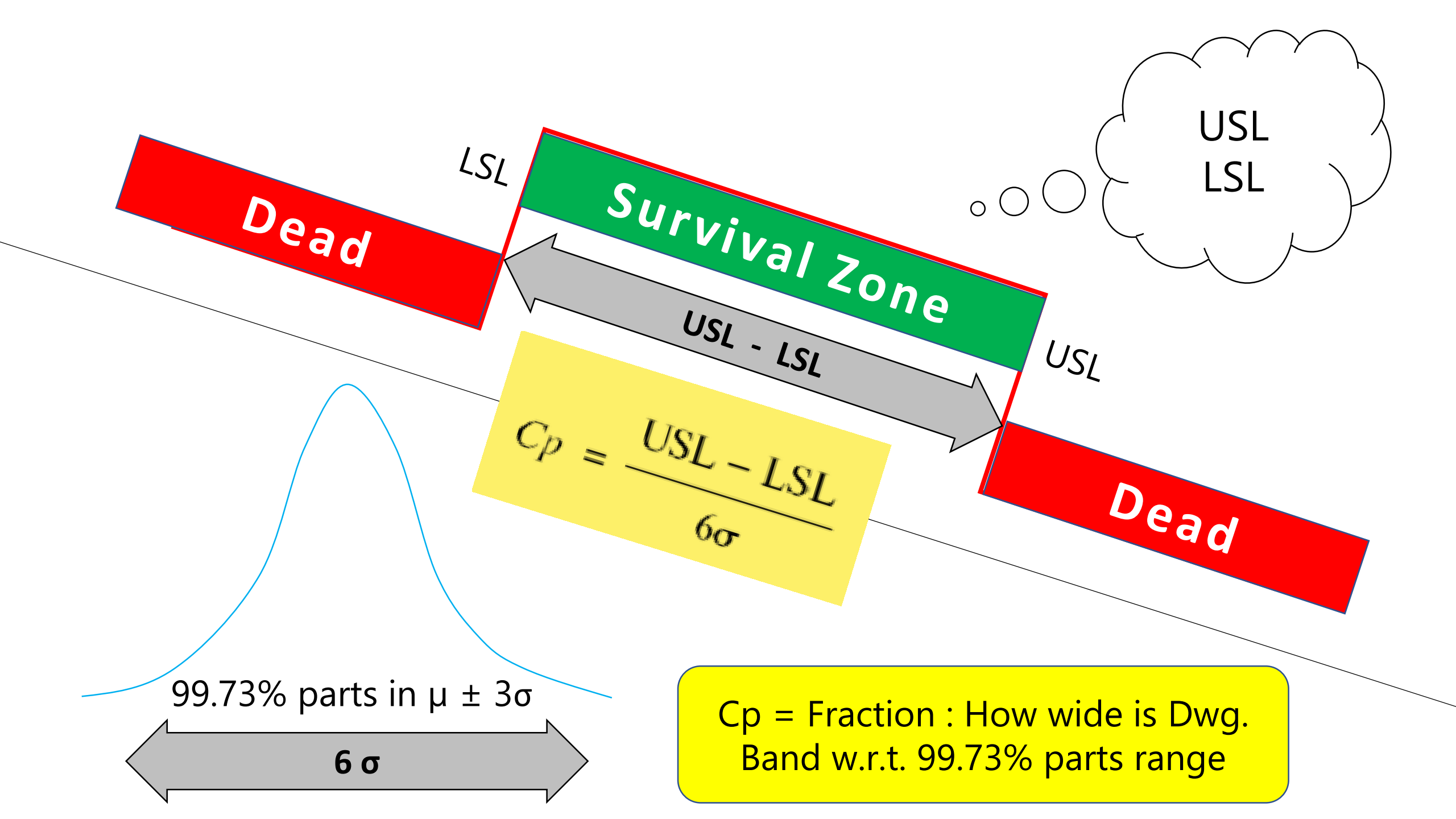

I have mean. I have sigma. I have area. I follow constant area under μ± zσ rule.
It makes no sense to speak of my Cp Cpk ！

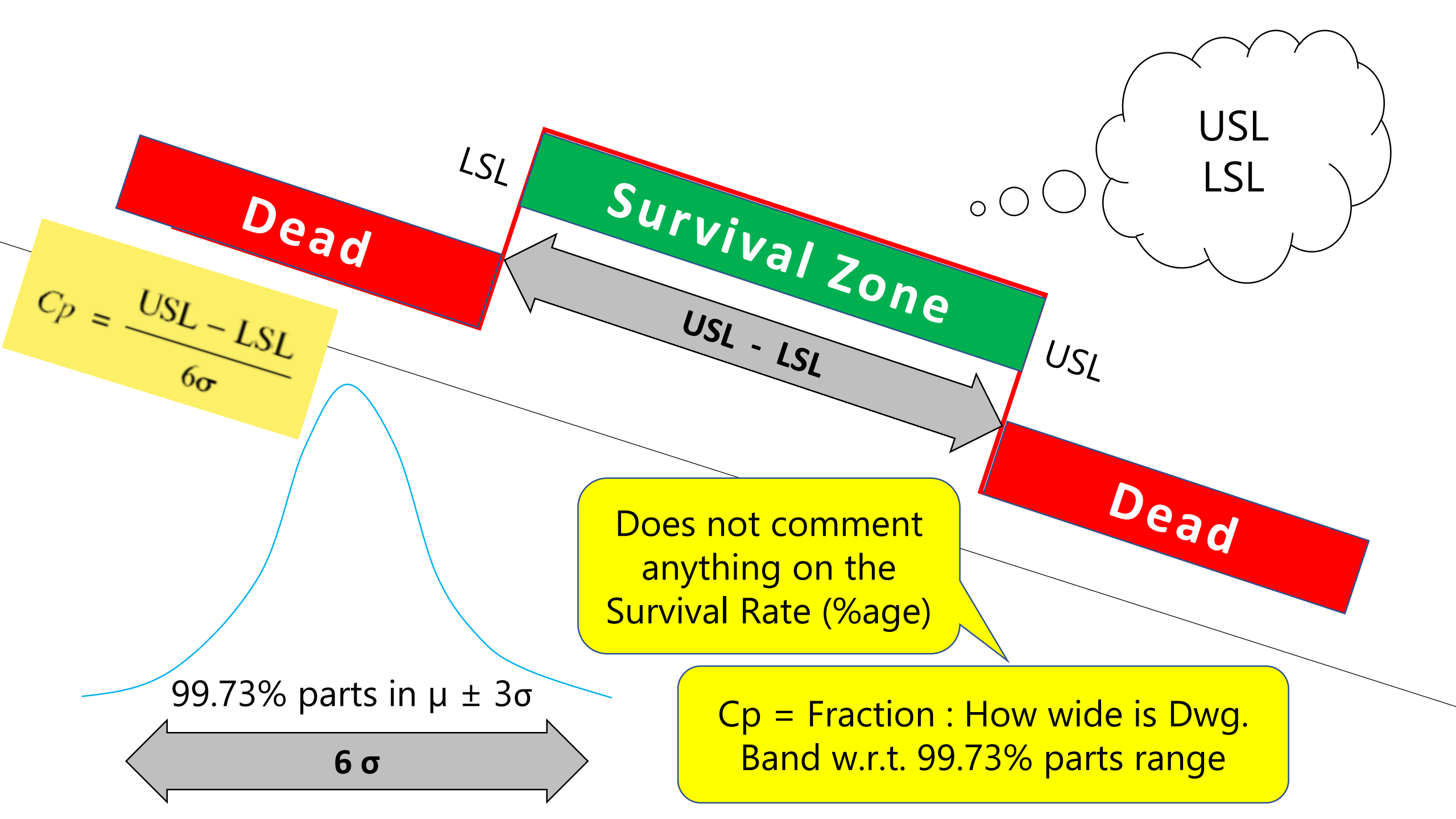
USL
LSL

Only if someone looks at me through the glasses of “Dwg. Spec” , Cp Cpk gain some meaning.

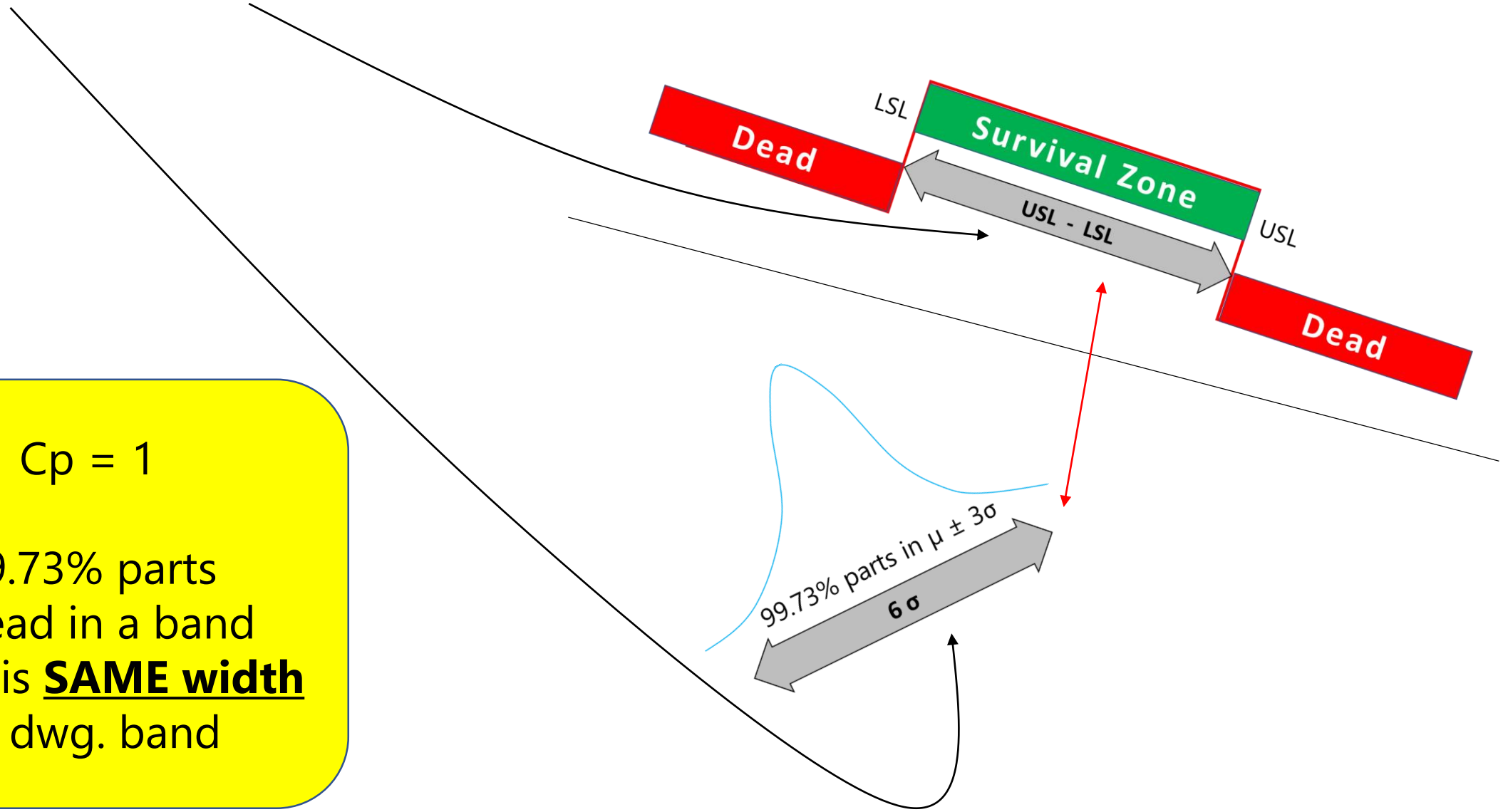
Cp - Relative Size Rule


USL
LSL
Dead
LSL
Survival Zone
USL - LSL
USL
Dead
99.73% parts in μ ± 3σ
6 σ


USL
LSL
Dead
LSL
Survival Zone
USL - LSL
USL
Dead
$Cp = \frac{USL - LSL}{6\sigma}$
99.73% parts in μ ± 3σ
6 σ
Cp = Fraction : How wide is Dwg.
Band w.r.t. 99.73% parts range


