

Discrete Mathematics

Coloring In Graphs

DPP-08

[MCQ]

1. Consider the following statements:

S_1 : If a connected graph G has a cut vertex, then G has a cut edge.

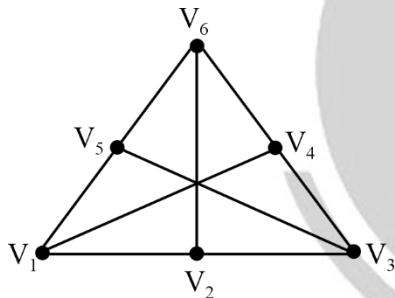
S_2 : If a connected graph G has a cut edge then G has a cut vertex.

Which of the following is true?

- (a) S_1 only (b) S_2 only
(c) Both S_1 and S_2 (d) Neither S_1 nor S_2

[NAT]

2. For the graph shown below, the chromatic number is _____.

**[NAT]**3. If G is a connected graph with 10 vertices and vertex connectivity is 3, then minimum number of edges necessary in G is _____.**[MSQ]**

4. which of the following options is/are correct?

- (a) The chromatic number of a graph with at least 1 edge is at least 2.

- (b) A graph is null graph if and only if its chromatic number is 2.
(c) For any graph, $K_G \leq 1 + \Delta(G) \leq n$, where $\Delta(G)$ is maximum degree and K_G is chromatic number.
(d) The chromatic number of a multi graph is equal to its equivalent simple graph chromatic number.

[MCQ]

5. Consider the following statements:

S_1 : A graph is bipartite graph if and only if its chromatic number is 2.

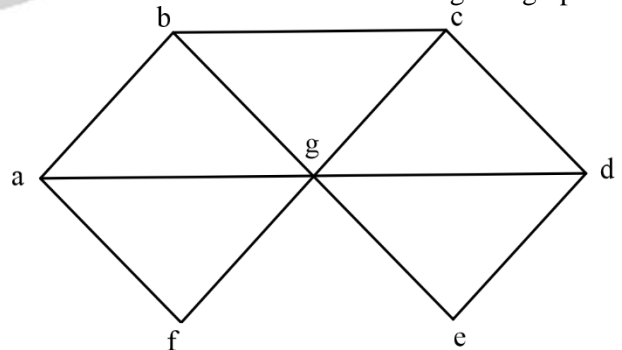
S_2 : The chromatic number of a tree is 2. Thus, every tree is bipartite graph.

Which of the following statement is False?

- (a) S_1 only
(b) S_2 only
(c) Both S_1 and S_2
(d) Neither S_1 nor S_2

[NAT]

6. What is the chromatic number of the given graph?



Answer Key

- | | |
|--------------|--------|
| 1. (d) | 5. (a) |
| 2. (2) | 6. (3) |
| 3. (15) | |
| 4. (a, c, d) | |

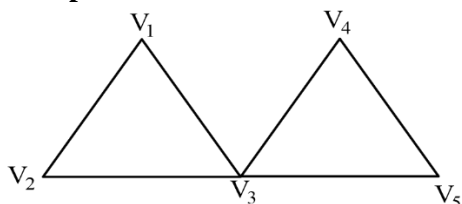


Hints and Solutions

1. (d)

Statement S1 : False

Example:

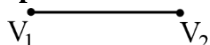


In the above graph G has cut vertex (V_3) but it does not have cut edge.

Hence, the statement S_1 is false.

Statement S_2 : False

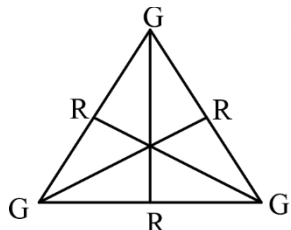
Example:



The complete graph with 2 vertices have only cut edge (V_1, V_2). It does not have cut vertex.

Hence, statement S_2 is also false.

2. (2)



Hence, we need only 2 color to completely cover the vertices. So, the chromatic number is 2.

3. (15)

In the problem, we have a connected graph G with 10 vertices and $VC = 3$.

