Data 609 - Module3

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Contents

Ex. 1

Write down Newton's formula for finding the minimum of $f(x) = (3x^4 - 4x^3)/12$ in the range [-10, 10]. Then, implement it in R.

Solution:

$f(x) = (3x^4 - 4x^3)/12$

$f'(x) = \frac{12x^3 - 12x^2}{12} = x^3 - x^2$

$f''(x) = 3x^2 - 2x$

Using Newton's formula

$x_{k+1} = x_k - \frac{x_k^3 - x_k^2}{3x_k^2 - 2x_k}$

```
f <- function(x) {
  x - (x^3 - x^2)/(3 * x^2 - 2*x)
}

fetch_values <- function(X_0) {
  val <- c()
  for(i in 1:10) {
    if(i==1) {
      val[[i]] <- f(X_0)
    } else {
      val[[i]] <- f(val[[i-1]])
    }
  }
  print(val)
}
```

```
fetch_values(3)
```

```
## [[1]]
## [1] 2.142857
##
## [[2]]
## [1] 1.589862
##
## [[3]]
## [1] 1.251256
##
## [[4]]
## [1] 1.071993
##
## [[5]]
## [1] 1.008525
##
## [[6]]
## [1] 1.000142
##
## [[7]]
## [1] 1
##
## [[8]]
## [1] 1
##
## [[9]]
## [1] 1
##
## [[10]]
## [1] 1
```

```
fetch_values(1.5)
```

```
## [[1]]
## [1] 1.2
##
## [[2]]
## [1] 1.05
##
## [[3]]
## [1] 1.004348
##
## [[4]]
## [1] 1.000037
##
## [[5]]
## [1] 1
##
## [[6]]
## [1] 1
##
## [[7]]
## [1] 1
##
## [[8]]
## [1] 1
```

```
##
## [[9]]
## [1] 1
##
## [[10]]
## [1] 1
```

It looks like the minimum is at 1 since given multiple starting values, it converges to 1.

Ex. 2

Explore optimize() in R and try to solve the previous problem.

Solution:

```r
f <- function(x) {
  (3*x^4 -  4*x^3)/12
}

x_min <- optimize(f, c(-10, 10))
x_min
```

```
## $minimum
## [1] 0.9999986
##
## $objective
## [1] -0.08333333
```

Here we can confirm that minimum is at 1 and value of given function is -0.0833.

Ex. 3

Use any optimization algorithm to find the minimum of $f(x,y) = (x-1)^2 + 100(y-x^2)^2$ in the domain $-10 \leq x, y \leq 10$. Discuss any issues concerning the optimization process.

Solution:

We will use Newton's method for optimization of multivariate function.

The equation is

$x_{t+1} = x_t - H^{-1} \bigtriangledown f(x,y)$

where $\bigtriangledown f$ is the gradient vector and H is Hessian matrix

Starting value.

$x_0 = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\bigtriangledown f(x,y) = \begin{bmatrix} (x-1)^2 \\ 100(y-x^2)^2 \end{bmatrix}$

We will start by finding Hessian matrix.

$f_{xx} = 1200x^2 + 2 - 400y$

$f_{xy} = -400x$

$f_{yy} = 200$

$f_{yx} = -400x$

$H = \bigtriangledown^2 f(x,y) = \begin{bmatrix} 2 & -0 \\ -0 & 200 \end{bmatrix}$

$H^{-1} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.005 \end{bmatrix}$

With t = 0 $x_1 = x_0 - H^{-1} \bigtriangledown f(x,y)$

$x_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0.5 & 0 \\ 0 & 0.005 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$x_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}$

$x_1 = \begin{bmatrix} -0.5 \\ 0 \end{bmatrix}$

Given this, we can say f(x,y) will converge into a single point as we go through further values of t.

Ex. 4

Explore the optimr package for R and try to solve the previous problem.

Solution:

```r
f_xy <- function(param) {
  (param[1] - 1)^2 + 100 * (param[2] - param[1]^2)^2
}

options(scipen = 999)
optimr(c(-10,10), f_xy, method = "L-BFGS-B")
```

```
## $par
## [1] 0.9997999 0.9995999
##
## $value
## [1] 0.00000004002673
##
## $counts
## function gradient
##       71       71
##
## $convergence
## [1] 0
##
## $message
## [1] "CONVERGENCE: REL_REDUCTION_OF_F <= FACTR*EPSMCH"
```