USL
LSL
Dead
LSL
Survival Zone
USL - LSL
USL
Dead
$Cp = \frac{USL - LSL}{6\sigma}$
99.73% parts in μ ± 3σ
6 σ
Does not comment anything on the Survival Rate (%age)
Cp = Fraction : How wide is Dwg. Band w.r.t. 99.73% parts range

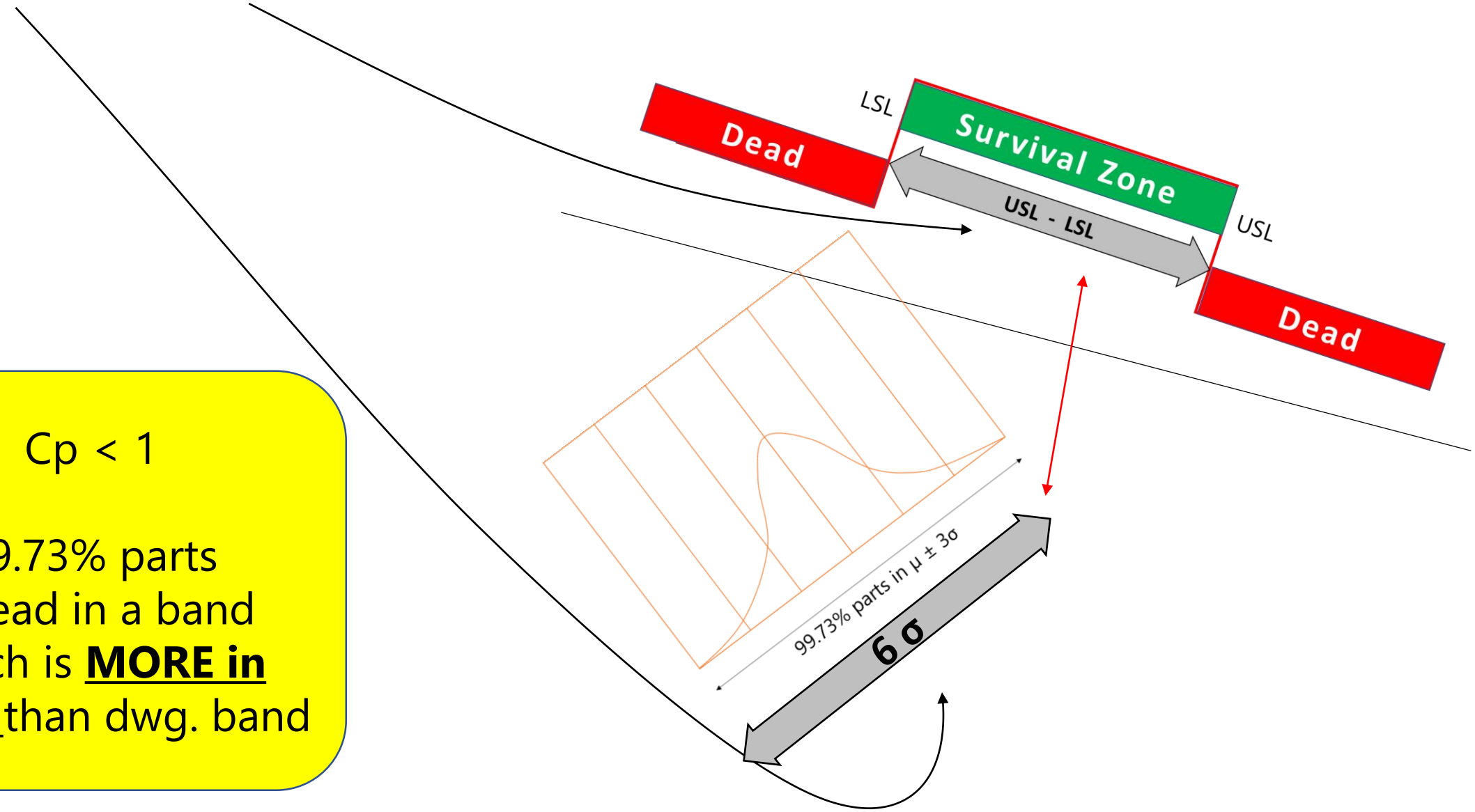
Meaning of Cp = 1



Cp = 1

99.73% parts spread in a band which is **<u>SAME width</u>** as dwg. band

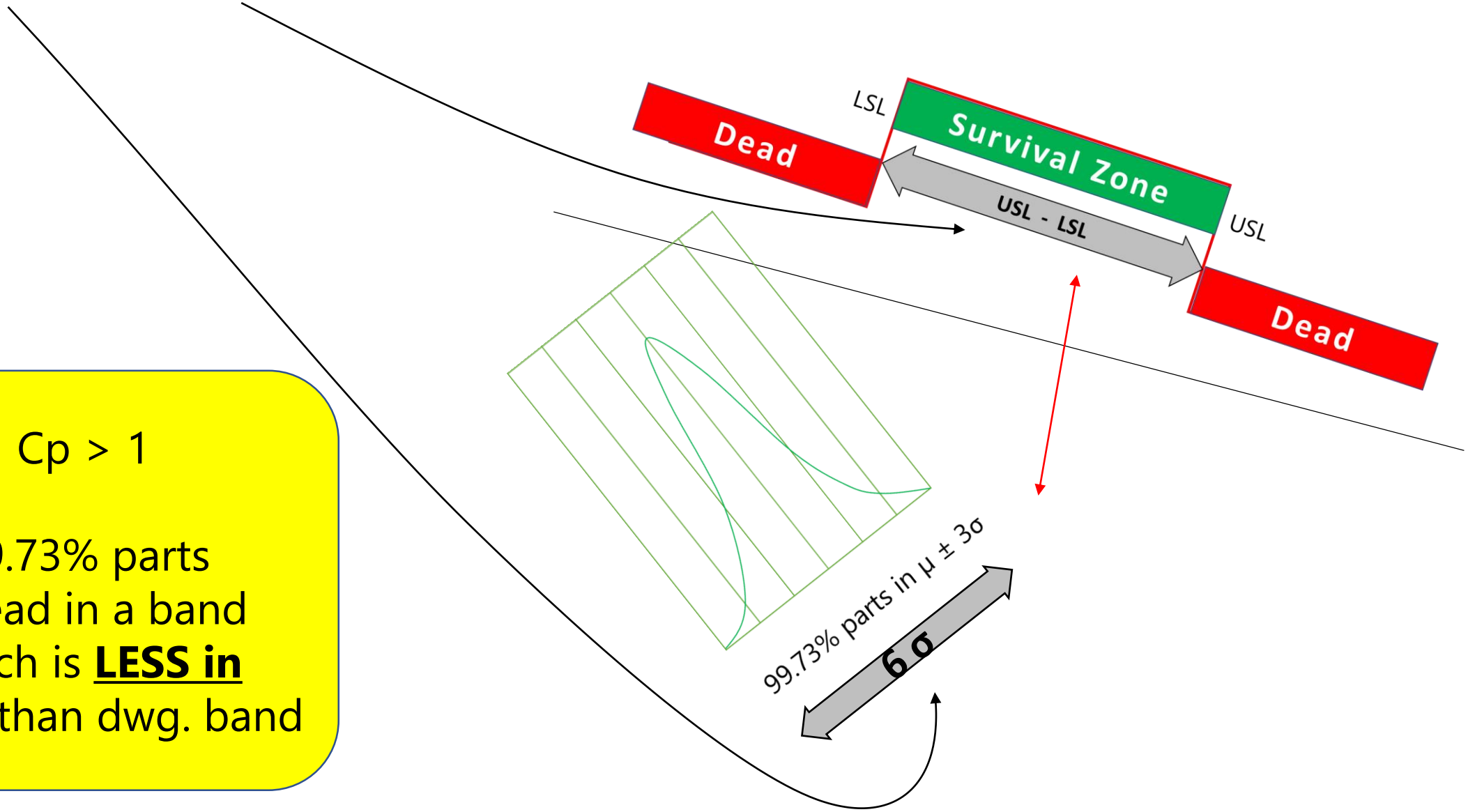
Meaning of Cp < 1



Cp < 1

99.73% parts spread in a band which is **<u>MORE in width</u>** than dwg. band

Meaning of Cp > 1



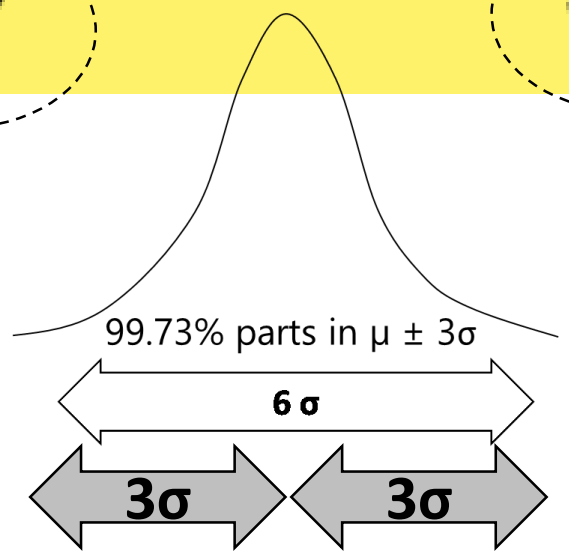
Cp > 1

99.73% parts spread in a band which is **LESS in width** than dwg. band

Cpk - (1)→(2)→(3) Rule

Cpk measures **position** of population wrt. dwg. band

$$Cpk = \text{lesser of } \frac{USL - \overline{\overline{X}}}{3\sigma} \text{ or } \frac{\overline{\overline{X}} - LSL}{3\sigma}$$



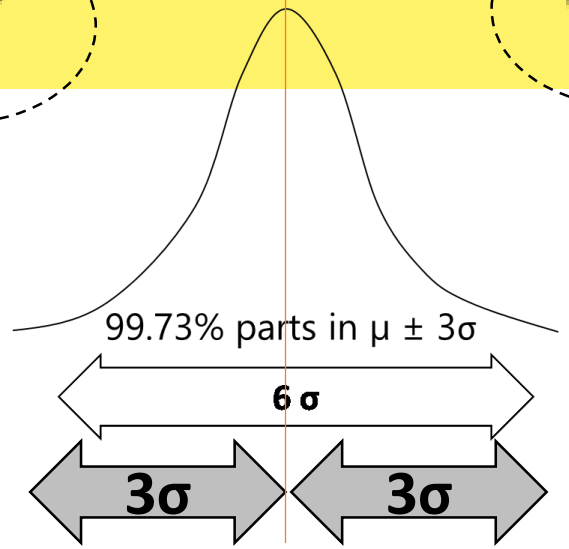
Cpk measures position of population wrt. dwg. band

Numerator : distance
Population mean w.r.t. (nearest)
Dwg. Extreme

w.r.t.

$$Cpk = \text{lesser of } \frac{USL - \bar{\bar{X}}}{3\sigma} \text{ or } \frac{\bar{\bar{X}} - LSL}{3\sigma}$$

Denominator : distance
Population mean w.r.t.
population Extreme



Cpk measures position of population wrt. dwg. band

Numerator : distance
Population mean w.r.t. (nearest)
Dwg. Extreme

w.r.t.

