CSE/IT

Discrete Mathematics

Connectivity Part – 3

DPP-07

[NAT]

1. If G is a bipartite graph with 9 vertices and maximum number of edges, then vertex connectivity of G = ____.

[MSQ]

2. Which of the following options is/are correct?

(a) A graph G is Euler iff it is connected and $\forall v \in G$ degree (v) = even.

(b) A K – regular graph is Euler iff K is even

(c) A wheel graph (w_n) can have Euler circuit.

(d) A graph will contain an Euler path if it contains at most two vertices of odd degree.

[MCQ]

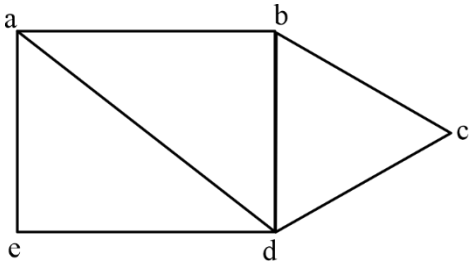
3. A forest is disconnected graph in which each component is a tree. Let F be a forest on 80 vertices with 21 connected components. Then number of edges in G is _____.

(a) 58 (b) 60

(c) 59 (d) 101

[MCQ]

4. For the graph shown below



Which of the following statements is/are true?

S_1 : Euler path exists

S_2 : Euler circuit exists

S_3 : Hamiltonian cycle exists

S_4 : Hamiltonian path exists

(a) S_1, S_3 and S_4

(b) S_1, S_2 and S_3

(c) S_1, S_2 and S_4

(d) S_2, S_3 and S_4

[MCQ]

5. Which of the following is Euler Graph ?

(a) K_{51}

(b) K_{50}

(c) $\overline{C_{60}}$

(d) 11- regular

[MCQ]

6. If G is not a simple connected graph with n vertices then maximum number of edges possible in G is ___.

(a) $\frac{n(n-2)}{2}$

(b) $\frac{(n-1)n}{2}$

(c) $\frac{(n-1)(n-2)}{2}$

(d) $\frac{(n-1)(n-2)}{4}$

Answer Key

1. **(4)**
2. **(a, d)**
3. **(c)**
4. **(a)**
5. **(a)**
6. **(c)**

Hints and Solutions

1. (4)

I. The number of vertices in bipartite graph is given as: m + n

where m = No of vertices in partition 1

n = No of vertices in partition 2

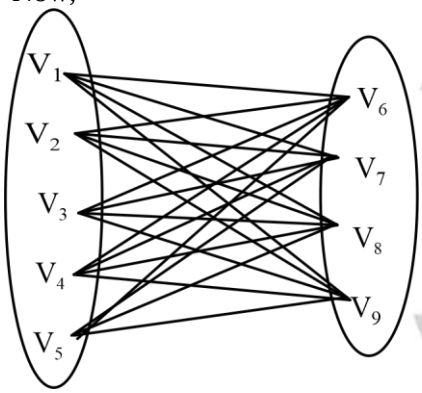
Now, we have 9 vertices in the given bipartite graph so, divide the vertices in such a way so, that we have maximum number of edges.

$\therefore$ Number of edges $= m * n$

$= 5 * 4 = 20$ edges

Here m = 5 is partition 1 vertices and n = 4 is partition 2 vertices.

II. Now,



If we remove partition 2 vertices then graph will disconnect. Hence, vertex connectivity for the given graph is 4.

2. (a, d)

Option a : correct

In the Euler graph every degree must be even because if we enter into a vertex in order to cover the edge then we should exit to complete the cycle or to reach the starting vertex.

Option b : Incorrect

K – regular graph with K is even mean all the degrees are even but the graph may itself not connected.

Hence, the statement is incomplete.

Option c : Incorrect

A wheel graph (W_n) is not Euler graph because the vertices at the outer edge (i.e n –1 vertices) will always have odd degree vertices that is degree 3.

Option d : Correct

To have a Euler path, the graph must have 2 odd degree vertices. Such Euler path will start form one of those odd degree vertex and ends at other odd degree vertex.

3. (c)

We know that a forest on n vertices with K. connected components have (n – K) number of edges.

Now, in the problem,

Number of vertices = 80

Number of Connected Component = 21

$\therefore$ Number of edges = 80 – 21

= 59 edges.

4. (a)

I. Euler path exists as in the graph 2 odd vertices are present.

Path : b – c – d – b – a – e – d – a

II. An Euler circuit exists in the graph iff all the degrees are even. In the given we have 2 odd degree vertices so, graph does not have Euler circuit.

III. If a graph have Hamiltonian cycle then it have also Hamiltonian path.

$\therefore$ Hamilton cycle : a b c d e a

Hence, option a is correct.

5. (a)

A graph G is Euler if and only if it is connected and all the vertices must have even degree.

Now, the degree of each vertex of K_n complete graph is (n – 1).

Option a : Correct

The degree of each vertex (K_{51})

$= (51 - 1)$

$= 50$

So, the degree of all the vertices is even. Hence, K_{51} is Euler graph

Option b : Incorrect

The complete graph K_{50} have odd degree vertices.

Option c : Incorrect

A cycle graph is always Euler graph mean C_{60} is euler graph but its complement $\overline{C_{60}}$ may be disconnect graph. Hence, it is also not a Euler graph.

Option d : Incorrect

It would not be Euler graph if 11 – regular is disconnected graph.

6. (c)

Here the graph is disconnected graph with n vertices. So, to get the maximum number of edges, try to make single component with maximum vertices.

$$\underbrace{V_1,\ V_2,\ V_3 \dots V_{n-1}}_{\text{Component 1}} \qquad \underbrace{V_n}_{\text{Component 2}}$$

So,

Now, The maximum number of edges with (n – 1) vertices is : $\frac{(n-1)(n-2)}{2}$.