Data 621

Group 4 - Homework1

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3/1/2020

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DATA EXPLORATION

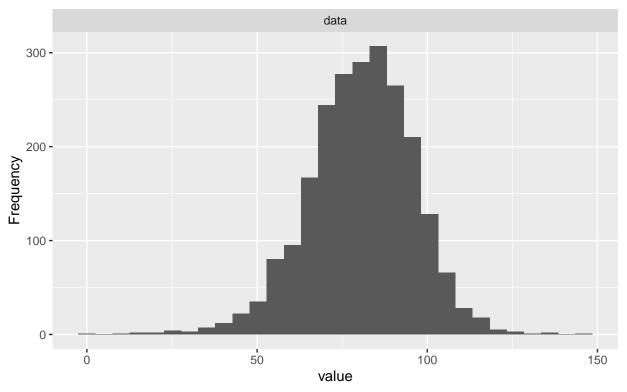
The data set contains approximately 2276 records. Each record represents a professional baseball team from the years 1871 to 2006 inclusive. Each record has the performance of the team for the given year, with all of the statistics adjusted to match the performance of a 162 game season. Below is a short description of the variables

- INDEX Identification Variable
- TARGET WINS Number of wins
- TEAM_BATTING_H Base Hits by batters (1B,2B,3B,HR)
- TEAM_BATTING_2B Doubles by batters (2B)
- TEAM_BATTING_3B Triples by batters (3B)
- TEAM BATTING HR Homeruns by batters (4B)
- TEAM_BATTING_BB Walks by batters

- TEAM BATTING HBP Batters hit by pitch (get a free base)
- TEAM_BATTING_SO Strikeouts by batters
- TEAM_BASERUN_SB Stolen bases
- TEAM_BASERUN_CS Caught stealing
- TEAM_FIELDING_E Errors
- TEAM_FIELDING_DP Double Plays
- TEAM_PITCHING_H Hits allowed
- TEAM_PITCHING_SO Strikeouts by pitchers

The wins distribution seems nearly normal distributed. It indicates that seasons do not have many too many high or low number of wins.

Distribution of TARGET_WINS



Objective To build a multiple linear regression model on the training data to predict *TARGET_WINS*, which is the number of wins for the team.

Summary

Table 1: Data summary

Name	baseball df
Number of rows	2276
Number of columns	17
Column type frequency:	
numeric	17

Group variables	None

Variable type: numeric

skim_variable	n_missing	complete_rate	mean	sd	p0	p25	p50	p75	p100
INDEX	0	1.00	1268.46	736.35	1	630.75	1270.5	1915.50	2535
TARGET_WINS	0	1.00	80.79	15.75	0	71.00	82.0	92.00	146
$TEAM_BATTING_H$	0	1.00	1469.27	144.59	891	1383.00	1454.0	1537.25	2554
$TEAM_BATTING_2B$	0	1.00	241.25	46.80	69	208.00	238.0	273.00	458
TEAM_BATTING_3B	0	1.00	55.25	27.94	0	34.00	47.0	72.00	223
TEAM_BATTING_HR	0	1.00	99.61	60.55	0	42.00	102.0	147.00	264
$TEAM_BATTING_BB$	0	1.00	501.56	122.67	0	451.00	512.0	580.00	878
$TEAM_BATTING_SO$	102	0.96	735.61	248.53	0	548.00	750.0	930.00	1399
TEAM_BASERUN_SB	131	0.94	124.76	87.79	0	66.00	101.0	156.00	697
TEAM_BASERUN_CS	772	0.66	52.80	22.96	0	38.00	49.0	62.00	201
TEAM_BATTING_HBP	2085	0.08	59.36	12.97	29	50.50	58.0	67.00	95
TEAM_PITCHING_H	0	1.00	1779.21	1406.84	1137	1419.00	1518.0	1682.50	30132
TEAM_PITCHING_HR	0	1.00	105.70	61.30	0	50.00	107.0	150.00	343
TEAM_PITCHING_BB	0	1.00	553.01	166.36	0	476.00	536.5	611.00	3645
TEAM_PITCHING_SO	102	0.96	817.73	553.09	0	615.00	813.5	968.00	19278
$TEAM_FIELDING_E$	0	1.00	246.48	227.77	65	127.00	159.0	249.25	1898
$TEAM_FIELDING_DP$	286	0.87	146.39	26.23	52	131.00	149.0	164.00	228

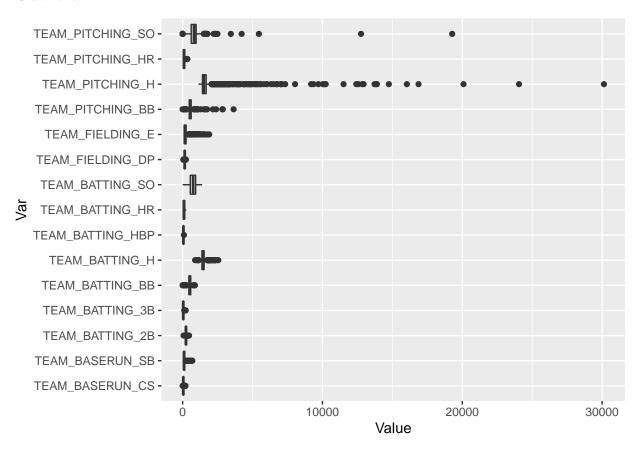
```
TARGET_WINS
                                      TEAM_BATTING_H TEAM_BATTING_2B
##
        INDEX
                          : 0.00
##
          :
                                             : 891
                                                            : 69.0
   Min.
               1.0
                     Min.
                                      Min.
                                                     Min.
   1st Qu.: 630.8
                     1st Qu.: 71.00
                                      1st Qu.:1383
                                                     1st Qu.:208.0
##
   Median :1270.5
                     Median: 82.00
                                      Median:1454
                                                     Median :238.0
##
   Mean
           :1268.5
                           : 80.79
                                             :1469
                                                             :241.2
                     Mean
                                      Mean
                                                     Mean
                     3rd Qu.: 92.00
##
   3rd Qu.:1915.5
                                      3rd Qu.:1537
                                                     3rd Qu.:273.0
##
   Max.
           :2535.0
                     Max.
                            :146.00
                                      Max.
                                             :2554
                                                     Max.
                                                             :458.0
##
##
   TEAM BATTING 3B
                     TEAM BATTING HR
                                      TEAM BATTING BB TEAM BATTING SO
##
   Min. : 0.00
                     Min. : 0.00
                                             : 0.0
                                                      Min. :
                                      Min.
                                                                  0.0
                     1st Qu.: 42.00
   1st Qu.: 34.00
                                      1st Qu.:451.0
                                                      1st Qu.: 548.0
##
   Median : 47.00
                     Median :102.00
                                      Median :512.0
                                                      Median : 750.0
##
   Mean
          : 55.25
                     Mean
                           : 99.61
                                      Mean
                                             :501.6
                                                      Mean
                                                            : 735.6
##
   3rd Qu.: 72.00
                     3rd Qu.:147.00
                                      3rd Qu.:580.0
                                                      3rd Qu.: 930.0
##
   Max.
           :223.00
                     Max.
                            :264.00
                                      Max.
                                             :878.0
                                                      Max.
                                                              :1399.0
##
                                                      NA's
                                                              :102
##
   TEAM_BASERUN_SB TEAM_BASERUN_CS TEAM_BATTING_HBP TEAM_PITCHING_H
##
         : 0.0
                    Min. : 0.0
                                    Min.
                                           :29.00
                                                     Min. : 1137
   1st Qu.: 66.0
                    1st Qu.: 38.0
                                    1st Qu.:50.50
                                                     1st Qu.: 1419
##
##
   Median :101.0
                    Median: 49.0
                                    Median :58.00
                                                     Median: 1518
                                           :59.36
##
   Mean
           :124.8
                    Mean
                          : 52.8
                                                            : 1779
                                    Mean
                                                     Mean
##
   3rd Qu.:156.0
                    3rd Qu.: 62.0
                                    3rd Qu.:67.00
                                                     3rd Qu.: 1682
##
   Max.
           :697.0
                    Max.
                           :201.0
                                    Max.
                                           :95.00
                                                     Max.
                                                             :30132
##
   NA's
           :131
                    NA's
                           :772
                                    NA's
                                           :2085
   TEAM_PITCHING_HR TEAM_PITCHING_BB TEAM_PITCHING_SO TEAM_FIELDING_E
##
   Min. : 0.0
                     Min. :
                                0.0
                                     \mathtt{Min.} :
                                                  0.0
                                                        Min. : 65.0
   1st Qu.: 50.0
                     1st Qu.: 476.0
                                      1st Qu.: 615.0
##
                                                        1st Qu.: 127.0
```

```
##
    Median :107.0
                      Median : 536.5
                                         Median :
                                                   813.5
                                                            Median: 159.0
##
            :105.7
                              : 553.0
    Mean
                      Mean
                                         Mean
                                                   817.7
                                                            Mean
                                                                    : 246.5
    3rd Qu.:150.0
                      3rd Qu.: 611.0
##
                                         3rd Qu.:
                                                   968.0
                                                            3rd Qu.: 249.2
            :343.0
                              :3645.0
                                                 :19278.0
                                                                    :1898.0
##
    Max.
                      Max.
                                         Max.
                                                            Max.
##
                                         NA's
                                                 :102
##
    TEAM_FIELDING_DP
##
    Min.
            : 52.0
    1st Qu.:131.0
##
##
    Median :149.0
##
    Mean
            :146.4
##
    3rd Qu.:164.0
            :228.0
##
    Max.
##
    NA's
            :286
   [1] "Observations per year, 1871 - 2006:"
## [1] 16.86
```

The summary views above gives a quick overview of the dataset in terms of missing observation (and subsequently the completion % out of 2276 records for each variable) averages, standard deviations, quartiles and percentiles, minimum and maximum values and distributions. All the datatypes seem to be numeric. Observations span 128 years, with an average of 17 teams playing per year.

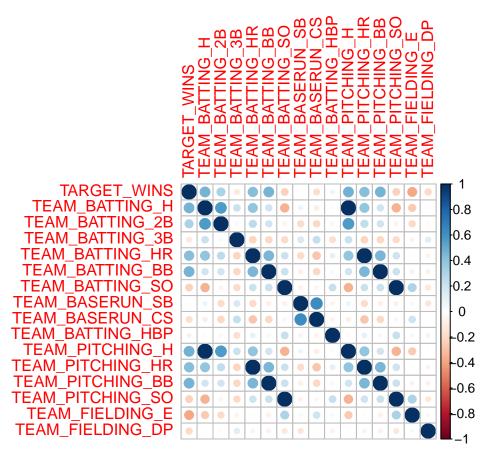
There are several variables with skewed distributions that could benefit from transformation. Additionally, there are a few variables with bi-modal distributions. Moreover, certain variables such as TEAM_BATTING_HBP have a lot of missing data (2085 out of 2276 obs.) which lowers its completion rate to about just 8%.

Outliers



From the boxplot above, we can see that several data columns like TEAM_PITCHING_H AND TEAM_PITCHING_SO have extreme outliers. The assignment mentions that some of the season records were adjusted to match the performance during a 162-game season.

Correlation and Collinearity



Looking at the correlation plot, there appear to be several statistically significant correlations between explanatory variables and the target.

From an initial inspection, it appears the team should focus on getting players on base through hits or walks. Contrary to what was expected, teams can still win if the pitchers allow homeruns, hits and walks to the other team.

Variables with Highest Positive Correlation with TARGET_WINS:

* TEAM_BATTING_H = 0.47 * TEAM_BATTING_HR = 0.42 * TEAM_BATTING_BB = 0.47 * TEAM_PITCHING_H = 0.47 * TEAM_PITCHING_HR = 0.42 * TEAM_PITCHING_BB = 0.47

To win more games it makes sense the team will need to make fewer errors.

Within this group, we detected collinearity between some of the variables:

Positive Correlations between predictors:

- * TEAM PITCHING H and TEAM BATTING H = 0.99
- * TEAM PITCHING HR and TEAM BATTING HR = 0.99
- * TEAM PITCHING BB and TEAM BATTING BB = 0.99

Negative Correlations between predictors:

* TEAM_PITCHING_SO and TEAM_BATTING_H = -0.34

DATA PREPARATION

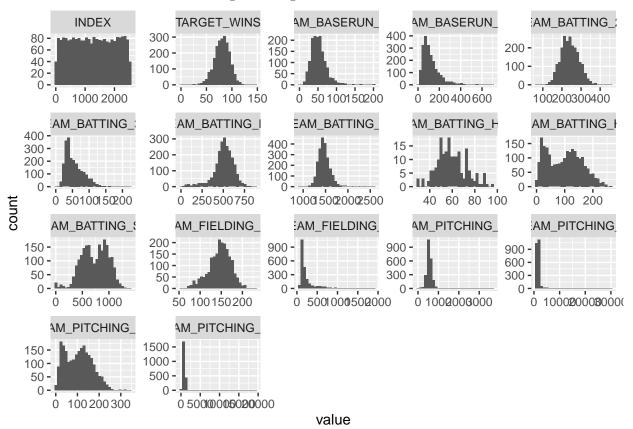
Missing values

In terms of missing values, there are two variables missing many observations. TEAM_BATTING_HBP is missing over 90% of its values, while TEAM_BASERUN_CS is missing just around 30%. Since TEAM_BATTING_HBP is barely complete and, deleting this variable would make sense.

