# CSE/IT

# Discrete Mathematics

## Logical Equivalence

**DPP-02**

**[MSQ]**

**1.** Which of the following is/are logical equivalence?

I. $\sim(p \to q)$

II. $(p \to q) \wedge (q \to r)$

III. $p \wedge \sim q$

IV. $(p \vee q) \to r$

(a) I and II (b) I and III

(c) II and IV (d) II and III

**[MSQ]**

**2.** Consider the following statement

$S_1$: $(p \to q) \wedge (p \to r)$

$S_2$: $p \to (q \wedge r)$

Which of the following is True?

(a) $S_1$ is tautology

(b) $S_1$ is contingency

(c) $S_1$ is logically equivalence to $S_2$

(d) None of these

**[MSQ]**

**3.** Which of the following is logically equivalence?

(a) $(p \to r) \vee (q \to r)$

(b) $(p \leftrightarrow q) \vee (q \to r)$

(c) $(p \wedge q) \wedge r$

(d) $(p \leftrightarrow r) \wedge (q \leftrightarrow r)$

**[MCQ]**

**4.** Consider the following statement

$S_1$: $\sim (p \leftrightarrow q)$

$S_2$: $p \leftrightarrow \sim q$

Which of the following is correct?

(a) $S_1$ is tautology

(b) $S_2$ is contradiction

(c) $S_1$ is equivalence to $S_2$

(d) None of these

**[MCQ]**

**5.** Consider the following statement

$S_1$: $\sim (p \vee (\sim p \wedge q))$

$S_2$: $\sim p \wedge \sim q$

Which of the following is correct?

(a) $S_1$ is tautology

(b) $S_2$ is contradiction

(c) $S_1$ is equivalence to $S_2$

(d) $S_1$ is not equivalence to $S_2$

# Answer Key

1. **(b, c)**
2. **(b, c)**
3. **(a, b, c)**
4. **(c)**
5. **(c)**

# Hints and solutions

**1. (b, c,)**

Two statements forms are logical equivalent if and only if their resulting truth values are identical for each variation of statement variables.

**I.** $\sim (p \to q)$

$= \sim(\sim p \vee q)$

$= p \wedge \sim q$

Hence, I is logically equivalent to III.

**II.** $(p \to r) \wedge (q \to r)$

$= (\bar{p} + r) \wedge (\bar{q} + r)$

$= \bar{p}\,\bar{q} + \bar{p}\,r + \bar{q}\,r + r$

$= \bar{p}\,\bar{q} \mid \bar{p}\,r + r$

$= \bar{p}\,\bar{q} + r$

$= \overline{(p \vee q)} + r \equiv (p \vee q) \to r$

Hence, II and IV are logically equivalence.

**2. (b, c)**

Statement $S_1$: $(p \to q) \wedge (p \to r)$

$= (\bar{p} + q) \wedge (\bar{p} + r)$

$= \bar{p} + \bar{p}\,r + \bar{p}\,q + qr$

$= \bar{p} + \bar{p}\,q + qr$

$= \bar{p} + qr$

$= p \to (q \wedge r) \neq 1$

Hence, $S_1$ is not tautology and $S_1$ is logically equivalent to $S_2$.

Statement $S_2$: $p \to (q \wedge r)$

$= \bar{p} + (q \wedge r)$

$= \bar{p} + qr \neq 1$ or $0$

Hence, statement $S_2$ is contingency.

**3. (a, b, c)**

Option A: $(p \to r) \vee (q \to r)$

$= (\bar{p} + r) \vee (\bar{q} + r)$

$= \bar{p} + r + \bar{q} + r$

$= \bar{p} + \bar{q} + r$

$= \overline{pq} + r \equiv \overline{(p \wedge q)} + r$

$\equiv (p \wedge q) \to r$

So, option A is logically equvalence to option C.

Option B: $(p \leftrightarrow r) \vee (q \to r)$

$= \bar{p}\,\bar{r} + pr + \bar{q} + r$

$= \bar{p}\,\bar{r} + \bar{q} + pr + r$

$= \bar{p}\,\bar{r} + \bar{q} + r$

$= \bar{p}\,\bar{r} + r + \bar{q}$

$= \bar{p} + r + \bar{q}$

$= \bar{p} + \bar{q} + r$

$= \overline{(p \wedge q)} + r = (p \wedge q) \to r$

So, option B is also logically equvalence to option A.

**4. (c)**

Statement $S_1$: $\sim (p \leftrightarrow q)$

$= \sim (\bar{p}\,\bar{q} + pq)$

$= (p + q)\,(\bar{p} + \bar{q})$

$= p\,\bar{q} + q\,\bar{p}$

Statement $S_2$: $p \leftrightarrow \sim q$

$= \bar{p}\,q + p\,\bar{q}$

Hence, $S_1$ and $S_2$ are equivalence to each other.

**5. (c)**

Statement $S_1$: $\sim (p \vee (\sim p \wedge q))$

$= \sim p \wedge [\sim (\sim p \wedge q)]$

$= \sim p \wedge [\sim (\sim p) \vee \sim q]$

$= \sim p \wedge [p \vee \sim q]$

$= (\sim p \wedge p) \vee (\sim p \wedge \sim q)$

$= F \vee (\sim p \wedge \sim q)$

$= (\sim p \wedge \sim q)$

$= \sim p \wedge \sim q)$

Hence, $S_1$ is equivalence to $S_2$.

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