Denominator : distance
Population mean w.r.t.
population Extreme

In plain English,

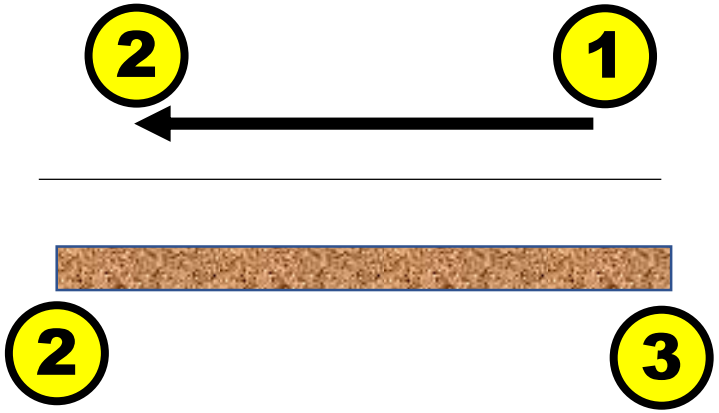
Sitting @ actual population mean,

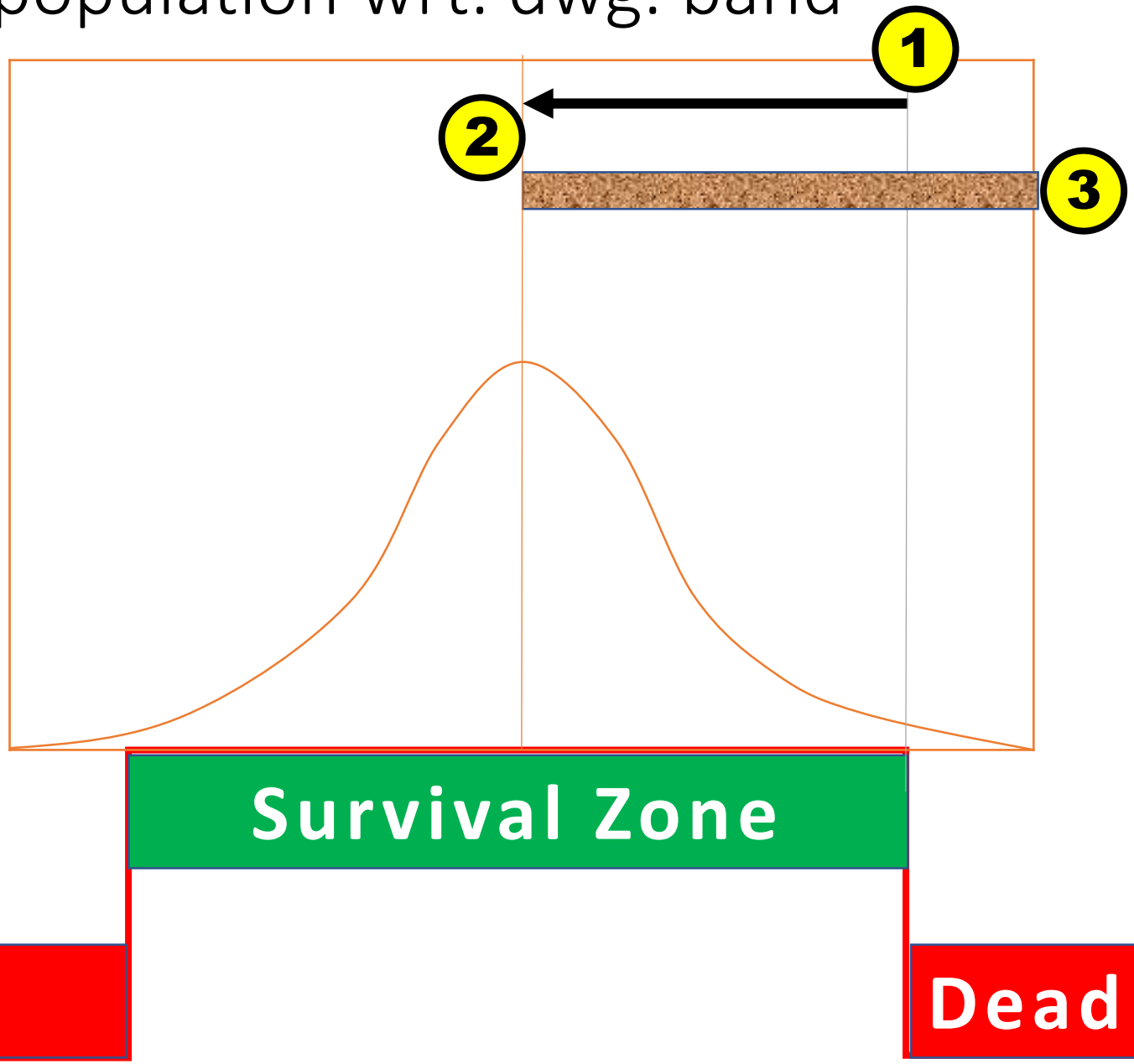
Which is more far away?

Dwg. Extreme ?
Or
Population Extreme?

Cpk measures position of population wrt. dwg. band

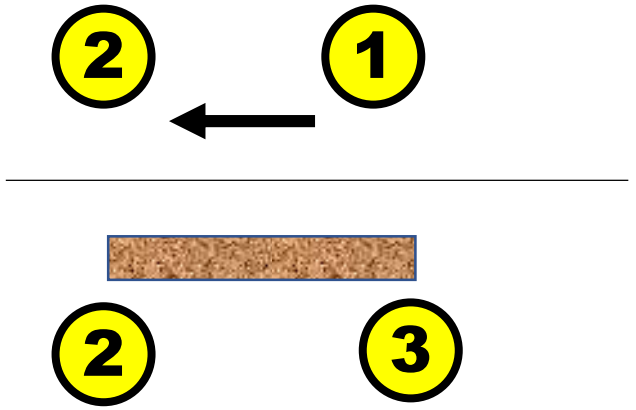
Numerator = Nearest Dwg. Limit to Actual mean
Denominator = Actual mean to Actual Extreme





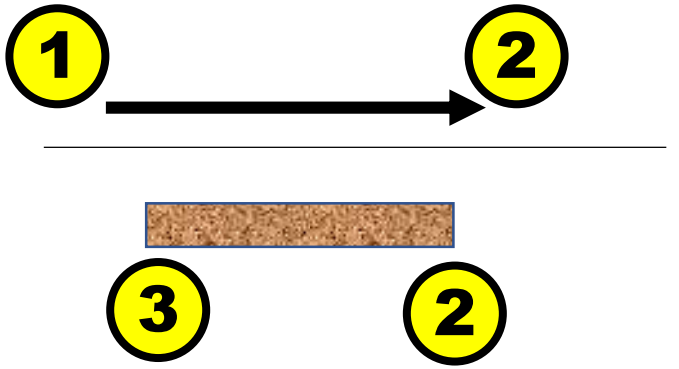
Cpk measures position of population wrt. dwg. band

Numerator = Nearest Dwg. Limit to Actual mean
Denominator = Actual mean to Actual Extreme



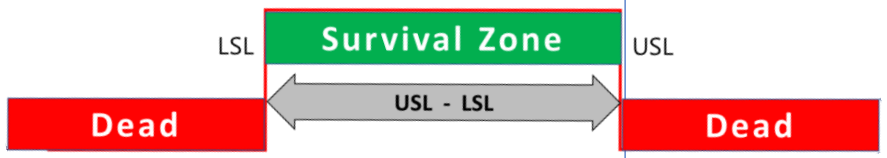
Cpk measures position of population wrt. dwg. band

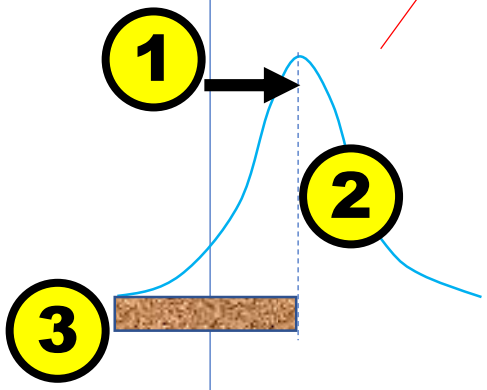
Numerator = Nearest Dwg. Limit to Actual mean
Denominator = Actual mean to Actual Extreme



Meaning of Cpk < 0

$$Cpk = \text{lesser of } \frac{USL - \overline{\overline{X}}}{3\sigma} \quad \text{or} \quad \frac{\overline{\overline{X}} - LSL}{3\sigma}$$





Cpk < 0

Numerator < 0

e.g. USL < Mean

=>

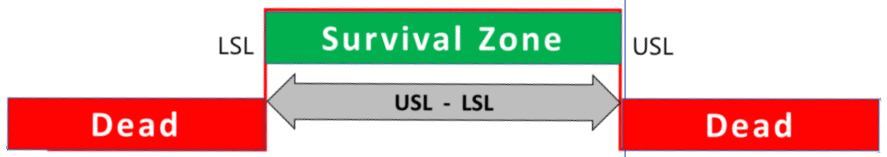
Mean outside Dwg. spec

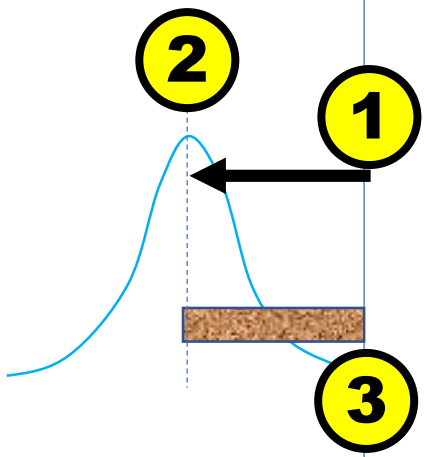
> 50% Rejection on one side

Since practically, situation is rarely so bad, so to keep the discussion relevant, further it is assumed that Population Mean is within Dwg. Spec

i.e. $C_{pk} > 0$

Meaning of Cpk = 1





Cpk = 1

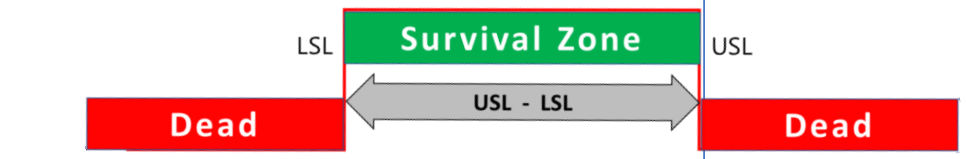
Population Extreme is <u>SAME POSITION AS</u> Dwg. Extreme