The rest of the variables with missing values are: TEAM_BATTING_SO TEAM_BASERUN_SB TEAM_BASERUN_CS TEAM_FIELDING_DP TEAM_PITCHING_SO

Multiple Imputation

We will also attempt multiple imputation on the original dataset. Multiple imputation assumes normality of data so let's check for skewness once again among the dataset:



It seems like TEAM_BASERUN_SB, TEAM_PITCHING_SO and TEAM_FIELDING_E are skewed. Let's confirm this using the skewness function. Anything that has a skewness above 1 is thought to be highly skewed.

[1] 22.17455

[1] 1.972414

[1] 2.990466

Let's log transform these variables prior to multiple imputation.

Now that we have log transformed most of the variable and all our data are numeric, let's impute the data.

```
-- Imputation 1 --
##
##
           3
             4
                5
                   6
                      7
                          8 9 10 11 12 13 14 15 16 17 18 19 20
##
   21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40
##
   41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60
##
   61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80
##
   81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100
    101 102 103 104 105 106 107 108 109 110 111 112 113 114 115 116 117 118 119 120
##
##
    121 122 123 124 125 126 127 128 129 130 131 132 133 134 135 136 137 138 139
##
    141 142 143 144 145 146 147 148 149 150 151 152 153 154 155 156 157 158 159
##
    161 162 163 164 165 166 167 168 169 170 171 172 173 174 175 176 177 178 179
##
    181 182 183 184 185 186 187 188 189 190 191 192 193 194 195 196 197 198 199
                                                                                 200
    201 202 203 204 205 206 207 208 209 210 211 212 213 214 215 216 217 218 219
    221 222 223 224 225 226 227 228 229 230 231 232 233 234 235 236 237 238
##
                                                                             239
                                                                                 240
##
    241 242 243 244 245 246 247 248 249 250 251 252 253 254 255 256 257 258 259
                                                                                 260
##
    261 262 263 264 265 266 267 268 269 270 271 272 273 274 275 276 277 278 279
                                                                                 280
    281 282 283 284 285 286 287 288 289 290 291 292 293 294 295 296 297 298 299
   301 302 303 304 305 306 307 308 309 310 311 312 313 314 315 316 317 318
##
                                                                                 320
##
    321 322 323 324 325 326 327 328 329 330 331 332 333 334 335 336 337 338 339
                                                                                 340
   341 342 343 344 345 346 347 348 349 350 351 352 353 354 355 356 357 358 359
##
                                                                                 360
##
    361 362 363 364 365 366 367 368 369 370 371 372 373 374 375 376 377 378 379
##
   381 382 383 384 385 386 387 388 389 390 391 392 393 394 395 396 397 398 399
   401 402 403 404 405 406 407 408 409 410 411 412 413 414 415 416 417 418 419
##
                                                                                 420
##
   421 422 423 424 425 426 427 428 429 430 431 432 433 434 435 436 437 438 439
##
   441 442 443 444 445 446 447 448 449 450 451 452 453 454 455 456 457 458 459
##
    461 462 463 464 465 466 467 468 469 470 471 472 473 474 475 476 477 478 479
    481 482 483 484 485 486 487 488 489 490 491 492 493 494 495 496 497 498 499
##
                                                                                 500
##
    501 502 503 504 505 506 507 508 509 510 511 512 513 514 515 516 517 518 519
   521 522 523 524 525 526 527 528 529 530 531 532 533 534 535 536 537 538 539
##
##
    541 542 543 544 545 546 547 548 549 550 551 552 553 554 555 556 557 558 559
##
    561 562 563 564 565 566 567 568 569 570 571 572 573 574 575 576 577 578 579
                                                                                 580
##
    581 582 583 584 585 586 587 588 589 590 591 592 593 594 595 596 597 598 599
    601 602 603 604 605 606 607 608 609 610 611 612 613 614 615 616 617 618 619
##
##
   621 622 623 624 625 626 627 628 629 630 631 632 633 634 635 636 637 638 639
##
   641 642 643 644 645 646 647 648 649 650 651 652 653 654 655 656 657 658 659
                                                                                 660
    661 662 663 664 665 666 667 668 669 670 671 672 673 674 675
                                                                676 677 678 679
    681 682 683 684 685 686 687 688 689 690 691 692 693 694 695 696 697 698 699
##
##
   701 702 703 704 705 706 707 708 709 710 711 712 713 714 715 716 717 718 719
                                                                                 720
##
   721 722 723 724 725 726 727 728 729 730 731 732 733 734 735 736 737 738 739
   741 742 743 744 745 746 747 748 749 750 751 752 753 754 755 756 757 758 759
##
##
   761 762 763 764 765 766 767 768 769 770 771 772 773 774 775
                                                                776
                                                                     777
                                                                         778
##
   781 782 783 784 785 786 787 788 789 790 791 792 793 794 795 796 797 798 799
                                                                                 800
##
   801 802 803 804 805 806 807 808 809 810 811 812 813 814 815 816 817 818 819
                                                                                 820
##
   821 822 823 824 825 826 827 828 829 830 831 832 833 834 835 836 837 838 839
   841 842 843 844 845 846 847 848 849 850 851 852 853 854 855 856 857 858 859
##
   861 862 863 864 865 866 867 868 869 870 871 872 873 874 875 876 877 878 879
##
                                                                                 880
    881 882 883 884 885 886 887 888 889 890 891 892 893 894 895 896 897 898 899
##
   901 902 903 904 905 906 907 908 909 910 911 912 913 914 915 916 917 918 919
##
   921 922 923 924 925 926 927 928 929 930 931 932 933 934 935 936 937 938 939
   941 942 943 944 945 946 947 948 949 950 951 952 953 954 955 956 957 958 959
##
    961 962 963 964 965 966 967 968 969 970 971 972 973 974 975 976 977 978 979 980
   981 982 983 984 985 986 987 988 989 990 991 992 993 994 995 996 997 998 999 1000
##
```

```
## 1001 1002 1003 1004 1005 1006 1007 1008 1009 1010 1011 1012 1013 1014 1015 1016 1017 1018 1019 1020
  1021 1022 1023 1024 1025 1026 1027 1028 1029 1030 1031 1032 1033 1034 1035 1036 1037 1038 1039 1040
  1041 1042 1043 1044 1045 1046 1047 1048 1049 1050 1051 1052 1053 1054 1055 1056 1057 1058 1059 1060
  1061 1062 1063 1064 1065 1066 1067 1068 1069 1070 1071 1072 1073 1074 1075 1076 1077 1078 1079 1080
  1081 1082 1083 1084 1085 1086 1087 1088 1089 1090 1091 1092 1093 1094 1095 1096 1097 1098 1099 1100
  1101 1102 1103 1104 1105 1106 1107 1108 1109 1110 1111 1112 1113 1114 1115 1116 1117 1118 1119 1120
  1121 1122 1123 1124 1125 1126 1127 1128 1129 1130 1131 1132 1133 1134 1135 1136 1137 1138 1139 1140
  1141 1142 1143 1144 1145 1146 1147 1148 1149 1150 1151 1152 1153 1154 1155 1156 1157 1158 1159 1160
   1161 1162 1163 1164 1165 1166 1167 1168 1169 1170 1171 1172 1173 1174 1175 1176 1177 1178 1179 1180
   1181 1182 1183 1184 1185 1186 1187 1188 1189 1190 1191 1192 1193 1194 1195 1196 1197 1198 1199 1200
   1201 1202 1203 1204
##
##
   -- Imputation 2 --
##
     1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
##
    21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40
   41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60
##
    61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80
   81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100
    101 102 103 104 105 106 107 108 109 110 111 112 113 114 115 116 117 118 119 120
##
    121 122 123 124 125 126 127 128 129 130 131 132 133 134 135 136 137 138 139 140
    141 142 143 144 145 146 147 148 149 150 151 152 153 154 155 156 157 158 159 160
   161 162 163 164 165 166 167 168 169 170 171 172 173 174 175 176 177 178 179
##
    181 182 183 184 185 186 187 188 189 190 191 192 193 194 195 196 197 198 199 200
##
##
   201 202 203 204 205 206 207 208 209 210 211 212 213 214 215 216 217 218 219 220
    221 222 223 224 225 226 227 228 229 230 231 232 233 234 235 236 237 238 239 240
   241 242 243 244 245 246 247 248 249 250 251 252 253 254 255 256 257 258 259
##
    261 262 263 264 265 266 267 268 269 270 271 272 273 274 275 276 277 278 279
   281 282 283 284 285 286 287 288 289 290 291 292 293 294 295 296 297 298 299 300
   301 302 303 304 305 306 307 308 309 310 311 312 313 314 315 316 317 318 319 320
##
   321 322 323 324 325 326 327 328 329 330 331 332 333 334 335 336 337 338 339
##
   341 342 343 344 345 346 347 348 349 350 351 352 353 354 355 356 357 358 359 360
   361 362 363 364 365 366 367 368 369 370 371 372 373 374 375 376 377 378 379 380
   381 382 383 384 385 386 387 388 389 390 391 392 393 394 395 396 397 398 399 400
##
    401 402 403 404 405 406 407 408 409 410 411 412 413 414 415 416 417 418 419 420
   421 422 423 424 425 426 427 428 429 430 431 432 433 434 435 436 437 438 439 440
##
   441 442 443 444 445 446 447 448 449 450 451 452 453 454 455 456 457 458 459 460
   461 462 463 464 465 466 467 468 469 470 471 472 473 474 475 476 477 478 479 480
##
    481 482 483 484 485 486 487 488 489 490 491 492 493 494 495 496 497 498 499 500
   501 502 503 504 505 506 507 508 509 510 511 512 513 514 515 516 517 518 519 520
##
   521 522 523 524 525 526 527 528 529 530 531 532 533 534 535 536 537 538 539 540
   541 542 543 544 545 546 547 548 549 550 551 552 553 554 555 556 557 558 559 560
##
   561 562 563 564 565 566 567 568 569 570 571 572 573 574 575 576 577 578 579 580
##
   581 582 583 584 585 586 587 588 589 590 591 592 593 594 595 596 597 598 599 600
   601 602 603 604 605 606 607 608 609 610 611 612 613 614 615 616 617 618 619 620
   621 622 623 624 625 626 627 628 629 630 631 632 633 634 635 636 637 638 639 640
##
##
   641 642 643 644 645 646 647 648 649 650 651 652 653 654 655 656 657 658 659 660
    661 662 663 664 665 666 667 668 669 670 671 672 673 674 675 676 677 678 679 680
    681 682 683 684 685 686 687 688 689 690 691 692 693 694 695 696 697 698 699 700
##
   701 702 703 704 705 706 707 708 709 710 711 712 713 714 715 716 717 718 719 720
   721 722 723 724 725 726 727 728 729 730 731 732 733 734 735 736 737 738 739 740
##
   741 742 743 744 745 746 747 748 749 750 751 752 753 754 755 756 757 758 759 760
   761 762 763 764 765 766 767 768 769 770 771 772 773 774 775 776 777 778 779 780
   781 782 783 784 785 786 787 788 789 790 791 792 793 794 795 796 797 798 799 800
```

```
801 802 803 804 805 806 807 808 809 810 811 812 813 814 815 816 817 818 819 820
    821 822 823 824 825 826 827 828 829 830 831 832 833 834 835 836 837 838 839 840
##
    841 842 843 844 845 846 847 848 849 850 851 852 853 854 855 856 857 858 859 860
    861 862 863 864 865 866 867 868 869 870 871 872 873 874 875 876 877 878 879 880
##
##
    881 882 883 884 885 886 887 888 889 890 891 892 893 894 895 896 897 898 899 900
    901 902 903 904 905 906 907 908 909 910 911 912 913 914 915 916 917 918 919 920
##
    921 922 923 924 925 926 927 928 929 930 931 932 933 934 935 936 937 938 939 940
##
    941 942 943 944 945 946 947 948 949 950 951 952 953 954 955 956 957 958 959 960
##
    961 962 963 964 965 966 967 968 969 970 971 972 973 974 975 976 977 978 979 980
    981 982 983 984 985 986 987 988 989 990 991 992 993 994 995 996 997 998 999 1000
   1001 1002 1003 1004 1005 1006 1007 1008 1009 1010 1011 1012 1013 1014 1015 1016 1017 1018 1019 1020
   1021 1022 1023 1024 1025 1026 1027 1028 1029 1030 1031 1032 1033 1034 1035 1036 1037 1038 1039 1040
   1041 1042 1043 1044 1045 1046 1047 1048 1049 1050 1051 1052 1053 1054 1055 1056 1057 1058 1059 1060
   1061 1062 1063 1064 1065 1066 1067 1068 1069 1070 1071 1072 1073 1074 1075 1076 1077 1078 1079 1080
  1081 1082 1083 1084 1085 1086 1087 1088 1089 1090 1091 1092 1093 1094 1095 1096 1097 1098 1099 1100
   1101 1102 1103 1104 1105 1106 1107 1108 1109 1110 1111 1112 1113 1114 1115 1116 1117 1118 1119 1120
   1121 1122 1123 1124 1125 1126 1127 1128 1129 1130 1131 1132 1133 1134 1135 1136 1137 1138 1139 1140
  1141 1142 1143 1144 1145 1146 1147 1148 1149 1150 1151 1152 1153 1154 1155 1156 1157 1158 1159 1160
  1161 1162 1163 1164 1165 1166 1167 1168 1169 1170 1171 1172 1173 1174 1175 1176 1177 1178 1179 1180
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  1161 1162 1163 1164 1165 1166 1167 1168 1169 1170 1171 1172 1173 1174 1175 1176 1177 1178 1179 1180
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Notice that we have imputed the entire dataset. This is because, although certain variables do not have any missing data, they maybe helpful in predicting missing values in the variables that do have them (Faraway, 2014)

BUILDMODELS

Model 1

We will build the initial models on the original dataset first (prior to multiple imputation) for comparison purposes.

We want to try creating a simple model with fewer predictors to see how it performs compared to our other models. To start, we chose a few variables that were highly positively and negatively correlated with TARGET WINS.

From there we removed multiple predictors at once. To do this we need to construct a null hypothesis test which states that removing the variables doesn't make a better model. We construct a F-test and compare both versions of the model. If the p-value is under 0.05 we reject the null hypothesis, which indicates our new model isn't different than the first model. If the p-value is greater than 0.05, the model isn't better with those variables, so we will remove them. The simpler the model the better.

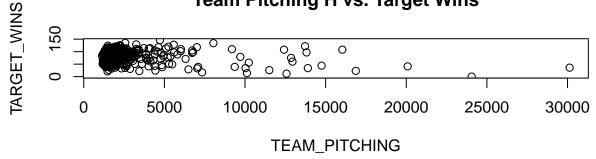
To determine which variables we removed, we chose the variable that was not proving to be significant in the linear regression (where the p-value was greater than 0.05). While this doesn't mean the variable itself isn't significant, it means the variable alongside the other combination of variables in the model is not significant.

