In this example the noise process will be called Z_n, as stated in the README this noise process is an AR process with an added white gaussian noise.

Here is the formal problem that will be solved.

X_n is an AR process that is generated by the following recursion formula:

$$X_0 \sim N\left(0, \frac{1}{1-\alpha^2}\right)$$

$$X_n = \alpha X_{n-1} + G_n$$

$\alpha < 1$ and G_n is a white gaussian noise with zero mean and unit variance.

$$N_n \sim N(0, \sigma^2)$$

$$Z_n = X_n + N_n$$

The processes $N_n, G_n and\ X_0$ are independent.

And now for the interesting part.

The auto correlation function of this AR process is:

$$R_{x,l} = \frac{\alpha^l}{1-\alpha^2}$$

And the auto correlation function of the complete noise process is:

$$R_{z,l} = \begin{matrix} \dfrac{1}{1-\alpha^2} + \sigma_N^2, & if\ l = 0 \\ \dfrac{\alpha^l}{1-\alpha^2}, & otherwise \end{matrix}$$

The goal here is to predict the next noise sample with our last samples.

In proper mathematical notation:

The desired signal is $D_n = Z_n$ and the measured signal is $U_n = z_{n-1}$

The cross-correlation function between D_n and U_n is as follows:

$$\vec{p} = \mathbb{E}[D_n U_n] = \mathbb{E}[Z_n Z_{n-1}] = \mathbb{E}\begin{Bmatrix} (X_n + N_n)(X_{n-1} + N_{n-1}) \\ (X_n + N_n)(X_{n-2} + N_{n-2}) \\ \vdots \\ (X_n + N_n)(X_{n-L} + N_{n-L}) \end{Bmatrix}$$

$$= \mathbb{E}\begin{Bmatrix} X_n X_{n-1} \\ X_n X_{n-2} \\ \vdots \\ X_n X_{n-L} \end{Bmatrix} = \begin{Bmatrix} R_{X,1} \\ R_{X,2} \\ \vdots \\ R_{X,L} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\alpha}{1-\alpha^2} \\ \dfrac{\alpha^2}{1-\alpha^2} \\ \vdots \\ \dfrac{\alpha^{|L|}}{1-\alpha^2} \end{Bmatrix}$$

And the auto correlation function of U_n is

$$R = \mathbb{E}[Z_{n-1}Z_{n-1}^T] = \mathbb{E}[(X_{n-1} + N_{n-1})(X_{n-1} + N_{n-1})^T]$$

$$= \begin{bmatrix} \frac{1}{1-\alpha^2} + \sigma_N^2 & \cdots & \frac{\alpha^{|L-1|}}{1-\alpha^2} \\ \vdots & \ddots & \vdots \\ \frac{\alpha^{|L-1|}}{1-\alpha^2} & \cdots & \frac{1}{1-\alpha^2} + \sigma_N^2 \end{bmatrix}$$

The ideal linear estimator of order l is $w_n = R_z^{-1}\vec{p}$.

Now I will compare several estimation filters to the ideal estimator

The first will be the steepest descent.

The second will LMS.

The third will be RLS.