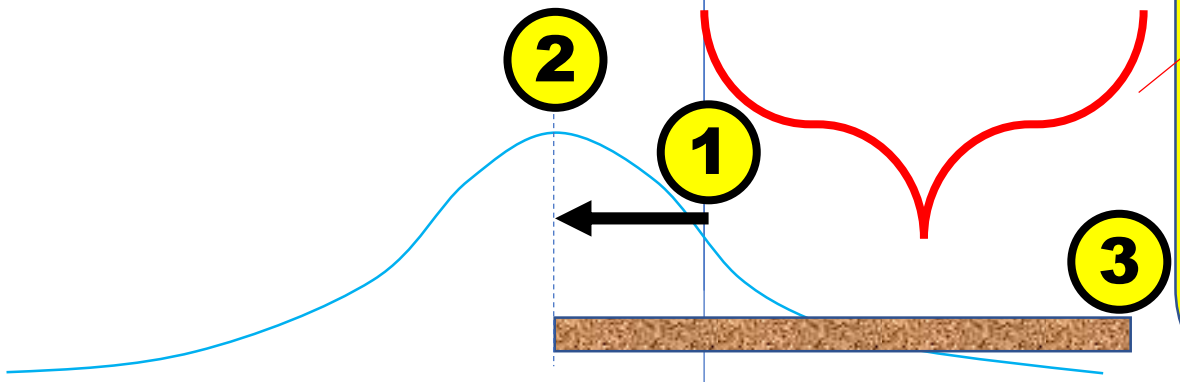
= >

Borderline OK

"Just In"

Meaning of Cpk < 1



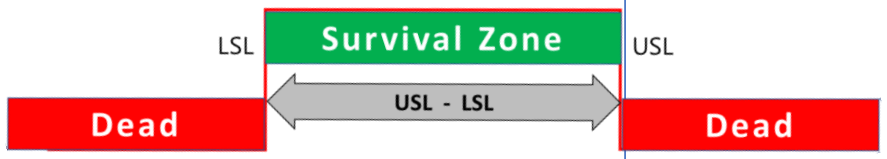


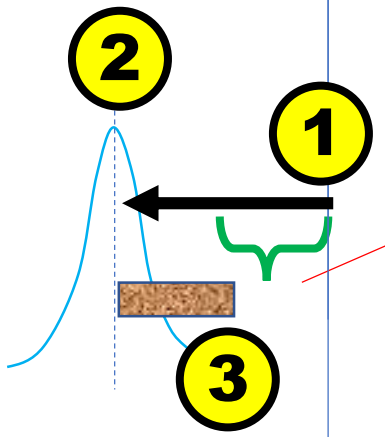
Cpk < 1

Population Extreme is
AWAY & BEYOND
Dwg. Extreme
= >

Sure Rejection

Meaning of Cpk > 1





Cpk > 1

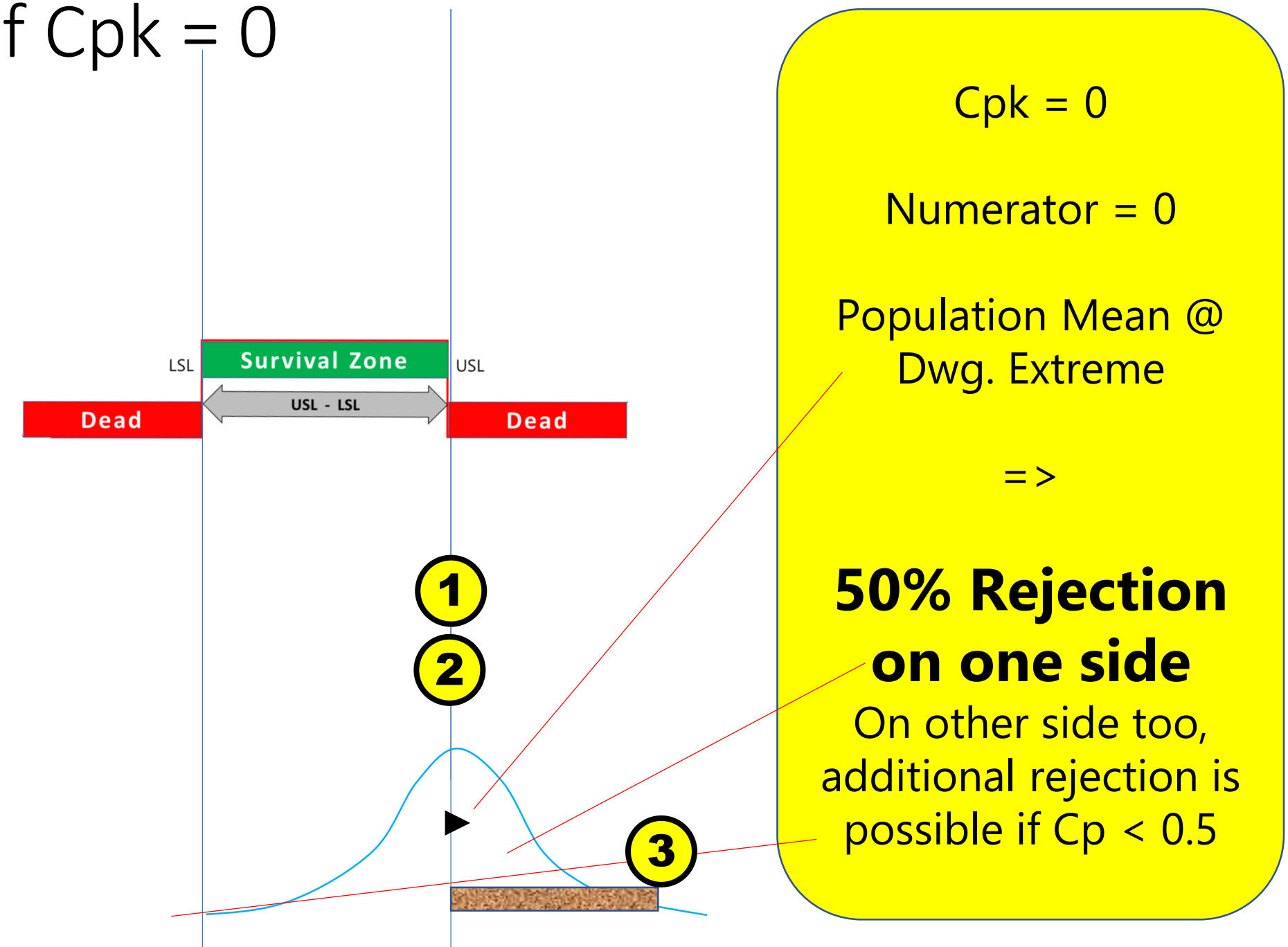
Population Extreme is <u>WELL WITHIN INSIDE</u> Dwg. Extreme

= >

Definite FOS

(“Factor of Safety”)

Meaning of Cpk = 0



Cpk = 0

Numerator = 0

Population Mean @ Dwg. Extreme

= >

50% Rejection on one side

On other side too, additional rejection is possible if Cp < 0.5

Cp Cpk seen Together

Meaning of Cp = Cpk

$$Cp = \frac{\text{USL} - \text{LSL}}{6\sigma}$$

$$Cpk = \text{lesser of } \frac{\text{USL} - \overline{\overline{X}}}{3\sigma} \quad \text{or} \quad \frac{\overline{\overline{X}} - \text{LSL}}{3\sigma}$$

$$\text{USL} - \overline{\overline{X}}$$

$$\overline{\overline{X}} - \text{LSL}$$

Meaning of Cp = Cpk

$$Cp = \frac{USL - LSL}{6\sigma}$$

Full / Full

Half 1 / Half **Half 2 / Half**

$$Cpk = \text{lesser of } \frac{USL - \overline{\overline{X}}}{3\sigma} \text{ or } \frac{\overline{\overline{X}} - LSL}{3\sigma}$$

Cp = Cpk Possible only if

Half 1 = Half 2

$$USL - \overline{\overline{X}} = \overline{\overline{X}} - LSL$$

Cp = Cpk

=>

Population Mean is at Dwg. Mean

One view Summary

Cp = spread : Dwg / Population

For : Cpk > 0

(w.r.t. Popⁿ. Mean)

Cpk = Extremes : Dwg / Population

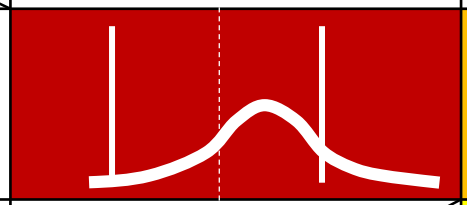
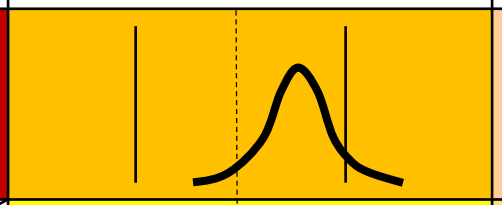
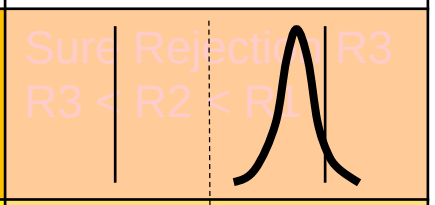
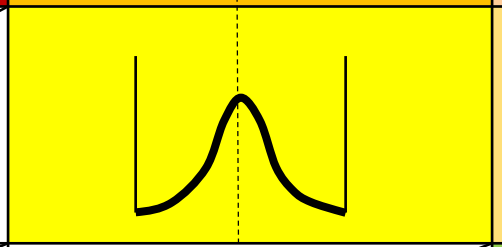
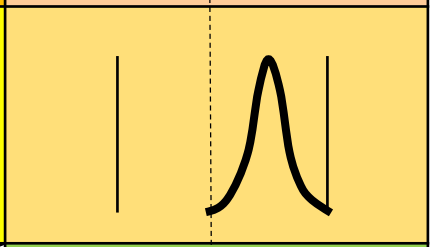
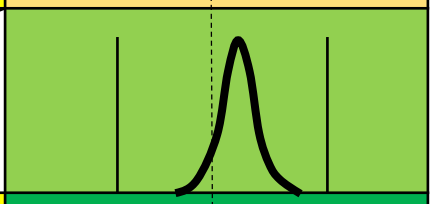
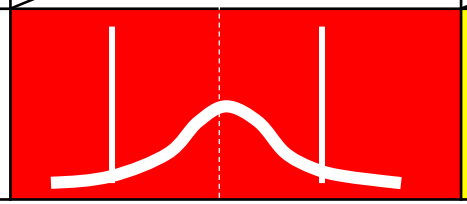
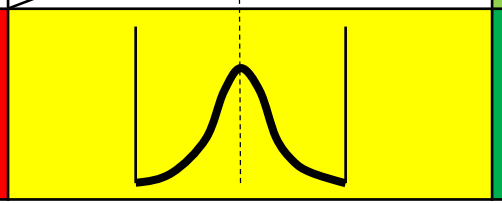
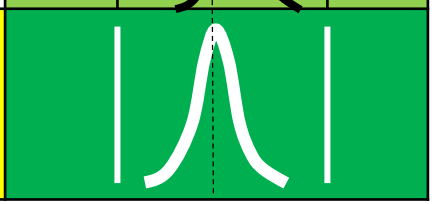
			Cp < 1	**Cp = 1**	**Cp > 1**
			Dwg spread < Population spread	Dwg spread = Population spread	Dwg spread > Population spread
			"Sure Rejection"	**"No FOS"**	**"FOS possible"**
Cpk < 1	Dwg. Extreme < Population Extreme	**"Sure Rejection"**	**More than or equal to Min. possible Rejection R1**	Sure Rejection R2 Possibility of R2 < R1	Sure Rejection R3 Possibility of R3 < R2 < R1
Cpk = 1	Dwg. Extreme = Population Extreme	**"Borderline OK"**		Just OK each side	Just OK on one side. More than Best FOS on other.
Cpk > 1	Dwg. Extreme > Population Extreme	**"Definite FOS"**			FOS on one side < FOS on other side
If Cp = Cpk	Dwg. Mean = Population Mean	**"Centering"**	**(Minimum Possible Rejection)**	(Just OK each side)	**(Equal & Best possible FOS each side)**