```
##
## Call:
  lm(formula = TARGET_WINS ~ TEAM_BATTING_H + TEAM_BATTING_HR +
##
       TEAM_BATTING_BB + TEAM_BATTING_SO + TEAM_PITCHING_H + TEAM_PITCHING_HR +
##
       TEAM_PITCHING_BB + TEAM_PITCHING_SO + TEAM_FIELDING_E + TEAM_FIELDING_DP,
##
       data = baseball_df_fix)
##
##
  Residuals:
##
       Min
                    Median
                                3Q
                10
                                       Max
                     0.200
                             7.831
                                    39.906
##
   -44.378
           -7.898
##
## Coefficients:
##
                     Estimate Std. Error t value Pr(>|t|)
                                           9.164 < 2e-16 ***
## (Intercept)
                    54.808616
                                5.980591
## TEAM BATTING H
                     0.016475
                                0.004154
                                           3.966 7.58e-05 ***
## TEAM BATTING HR
                     0.266831
                                           4.654 3.49e-06 ***
                                0.057338
## TEAM_BATTING_BB
                     0.042737
                                0.019148
                                           2.232 0.025733 *
## TEAM_BATTING_SO
                    -0.028802
                                0.009339
                                          -3.084 0.002071 **
## TEAM_PITCHING_H
                     0.016827
                                0.002304
                                           7.303 4.13e-13 ***
## TEAM_PITCHING_HR -0.207321
                                0.054299
                                          -3.818 0.000139 ***
## TEAM PITCHING BB -0.010049
                                0.017800
                                          -0.565 0.572447
## TEAM_PITCHING_SO 0.011148
                                0.008467
                                           1.317 0.188131
## TEAM_FIELDING_E -0.057509
                                0.005488 -10.478
                                                  < 2e-16 ***
## TEAM_FIELDING_DP -0.158032
                                0.012962 -12.192 < 2e-16 ***
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 11.42 on 1877 degrees of freedom
     (388 observations deleted due to missingness)
## Multiple R-squared: 0.2991, Adjusted R-squared: 0.2954
## F-statistic: 80.1 on 10 and 1877 DF, p-value: < 2.2e-16
##
## Call:
## lm(formula = TARGET_WINS ~ TEAM_BATTING_H + TEAM_BATTING_HR +
```

```
##
      TEAM_BATTING_BB + TEAM_BATTING_SO + TEAM_PITCHING_H + TEAM_PITCHING_HR +
##
      TEAM_FIELDING_E + TEAM_FIELDING_DP, data = baseball_df_fix)
##
## Residuals:
##
      Min
               1Q Median
                               3Q
                                      Max
## -44.327 -7.799
                    0.209
                            7.875 40.210
## Coefficients:
##
                    Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                   54.574784
                               5.954596
                                          9.165 < 2e-16 ***
## TEAM_BATTING_H
                    0.015205
                               0.003802
                                          4.000 6.59e-05 ***
## TEAM_BATTING_HR
                    0.243859
                               0.032695
                                          7.459 1.33e-13 ***
## TEAM_BATTING_BB
                    0.032029 0.003446
                                         9.294 < 2e-16 ***
                               0.002207 -7.646 3.29e-14 ***
## TEAM_BATTING_SO
                  -0.016874
## TEAM_PITCHING_H
                    0.017731
                               0.001833
                                          9.671 < 2e-16 ***
## TEAM_PITCHING_HR -0.184892
                               0.031112 -5.943 3.33e-09 ***
                               0.005293 -10.535 < 2e-16 ***
## TEAM_FIELDING_E -0.055757
## TEAM_FIELDING_DP -0.156392
                               0.012867 -12.155 < 2e-16 ***
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 11.42 on 1879 degrees of freedom
    (388 observations deleted due to missingness)
## Multiple R-squared: 0.2984, Adjusted R-squared: 0.2955
## F-statistic: 99.92 on 8 and 1879 DF, p-value: < 2.2e-16
## Analysis of Variance Table
##
## Model 1: TARGET_WINS ~ TEAM_BATTING_H + TEAM_BATTING_HR + TEAM_BATTING_BB +
      TEAM BATTING SO + TEAM PITCHING H + TEAM PITCHING HR + TEAM PITCHING BB +
      TEAM_PITCHING_SO + TEAM_FIELDING_E + TEAM_FIELDING_DP
##
## Model 2: TARGET_WINS ~ TEAM_BATTING_H + TEAM_BATTING_HR + TEAM_BATTING_BB +
      TEAM_BATTING_SO + TEAM_PITCHING_H + TEAM_PITCHING_HR + TEAM_FIELDING_E +
##
##
      TEAM_FIELDING_DP
    Res.Df
              RSS Df Sum of Sq
                                    F Pr(>F)
##
      1877 244715
## 1
      1879 244944 -2 -229.32 0.8794 0.4152
## 2
```

• Took the log of TEAM_PITCHING_H it's relationship to TARGET_WINS more linear

Team Pitching H vs. Target Wins



```
lm(formula = TARGET_WINS ~ TEAM_BATTING_H + TEAM_BATTING_HR +
       TEAM_BATTING_BB + TEAM_BATTING_SO + log(TEAM_PITCHING_H) +
##
##
       TEAM_PITCHING_HR + TEAM_FIELDING_E + TEAM_FIELDING_DP, data = baseball_df_fix)
##
## Residuals:
##
      Min
                1Q
                   Median
                                3Q
                                       Max
  -43.996
           -7.809
                     0.122
                             7.874
                                    37.186
##
##
## Coefficients:
##
                          Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                        -3.017e+02
                                    3.377e+01
                                               -8.933
                                                      < 2e-16 ***
## TEAM_BATTING_H
                        -1.227e-03
                                    4.861e-03
                                               -0.252
                                                         0.801
## TEAM_BATTING_HR
                         3.815e-01
                                    4.108e-02
                                                9.286
                                                       < 2e-16 ***
                                                9.421
## TEAM_BATTING_BB
                         3.238e-02
                                    3.437e-03
                                                       < 2e-16 ***
## TEAM_BATTING_SO
                        -1.557e-02
                                    2.183e-03
                                               -7.132 1.40e-12 ***
## log(TEAM_PITCHING_H)
                                    5.435e+00
                                               10.243 < 2e-16 ***
                        5.567e+01
## TEAM_PITCHING_HR
                        -3.208e-01
                                    3.984e-02
                                               -8.051 1.44e-15 ***
## TEAM_FIELDING_E
                        -6.390e-02
                                    5.740e-03 -11.133
                                                      < 2e-16 ***
## TEAM_FIELDING_DP
                        -1.564e-01
                                    1.283e-02 -12.194
                                                      < 2e-16 ***
##
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 11.38 on 1879 degrees of freedom
     (388 observations deleted due to missingness)
## Multiple R-squared: 0.3025, Adjusted R-squared: 0.2995
## F-statistic: 101.8 on 8 and 1879 DF, p-value: < 2.2e-16
```

##

```
##
## Call:
## lm(formula = TARGET WINS ~ TEAM BATTING HR + TEAM BATTING BB +
       TEAM_BATTING_SO + log(TEAM_PITCHING_H) + TEAM_PITCHING_HR +
##
       TEAM_FIELDING_E + TEAM_FIELDING_DP, data = baseball_df_fix)
##
## Residuals:
      Min
##
               1Q Median
                               3Q
                                      Max
## -43.997 -7.787
                    0.124
                            7.878 37.288
##
## Coefficients:
                         Estimate Std. Error t value Pr(>|t|)
##
## (Intercept)
                        -2.957e+02 2.409e+01 -12.274 < 2e-16 ***
## TEAM_BATTING_HR
                        3.747e-01 3.103e-02 12.073 < 2e-16 ***
## TEAM_BATTING_BB
                        3.231e-02 3.423e-03
                                               9.438 < 2e-16 ***
## TEAM_BATTING_SO
                        -1.535e-02
                                   1.994e-03 -7.696 2.25e-14 ***
                                   3.284e+00 16.618 < 2e-16 ***
## log(TEAM_PITCHING_H) 5.458e+01
## TEAM PITCHING HR
                       -3.146e-01
                                   3.134e-02 -10.037 < 2e-16 ***
                        -6.300e-02 4.494e-03 -14.018 < 2e-16 ***
## TEAM_FIELDING_E
## TEAM FIELDING DP
                        -1.564e-01 1.282e-02 -12.194 < 2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 11.38 on 1880 degrees of freedom
     (388 observations deleted due to missingness)
## Multiple R-squared: 0.3024, Adjusted R-squared: 0.2998
## F-statistic: 116.4 on 7 and 1880 DF, p-value: < 2.2e-16
## Analysis of Variance Table
## Model 1: TARGET_WINS ~ TEAM_BATTING_H + TEAM_BATTING_HR + TEAM_BATTING_BB +
##
       TEAM_BATTING_SO + log(TEAM_PITCHING_H) + TEAM_PITCHING_HR +
##
       TEAM_FIELDING_E + TEAM_FIELDING_DP
## Model 2: TARGET_WINS ~ TEAM_BATTING_HR + TEAM_BATTING_BB + TEAM_BATTING_SO +
       log(TEAM_PITCHING_H) + TEAM_PITCHING_HR + TEAM_FIELDING_E +
##
##
       TEAM FIELDING DP
##
    Res.Df
              RSS Df Sum of Sq
                                    F Pr(>F)
## 1
      1879 243539
      1880 243547 -1
                       -8.2595 0.0637 0.8007
## 2
```

Model 2

This model eliminates several features altogether from Model 1 including those with missing values, transforms three, and considers four different interaction effects.

