CS & IT ENGINEERING

Logical Equivalence

Lecture No. 02

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TOPICS TO BE COVERED

01 Type 1

02 Logical equivalence

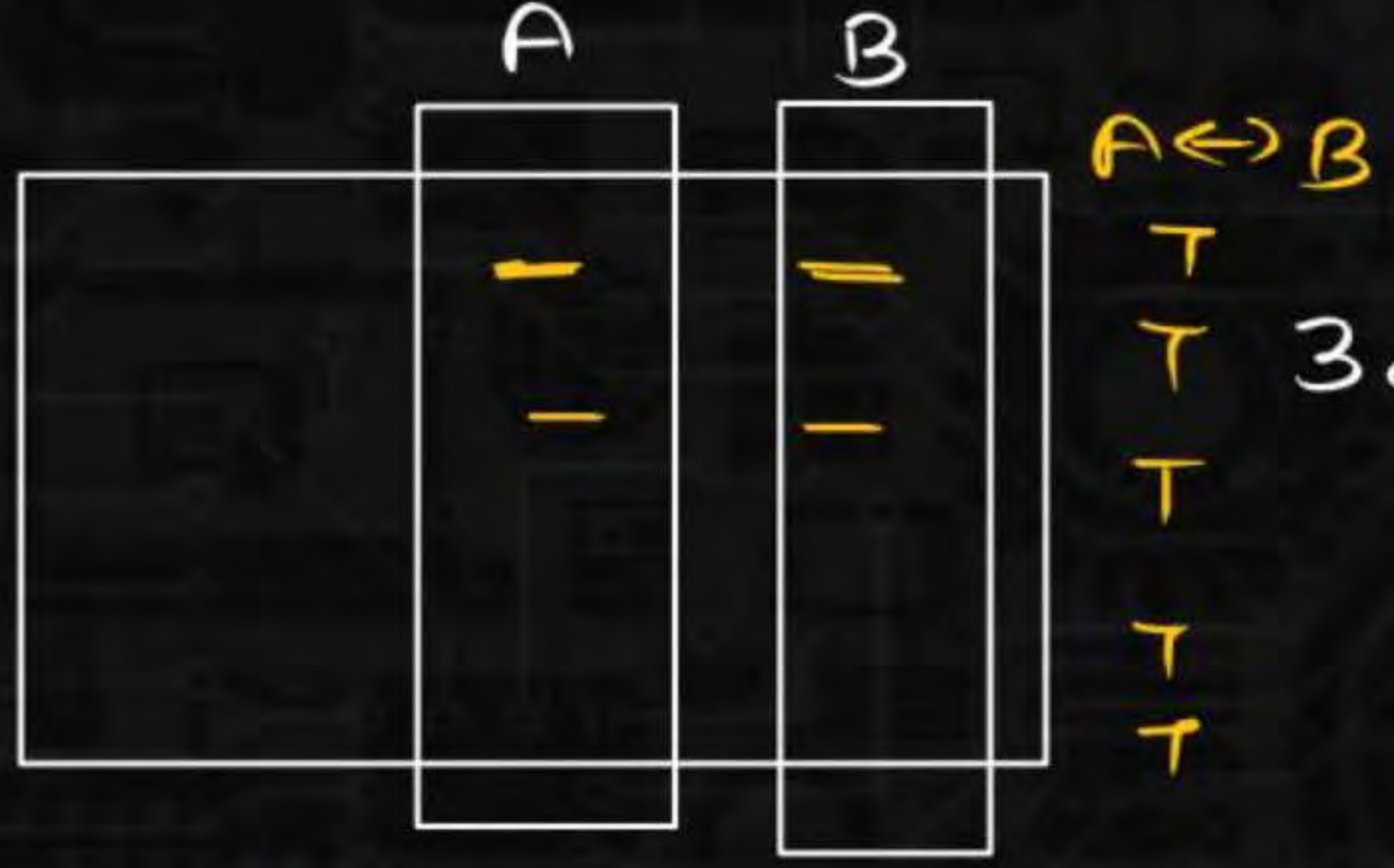
03 Conditional Equivalence

04 Biconditional equivalence

05 Type 2 Questions in Logic

* $\boxed{A \equiv B}$

1. A, B are having same behaviour =



2. A, B are having same column

3. $\boxed{A \longleftrightarrow B}$

tautology

valid.

we will try → false

Type-1 :

$$\overbrace{\boxed{p} \wedge \underbrace{\boxed{(p \rightarrow q)}}_{\substack{(T \rightarrow F) \\ \text{false}}} \wedge \overbrace{\boxed{(\neg q \vee r)}}^{T}}^{T} \rightarrow \overbrace{r}^{F}$$

$p \rightarrow T$

$(\neg q \vee r) \rightarrow (T \vee F)$

$r = F$

$p = T$

$\neg q = T$

$\rightarrow q = F$

false

True

$$\overbrace{\left[\,\boxed{p} \wedge \overbrace{\boxed{(p \to q)}}^{T} \wedge \overbrace{\boxed{(\neg q \vee r)}}^{T}\,\right]}^{T} \longrightarrow \overbrace{r}^{F}$$

$$p \to T \qquad (T \to T) \qquad \underbrace{(F \vee F)}_{F}$$

$r = F$

$p = T$

$q = T$

$\neg q = F$

false

True.

$$\overline{[m \wedge (\neg m \vee p) \wedge (p \to w) \wedge (\neg w \vee x)]}^{\,T} \to \overline{x}^{\,F}$$

m: T.