As we know that the relation between VC, EC and minimum degree is:

$$VC \leq EC \leq \delta(G)$$

$$\therefore 3 \leq EC \leq \delta(G)$$

From the above equation, we can conclude that the minimum degree of vertex would be 3.

$$\therefore \delta(G) = 3$$

Now, by using handshaking lemma:

$$\text{Sum of degree} = 2 * |E|$$

$$\therefore 10 * 3 = 2 * |E|$$

$$\therefore |E| = \frac{30}{2} = 15 \text{ edges}$$

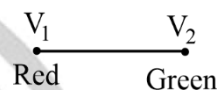
Hence, 15 edges necessary in graph G to have vertex connectivity = 3.

4. (a, c, d)

Option A : Correct

The chromatic number of a graph with at least 1 edge is at least 2.

Example,



Option B: Incorrect

A graph is null graph if and only if its chromatic number is 1.

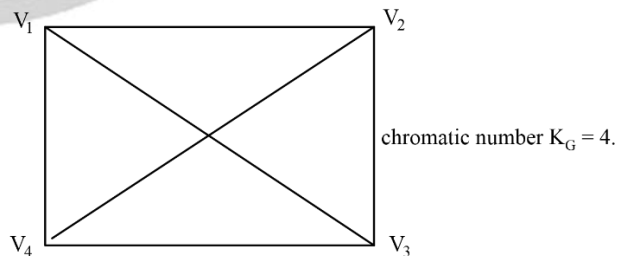
Example:



Option C : Correct

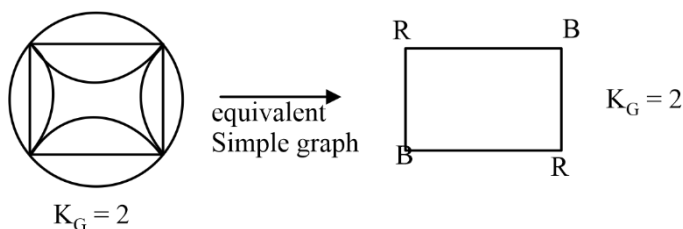
The chromatic number will be less than or equal to " $1 + \Delta(G)$ ".

Example: The complete graph (K_4) have $\Delta(G) = 3$.



Option d : Correct

Example



5. (a)

Statement S_1 : False

Null graph is a graph with no edges and 1 or n vertices and every null graph is bipartite graph.

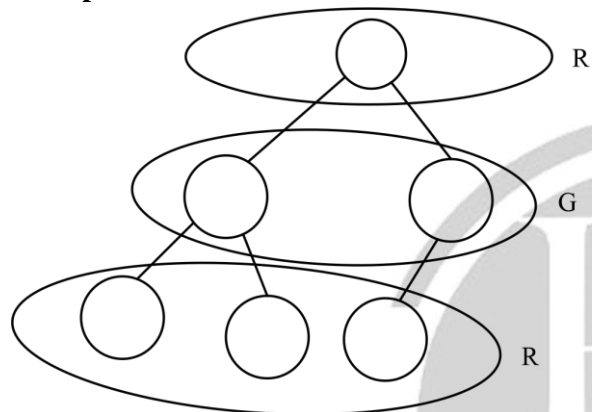
As we know that a null graph is 1 colorable.

Note: A non – null graph is bipartite graph iff its chromatic number is 2.

Statement S_2 : True

Every tree is a bipartite graph. So, the chromatic number is 2.

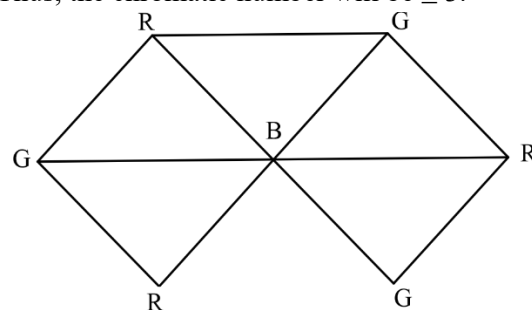
Example



6. (3)

The given graph have complete graph with 3 vertices (K_3).

Thus, the chromatic number will be ≥ 3 .



Hence, the chromatic number is 3.



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