Cp = spread : Dwg / Population

For : Cpk > 0

(w.r.t. Popⁿ. Mean)

Cpk = Extremes : Dwg / Population

			Cp < 1	**Cp = 1**	**Cp > 1**
			Dwg spread < Population spread	Dwg spread = Population spread	Dwg spread > Population spread
			"Sure Rejection"	**"No FOS"**	**"FOS possible"**
Cpk < 1	Dwg. Extreme < Population Extreme	**"Sure Rejection"**			
Cpk = 1	Dwg. Extreme = Population Extreme	**"Borderline OK"**			
Cpk > 1	Dwg. Extreme > Population Extreme	**"Definite FOS"**			
If Cp = Cpk	Dwg. Mean = Population Mean	**"Centering"**			

Rejections are shown at USL as example. It is possible at LSL too.

Graphing Quickly from Values

Before Graphing

See Cp Value

- $Cp \leq 1$
- $Cp > 1$

→ Just Recall
Cp : Relative Size Rule
DWG BAND Vs. Population Spread

See Cpk Value

- $Cpk \leq 1$
- $Cpk > 1$

→ Just Recall
Cpk : (1) → (2) → (3) Rule
USL (or LSL) → Pop. Mean → Pop. Extreme

See if $Cp \approx Cpk$

→ Just Recall
If Cp = Cpk
Pop. Mean = Dwg. Mean

Before Graphing

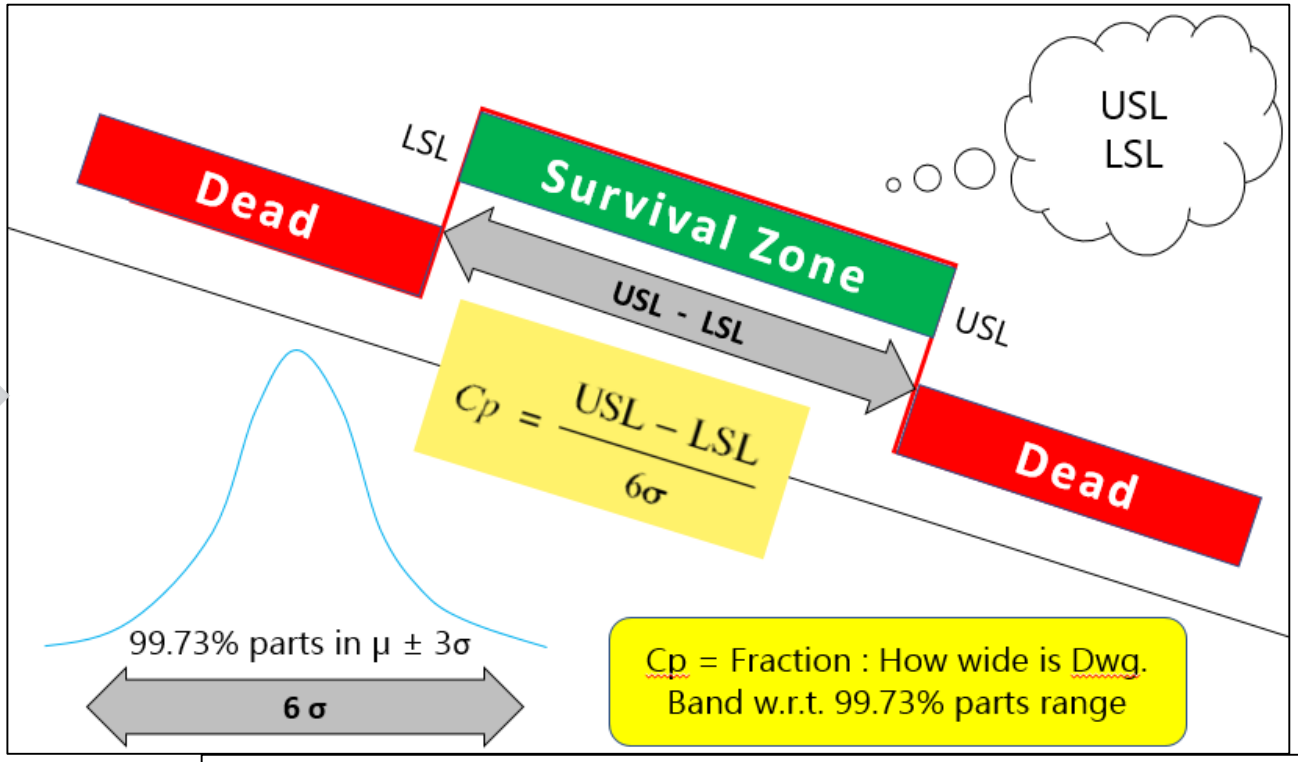
See Cp Value

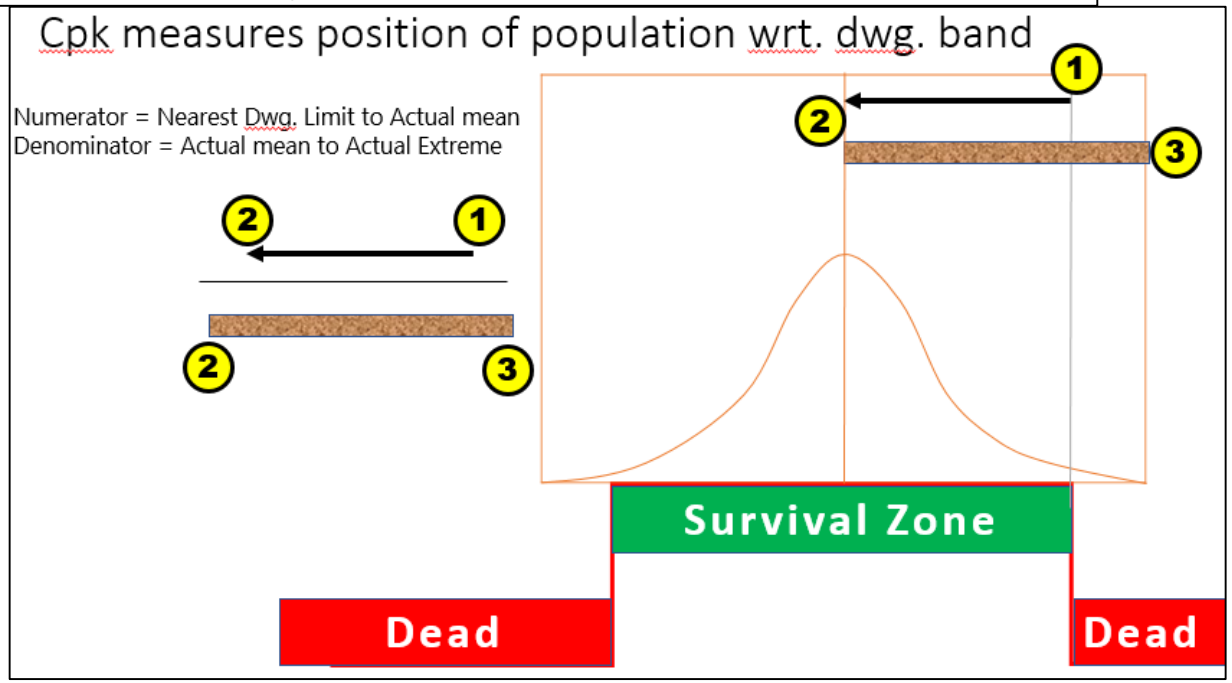
- Cp ≤ 1
- Cp > 1

See Cpk Value

- Cpk ≤ 1
- Cpk > 1

See if Cp≈Cpk





Start Graphing

Make a random N-dist curve

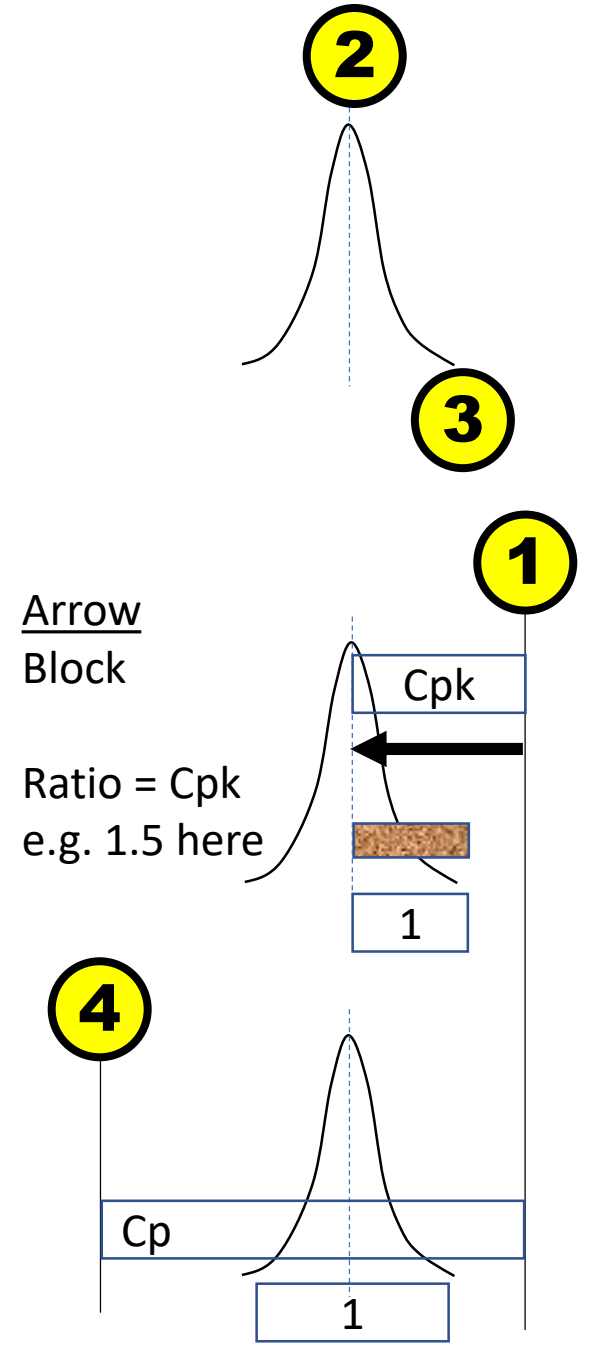
(2) : Pop. Mean | (3) : Pop. Extreme

As per Cpk Value : Make a vertical Line = (1)

Representing USL | Or representing LSL

As per Cp Value : make 2nd vertical Line (4) **wrt (1)**

Representing LSL | Or representing USL



Deciding Starting Line : USL or LSL ?

