# CSE/IT

# Discrete Mathematics
## Connectivity Part – 1

DPP-05

**[NAT]**

1. Consider an undirected graph G, which is connected and have 8 vertices and 13 edges. Find the minimum number edges, whose deletion from graph G is always guarantee that it would be disconnected graph.

**[MCQ]**

2. The order (Number of vertices) of a complete bipartite graph in which there are 162 edges and one of the partitions has twice the number of vertices as of other ____?
   (a) 20
   (b) 25
   (c) 27
   (d) 29

**[NAT]**

3. Consider a simple graph of 10 vertices. If the graph is disconnected, then the maximum number of edges, it can have is _____ ?

**[NAT]**

4. Consider a wheel graph ($w_n$) of 9 vertices. Find the number of edges to be deleted from the above graph, such that the resultant graph must be minimal connected graph ____?

**[MCQ]**

5. If G is a simple disconnected graph with 16 vertices and 3 components, then maximum number of edges possible in G is _____?
   (a) 90
   (b) 91
   (c) 92
   (d) 93

**[MCQ]**

6. Which of the following statements is/are true?

   $S_1$: A graph $G(V, E)$ is called tree if there is an exactly one path between every 2 vertices.

   $S_2$: A graph $G(V, E)$ is tree iff it is connected, and it does not contain cycle.
   (a) $S_1$ only
   (b) $S_2$ only
   (c) Both $S_1$ and $S_2$
   (d) None of these.

# Answer Key

1. 7
2. (c)
3. 36
4. 8

5. (b)
6. (c)

# Hints and solutions

**1. (7)**

Here, we have 8-vertices and 13 edges.

Now, a connected graph with n vertices has atleast $(n-1)$ edges.

So, $(n-2)$ edges will lead to disconnected graph and thus we need to delete:

$$e - (n-2) = 13 - (8-2)$$
$$= 13 - 6$$
$$= 7 \text{ edges}$$

Hence to get the disconnected graph we need to delete 7 edges.

**2. (c)**

As we know that, the total number of edges in a complete bipartite graph is "$m * n$" and number of vertices is "$m + n$".

Now,

$$m * n = 162 - - - - - (I)$$

Where

m = Number at vertices in partition 1

n = Number of vertices in partition 2

Also, in the problem it is given that: $m = 2n$

So, after substitution into eq (I):

$$2n \times n = 162$$
$$2n^2 = 162$$
$$n^2 = 81$$
$$n = 9$$

Now, the order of the graph $= m + n$
$$= 2n + n$$
$$= 3n$$
$$= 3 \times 9$$
$$= 27$$

Hence, the order of the given graph is 27.

**3. (36)**

In the problem, we, have 10 vertices in the graph. To make it disconnected divide the 10 vertices into partitions

Partition 1: $\underbrace{V_1, V_2, V_3, ..... V_9}_{\substack{\text{connected graph} \\ \text{with 9 vertices}}}$

Partition 2: $\underbrace{V_{10}}_{\substack{\text{Single/Isoloted} \\ \text{vertices}}}$

So, the maximum number of edges with 9 vertices is possible with complete graph:

∴ Complete graph $(k_9) = {}^nC_2$

$$= \frac{9 \times 8}{2}$$
$$= 36 \text{ edges}$$

**4. (8)**

In the problem, a wheel graph $(w_n)$ of 9 vertices. The number of edges in $(w_9)$; $2(n-1) = 2(9-1) = 2 \times 8 = 16$ edges.

Now, the minimal connected graph means the graph is connecte with minimum number of edges.

We know that a graph of $n$ vertices is minimal connected with $(n-1)$ edges.

Number of edges $= 16 - (n-1)$
$$= 16 - (9-1)$$
$$= 16 - 8 = 8 \text{ edges.}$$

**5. (b)**

Here, a simple disconnected graph with 16 vertices and 3 component is given.

Now, to get the maximum number of edges, try to distribute the maximum vertices in one component. as follows:

Component 1 :1 vertex $(V_1)$
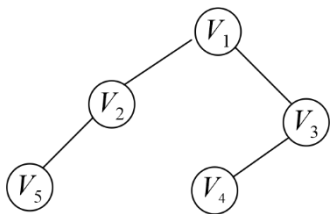
Component 2 :1 vertex $(V_2)$

Component 3 :14 vertices $(V_3 … V_{16})$

∴ Maximum number of edges

$$= \frac{14 \times 13}{2} = 91 \text{ edges}$$

**6.** **(C)**

$S_1$ : Consider the given graph G:

In the above graph, we have exactly one path between every 2 vertices, that is graph is connected and does not contain cycles.

Hence, $S_1$ is True.

$S_2$ : It is the definition of tree. So, statement $S_2$ is also True.