```
##
## Call:
## lm(formula = TARGET_WINS ~ . - INDEX + log(TEAM_FIELDING_E) +
## log(TEAM_PITCHING_BB) - TEAM_PITCHING_H - TEAM_BATTING_BB -
## TEAM_PITCHING_HR - TEAM_PITCHING_BB - TEAM_FIELDING_E + log(TEAM_FIELDING_E) +
## TEAM_BATTING_3B:TEAM_BATTING_HR + TEAM_BATTING_2B:TEAM_BATTING_HR +
## TEAM_BATTING_H:TEAM_BATTING_HR + TEAM_BATTING_H:TEAM_BATTING_3B -
## TEAM_BATTING_3B - TEAM_BATTING_SO - TEAM_BATTING_2B - TEAM_BATTING_BB -
## TEAM_BATTING_HR - TEAM_BATTING_H - TEAM_BATTING_HR - TEAM_PITCHING_HR -
```

```
##
       TEAM_BATTING_HBP - TEAM_FIELDING_DP - TEAM_PITCHING_SO -
##
       TEAM_BASERUN_SB - TEAM_BASERUN_CS, data = baseball_df_fix)
##
##
  Residuals:
##
        Min
                  1Q
                       Median
                                    3Q
                                            Max
                      -0.1331
   -20.8825
            -5.7136
                                6.3792
                                        22.9085
##
##
## Coefficients:
##
                                     Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                                    2.143e+00
                                               4.139e+01
                                                           0.052 0.95876
## log(TEAM_FIELDING_E)
                                   -2.392e+01
                                               4.325e+00
                                                           -5.530 1.09e-07 ***
## log(TEAM_PITCHING_BB)
                                    2.508e+01
                                               5.440e+00
                                                           4.611 7.49e-06 ***
## TEAM_BATTING_3B:TEAM_BATTING_HR -4.306e-03
                                               1.427e-03
                                                           -3.018 0.00290 **
## TEAM_BATTING_2B:TEAM_BATTING_HR -3.449e-05
                                               1.703e-04
                                                           -0.203 0.83971
## TEAM_BATTING_H:TEAM_BATTING_HR
                                    1.434e-04
                                               4.261e-05
                                                           3.366 0.00093 ***
## TEAM_BATTING_H:TEAM_BATTING_3B
                                    4.332e-04
                                               1.556e-04
                                                           2.783 0.00594 **
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 9.06 on 184 degrees of freedom
##
     (2085 observations deleted due to missingness)
## Multiple R-squared: 0.4584, Adjusted R-squared: 0.4408
## F-statistic: 25.96 on 6 and 184 DF, p-value: < 2.2e-16
```

The R-squared statistic indicates that this model predicts about half of the variation in wins with the included features. The model is statistically significant at p<.05, however the F-Statistic seems to have fallen quite a bit from the initial model.

Model 3

This model takes into account the imputed dataset and models TARGET_WINS against all variables present in the dataset except for INDEX (since this is the id variables and had a very weak negative association with TARGET_WINS)

```
##
## Call:
## lm(formula = TARGET_WINS ~ . - INDEX, data = missmod$imputations[[i]])
##
## Residuals:
##
       Min
                1Q
                    Median
                                 3Q
                                        Max
  -57.105 -7.879 -0.086
                              8.055
                                     43.164
##
## Coefficients:
##
                      Estimate Std. Error t value Pr(>|t|)
                                            -0.838 0.402140
## (Intercept)
                    -23.836016
                                 28.445175
## TEAM_BATTING_H
                      0.153522
                                  0.063906
                                              2.402 0.016371 *
## TEAM_BATTING_2B
                      -0.033816
                                  0.008848
                                            -3.822 0.000136 ***
## TEAM_BATTING_3B
                      0.079610
                                  0.016635
                                              4.786 1.82e-06 ***
## TEAM_BATTING_HR
                      0.368897
                                  0.165439
                                              2.230 0.025858 *
                      -0.344387
                                            -1.483 0.138134
## TEAM_BATTING_BB
                                  0.232176
## TEAM_BATTING_SO
                      -0.050566
                                  0.006119
                                             -8.265 2.36e-16 ***
## TEAM_BASERUN_SB
                      3.987815
                                  0.771488
                                             5.169 2.56e-07 ***
## TEAM_BASERUN_CS
                      0.074637
                                  0.017358
                                              4.300 1.78e-05 ***
## TEAM_BATTING_HBP
                                             1.584 0.113289
                      0.033273
                                  0.021003
```

```
## TEAM PITCHING H
                     -0.103930
                                           -1.599 0.109852
                                 0.064977
## TEAM_PITCHING_HR
                     -0.291142
                                           -1.764 0.077813
                                 0.165017
## TEAM PITCHING BB
                      0.363834
                                 0.232686
                                            1.564 0.118045
## TEAM_PITCHING_SO
                     23.548478
                                 4.552359
                                            5.173 2.51e-07 ***
## TEAM FIELDING E
                   -19.679926
                                 1.056598 -18.626
                                                   < 2e-16 ***
## TEAM FIELDING DP
                     -0.159912
                                 0.012724 - 12.567
                                                   < 2e-16 ***
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 12.14 on 2260 degrees of freedom
## Multiple R-squared: 0.4096, Adjusted R-squared: 0.4057
## F-statistic: 104.5 on 15 and 2260 DF, p-value: < 2.2e-16
```

The model above is for all of the predictors in the dataset (except for Index). It is statistically significant at p<.05 and the adjusted r squared for the model is 0.405, which means that about 40.5% of the variance in the dataset is explained by the model.

Model 4

We will modify the model a bit and eliminate variables that we had previously flagged for multicollinearity such as TEAM_PITCHING_HR,TEAM_PITCHING_BB and TEAM_PITCHING_H. This is important since multicollinearity can significantly reduce model performance. Out of these predictors, TEAM_PITCHING_H also had extreme outliers, along with TEAM_PITCHING_SO. Since the r-square is computed using the mean, variables with outliers will throw off this value. Therefore, although we have transformed TEAM_PITCHING_SO, it maybe best to still remove this variable from the model.

```
##
##
  Call:
##
   lm(formula = TARGET_WINS ~ . - INDEX - TEAM_PITCHING_HR - TEAM_PITCHING_BB -
       TEAM_PITCHING_H - TEAM_PITCHING_SO, data = missmod$imputations[[i]])
##
##
## Residuals:
##
       Min
                1Q
                    Median
                                3Q
                                        Max
   -54.237
            -7.926
                    -0.228
                             7.878
                                    46.116
##
## Coefficients:
                      Estimate Std. Error t value Pr(>|t|)
##
## (Intercept)
                     1.110e+02
                                7.640e+00
                                            14.532
                                                    < 2e-16 ***
## TEAM_BATTING_H
                     4.342e-02
                                3.479e-03
                                            12.480
                                                    < 2e-16 ***
## TEAM BATTING 2B
                    -2.566e-02
                                8.537e-03
                                            -3.006 0.002678 **
## TEAM_BATTING_3B
                     5.995e-02
                                1.616e-02
                                             3.709 0.000213 ***
## TEAM BATTING HR
                     8.397e-02
                                9.464e-03
                                             8.872
                                                    < 2e-16 ***
## TEAM_BATTING_BB
                     1.501e-02
                                2.939e-03
                                             5.106 3.56e-07 ***
## TEAM_BATTING_SO
                    -2.028e-02
                                2.310e-03
                                            -8.781
                                                    < 2e-16 ***
## TEAM_BASERUN_SB
                     3.505e+00
                                7.579e-01
                                             4.624 3.97e-06 ***
## TEAM_BASERUN_CS
                     9.757e-02
                                1.677e-02
                                             5.819 6.74e-09 ***
## TEAM_BATTING_HBP -7.407e-05
                                1.069e-04
                                            -0.693 0.488362
## TEAM_FIELDING_E -1.770e+01
                                9.918e-01 -17.849
                                                    < 2e-16 ***
## TEAM_FIELDING_DP -1.576e-01
                                1.277e-02 -12.340
                  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
  Signif. codes:
## Residual standard error: 12.26 on 2264 degrees of freedom
## Multiple R-squared: 0.3968, Adjusted R-squared: 0.3938
```

```
## F-statistic: 135.4 on 11 and 2264 DF, p-value: < 2.2e-16
```

After removing variables that we had previously flagged for multicollinearity and outliers, we can see that the adjusted r-squared for the model drops a bit. However the F-Statistic seems to have improved.

Model 5

In addition to the above, this model considers the four different interaction effects from Model 2 above.