Make a random N-dist curve

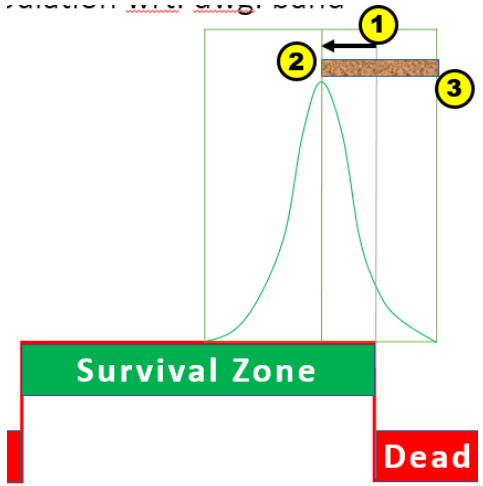
(2) : Pop. Mean | (3) : Pop. Extreme

As per Cpk Value : Make a vertical Line = (1)

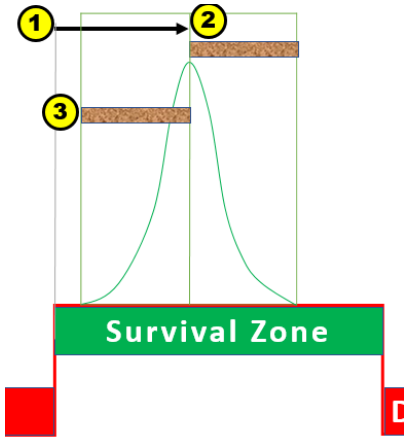
Representing USL | Or representing LSL

As per Cp Value : make 2nd vertical Line (4) wrt (1)

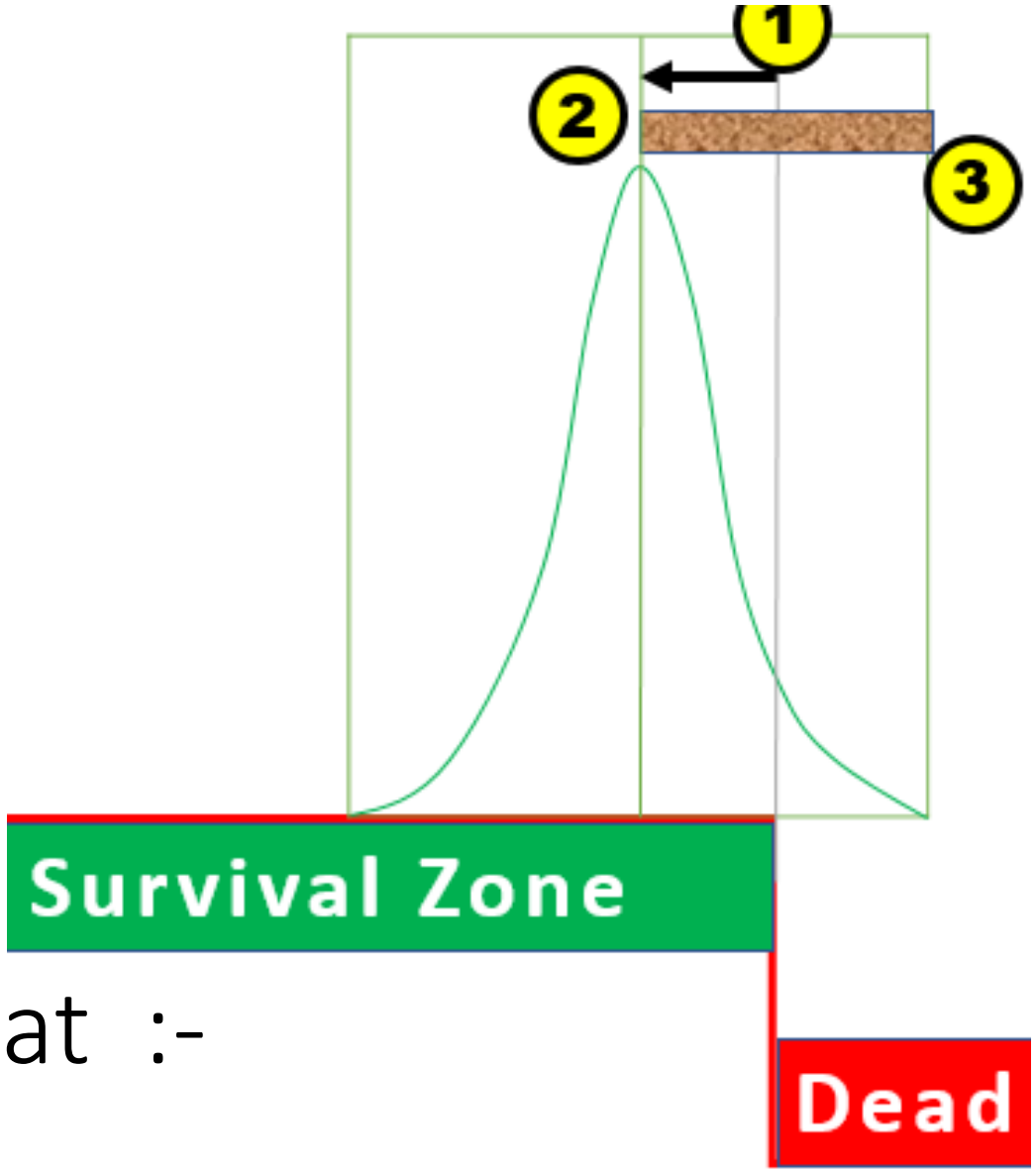
Representing LSL | Or representing USL



Decide by comparing Dwg. Mean vs. Actual Mean

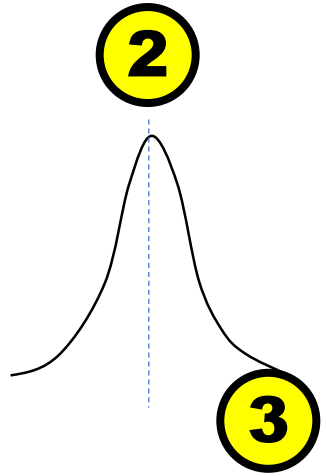


Practice



(we will assume we know that :-
Actual Mean > Dwg Mean
=> Start Point = USL)

Practice : Cp = 1.7 , Cpk = 0.9



Make a random N-dist curve	
(2) : Pop. Mean	(3) : Pop. Extreme

As per Cpk Value : Make a vertical Line = (1)	
Representing USL	Or representing LSL

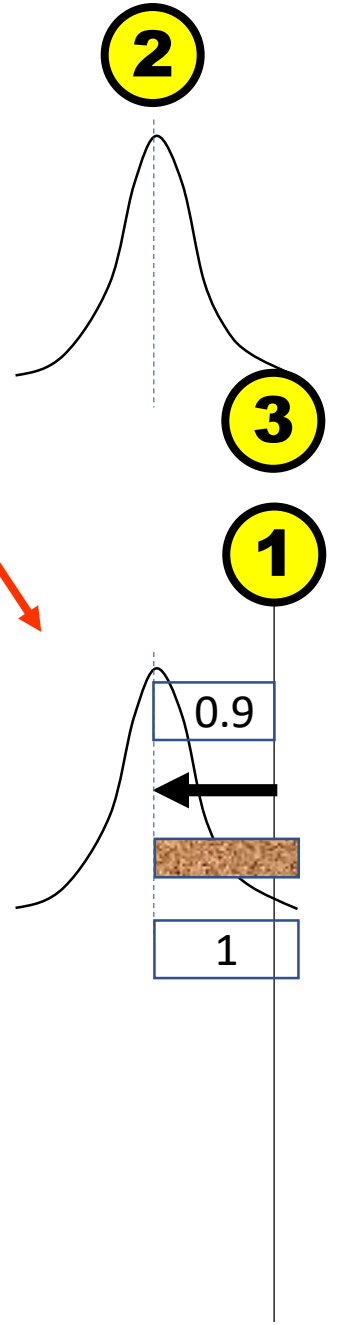
As per Cp Value : make 2nd vertical Line (4) wrt (1)	
Representing LSL	Or representing USL

Practice : Cp = 1.7 , Cpk = 0.9

Make a random N-dist curve	
(2) : Pop. Mean	(3) : Pop. Extreme

As per Cpk Value : Make a vertical Line = (1)	
Representing USL	Or representing LSL

As per Cp Value : make 2nd vertical Line (4) wrt (1)	
Representing LSL	Or representing USL



Practice : Cp = 1.7 , Cpk = 0.9

Make a random N-dist curve

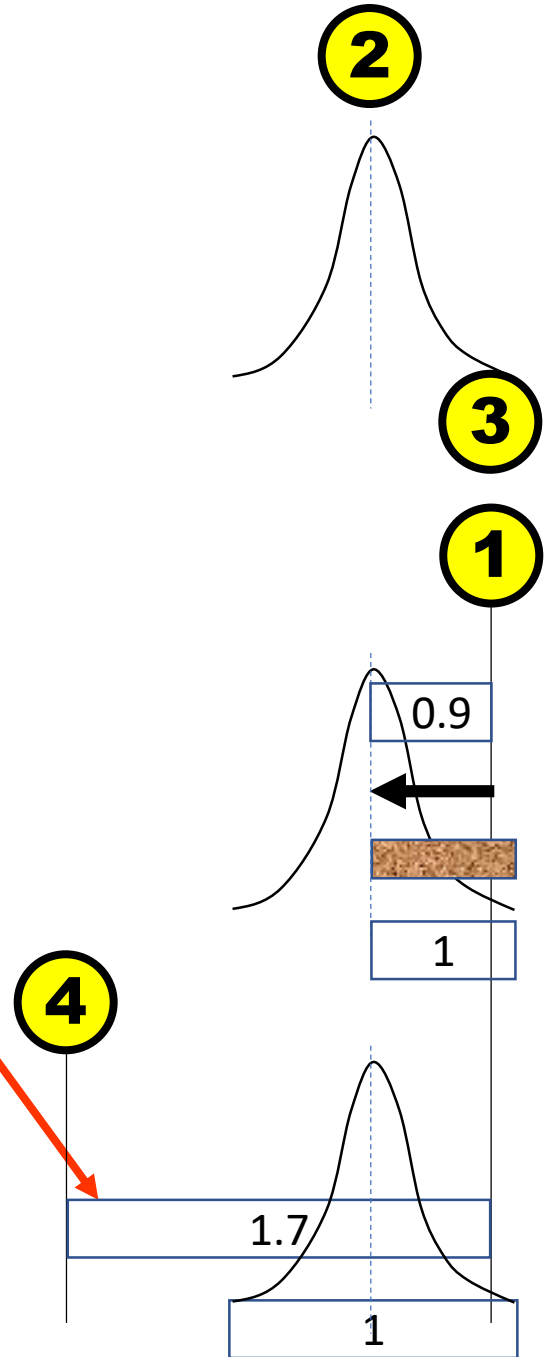
(2) : Pop. Mean | (3) : Pop. Extreme

As per Cpk Value : Make a vertical Line = (1)

