MATH 413 **Name:** *Amit Sawhney*
Homework 3
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1. Hotel Problem

   Solution: 30159509225913491526359449 6.0

   Order of Magnitude: 26

```
import scipy.special as spys
import numpy as np
import math

NUM_ROOMS = 8
NUM_PEOPLE = 32
NUM_PER_ROOM = 4

dp = np.zeros((NUM_ROOMS + 1, NUM_PEOPLE + 1))

for person in range(0, NUM_PEOPLE + 1):
  # traverse from NUM_ROOMS to 0
  for room in range(NUM_ROOMS, -1, -1):
    if room == NUM_ROOMS and (person >= 0 or person <= 4):
      dp[room][person] = 1
    elif person == 0:
      dp[room][person] = 1
    else:
      for occupant in range(0, NUM_PER_ROOM + 1):
        dp[room][person] += spys.comb(person, occupant) *
        dp[room + 1][person - occupant]

total = np.sum(dp[1:])
```

   Essentially, we are counting how many ways we can put 1 person into 1 room, 2 rooms, 3 rooms, . . . , 8 rooms. And then doing the same for 2 people, 3 people, . . . , 32 people. By caching the amounts of previous counts using a DP algorithm, we can easily calculate later probabilities for by referencing cached values.

2. Q57

$$\frac{\binom{13}{1}\cdot\binom{4}{2}\cdot\binom{12}{3}\cdot\binom{4}{1}\cdot\binom{4}{1}\cdot\binom{4}{1}\cdot}{\binom{52}{5}} = 0.4226$$

   There are clearly $\binom{52}{5}$ ways to select 5 cards from a deck. There are $\binom{13}{1}$ ways to pick which card will have a pair. There are $\binom{4}{2}$ ways to pick what specific cards of that rank will make up the pair. Then there are $\binom{12}{3}$ ways to choose what the remaining three cards will be. And for each of these cards there are $\binom{4}{1}$ ways to pick which card each will be.

3. Q60

   For the first question,

$$\frac{\binom{9+6-1}{9}}{\binom{15+6-1}{15}} = 0.1291$$

   This makes sense because there are $\binom{15+6-1}{15}$ ways to 15 bagels from 6 types of bagels and there are $\binom{9+6-1}{9}$ ways to select 9 free degrees of bagel from 6 types of bagel (which ensures that there is 1 type of each bagel).

$$\frac{\binom{7+6-1}{7}}{\binom{15+6-1}{15}} = 0.0511$$

   This makes sense in the denominator for the same reason as above. The numerator becomes $\binom{7+6-1}{7}$ because there are only 7 bagels of freedom (1 sesame bagel is already accounted for in the 1 bagel of each kind)

4. Q64

$$\frac{\binom{n}{1} \cdot \binom{n-1}{n-3} \cdot \frac{n!}{3!} + \binom{n}{2} \cdot \binom{n-2}{n-4} \cdot \frac{n!}{2! \cdot 2!}}{n^n}$$

   There are two cases:

   (a) There is 1 number repeated 3 times. In this case there are $\binom{n}{1}$ ways to pick the number that is repeated. There are $\binom{n-1}{n-3}$ ways to pick the rest of the numbers. Lastly, there are $\frac{n!}{3!}$ ways to arrange this sequence.

   (b) There are two numbers repeated 2 times. In this case there are $\binom{n}{2}$ ways to choose the numbers that are repeated twice. There are $\binom{n-2}{n-4}$ ways to pick the remaining numbers. Lastly, there are $\frac{n!}{2! \cdot 2!}$ ways to arrange this sequence

   So the probability is the total number of possibilities between the two cases over the total number of ways to create a sequence this way, which is n^n.

5. $$\binom{n+q-1}{n}$$

   This is simply a a question of how you can partition n people among q clubs.