```
## Call:
  lm(formula = TARGET_WINS ~ . - INDEX - TEAM_PITCHING_HR - TEAM_PITCHING_BB -
       TEAM_PITCHING_H - TEAM_PITCHING_SO + (TEAM_BATTING_H * TEAM_BATTING_2B +
       TEAM_BATTING_H * TEAM_BATTING_3B + TEAM_BATTING_H * TEAM_BATTING_HR),
##
       data = missmod$imputations[[i]])
##
##
## Residuals:
##
      Min
                1Q Median
                                3Q
                                       Max
## -54.645 -7.961 -0.071
                            7.958 53.271
##
## Coefficients:
##
                                    Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                                   1.181e+02 1.352e+01
                                                          8.734 < 2e-16 ***
## TEAM_BATTING_H
                                                          4.285 1.91e-05 ***
                                   3.572e-02 8.336e-03
## TEAM_BATTING_2B
                                  -1.802e-01 5.069e-02 -3.555 0.000385 ***
                                                          3.086 0.002052 **
## TEAM BATTING 3B
                                   2.834e-01 9.183e-02
## TEAM_BATTING_HR
                                   4.098e-01 6.257e-02
                                                          6.550 7.10e-11 ***
## TEAM_BATTING_BB
                                   1.596e-02
                                             2.930e-03
                                                          5.448 5.64e-08 ***
## TEAM_BATTING_SO
                                  -2.250e-02
                                             2.349e-03
                                                         -9.580 < 2e-16 ***
## TEAM_BASERUN_SB
                                             7.630e-01
                                   3.820e+00
                                                          5.006 5.97e-07 ***
## TEAM BASERUN CS
                                             1.695e-02
                                                          5.978 2.62e-09 ***
                                  1.013e-01
## TEAM_BATTING_HBP
                                  -9.007e-05
                                              1.084e-04
                                                         -0.831 0.405920
                                              9.863e-01 -18.029
## TEAM_FIELDING_E
                                  -1.778e+01
                                                                 < 2e-16 ***
## TEAM_FIELDING_DP
                                  -1.561e-01
                                              1.274e-02 -12.252
                                                                 < 2e-16 ***
## TEAM_BATTING_H:TEAM_BATTING_2B 1.105e-04
                                              3.377e-05
                                                          3.272 0.001083 **
## TEAM_BATTING_H:TEAM_BATTING_3B -1.431e-04
                                             5.800e-05
                                                         -2.468 0.013673 *
## TEAM_BATTING_H:TEAM_BATTING_HR -2.143e-04 4.088e-05
                                                        -5.242 1.74e-07 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 12.19 on 2261 degrees of freedom
## Multiple R-squared: 0.4047, Adjusted R-squared: 0.4011
## F-statistic: 109.8 on 14 and 2261 DF, p-value: < 2.2e-16
```

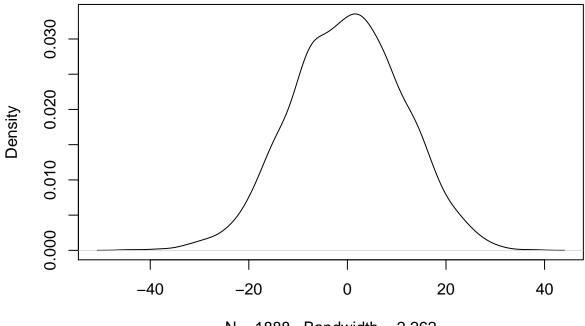
After adding the interaction effects, it seems that out adjusted r-squared has gone up to 0.40 (model explains 40% of the variance in the data). The model is statistically significant at p<0.05.

SELECT MODELS

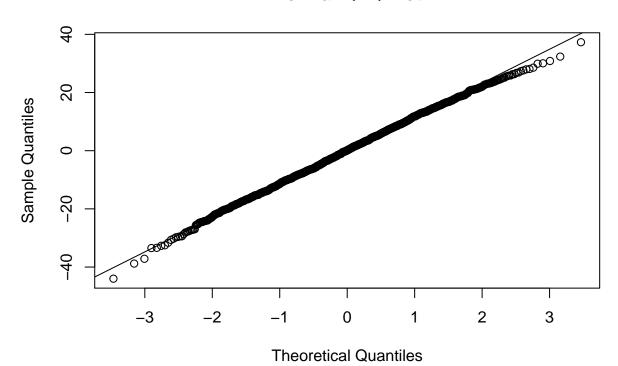
Diagnostics

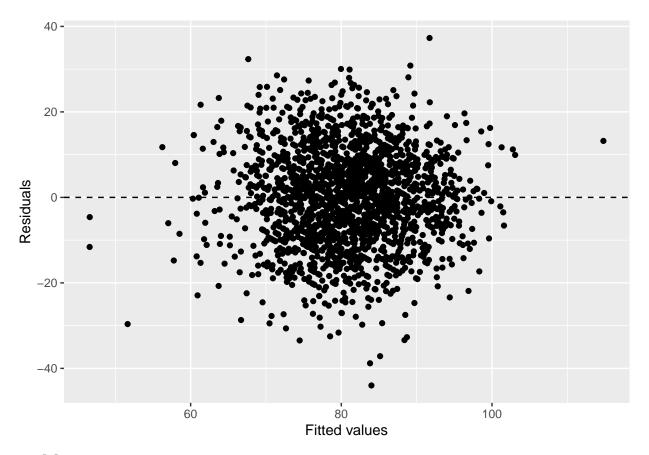
We will look at the residual plots and model performance statistics (MSE and RMSE) for each of the models.

density.default(x = res0)



N = 1888 Bandwidth = 2.262 Normal Q-Q Plot



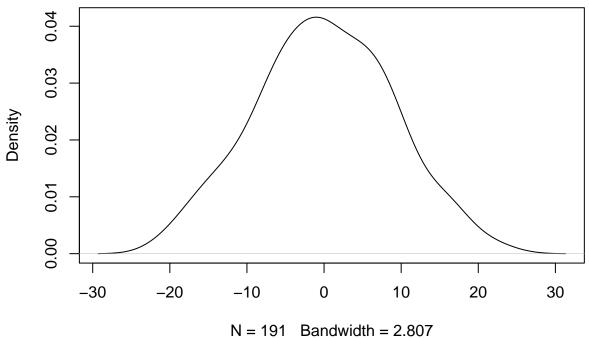


[1] "Mean Squared Error: 128.997300425331"

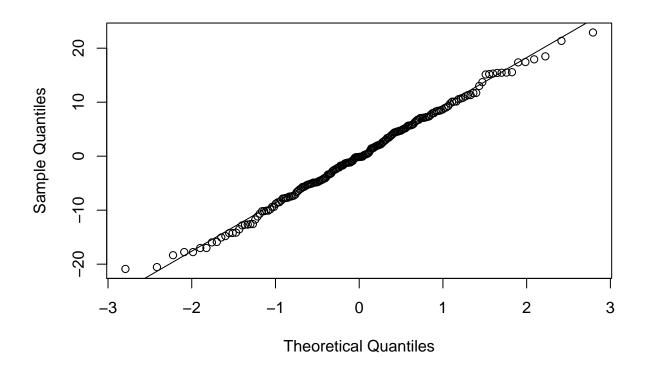
[1] "Root MSE: 11.3576978488306"

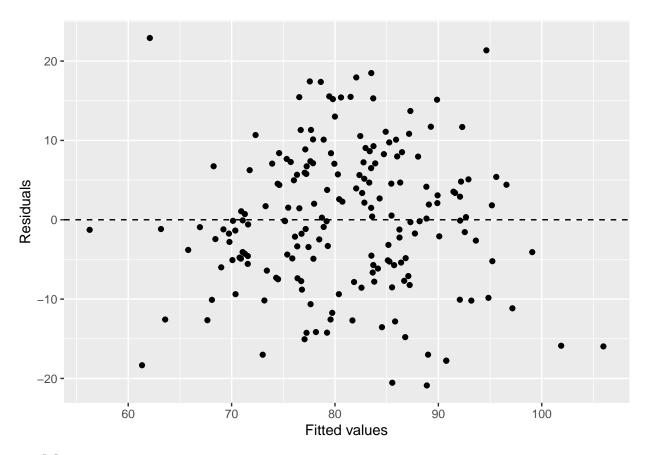
Model~2

density.default(x = resx)



N = 191 Bandwidth = 2.807 Normal Q-Q Plot



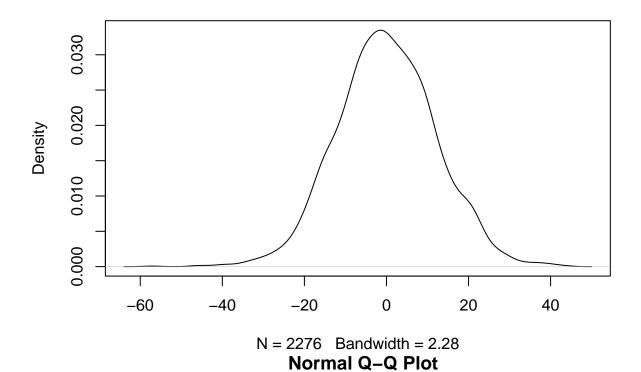


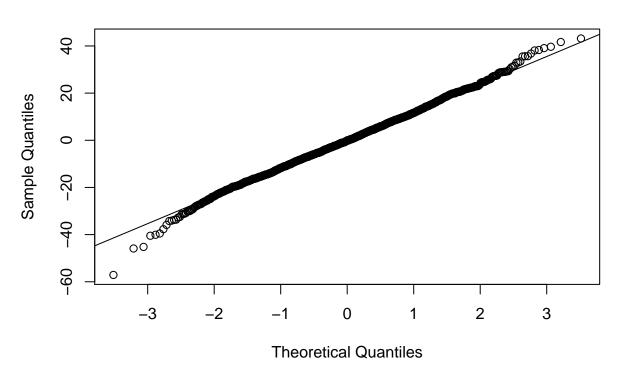
[1] "Mean Squared Error: 79.0711797402695"

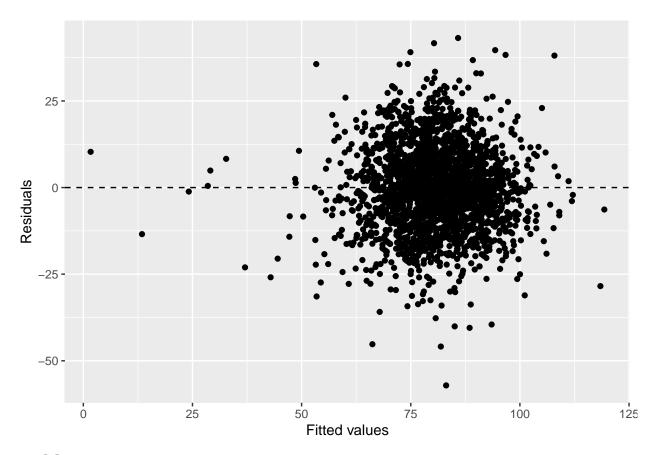
[1] "Root MSE: 8.8921976890007"

 $Model \ 3$

density.default(x = res)





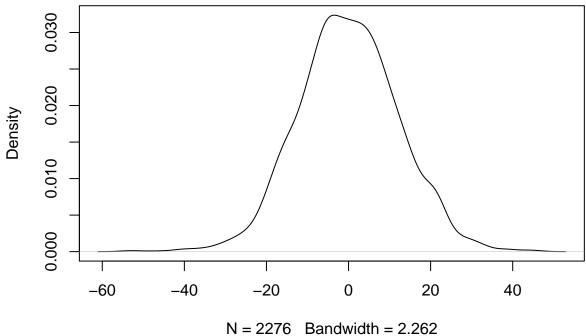


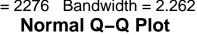
[1] "Mean Squared Error: 146.432480093758"

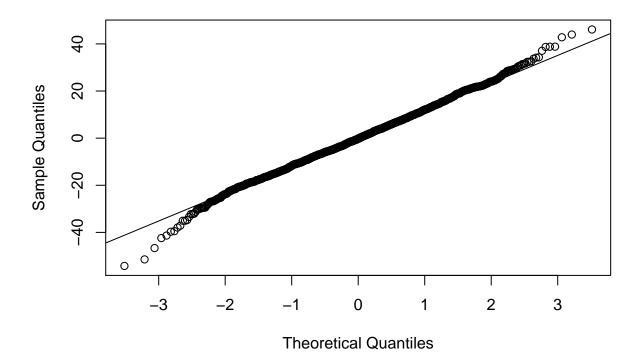
[1] "Root MSE: 12.1009288938395"

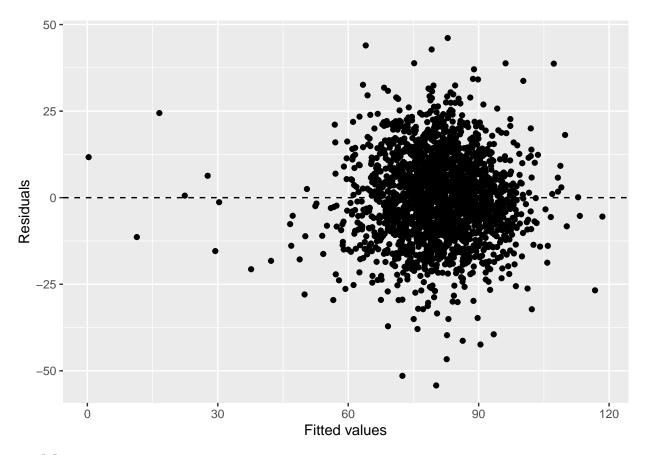
Model 4

density.default(x = res2)







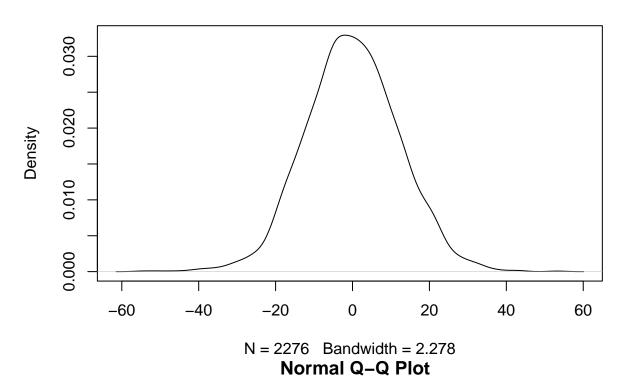


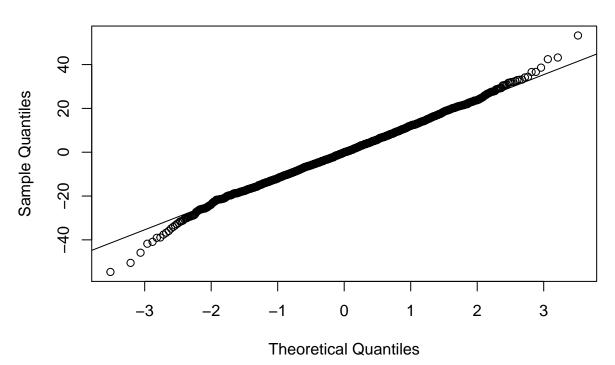
[1] "Mean Squared Error: 149.617687432938"

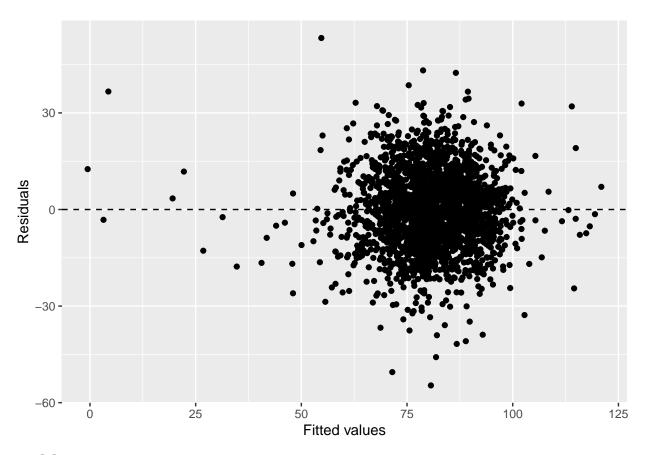
[1] "Root MSE: 12.2318309109036"

Model 5

density.default(x = res3)







[1] "Mean Squared Error: 147.636505909219"

[1] "Root MSE: 12.1505763611945"

The diagnostic plots illustrate that our residuals for all 5 models are normally distributed. However, in terms of the residual abline plot, Model 2's residual plot seems the best compared to the other 4, which seem to have a pattern to them. In addition, Model 2 also had the lowest MSE/RMSE and highest adjusted R^2 (0.44)

Therefore, $Model\ 2$ is the best model thus far. We will further test out this model on the evaluation dataset to see if this hold.

FURTHER EVALUATION

To ensure the model's efficacy when applied to the evaluation data set, we apply the same set of transformations used on the Training data set. Since the actual wins are withheld, we compared the distribution of predictions to the actual wins in the training set. The means were similar but the training data included much more variation between teams. It's also worth mentioning as well that using the predict function creates missing values as the evaluation data is missing. In fact, for TEAM_BATTING_HBP, over 90% of rows are missing entries.

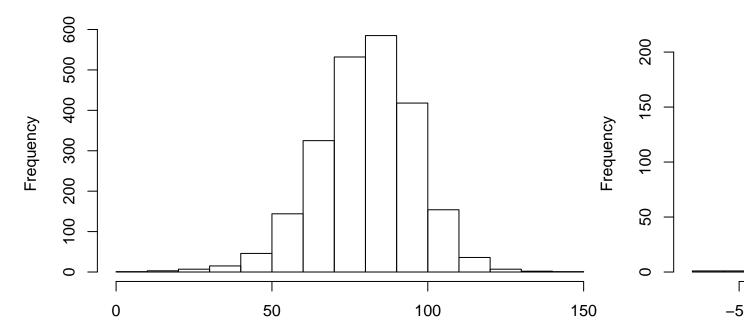
##	INDEX	TEAM_BATTING_H	TEAM_BATTING_2B	TEAM_BATTING_3B
##	0.00	0.00	0.00	0.00
##	TEAM_BATTING_HR	TEAM_BATTING_BB	TEAM_BATTING_SO	TEAM_BASERUN_SB
##	0.00	0.00	6.95	5.02
##	TEAM_BASERUN_CS	TEAM_BATTING_HBP	TEAM_PITCHING_H	TEAM_PITCHING_HR
##	33.59	92.66	0.00	0.00