Representing USL | Or representing LSL

As per Cp Value : make 2nd vertical Line (4) **wrt (1)**

Representing LSL | Or representing USL

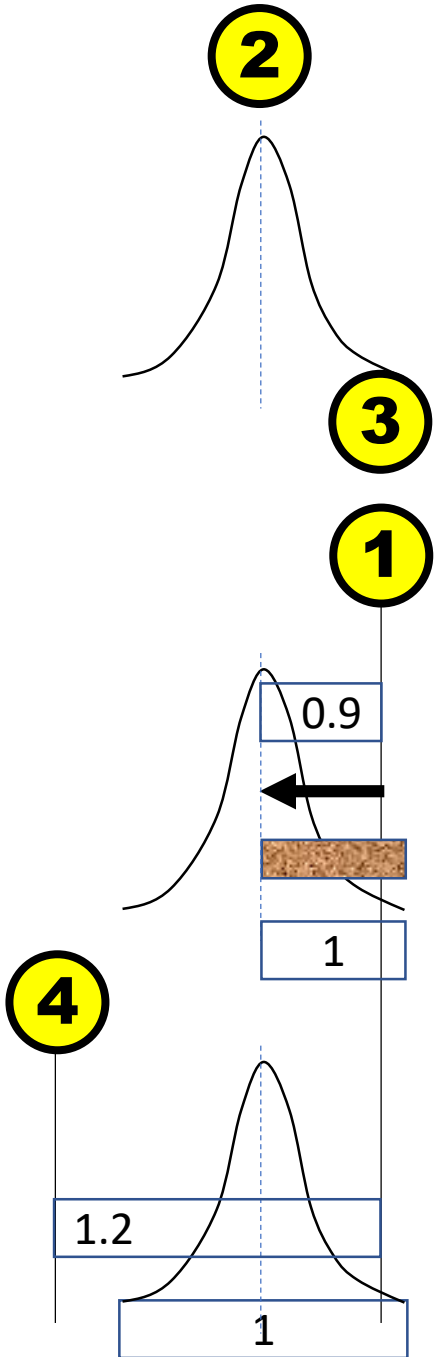


Practice : Cp = 1.2 , Cpk = 0.9

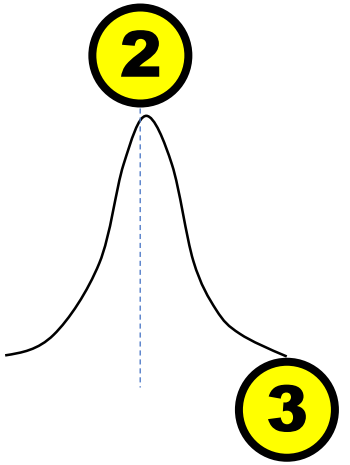
Make a random N-dist curve	
(2) : Pop. Mean	(3) : Pop. Extreme

As per Cpk Value : Make a vertical Line = (1)	
Representing USL	Or representing LSL

As per Cp Value : make 2nd vertical Line (4) wrt (1)	
Representing LSL	Or representing USL



Practice : Cp = 0.8 , Cpk = 0.6



Make a random N-dist curve	
(2) : Pop. Mean	(3) : Pop. Extreme

↓

As per Cpk Value : Make a vertical Line = (1)	
Representing USL	Or representing LSL

↓

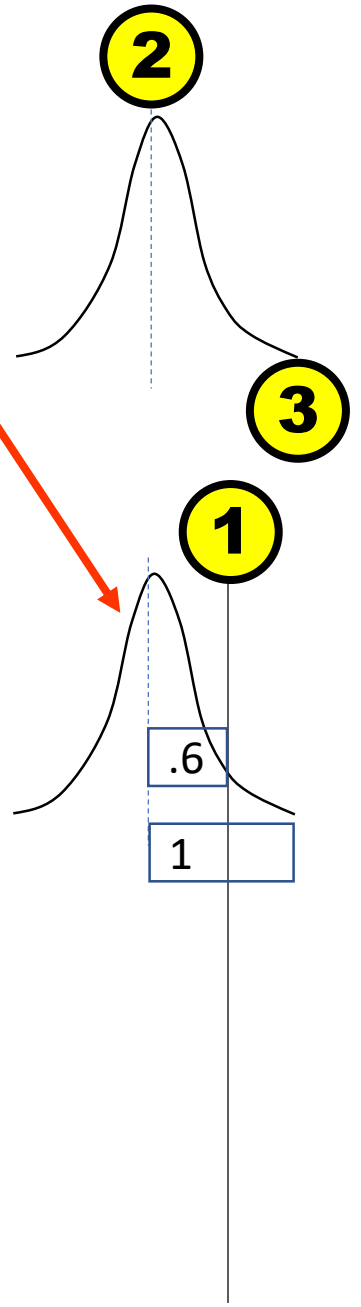
As per Cp Value : make 2nd vertical Line (4) wrt (1)	
Representing LSL	Or representing USL

Practice : Cp = 0.8 , Cpk = 0.6

Make a random N-dist curve	
(2) : Pop. Mean	(3) : Pop. Extreme

As per Cpk Value : Make a vertical Line = (1)	
Representing USL	Or representing LSL

As per Cp Value : make 2nd vertical Line (4) wrt (1)	
Representing LSL	Or representing USL

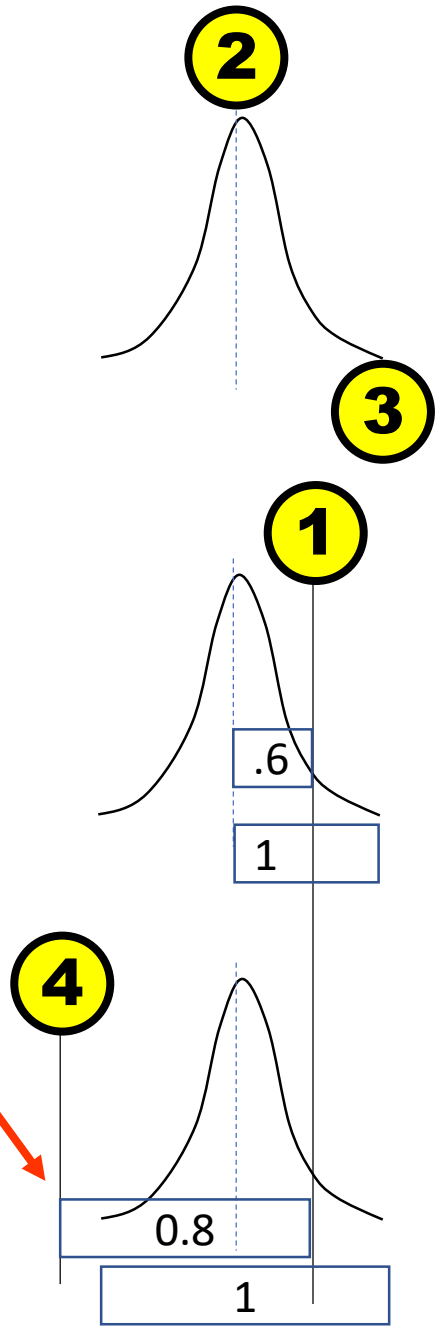


Practice : Cp = 0.8 , Cpk = 0.6

Make a random N-dist curve	
(2) : Pop. Mean	(3) : Pop. Extreme

As per Cpk Value : Make a vertical Line = (1)	
Representing USL	Or representing LSL

As per Cp Value : make 2nd vertical Line (4) **wrt (1)**	
Representing LSL	Or representing USL

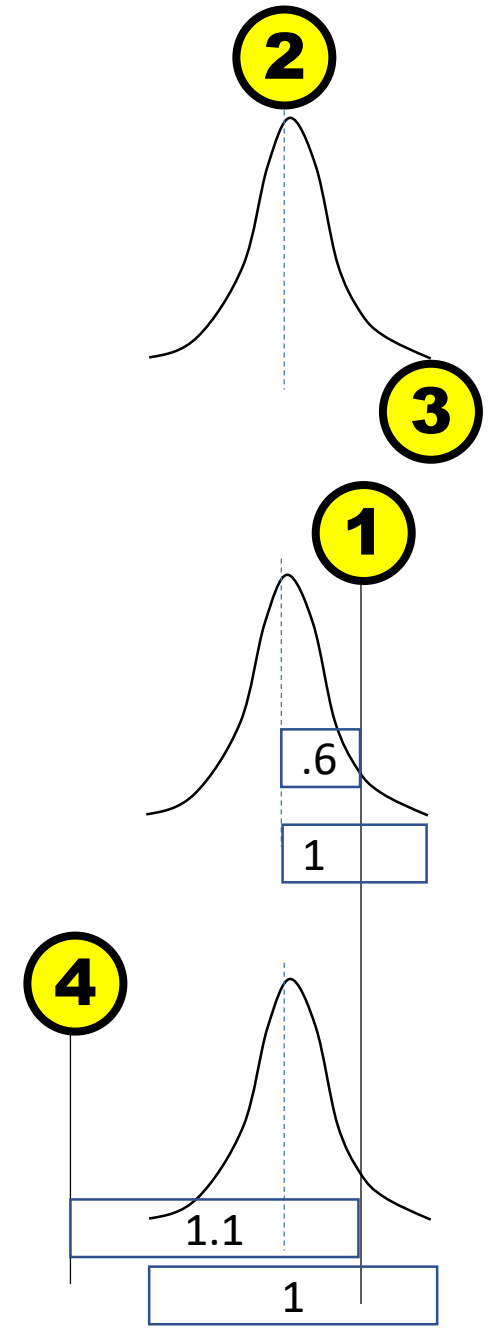


Practice : Cp = 1.1 , Cpk = 0.6

Make a random N-dist curve	
(2) : Pop. Mean	(3) : Pop. Extreme

As per Cpk Value : Make a vertical Line = (1)	
Representing USL	Or representing LSL

As per Cp Value : make 2nd vertical Line (4) wrt (1)	
Representing LSL	Or representing USL



Practice : Cp = 0.6 , Cpk = 0.6

Make a random N-dist curve

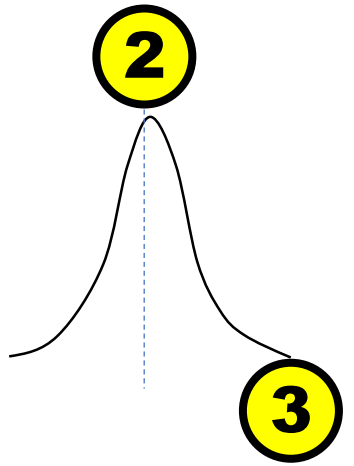
(2) : Pop. Mean | (3) : Pop. Extreme

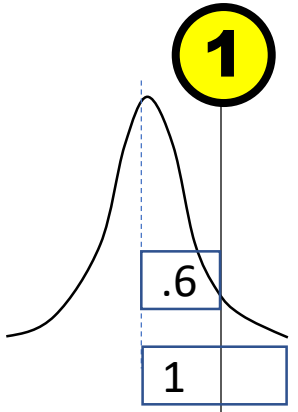
As per Cpk Value : Make a vertical Line = (1)

