CSE/IT

Discrete Mathematics

Quantifier With Inference Rule

DPP-08

[MSQ]

1. Consider the given logical statement.

S: "There is exactly one apple on the table"

Domain is all objects on the table and A(x): x is an apple.

Which of the following is/are correct predicate logic represent the statement?

(a) $\exists x[A(x) \wedge \forall y(y \neq x \rightarrow \sim A(y))]$

(b) $\exists x[A(x) \wedge \forall y(A(y) \rightarrow y = x)]$

(c) $\exists x\ A(x) \wedge \forall x\ \forall y\ ((A(x) \wedge A(y) \rightarrow x = y))$

(d) $\forall y\ \forall x\ (A(x) \wedge \forall(y) \rightarrow x = y)$

[MCQ]

2. Which of the following is true about below predicate logic p?

p: $\sim\forall z\ [p(z) \rightarrow (\sim Q(z) \rightarrow p(z)]$

(a) p is contradiction

(b) p is tautology

(c) p is satisfiable

(d) none

[MCQ]

3. Consider the given premises:

P_1: Everyone in this discrete mathematics class has taken a course in computer science.

P_2: Madhu is a student in this class.

Which of the following is conclusion of the given premises?

(a) Madhu has not taken a course in computer science.

(b) Madhu has taken a course in computer science.

(c) Madhu has not studied discrete mathematics.

(d) None of these

[MCQ]

4. Consider the given premises

P_1: A student in this class has not read the book

P_2: Everyone in this class passed the first exam.

Which of the following is conclusion of the given premises?

(a) Someone who passed the first exam has read the book.

(b) Someone who passed the first exam has not read the book.

(c) Everyone read the book.

(d) None of the above

[MSQ]

5. Let the premises are as follows:

P: $\exists x\ p(x) \wedge \sim (\exists y\ (P(y) \wedge Q(y)))$

Which of the following can be concluded?

(a) $\sim \exists x\ Q(x)$

(b) $\sim \forall x\ Q(x)$

(c) $\forall x \sim Q(x)$

(d) None of these

Answer Key

1. **(a, b, c)**
2. **(a)**
3. **(b)**
4. **(b)**
5. **(a, c)**

Hints and Solutions

1. (a, b, c)

Option a: correct

The given predicate logic can be stated that there exists x which is an apple and all other object y on the table can not be the apple.

Option b: correct

$$\exists x\ \underbrace{[A(x)}_{\text{Apple on table}} \wedge \underbrace{\forall y\,(A(y) \to y = x)}_{\text{other apple(y) on the table are apple (x) only.}}$$

Option c: correct

$$\underbrace{\exists x A(x)}_{\text{Atleast one apple}} \wedge \underbrace{\forall y \forall y\,((A(x) \wedge A(y) \to x = y))}_{\text{Atmost one apple}}$$

Hence, the correct options are a, b and c.

2. (a)

I: The given predicate logic P:

$\sim\forall z[P(z) \to (\sim Q(z) \to P(z))]$

This evaluate to there exists a z for which $\sim(P(z) \to (\sim Q(z) \to P(z))$ is true.

II: Now, $\sim(P(z) \to (\sim Q(z) \to P(z)))$

$$\begin{cases} Q(z): \text{False} \\ p(z): \text{False} \end{cases}$$

$\therefore\ \sim(\text{False} \to (\sim\text{False} \to \text{False}))$

$\therefore\ \sim(\text{False} \to (\text{True} \to \text{False}))$

$\therefore\ \sim(\text{False} \to \text{False}) \equiv \sim(\text{True}) \equiv \text{False}$

III: Therefore, the predicate:

$\exists z \sim [P(z) \to (\sim Q(z) \to P(z))]$

is false for all cases. Therefore this is a contradiction.

3. (b)

I. Let D(x) denotes "x is in this discrete mathematic class".

C(x) denotes "x has taken a course in computer science.

Then:

P_1: $\forall x(D(x) \to C(x))$

P_2: D(Madhu)

II: Now, following steps can be used to establish the conclusion from the premises.

Step	Reason
1. $\forall x(D(x) \to C(x))$	Premise
2. $D(\text{Madhu}) \to C(\text{Madhu})$	Universal instantiation (1)
3. D(Madhu)	Premise
4. C(Madhu)	Modus ponens from (2) and (3)

Hence, option B is correct.

4. (b)

I: Let C(x) be "x is in this class".

B(x) denotes "x has read the book".

P(x) denotes "x passed the first exam".

Then:

P_1: $\exists x\,(C(x) \wedge \sim B(x))$

P_2: $\forall x\,(C(x) \to B(x))$

II: Now, these steps can be used to establish the conclusion from the premises.

Steps	Reason
1. $\exists x\,(C(x) \wedge \sim B(x))$	Premise
2. $C(a) \wedge \sim B(a)$	Existential instantiation (1)
3. C(a)	Simplification form (2)
4. $\forall x\,(C(x) \to P(x))$	Premise
5. $C(a) \to P(a)$	Universal instantiation (4)
6. P(a)	Modus ponens from (3) and (5)
7. $\sim B(a)$	Simplification from (2)
8. $P(a) \wedge \sim B(a)$	Conjunction from (6) and (7)
9. $\exists x\,(P(x) \wedge \sim B(x))$	Existential generalization from (8)

Hence, the conclusion "someone who passed the first exam has not read the book."

5. (a, c)

These steps can be used to establish the conclusion from the premises.

Steps	**Reason**
1. $\exists x\ p(x) \wedge \sim(\exists y\ (P(y) \wedge Q(y)))$	Premise
2. $\exists x\ p(x)$	Specialization (1)
3. $\sim(\exists y\ (P(y) \wedge Q(y))$	Specialization (1)
4. $\forall y \sim (P(y) \wedge Q(y))$	Negation of $\exists$ (3)
5. $\forall y\ (\sim P(y) \vee \sim Q(y))$	De Morgan (4)
6. $P(x)$	E.I (2)
7. $\sim P(x) \vee \sim Q(x)$	U.I (5)
8. $\sim Q(x)$	Elimination ()
9. $\forall x \sim Q(x)$	U.G (8)
10. $\sim\exists x\ Q(x)$	Negation of $\exists$ (9)

Hence, option a and c are equivalent.