$(\neg m \vee p)$: $(F \vee F)$ = F

$(p \to w)$: $(F \to F)$

$(\neg w \vee x)$: $(T \vee F)$

$x = F$.

$m = T$.

$\neg w = T$.

$w = F$

$\to p = F$

false

True

$m = F$

$p = F$ $\quad \underline{z = F}$ $\quad \neg w = T$

$x = T$ $\quad w = F$

$$\overline{\left[(x \to m) \wedge \overbrace{(m \to p)}^{T} \wedge \overbrace{(p \to w)}^{T} \wedge \overbrace{(\neg w \vee z)}^{T} \wedge x\right]}^{T} \to \overset{F}{Z}$$

$(T \to F)$ $\quad (F \to F$ $\quad F \to F$ $\quad (T \vee F)$

F $\quad T$

false

$$\overbrace{(n \to m) \land (m \to p) \land (p \to w) \land \overbrace{(\neg w \lor z)}^{T} \land \overset{T}{n}}^{T}\,] \to \overset{f}{z}$$

$(T \to f)$ $(f \to f)$ $(f \to f)$ $(T \lor f)$

F

$z = f$

$n = T$

$w = f$

$p = f$

$m = f$

false $\longrightarrow$ $m = f$

$$\overbrace{\left[\overset{T}{x} \wedge \overset{T}{(x \to p)} \wedge \overset{T}{(m \to p)}\right]}^{T} \to \overset{F}{m}$$

$m = f$

$x = T$

$p = T$

$$(T \to T) \qquad (f \to T)$$

$$T \wedge T \wedge T$$

$$T \longrightarrow f$$

false

Type-1 : Subst^n method :

$\left(\quad \wedge \quad \wedge \quad \wedge \quad \right) \longrightarrow$

Check valid?

Type-2 logical equivalence:

2: $P \wedge T \equiv P$

$P \vee F \equiv P$

1.

$P \wedge P \equiv P$

$P \vee P \equiv P$

2.

$P \wedge T \equiv P$

$P \vee F \equiv P$

$P = T$ $\quad$ $P = F$

$T \wedge T \equiv T$ $\quad$ $F \wedge T \equiv F$

$P = T$ $\quad$ $P = F$

$T \vee F \equiv T$ $\quad$ $F \vee F \equiv F$

3:

$p \vee T \equiv T$.

$p \wedge f \equiv f$

4. $p \vee q = q \vee p$

$p \wedge q = q \wedge p$.

Associative. $\rightarrow$ same operator.

5. $p \wedge (q \wedge R) \equiv (p \wedge q) \wedge R$.

$p \vee (q \vee R) \equiv (p \vee q) \vee R$.

6: Distributive laws. : diff operator.

$p \wedge (q \vee R) \equiv (\boxed{p \wedge} q) \vee (\boxed{p \wedge} R)$

$p \vee (q \wedge R) \equiv (p \vee q) \wedge (p \vee R)$

$p \wedge (q \vee R) \equiv (p \wedge q) \vee (p \wedge R)$ $\quad F \vee (\underbrace{F \wedge -}_{F}) \equiv F$

Absorption law :

$p \vee (p \wedge q) \equiv p$

$p \wedge (p \vee q) \equiv p$

$p \vee (p \wedge q) \equiv p$

$p = T$

$\dfrac{T \vee (T \wedge -)}{T}$

$$p \quad 100V \;\vee\; \left(100V \;\wedge\; \boxed{500V} \right) \equiv$$

$$(p \to Q) \vee R) \vee (((p \to Q) \vee R) \wedge (t \to (m \vee n))] \equiv (p \to Q) \vee R.$$

$$\neg(a \vee b) \equiv \neg a \wedge \neg b.$$

$$\neg(a \wedge b) \equiv \neg a \vee \neg b.$$

$$\boxed{\begin{gathered} p \rightarrow q \equiv \neg q \rightarrow \neg p. \\ p \rightarrow q \equiv \neg p \vee q. \end{gathered}}$$

$$\neg(p \rightarrow q) \equiv \neg(\neg p \vee q)$$

$$\neg(p \rightarrow q) \equiv p \wedge \neg q$$

God's Rule.