Representing USL | Or representing LSL

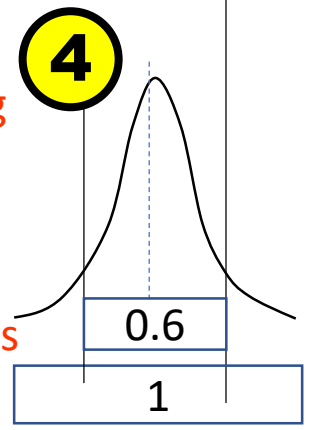
As per Cp Value : make 2nd vertical Line (4) wrt (1)

Representing LSL | Or representing USL





Cp=Cpk,
Centering
+
Cp < 1
Rejection
Both sides

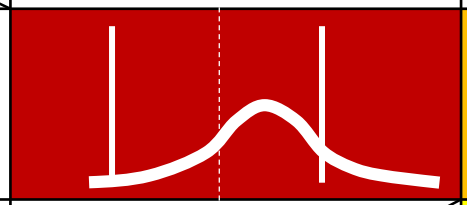
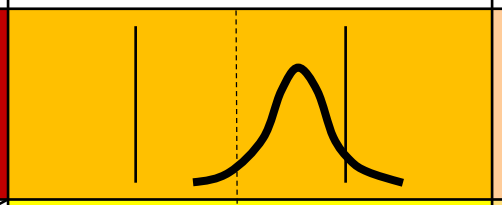
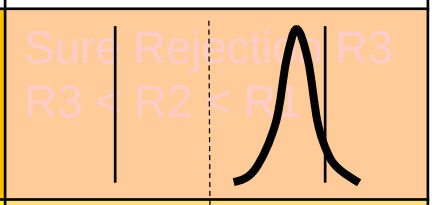
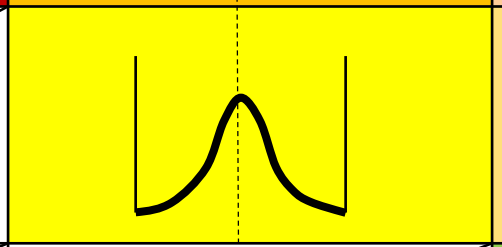
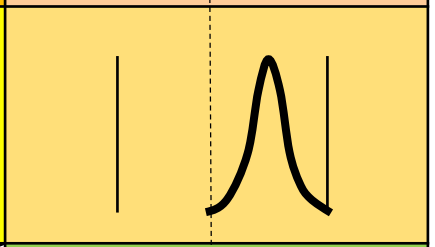
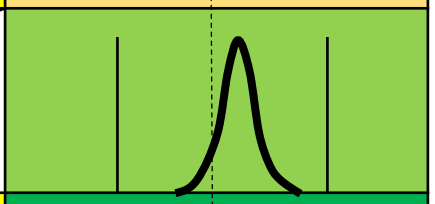
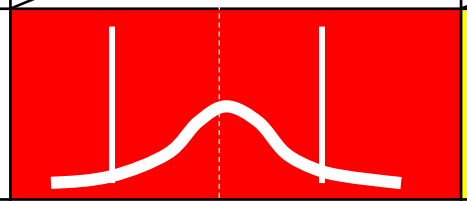
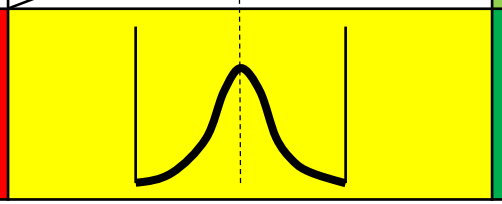
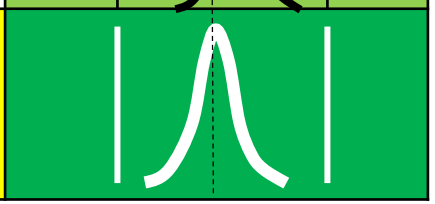


Cp = spread : Dwg / Population

For : Cpk > 0

(w.r.t. Popⁿ. Mean)

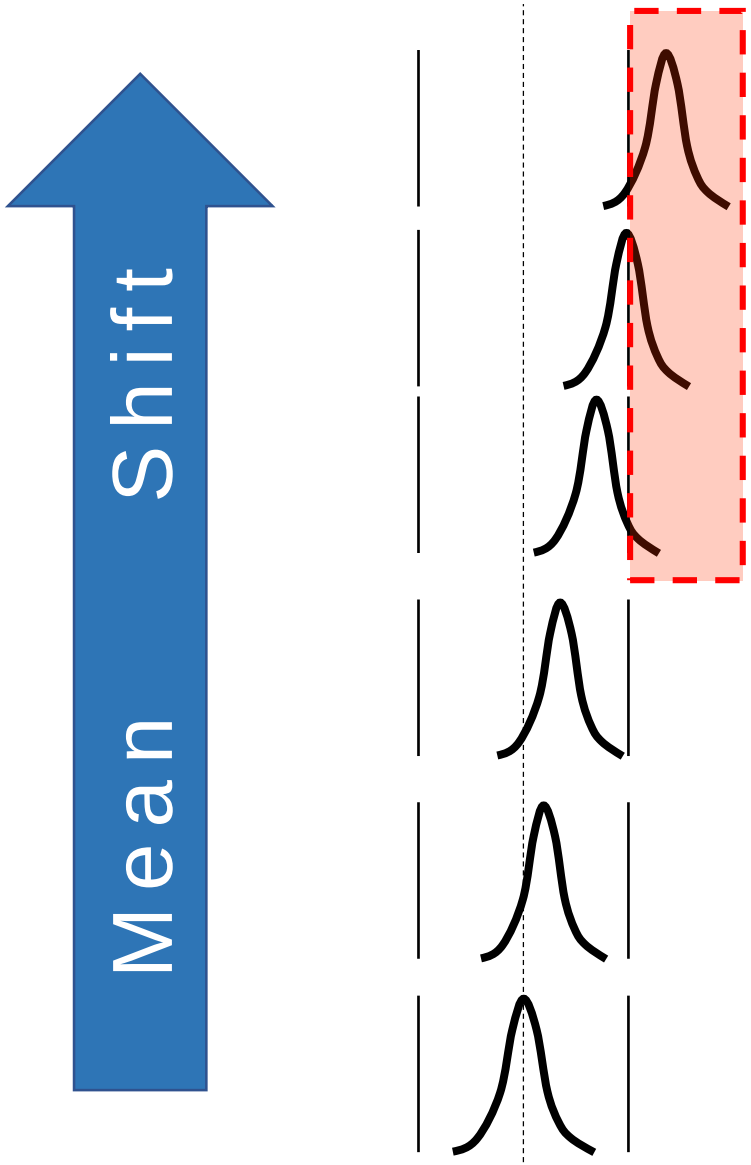
Cpk = Extremes : Dwg / Population

			Cp < 1	**Cp = 1**	**Cp > 1**
			Dwg spread < Population spread	Dwg spread = Population spread	Dwg spread > Population spread
			"Sure Rejection"	**"No FOS"**	**"FOS possible"**
Cpk < 1	Dwg. Extreme < Population Extreme	**"Sure Rejection"**			
Cpk = 1	Dwg. Extreme = Population Extreme	**"Borderline OK"**			
Cpk > 1	Dwg. Extreme > Population Extreme	**"Definite FOS"**			
If Cp = Cpk	Dwg. Mean = Population Mean	**"Centering"**			

Rejections are shown at USL as example. It is possible at LSL too.

With great powers come great responsibility

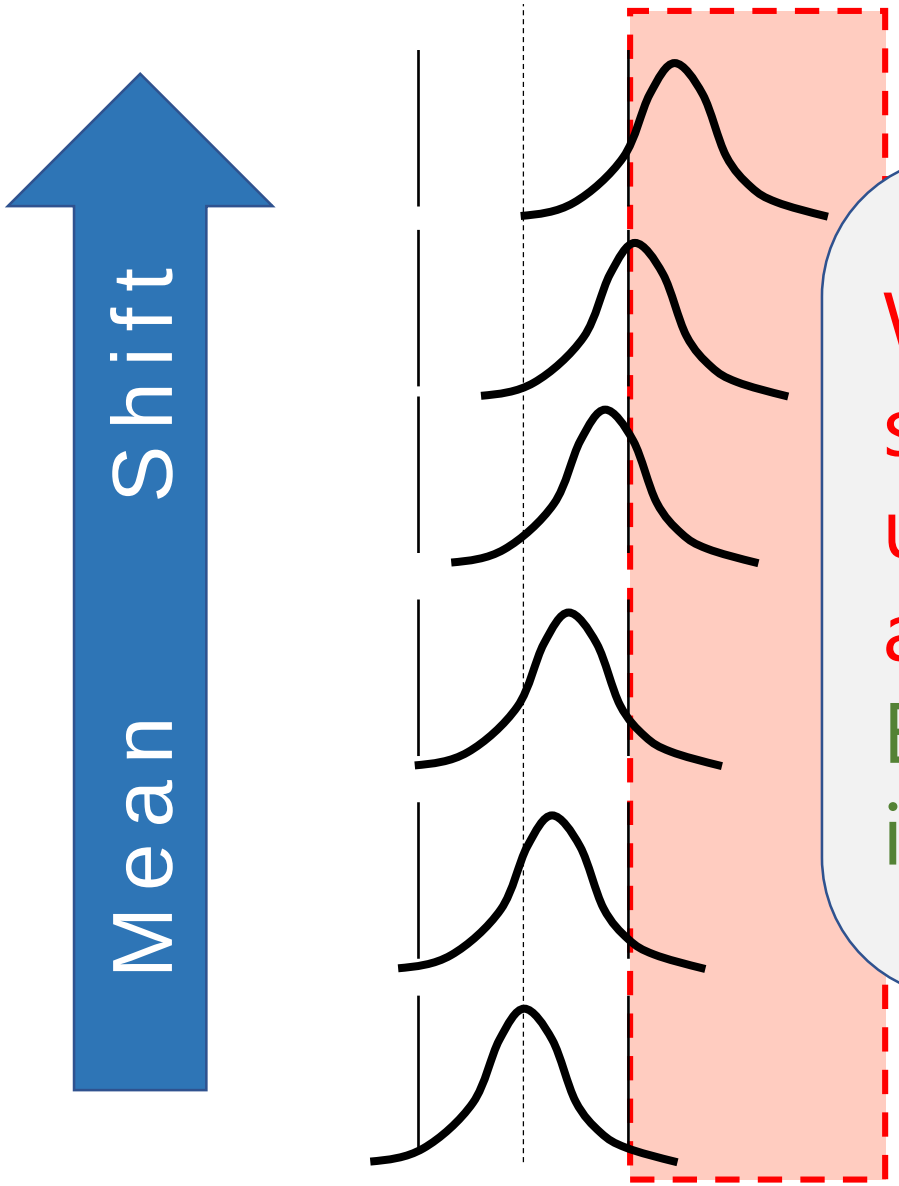
When mean shift happens to good Cp…



When a **good** Cp population shifts mean, it remains unrejected for a **longer** amount of shift.

But when it starts becoming NG, it becomes NG very **fast**

When mean shift happens to poor Cp…



When a **poor** Cp population shifts mean, it remains unrejected for a **not even slight** amount of shift.

But when it starts becoming NG, it becomes NG **relatively slowly**

FINISH