```
## TEAM_PITCHING_BB TEAM_PITCHING_SO TEAM_FIELDING_E TEAM_FIELDING_DP
## 0.00 6.95 0.00 11.97
```

The prediction data also has missing values, which are approximately the same as the training data.

We will run our selected model on the evaluation dataset and look at the summary.

Histogram of baseball_df\$TARGET_WINS



baseball_df\$TARGET_WINS

```
##
         fit
                            lwr
                                              upr
            :-34.51
                              :-65.85
                                                 : -3.161
##
    Min.
                      Min.
                                         Min.
##
    1st Qu.: 62.43
                      1st Qu.: 38.50
                                         1st Qu.: 83.060
    Median: 71.55
                      Median : 50.38
                                         Median: 91.999
##
                                                 : 93.867
##
    Mean
            : 69.32
                      Mean
                              : 44.78
                                         Mean
    3rd Qu.: 78.39
                       3rd Qu.: 56.34
                                         3rd Qu.: 99.615
##
    Max.
            :127.32
                              : 71.53
                                         Max.
                                                 :206.949
##
                      Max.
##
   [1] 27.97456
##
      Min. 1st Qu.
                     Median
                                Mean 3rd Qu.
                                                  Max.
##
      0.00
              71.00
                      82.00
                               80.79
                                        92.00
                                                146.00
## [1] 15.75215
```

Here are the predicted values for Target Wins, based on our model for the teams in the evaluation dataset.

Table 3: Predicted Target Wins

IN	DEKAI	VITIBAN	NIT <u>ITHE</u> SSAY	NTTT HE SAX	NTTP#BXX	DAKKHETITIV	ATTT HESKEY	NTTI HEBARY	SERV	NATE IRREVI	NITIVE S		KCTHHAV	CCT HHAN	KCCC HIPBAN	
9	1209	170	33	83	447	1080	62	50	NA	1209	83	447	1080	140	156	56
10	1221	151	29	88	516	929	54	39	NA	1221	88	516	929	135	164	61
14	1395	183	29	93	509	816	59	47	NA	1395	93	509	816	156	153	62
47	1539	309	29	159	486	914	148	57	42	1539	159	486	914	124	154	75
60	1445	203	68	5	95	416	NA	NA	NA	3902	14	257	1123	616	130	30

INDEXA	VI <u>T</u> IBAA	ANT IT HE SAX	ONT IT HEBARA			OTT THE SARK	NTTI IBBAN			NITTICESSO	CCTHAN	KCC HHAN	(CETHERAX	K(GT)HIBIN	BAGILAN BRACHELDH	
63 1431	236	53	10	215	377	NA	NA	NA	2793	20	420	736	572	105	34	
74 1430	219	55	37	568	527	365	NA	NA	1544	40	613	569	490	NA	48	
83 1385	158	42	33	356	609	185	NA	NA	1626	39	418	715	328	104	41	
98 1259	177	78	23	466	689	150	NA	NA	1342	25	497	734	226	132	67	
120 1397	212	42	58	452	584	52	NA	NA	1489	62	482	622	184	145	58	
$123\ 1427$	243	40	50	495	640	64	NA	NA	1501	53	521	673	200	183	58	
$135\ 1496$	239	55	164	462	670	48	28	NA	1574	173	486	705	150	178	68	
$138\ 1420$	223	57	186	511	751	31	21	NA	1494	196	538	790	137	167	68	
$140\ 1460$	232	22	176	503	680	27	8	NA	1536	185	529	715	125	160	77	
$151\ 1411$	195	22	141	485	665	59	48	NA	1411	141	485	665	115	114	71	
$153\ 1434$	192	30	153	434	747	57	46	NA	1434	153	434	747	146	180	65	
$171\ 1297$	204	22	130	491	1008	84	55	NA	1313	132	497	1021	154	126	61	
$184\ 1446$	284	25	166	565	1041	77	39	55	1464	168	572	1054	115	172	78	
$193\ 1276$	162	52	17	383	NA	138	NA	NA	1351	18	406	NA	301	83	44	
$213\ 1715$	322	72	116	527	397	90	83	NA	1816	123	558	420	232	174	75	
$217\ 1520$	295	68	49	628	459	77	49	NA	1620	52	669	489	166	158	84	
$226\ 1597$	291	38	98	629	563	54	43	NA	1702	104	670	600	155	174	76	
$230\ 1453$	256	67	105	653	651	40	41	NA	1559	113	701	698	179	153	75	
$241\ 1378$	225	26	118	533	677	18	36	NA	1450	124	561	712	160	174	64	
$291\ 1516$	277	24	152	431	902	89	36	54	1516	152	431	902	105	164	75	
$294\ 1556$	288	20	164	474	878	121	32	73	1556	164	474	878	102	156	80	
300 1499	183	28	3	83	0	NA	NA	NA	5167	10	286	0	1224	NA	-8	
$348\ 1464$	263	58	47	385	479	63	66	NA	1540	49	405	504	232	146	57	
$350\ 1558$	318	66	32	634	439	83	64	NA	1639	34	667	462	218	130	79	
$357\ 1502$	308	36	39	432	602	45	46	NA	1601	42	460	642	199	135	55	
$367\ 1596$	320	58	130	718	596	70	54	NA	1679	137	755	627	178	146	80	
$368\ 1546$	260	59	110	630	541	72	65	NA	1648	117	671	577	167	166	78	
$372\ 1516$	282	53	115	723	695	47	38	NA	1595	121	761	731	146	174	82	
$382\ 1550$	275	47	146	765	723	29	20	NA	1631	154	805	761	178	177	79	
$388\ 1447$	260	54	148	532	935	39	33	NA	1465	150	539	947	130	154	72	
$396\ 1450$	252	28	203	594	855	50	48	NA	1450	203	594	855	156	131	75	
$398\ 1347$	239	36	130	546	897	69	31	NA	1408	136	571	938	136	147	69	
$403\ 1561$	260	56	214	531	911	66	47	NA	1571	215	534	917	133	163	75	
$407\ 1578$	252	26	135	567	780	48	47	NA	2367	203	851	1170	137	162	86	
$410\ 1598$	259	45	181	500	842	38	25	NA	1598	181	500	842	143	128	75	
$412\ 1497$	322	21	145	599	711	41	34	NA	1506	146	603	715	130	147	76	
$414\ 1569$	310	39	124	623	728	65	36	NA	1569	124	623	728	93	123	87	
$436\ 1119$	118	33	7	37	0	NA	NA	NA	4120	26	136	0	1568	NA	-35	
$440\ 1609$	196	120	62	781	599	536	NA	NA	1931	74	937	719	470	NA	92	
$476\ 1514$	175	70	80	615	612	392	NA	NA	1803	95	733	729	413	NA	62	
$479\ 1657$	237	119	41	593	334	325	NA	NA	2114	52	756	426	537	NA	92	
$481\ 1746$	213	106	69	526	429	324	NA	NA	2176	86	655	535	500	NA	82	
501 1319	224	70	56	416	677	176	131	NA	1397	59	440	717	284	100	53	
$503\ 1293$	204	70	18	437	630	134	NA	NA	1360	19	460	663	281	127	58	
$506\ 1420$	235	70	36	450	443	121	136	NA	1494	38	473	466	237	118	65	
519 1496	269	54	76	412	500	55	NA	NA	1574	80	433	526	177	171	64	
$522\ 1625$	289	38	80	517	486	72	NA	NA	1709	84	544	511	154	164	71	
$550\ 1391$	239	50	145	499	1041	70	49	NA	1391	145	499	1041	162	147	63	
$554\ 1319$	203	43	130	415	854	41	30	NA	1319	130	415	854	119	149	63	
566 1411	251	35	107	471	912	93	64	NA	1411	107	471	912	174	149	59	
578 1420	221	41	104	417	816	77	51	NA	1420	104	417	816	114	142	67	
$596\ 1552$	206	106	38	566	401	334	NA	NA	1849	45	674	478	411	119	84	

INDEXAM	VITEAN	NITITIESAN	(NITTI HEXXX			ATTE HEAR					TS COTHERNS	KGTHHAN	XCCTHPAN	K GETHERN		RALL WI
599 1280	203	72	15	392	616	227	NA	NA	1346	16	412	648	250	100	59	
605 1120	122	61	7	427	NA	194	NA	NA	1186	7	452	NA	332	106	46	
607 1390	183	84	18	445	NA	216	NA	NA	1462	19	468	NA	304	107	67	
614 1554	252	81	29	494	414	174	NA	NA	1798	34	572	479	200	134	85	
644 1410	218	69	45	738	627	65	58	NA	1483	47	776	660	142	189	88	
692 1507	262	28	159	573	907	107	52	NA	1516	160	577	913	126	132	78	
699 1481	284	19	242	499	1030	78	51	63	1481	242	499	1030	100	167	89	
700 1450	253	23	200	435	1002	137	67	79	1450	200	435	1002	94	166	80	
716 1637	260	93	26	487	288	446	NA	NA	2088	33	621	367	321	NA	87	
721 1436	202	82	44	376	681	160	NA	NA	1674	51	438	794	414	119	55	
$722\ 1600$	218	89	21	344	538	152	NA	NA	1851	24	398	623	373	137	69	
729 1348	168	76	23	506	NA	296	NA	NA	1427	24	536	NA	327	127	62	
731 1460	191	111	22	612	629	306	NA	NA	1546	23	648	666	314	114	91	
$746\ 1621$	255	126	37	478	350	54	NA	NA	1705	39	503	368	193	168	109	
$763\ 1433$	241	49	45	468	501	52	NA	NA	1507	47	492	527	127	203	72	
774 1440	232	48	155	586	679	49	32	NA	1515	163	616	714	144	204	73	
776 1479	211	34	232	555	799	47	23	NA	1556	244	584	841	119	155	83	
788 1573	281	36	106	379	938	59	55	NA	1573	106	379	938	144	144	63	
789 1558	224	42	171	474	1042	79	56	NA	1558	171	474	1042	168	158	68	
792 1385	225	46	130	637	961	147	66	NA	1457	137	670	1011	116	150	78	
811 1419	250	27	164	488	1006	124	56	NA	1419	164	488	1006	125	131	71	
835 1284	198	61	19	383	NA	186	NA	NA	1351	20	403	NA	270	100	51	
837 1403	200	68	10	390	NA	201	NA	NA	1495	11	416	NA	262	119	61	
861 1631	358	48	105	553	455	55	34	NA	1716	110	582	479	179	173	73	
862 1666	343	82	98	487	600	67	57	NA	1764	104	516	635	184	156	81	
863 1804	376	86	129	541	494	69	56	NA	1898	136	569	520	191	162	87	
$871\ 1534$	284	53	74	539	624	50	44	NA	1614	78	567	656	173	202	72	
$879\ 1472$	222	52	156	659	788	48	41	NA	1548	164	693	829	163	148	74	
887 1489	229	21	134	467	603	61	26	NA	1566	141	491	634	133	174	70	
$892\ 1367$	198	21	156	506	857	109	46	NA	1367	156	506	857	114	127	73	
$904\ 1485$	222	46	101	534	692	88	88	NA	1494	102	537	696	131	146	74	
$909\ 1458$	225	32	109	651	625	151	68	NA	1458	109	651	625	123	129	77	
$925\ 1530$	334	30	198	630	1061	143	60	NA	1530	198	630	1061	110	146	87	
$940\ 1421$	160	72	30	523	508	289	NA	NA	1731	37	637	619	445	NA	59	
$951\ 1869$	301	122	58	347	127	399	NA	NA	10814	336	2008	735	1261	NA	105	
$976\ 1400$	169	66	26	431	344	156	NA	NA	1680	31	517	413	398	133	53	
981 1494	193	81	12	340	NA	207	NA	NA	1614	13	367	NA	285	85	66	
983 1449	223	62	20	423	NA	298	NA	NA	1544	21	451	NA	286	93	58	
$984\ 1385$	200	76	29	483	NA	262	NA	NA	1457	31	508	NA	296	83	64	
$989\ 1443$	218	99	24	716	554	254	154	NA	1518	25	753	583	271	113	91	
$995\ 1825$	284	106	61	616	398	101	94	NA	1932	65	652	421	245	113	104	
10001627	296	95	38	630	445	93	76	NA	1712	40	663	468	207	159	97	
10011623	299	106	54	622	445	149	77	NA	1718	57	659	471	221	183	98	
10071556	298	82	60	500	550	72	53	NA	1637	63	526	579	187	176	81	
10161381	228	39	80	535	501	41	42	NA	1453	84	563	527	203	149	59	
10271556	272	46	114	532	634	32	37	NA	1637	120	560	667	138	157	76	
10331416	206	32	168	610	775	36	18	NA	1490	177	642	815	130	138	77	
10701413	257	21	204	546	1268	87	50	NA	1413	204	546	1268	135	157	77	
10811504	253	102	33	262	482	NA	NA	NA	2901	64	505	930	652	154	62	
10841193	165	68	45	299	1011	NA	NA	NA	1726	65	432	1462	743	NA	26	
10981461	325	30	166	470	1145	89	40	67	1461	166	470	1145	103	174	76	
11501458	294	36	187	590	999	89	30	61	1458	187	590	999	101	136	83	

INDEXAN	VITIBAN	NTTFHEXX	(SYTTT HIBSAY)			ATTI HESIAR	(SYTTI HB3KSX)			NTT NEXS		Jege Heavi	VOET HERAN	K GETHIBA	BOLIDAS ROCLEDEN VS	ZINDE
11601295	237	64	25	360	814	129	NA	NA	1734	33	482	1090	609	NA	<u> </u>	_
11691431	263	58	118	591	675	155	75	NA	1431	118	591	675	155	151	71	
11721469	305	59	98	498	644	216	84	NA	1469	98	498	644	150	153	70	
1174633	266	59	115	508	709	185	43	NA	1633	115	508	709	141	150	78	
11761603	$\frac{200}{295}$	58	132	442	758	133	48	NA	1603	132	442	758	127	140	75	
11781487	$\frac{250}{269}$	52	$132 \\ 117$	400	832	106	64	NA	1487	$132 \\ 117$	400	832	129	157	67	
11841474	$\frac{209}{318}$	$\frac{32}{44}$	101	501	884	108	62	NA	1483	102	504	889	129 123	162	72	
11931594	296	52	151	538	938	128	39	NA	1604	153	$504 \\ 541$	944	126	190	79	
11961415	$\frac{290}{285}$	$\frac{32}{42}$	140	524	938 921	140	65	52	1415	140	524	921	130	150	79 70	
								63						163		
11991445	289	34	126	424	1008	53	33		1445	126	424	1008	125		66	
12071362	199	81	29	408	508	386	NA	NA	1576	34	472	588	581	NA	47	
12181572	195	106	30	522	288	297	NA	NA	1721	33	571	315	344	NA	87	
12231209	168	56	16	435	NA	217	NA	NA	1280	17	461	NA	363	92	43	
