CSE/IT

Discrete Mathematics

Predicate Logic

DPP-04

[MSQ]

1. Let $R(x, y, z)$ denote the statement

"$x + y = z$"

Which of the following proposition will evaluate truth value True?

(a) $R(1, 2, 3)$ (b) $R(0, 0, 1)$

(c) $R(1, 1, 2)$ (d) $R(2, 3, 4)$

[MCQ]

2. Let $p(x)$, $q(x)$ denote the following open statements.

$p(x)$: $x \le 3$ $q(x)$: $x + 1$ is odd

If the universe consists of all integers, what are the truth values of the following statements?

S_1: $\sim(p(-4) \vee q(-3))$

S_1: $\sim(p(-4) \wedge \sim q(-3))$

(a) S_1 : True, S_2 : False

(b) S_1 : False, S_2 : True

(c) S_1 : True, S_2 : True

(d) S_1 : False, S_2 : False

[NAT]

3. Let $p(x)$, $q(x)$ denote the following open statements.

$p(x)$: $x+1>x$ $q(x)$: $x^2>0$

How many expressions evaluate to True?

I. $p(3) \vee [q(3) \vee \sim p(3)]$

II. $p(2) \rightarrow [q(2) \rightarrow p(2)]$

III. $[p(2) \rightarrow q(2)] \wedge p(-3)]$

[MSQ]

4. Consider the english sentence

"You can not ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old".

Which of the following correctly respresent the logical expression for the sentence?

(a) $q \rightarrow \sim(r \wedge \sim S)$

(b) $(r \vee \sim S) \rightarrow q$

(c) $(r \wedge \sim S) \rightarrow \sim q$

(d) None of these

[MCQ]

5. Let $p(x)$ be the statement

"$x + 1 > x$"

Now, consider the truth value of quantification, where the domain consists of all real number.

$L_1 = \forall x\ p(x)$

$L_2 = \exists x\ p(x)$

Which of the following evaluate to True?

(a) L_1 only

(b) L_2 only

(c) Both L_1 and L_2 are True

(d) Neither L_1 nor L_2

Answer Key

1. **(a, c)**
2. **(d)**
3. **(3)**
4. **(a, c)**
5. **(c)**

Hints and Solutions

1. (a, c)

I. The proposition $R(1, 2, 3)$ is obtained by setting $x = 1$, $y = 2$ and $z = 3$ in the statement $R(x, y, z)$

So, $R(1, 2, 3) \equiv 1 + 2 = 3 \equiv$ True

II. $R(1, 1, 2) \equiv 1 + 1 = 2 \equiv$ True

Hence, option a and c is correct.

2. (d)

Statement S_1:

$$\sim (p(-4) \vee q(-3))$$
$$\Downarrow \quad \Downarrow$$
$$-4 \le 3 \qquad -3+1 = -2 \text{ is not odd}$$
$$\Downarrow$$

$\therefore \sim (\text{True} \vee \text{False})$

$\therefore \sim (\text{True}) \equiv \text{False}$

Statement S_2:

$$\sim p(-4) \wedge \sim q(-3)$$
$$\Downarrow$$
$$\sim (\text{True}) \wedge \sim (\text{False})$$

$\therefore$ False $\wedge$ True = False

Hence, option d is correct

3. (3)

I.

$$p(3) \quad \vee \quad [q(3) \vee \sim p(3)]$$
$$\Downarrow \qquad \Downarrow \qquad \Downarrow$$
$$3+1>3 \quad 3^2>0 \quad \text{True}$$
$$\Downarrow \qquad \Downarrow$$
$$\text{True} \qquad \text{True}$$

$\therefore$ True $\vee$ [True $\vee \sim$True]

$\therefore$ True $\vee$ True = True

II.

$$p(2) \rightarrow [q(2) \rightarrow p(2)]$$
$$\Downarrow \qquad \Downarrow \qquad \Downarrow$$
$$\text{True} \qquad \text{True} \qquad \text{True}$$

$\therefore$ True $\rightarrow$ [True $\rightarrow$ True] $\equiv$ True

III.

$$[p(2) \rightarrow q(2)] \wedge p(-3)$$
$$\Downarrow \qquad \Downarrow \qquad \Downarrow$$
$$\text{True} \qquad \text{True} \qquad \text{True}$$

$\therefore$ [True $\rightarrow$ True] $\wedge$ True $\equiv$ True

4. (a, c)

I. Let q, r, and s represents:

q: you can ride the roller coaster

r: you are under 4 feet tall

s: you are older than 16 years old.

$\therefore$ The sentence can be translated to

$(r \wedge \sim s) \rightarrow \sim q$

II. An implication and its contrapositive always have the same truth value.

So, $q \rightarrow \sim (r \wedge \sim s)$ also represent the sentence.

5. (c)

I. $L_1 = \forall x\ p(x) : \text{True}$

Here $p(x)$ is true for all real number x, so, the quantification $\forall x\ p(x)$ is True.

II. $L_2 = \exists x\ p(x) : \text{True}$

Here $p(x)$ is true for all real number

Thus, it will also true for same.