$$\boxed{\begin{aligned} p \to q &\equiv \neg q \to \neg p \\ p \to q &\equiv \neg p \vee q \end{aligned}}$$

$$\neg p \to q \equiv p \vee q$$

$$\neg(p \to q)$$

$$\neg(\neg p \vee q)$$

$$p \wedge \neg q$$

$$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p) \equiv (p \to q) \wedge (q \to p)$$

$$\equiv (\neg p \vee q) \wedge (q \to p) \equiv (p \to q) \wedge (\neg q \vee p)$$

$$\equiv (\neg p \vee q) \wedge (\neg p \to \neg q) \equiv (\neg q \to \neg p) \wedge (\neg q \vee p)$$

$$(p \to q) \wedge (p \to R) \equiv p \to (q \wedge R)$$

$$(\boxed{\neg p \vee}\, q) \wedge (\boxed{\neg p}\, \vee R)$$

$$\neg p \vee (q \wedge R)$$

$$p \to (q \wedge R)$$

$$\left\{ \begin{array}{l} \underline{(p \to q) \wedge (p \to R)} \equiv p \to (q \wedge R) \\ (p \to q) \vee (p \to R) \equiv p \to (q \vee R) \end{array} \right.$$

$$(P \to R) \land (Q \to R)$$

$$(\neg P \boxed{\vee R}) \land (\neg Q \boxed{\vee R})$$

$$(\neg P \land \neg Q) \vee R$$

$$\neg (P \vee Q) \vee R$$

$$(P \vee Q) \to R$$

$$(P \to R) \land (P \to Q) \equiv P \to (Q \land R)$$

$$(P \to R) \vee (P \to Q) \equiv P \to (Q \vee R)$$

$$(P \to R) \land (Q \to R) \equiv (P \vee Q) \to R$$

$$(P \to R) \vee (Q \to R) \equiv (P \land Q) \to R$$

$$(p \rightarrow R) \wedge (Q \rightarrow R)$$

$$(\neg p \vee R) \wedge (\neg Q \vee R)$$

$$\vee R$$

$$(\neg p \wedge \neg Q) \vee R$$

$$\neg(p \vee Q) \vee R \equiv (p \vee Q) \rightarrow R$$

$$\boxed{(P \to} Q) \wedge \boxed{(P \to} R) \equiv P \to (Q \boxed{\wedge} R)$$

$$(\boxed{P \to} Q) \vee (\boxed{P \to} R) \equiv P \to (Q \boxed{\vee} R)$$

$$(P \boxed{\to R}) \wedge (Q \boxed{\to R}) \equiv (P \boxed{\vee} Q) \to R.$$

$$(P \boxed{\to R}) \vee (Q \boxed{\to R}) \equiv (P \boxed{\wedge} q) \to R.$$

God Rule
&
Demorgan's.

$$(P \to R) \vee (Q \to R)$$

$$(\neg P \boxed{\vee R}) \vee (\neg Q \boxed{\vee R})$$

$$\underline{(\neg P \vee \neg Q)} \vee R$$

$$\neg (P \wedge Q) \vee R \equiv (P \wedge Q) \to R.$$

The Simplest form of $(p \wedge (\sim r \vee \boxed{q \vee \sim q})) \vee ((r \vee t \vee \sim r) \wedge \sim q)$ is

(a) $p \wedge \sim q$

(b) $p \vee \sim q$

(c) t

(d) $(p \rightarrow \sim q)$

The Simplest form of

$(p \vee (p \wedge q) \vee (p \wedge q \wedge \sim r)) \wedge ((p \wedge r \wedge t) \vee t)$ is

(a) $p \wedge t$

(b) $q \wedge t$

(c) $p \wedge r$

(d) $p \wedge q$

Which one of the following is NOT equivalent to $p \leftrightarrow q$?

(GATE-15-Set1)

(a) $(\sim p \vee q) \wedge (p \vee \sim q)$

(b) $(\sim p \vee q) \wedge (q \rightarrow p)$

(c) $(\sim p \wedge q) \vee (p \wedge \sim q)$

(d) $(\sim p \wedge \sim q) \vee (p \wedge q)$

P and Q are two propositions. Which of the following logical expressions are equivalent?

I. $P \vee \sim Q$

II. $\sim(\sim P \wedge Q)$

III. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$

IV. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge Q)$

(GATE-08)

(a) Only I and II

(b) Only I, II and III

(c) Only I, II and IV

(d) All of I, II, III and IV

$\wedge$	$\cap$	$\cdot$
$\vee$	$\cup$	$+$
T	U	1
F	$\emptyset$	0

$$p \wedge (p \vee q) \equiv p.$$

$$a \cap (a \cup b) = a.$$

$$a \cup (a \cap b) = a.$$


THANK YOU GW
SOLDIERS !