12261242	155	69	20	368	NA	132	NA	NA	1359	22	403	NA	287	103	52	
12271098	116	63	29	340	NA	119	NA	NA	1155	31	358	NA	254	69	44	
12291235	175	77	26	457	743	159	NA	NA	1299	27	481	782	246	131	62	
12411651	247	80	59	357	335	83	63	NA	1737	62	376	352	219	146	72	
12441712	265	85	68	463	406	39	32	NA	1813	72	490	430	221	138	83	
12461391	206	78	41	390	523	112	NA	NA	1473	43	413	554	239	124	63	
12481625	299	73	105	534	481	85	NA	NA	1721	111	565	509	203	120	76	
12491740	319	77	128	506	569	56	NA	NA	1830	135	532	599	178	176	82	
12531626	303	55	84	584	592	59	NA	NA	1733	90	622	631	192	150	75	
12611471	277	36	65	602	509	85	NA	NA	1547	68	633	535	145	158	71	
13051373	232	14	130	478	966	155	67	NA	1373	130	478	966	179	118	58	
13141466	215	35	158	527	1151	143	51	NA	1649	178	593	1295	146	135	74	
13231450	226	30	203	536	1092	102	41	62	1450	203	536	1092	73	145	90	
13281474	223	57	18	259	391	NA	NA	NA	2985	36	524	792	780	75	36	
13531335	228	49	120	500	909	106	75	NA	1335	120	500	909	127	168	67	
13631455	233	36	97	435	677	52	57	NA	1464	98	438	681	137	157	64	
13711477	272	35	82	511	779	256	115	NA	1477	82	511	779	89	146	78	
13721426	240	25	125	555	932	138	93	NA	1426	125	555	932	131	148	71	
13891255	183	61	11	304	814	161	NA	NA	1346	12	326	873	336	104	40	
13931264	141	79	9	392	NA	181	NA	NA	1347	10	418	NA	294	95	59	
14211695	310	89	66	610	421	110	44	NA	1795	70	646	446	193	173	94	
14311460	274	66	63	538	674	54	53	NA	1536	66	566	709	$\frac{133}{222}$	170	68	
	$\frac{214}{237}$	46	53	610	639	50	39	NA	1419	56	642	672	137	160	73	
14371349 14421340	226				771			NA	1419 1410	123	583	811	137	167	69	
		40	117	554		14	40									
14501396	257	42	150	554	969	92	$\frac{33}{5c}$	NA	1396	150	554	969	172	158	64	
14631472	$\frac{259}{256}$	47	82	604	684	99 cc	56	NA	1472	82	604	684	146	171	74	
14641544	256	46	112	526	693	66	45	NA	1544	112	526	693	134	203	75 70	
14701453	282	41	141	502	779	68	44	NA	1453	141	502	779	120	139	73	
14711446	257	39	196	501	977	81	61	NA	1446	196	501	977	118	168	74	
14841468	289	30	106	506	990	119	61	NA	1486	107	512	1002	93	146	77	
14951546	44	29	0	15	44	0	0	NA	22768		221	648	1473	NA	-18	
15071372	195	31	103	353	932	36	31	NA	1372	103	353	932	166	154	51	
15141365	203	29	98	547	958	89	43	NA	1365	98	547	958	112	135	71	
15261314	172	26	112	436	1031	141	64	NA	1314	112	436	1031	151	171	57	
15491469	323	41	200	547	1071	146	35	62	1469	200	547	1071	104	131	80	
15521382	185	86	32	326	642	NA	NA	NA	2073	48	489	963	680	NA	47	
15561642	218	135	29	449	459	252	NA	NA	2000	35	547	559	488	93	98	
15641324	153	65	17	437	NA	201	NA	NA	1420	18	469	NA	352	101	52	
15851770	313	116	160	677	599	96	63	NA	1862	168	712	630	219	139	86	

INDEXAN	MT IBAN	NITEM	NTTTHBAX			ATTI HBAR		OSTESBRO		SATITIVE: SAS		KCCTHHAN	XCCTHHAN	CCC HIBBAN	BYTELL ZIERROCCE	
15861765	293	83	164	792	587	146	72	NA	1869	174	839	622	177	139	92	
15901590	277	76	113	657	510	74	50	NA	1729	123	714	554	164	124	85	
15911775	334	88	193	741	629	82	42	NA	1879	204	785	666	173	157	88	
15921635	297	77	183	746	639	63	38	NA	1720	193	785	672	178	141	80	
16031557	$\frac{1}{264}$	79	146	655	503	25	25	NA	1638	154	689	529	126	169	85	
16121485	210	57	153	591	746	52	40	NA	1562	161	622	785	129	193	78	
16341461	229	41	152	515	597	66	47	NA	1479	154	521	604	124	185	74	
16451322	208	19	147	427	1027	119	45	NA	1322	147	427	1027	126	164	64	
16471462	281	18	163	536	903	78	37	NA	1462	163	536	903	112	165	78	
16731537	217	115	23	517	NA	275	NA	NA	1638	25	551	NA	280	123	96	
16741495	236	85	35	565	579	234	NA	NA	1583	37	598	613	224	114	83	
16871468	280	70	66	565	488	60	50	NA	1585	71	610	527	187	141	76	
16881689	296	74	59	580	343	103	66	NA	1777	62	610	361	204	130	85	
17001533	301	59	104	536	567	64	36	NA	1634	111	571	604	224	140	66	
17081379	229	55	64	636	592	39	35	NA	1451	67	669	623	150	169	75	
17131373	223	37	94	718	590	55	45	NA	1444	99	755	621	147	156	74	
17171394	215	43	118	505	765	42	32	NA	1466	124	531	805	175	197	63	
17211371	223	36	116	540	783	17	12	NA	1442	122	568	824	134	157	69	
17301400	210	28	148	617	953	100	39	NA	1400	148	617	953	137	162	73	
17371327	209	33	114	596	823	343	124	NA	1335	115	600	828	145	131	67	
17481432	263	33	199	593	1056	140	63	NA	1432	199	593	1056	142	122	75	
17491474	251	22	156	580	926	129	54	NA	1474	156	580	926	105	151	81	
17631450	279	28	205	609	1008	46	20	68	1450	205	609	1008	102	144	86	
176 & 025	292	140	32	259	70	259	NA	NA	10935	173	1399	378	1172	NA	127	
17781669	281	102	35	391	473	580	NA	NA	2033	43	476	576	643	NA	69	
17801631	291	79	52	650	604	307	NA	NA	1987	63	792	736	566	NA	68	
17821420	299	79	5	233	587	NA	NA	NA	2347	8	385	970	1056	NA	33	
17841312	230	52	29	324	591	NA	NA	NA	1932	43	477	870	658	NA	30	
17942058	336	90	75	573	324	341	NA	NA	2545	93	709	401	456	NA	93	
18031351	181	58	25	402	NA	169	NA	NA	1440	27	428	NA	427	99	42	
18041452	199	87	17	433	NA	192	NA	NA	1548	18	461	NA	293	106	72	
18191466	242	57	68	300	562	106	88	NA	1552	72	318	595	246	143	48	
18321534	256	44	64	406	511	59	NA	NA	1635	68	433	545	195	166	59	
18331609	311	38	61	433	581	57	NA	NA	1749	66	471	632	214	152	58	
18441344	207	28	59	472	527	57	NA	NA	1414	62	497	554	246	158	46	
18471438	239	41	96	463	629	72	NA	NA	1513	101	487	662	221	133	56	
18541368	225	53	139	686	708	46	34	NA	1439	146	722	745	116	123	79	
18551381	218	52	127	615	708	47	24	NA	1453	134	647	745	151	147	71	
18571498	250	59	130	603	916	54	35	NA	1576	137	634	964	136	143	79	
18641389	206	53	145	497	1098	46	32	NA	1398	146	500	1105	158	154	64	
18651448	224	49	117	510	969	56	42	NA	1448	117	510	969	113	147	75	
18691307	225	58	102	522	1073	72	64	NA	1315	103	525	1080	113	135	72	
18801517	250	38	104	563	654	156	70	NA	2297	157	852	990	130	136	85	
18811417	245	25	112	506	831	128	76	NA	1417	112	506	831	121	138	69	
18821352	209	45	125	640	906	143	75	NA	1352	125	640	906	152	117	70	
18941458	296	34	106	559	995	81	28	NA	1640	119	629	1119	108	156	79	
18961390	290	35	116	519	1032	92	56	NA	1390	116	519	1032	105	134	73	
19161475	257	80	52	515	573	284	NA	NA	1810	64	632	703	471	NA	60	
19181378	178	85	35	512	604	246	NA	NA	1654	42	614	725	570	NA	56	
19211817	277	155	60	541	259	319	NA	NA	2264	75	674	323	441	NA	117	
19261711	213	133	29	418	375	195	NA	NA	1860	32	454	408	392	NA	102	
19381415	217	112	52	552	613	168	NA	NA	1489	55	581	645	243	138	84	

INDEXAN	Λ <u>ΓΙΒΑ</u> Α	NITE EXX	NATITI HIBAN	NITTE BEEN		ATTI HEXX		ISTESBRAY		NTT ICESS		RCCTHHAN	VCCT HEAR	XCCTHIBBN	MELIZAS RACCIEDEN VV
19791263	190	32	97	511	762	45	43	NA	1329	102	538	802	190	176	 55
19821328	221	63	96	495	686	23	23	NA	1397	101	521	722	175	184	63
19871571	248	59	126	511	786	36	25	NA	1653	133	538	827	135	171	78
19971522	235	70	130	444	871	66	34	NA	1522	130	444	871	137	195	72
20041550	278	57	133	474	878	260	120	NA	1550	133	474	878	145	137	72
20111412	237	33	98	438	841	96	62	NA	1412	98	438	841	128	142	64
20151344	243	46	111	560	959	120	61	NA	1361	112	567	971	126	130	71
20221441	276	30	141	513	1094	95	62	NA	1621	159	577	1231	136	155	72
20251395	271	35	107	393	1060	159	51	NA	1395	107	393	1060	140	161	59
20271506	320	31	168	564	1032	86	40	66	1506	168	564	1032	132	169	76
20311437	269	39	143	418	1073	63	40	96	1446	144	421	1080	104	190	71
20362170	241	70	13	111	102	92	76	NA	6893	41	353	324	1217	NA	45
20661324	194	53	94	537	775	101	58	NA	1332	95	540	780	141	155	68
20731442	239	25	136	484	917	96	68	NA	1442	136	484	917	135	135	68
20871413	279	37	157	602	1177	131	53	46	1413	157	602	1177	141	155	72
20921416	269	39	130	600	977	99	44	49	1416	130	600	977	109	136	78
21251523	216	97	33	360	712	NA	NA	NA	2203	48	521	1030	743	NA	58
21481294	169	51	24	546	NA	217	NA	NA	1370	25	578	NA	244	79	58
21621668	251	98	79	497	413	145	121	NA	1766	84	526	437	198	164	89
21911422	215	53	140	660	662	44	NA	NA	1496	147	694	696	144	190	76
22031524	231	31	200	513	807	72	49	NA	1496	196	504	792	139	150	76
22181392	227	41	134	568	842	90	59	NA	1392	134	568	842	178	136	64
22211318	200	$\overline{44}$	80	512	845	101	58	NA	1326	80	515	850	157	125	62
22251499	229	26	112	528	980	126	76	NA	1499	112	528	980	169	134	64
22321345	215	48	141	471	973	95	57	NA	1345	141	471	973	108	151	70
22671620	210	139	66	542	355	233	NA	NA	1988	81	665	436	523	NA	88
22911339	185	80	34	413	579	149	NA	NA	1682	43	519	727	276	146	66
22991621	272	86	95	503	545	87	NA	NA	1705	100	529	573	208	148	78
23171585	288	62	105	572	498	39	NA	NA	1667	110	602	524	118	170	86
23181576	269	46	67	542	513	58	NA	NA	1658	70	570	540	143	158	75
23531541	300	49	101	451	781	117	54	NA	1541	101	451	781	122	174	73
24031149	175	18	59	529	974	133	77	NA	1209	62	556	1025	175	155	51
24111626	265	27	125	483	593	92	49	NA	1636	126	486	597	148	170	70
24151461	$\frac{200}{228}$	29	121	423	812	82	50	NA	1470	122	426	817	139	139	64
24241472	284	39	181	483	984	113	67	NA	1472	181	483	984	130	145	72
24411366	218	39	99	451	649	28	52	NA	1374	100	454	653	131	164	64
24641489	287	36	195	470	1094	156	55	74	1489	195	470	1094	97	184	80
24651457	305	38	187	522	1142	71	18	53	1457	187	522	1142	107	159	78
24721454	220	52	9	97	393	NA	NA	NA	3141	19	210	849	994	95	4
24811642	$\frac{220}{221}$	98	56	638	451	319	NA	NA	2031	69	789	558	492	NA	80
2487819	72	72	18	198	1107	NA	NA	NA	7371	162	1782	9963	936	NA	48
25001251	162	23	95	492	860	71	69	NA	1299	99	511	893	139	146	60
25011345	190	$\frac{23}{23}$	125	695	777	77	68	NA	1345	125	695	777	163	156	69
25201381	263	$\frac{25}{37}$	102	463	976	196	63	NA	1381	102	463	976	124	113	66
25211410	270	36	122	542	860	$\frac{130}{228}$	56	NA	1410	102 122	542	860	159	144	65
25251423	339	34	172	420	1084	75	46	NA	1423	172	420	1084	131	150	66

CONCLUSION

Our final model predicts target wins for the team as follows:

$$\label{eq:target_wins} \begin{split} & \text{Target Wins} = 2.143\text{e} + 00\text{-}2.392\text{e} + 01(\log(\text{TEAM_FIELD_E})) + 2.508\text{e} + 01(\log(\text{TEAM_PITCHING_BB})) - \\ & 4.306\text{e} - 03(\text{TEAM_BATTING_3B}: \text{TEAM_BATTING_HR}) + 3.449\text{e} - 05(\text{TEAM_BATTING_2B}: \text{TEAM_BATTING_HR}) + 1 \\ & 04(\text{TEAM_BATTING_H}: \text{TEAM_BATTING_HR}) + 4.332\text{e} - 04(\text{TEAM_BATTING_H}: \text{TEAM_BATTING_3B}) \end{split}$$

The model seems to suggest that in order to maximize a teams chances of winning they should focus on reducing fielding errors which makes sense. However, what is interesting from our model is the positive association between walks allowed and target wins. Moreover, the model also seems to suggest that some of the batting interaction effects may slightly lower your chances of winning. While this is definitely an interesting finding, this may just as well be because observations suggest that one may lower the chances or another and vice versa.

REFERENCES

An Introduction to Statistical Learning with Applications in R Springer Linear Models with R (2014), Julian J, Faraway

CODE APPENDIX

The code chunks below shows the R code called above throughout the analysis. They are being reproduced in the appendix for review and feedback.

