CSE/IT

Discrete Mathematics

Independence Number & Domination Number

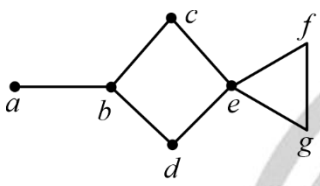
DPP-09

[NAT]

1. If G is a bipartite graph with 6 vertices and 9 edges then the chromatic number of $\overline{G}$ =_____.

[MSQ]

2. Consider the graph shown below.



Which of the following option is correct?

(a) Dominating set = {*e*, *b*} and Domination no = 2

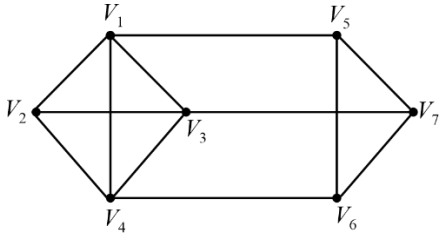
(b) Dominating set = {*a*, *c*,*d*, *f*} and Domination no = 4

(c) Dominating set = {*b*, *f*} and Domination no = 2

(d) None of these

[NAT]

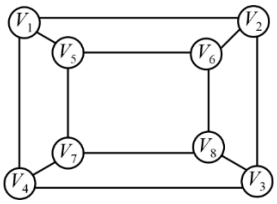
3. For the graph shown below.



Assume *x* is the chromatic number of the graph and *y* is the domination number then find *x* + *y*?

[MCQ]

4. Which of the following is/are a independent set for the graph shown below?



(a) $\{V_1,V_8,V_2\}$

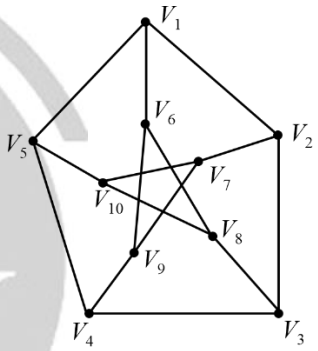
(b) $\{V_1,V_8\}$

(c) $\{V_2,V_4,V_5,V_8\}$

(d) $\{V_1,V_3,V_6,V_7\}$

[MCQ]

5. Consider the given graph G.



Which of the following option is correct ?

S_1 : The chromatic number for the given graph is 3.

S_2 : The independence number of the graph is 4.

(a) S_1 only

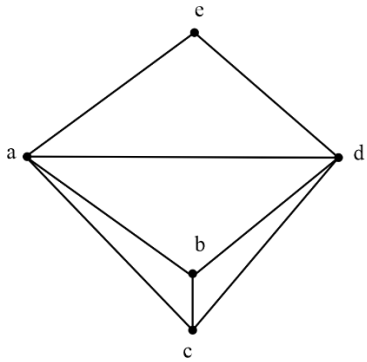
(b) S_2 only

(c) S_1 and S_2 both

(d) Neither S_1 nor S_2

[NAT]

6. For the graph shown below, the chromatic number is ____.



Answer Key

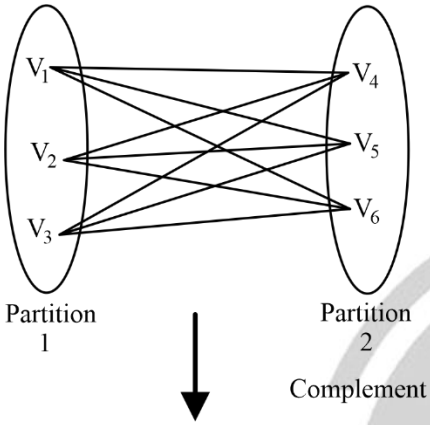
1. **(3)**
2. **(a, c)**
3. **6**
4. **(b, c, d)**
5. **(c)**
6. **(4)**

Hints and solutions

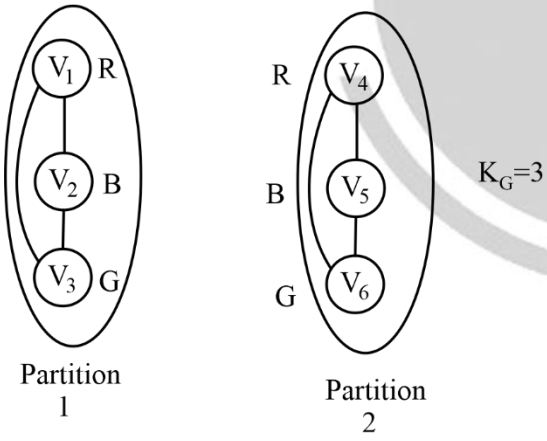
1. (3)

In the problem we have a bipartite graph with 6 vertices and 9 edges

So,



The complement of the bipartite graph will have complete graph with 3 vertices within partition 1 and partition 2.



Hence, the chromatic number of $\overline{G}$ is 3

2. (a, c)

I. Dominating Set: The set of vertices from which the whole graph can be covered in the single move.

II. Domination No: The smallest/minimal dominating set.

Option a and c: correct

The vertex sets {e, b} and {b, f} covers the complete graph and it is also the minimal dominating set with Domination number is 2.

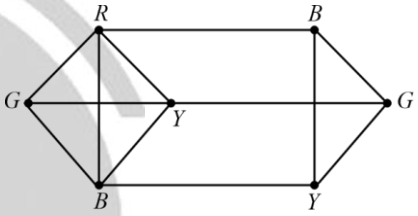
Option b: Incorrect

As the vertex set {a, c, d, f} is the dominating set but it is not the minimal dominating set.

Hence, the domination number 4 is incorrect

3. (6)

I. Chromatic number: The above graph have complete graph (K_4) so, the '$K_G \le 1 + \Delta(G)$' and due to complete graph 'K_4', we need at least '4' color for K_4.



Hence, the chromatic number is 4 so, $x = 4$.

II. Domination Number: To find the minimal dominating set, always start with maximum degree vertex.

$\therefore$ Dominating set = $\{V_1, V_6\}$

Hence, the domination No. is 2 so, $y = 2$

$\therefore x + y = 4 + 2 = 6$

4. (b, c, d)

Independent Set : The set of vertices which are not adjecent to each other.

Option a : Incorrect

As vertex V_1 is adjecnt to vertex V_2.

Hence, given vertex set is not independent set.

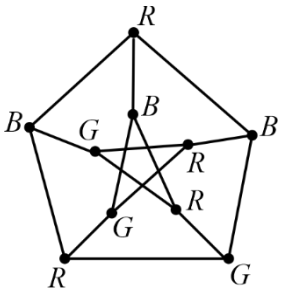
Option b, c, d : Correct

All the vertices of option b, c, d are adjecent to none within the set.

Hence, b, c, d are the independent set of given graph.

5. (c)

Statement S_1 : True



Hence, the chromatic number of the graph is 3.

Statement S_2 : True

All the red (R) colored vertices could be the maximal independent set.

Independent Set : $\{V_1, V_4, V_7, V_8\}$

Hence, the independence number of the graph is 4

6. (4)

The graph contains complete graph of 3 vertices (K_3) so, the chromatic number will be at least 3.

Hence, the chromatic number is 4.