CSE/IT

Discrete Mathematics

Nested Quantifiers

DPP-07

[MCQ]

1. Consider
Actor (x) = x is an actor
Smart (x) = x is smart
and the well-formed formula:
fx (Actor(x) $\wedge$ Smart (x))
Choose the correct representation of above in english sentence.

(a) Some Actor is smart.
(b) Some Actor is not smart.
(c) All actors are smart.
(d) All smart are actors.

[MCQ]

2. Consider the following statement
"There is exactly one apple".
Let G(x) : x is an apple.
Now consider the predicate logic statements:
I. $\exists x$ apple (x) $\wedge \forall_y$ (apple (y)) $\Rightarrow$ x = y)
II. $\exists x$ apple (x)
The correct representation in predicate logic is ?
(a) Only I
(b) Only II
(c) Both I and II
(d) Neither I and II

[NAT]

3. Consider the following statements:
I : $\exists x \{p(x) \rightarrow \{\forall x P(x) \rightarrow$ fx $Q(x)\}$
II. $\exists x \forall_y P(x, y) \rightarrow \forall_y \exists x P(x, y)]$
The number of valid statements is/are _____.

[MSQ]

4. Choose among the following that are not equivalent to the given first order logic statement:
$(\exists x)(\forall y)[p(x, y) \wedge q(x, y) \wedge \neg r(x, y)]$

(a) $(\forall x)(\exists y)[p(x, y) \wedge q(x, y) \rightarrow r(x, y)]$
(b) $(\exists x)(\forall y)[p(x, y) \vee q(x, y) \wedge \neg r(x, y)]$
(c) $\neg (\forall x)(\exists y)[p(x, y) \vee q(x, y) \rightarrow r(x, y)]$
(d) $\neg (\forall x)(\exists y)[p(x, y) \wedge q(x, y) \rightarrow r(x, y)]$

[MCQ]

5. Choose the correct representation for the below statement:
"Every player is liked by some coach"
(a) $\forall(x)$ [player (x) $\rightarrow \exists y$ [coach (y) $\wedge$ likes (y, x)]]
(b) $\forall(x)$ [player (x) $\rightarrow \exists y$ [coach (y) $\rightarrow$ likes (y, x)]]
(c) $\exists$ (x) [player (x) $\rightarrow \forall y$ [coach (y) $\rightarrow$ likes (y, x)]]
(d) $\exists$ (x) [player (x) $\rightarrow \forall y$ [coach (y) $\wedge$ likes (y, x)]]

Answer Key

1. **(a)**
2. **(a)**
3. **(2)**
4. **(a, b, c)**
5. **(a)**

Hints and Solutions

1. (a)

- f(x) represents some/any/atleast one:
- Actor (x) $\wedge$ Smart (x) means x is an actor and smart.
- Therefore fx (Actor (x) $\wedge$ smart (x)) represents some actor is smart.

2. (a)

I is the correct representation as it reads "there exist an apple x and there exist an apple y and if apple y exists then it is equal to x" that means there is only one apple (exactly one).

II is absolutely incorrect as it says "some apple or at least one apple" instead of exactly one apple.

3. (2)

I: $\exists x \{P(x) \rightarrow Q(x)\} = \exists x \{\neg (P(x)) \vee Q(x)\}$

$= \{\exists x \neg (P(x)) \vee \exists x\, Q(x)\}$

$= \forall x\, P(x) \rightarrow \exists x\, Q(x)\}$ $\therefore$ True.

II. $\exists x \forall_y P(n, y)$

$\forall_y \; P(a, y)$ for same a

$P(a, b)$ is true for $\forall_b = \exists x\, P(x, b)$

$\forall_y \; \exists x\, P(x, y)$ is ture.

4. (a, b, c,)

Two points/rules to solve the question:

I. $\exists x\, f(x) \equiv \neg \forall (x)\, \neg f(x)$

II. $\forall x\, f(x) \equiv \neg\, \exists (x\,)\, \neg f(x)$

The given statement:

$(\exists x)\, (\forall y)\, [p(x, y) \wedge q(x, y) \wedge \neg r(x, y)]$

Can be written as: -

$\neg\, (\forall x)\, \exists (y)\, [p(x, y) \wedge q(x, y) \rightarrow r(x, y)]$

NOTE: $[\neg\, (x \wedge y) \vee 3 \equiv x \wedge y \rightarrow 3\,]$

5. (a)

We write "Every player" as $\forall x$[player (x) $\rightarrow$]

'There is some coach who likes x" as $\exists y$ [coach (y) p(x)]

Where P is the property.

Therefore we can write the first order logic for the given statement as

$\forall (x)$ [player (x) $\rightarrow \exists y$ [coach (y) $\wedge$ likes (y, x)]]