```
# Libraries
library(dplyr)
library(GGally)
library(DataExplorer)
library(ggplot2)
library(readr)
library(reshape2)
library(purrr)
library(tidyr)
library(corrplot)
library(MASS)
library(caret)
library(Hmisc)
library(e1071)
library(Amelia)
set.seed(2012)
# read data
baseball df <- read.csv('https://raw.githubusercontent.com/hillt5/DATA 621/master/HW1/moneyball-trainin
baseball_eval <- read.csv('https://raw.githubusercontent.com/hillt5/DATA_621/master/HW1/moneyball-evalu
plot_histogram(baseball_df$TARGET_WINS,
               title="Distribution of TARGET WINS")
skimr::skim(baseball_df)
summary(baseball_df)
print('Observations per year, 1871 - 2006:')
round(nrow(baseball_df)/(2006-1871),2)
baseball_df %>%
  dplyr::select(-INDEX, -TARGET_WINS) %>%
  pivot_longer(everything(), names_to = 'Var', values_to='Value') %>%
```

```
ggplot(aes(x = Var, y = Value)) +
    geom_boxplot() +
    coord_flip()
corrplot(cor(baseball_df[,2:17], use = 'complete.obs'))
baseball_df %>%
    keep(is.numeric) %>%
    gather() %>%
    ggplot(aes(value)) +
    facet_wrap(~key,scales="free")+
    geom_histogram()
skewness(baseball_df$TEAM_PITCHING_SO, na.rm=TRUE)
skewness(baseball_df$TEAM_BASERUN_SB,na.rm=TRUE)
skewness(baseball_df$TEAM_FIELDING_E, na.rm=TRUE)
# Log Transformation
baseball_df2 <- baseball_df</pre>
baseball_df2$TEAM_PITCHING_SO <- log(baseball_df2$TEAM_PITCHING_SO)</pre>
baseball_df2$TEAM_BASERUN_SB <- log(baseball_df2$TEAM_BASERUN_SB)</pre>
baseball_df2$TEAM_FIELDING_E <- log(baseball_df2$TEAM_FIELDING_E)
#Certain values changed to -lnf afte transformation. This throws an error during imputation so we will
baseball_df2$TEAM_PITCHING_SO <- ifelse(baseball_df2$TEAM_PITCHING_SO=="-Inf", NA, baseball_df2$TEAM_PITC
baseball_df2$TEAM_BASERUN_SB <- ifelse(baseball_df2$TEAM_BASERUN_SB == "-Inf", NA, baseball_df2$TEAM_BASE
baseball_df2$TEAM_FIELDING_E <-ifelse(baseball_df2$TEAM_FIELDING_E=="-Inf",NA,baseball_df2$TEAM_FIELDIN
require(Amelia)
set.seed(123)
missmod<- amelia(baseball_df2)</pre>
baseball_df_fix <- baseball_df
m1 <- lm(TARGET_WINS ~ TEAM_BATTING_H + TEAM_BATTING_HR +TEAM_BATTING_BB + TEAM_BATTING_SO + TEAM_PITC
summary(m1)
#remove TEAM PITCHING BB & TEAM PITCHING SO
m2<- lm(TARGET_WINS ~ TEAM_BATTING_H + TEAM_BATTING_HR +TEAM_BATTING_BB + TEAM_BATTING_SO + TEAM_PITCH
summary(m2)
anova(m1, m2)
par(mfrow=c(2,1))
plot(baseball_df_fix$TEAM_PITCHING_H,baseball_df_fix$TARGET_WINS,xlab = 'TEAM_PITCHING',ylab = 'TARGET_
plot(log(baseball_df_fix$TEAM_PITCHING_H),baseball_df_fix$TARGET_WINS,xlab = 'LOG(TEAM_PITCHING)',ylab = 'LOG(TEAM
#log TEAM_PITCHING_H
m3 <- lm(TARGET_WINS ~ TEAM_BATTING_H + TEAM_BATTING_HR +TEAM_BATTING_BB + TEAM_BATTING_SO + log(TEAM_
summary(m3)
#Remove TEAM_BATTING_H
m4 <- lm(TARGET_WINS ~ TEAM_BATTING_HR +TEAM_BATTING_BB + TEAM_BATTING_SO + log(TEAM_PITCHING_H) + TEA
summary(m4)
anova(m3, m4)
```

```
baseball_lm2 <- lm (baseball_df_fix, formula = TARGET_WINS ~.-INDEX+log(TEAM_FIELDING_E) + log(TEAM_PIT
summary(baseball_lm2)
betas <- NULL
ses <- NULL
for(i in 1:missmod$m)
  lmod <- lm (TARGET_WINS ~.-INDEX, missmod$imputations[[i]])</pre>
  betas <- rbind(betas ,coef(lmod))</pre>
  ses <- rbind(ses,coef(summary(lmod))[,2])</pre>
}
summary(lmod)
betas <- NULL
ses <- NULL
for(i in 1:missmod$m)
  lmod2 <- lm (TARGET_WINS ~.-INDEX-TEAM_PITCHING_HR-TEAM_PITCHING_BB-TEAM_PITCHING_H-TEAM_PITCHING_SO</pre>
  betas <- rbind(betas ,coef(lmod2))</pre>
  ses <- rbind(ses,coef(summary(lmod2))[,2])</pre>
}
summary(lmod2)
betas <- NULL
ses <- NULL
for(i in 1:missmod$m)
  lmod3 <- lm (TARGET_WINS ~.-INDEX-TEAM_PITCHING_HR-TEAM_PITCHING_BB-TEAM_PITCHING_H-TEAM_PITCHING_SO</pre>
  betas <- rbind(betas ,coef(lmod3))</pre>
  ses <- rbind(ses,coef(summary(lmod3))[,2])</pre>
}
summary(lmod3)
res0 <- resid(m4)
plot(density(res0))
qqnorm(res0)
qqline(res0)
ggplot(data = m4, aes(x = .fitted, y = .resid)) +
  geom_jitter() +
  geom_hline(yintercept = 0, linetype = "dashed") +
  xlab("Fitted values") +
 ylab("Residuals")
RSS <- c(crossprod(m4$residuals))
MSE <- RSS/length(m4$residuals)</pre>
print(paste0("Mean Squared Error: ", MSE))
print(pasteO("Root MSE: ", sqrt(MSE)))
resx <- resid(baseball_lm2)</pre>
plot(density(resx))
qqnorm(resx)
qqline(resx)
ggplot(data = baseball_lm2, aes(x = .fitted, y = .resid)) +
  geom_jitter() +
  geom_hline(yintercept = 0, linetype = "dashed") +
```

```
xlab("Fitted values") +
  ylab("Residuals")
RSS <- c(crossprod(baseball_lm2$residuals))</pre>
MSE <- RSS/length(baseball_lm2$residuals)</pre>
print(paste0("Mean Squared Error: ", MSE))
print(paste0("Root MSE: ", sqrt(MSE)))
res <- resid(lmod)
plot(density(res))
qqnorm(res)
qqline(res)
ggplot(data = lmod, aes(x = .fitted, y = .resid)) +
  geom_jitter() +
  geom_hline(yintercept = 0, linetype = "dashed") +
 xlab("Fitted values") +
 ylab("Residuals")
RSS <- c(crossprod(lmod$residuals))
MSE <- RSS/length(lmod$residuals)</pre>
print(paste0("Mean Squared Error: ", MSE))
print(paste0("Root MSE: ", sqrt(MSE)))
res2 <- resid(lmod2)
plot(density(res2))
qqnorm(res2)
qqline(res2)
ggplot(data = lmod2, aes(x = .fitted, y = .resid)) +
 geom_jitter() +
  geom_hline(yintercept = 0, linetype = "dashed") +
 xlab("Fitted values") +
 ylab("Residuals")
RSS <- c(crossprod(lmod2$residuals))</pre>
MSE <- RSS/length(lmod2$residuals)</pre>
print(paste0("Mean Squared Error: ", MSE))
print(paste0("Root MSE: ", sqrt(MSE)))
res3 <- resid(lmod3)</pre>
plot(density(res3))
qqnorm(res3)
qqline(res3)
ggplot(data = lmod3, aes(x = .fitted, y = .resid)) +
  geom_jitter() +
  geom_hline(yintercept = 0, linetype = "dashed") +
  xlab("Fitted values") +
 ylab("Residuals")
RSS <- c(crossprod(lmod3$residuals))</pre>
MSE <- RSS/length(lmod3$residuals)</pre>
print(paste0("Mean Squared Error: ", MSE))
print(paste0("Root MSE: ", sqrt(MSE)))
round(100*colSums(is.na(baseball_eval))/nrow(baseball_eval),2)
```

```
eval_predict <- predict(baseball_lm2, newdata = baseball_eval, interval="prediction")
hist(baseball_df$TARGET_WINS)
hist(eval_predict)
summary(eval_predict)
summary(baseball_df$TARGET_WINS)
sd(baseball_df$TARGET_WINS)

pred.TW <- round(predict(baseball_lm2, baseball_eval))
baseball_eval$TARGET_WINS <- pred.TW</pre>
knitr::kable(baseball_eval,caption="Predicted Target Wins")
```