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1. Section 2.7 Question 18.

   Based on the formula provided in the chapter,

   $$\frac{(\frac{8!}{2!})^2}{2! \cdot 4!} = 8467200$$

   Similarly you can construct an algorithm that says there are 64 choices for the first rook, 49 for the next, then 26, and so on (each rook essentially removes a dimension off the board), and then divide out the repeated blue and red rooks to avoid for duplicated arrangements. This shows:

   $$\frac{64 \cdot 49 \cdot 36 \cdot 25 \cdot 16 \cdot 9}{4! \cdot 2!} = 8467200$$

2. Section 2.7 Question 48

   This can be represented by counting the number of lattice paths from $(0,0)$ to $(m+1,n)$. First let's decompose this as there are at most n Bs. Consider how many lattice paths there are from $(0,0)$ to $(m,0)$ which covers the case of m As and 0 Bs, and there are $\binom{m+0}{0}$ ways to form n As and 0 Bs. Similarly, consider the number of paths from $(0,0)$ to $(m,1)$ which represents m As and 1 B and thus, there can be $\binom{m+1}{1}$ possibilities. Continuing this to m As and n Bs we find that the number of possibilities can be represented by $\binom{m+n}{n}$. So, the total number of possibilities for at most n Bs is:

   $$\binom{m+0}{0} + \binom{m+1}{1} + \binom{m+2}{2} + \cdots \binom{m+n}{n} = \binom{m+n+1}{n} = \binom{m+n+1}{m+1}$$

   Citation: Consulted Eric Liu about the approach that was talked about in lecture as I missed it.

3. Section 2.7 Question 54

   Number of towers is 3^n by some simple number analysis. Justification for this: Each number in the set has 3 choices:

   (a) It is in B
   (b) It is in A
   (c) It is in neither

   For fun, when I was attempting to derive a formula I came up with this,

   $$2^n + (\sum_{i=1}^{n-1} n \cdot 2^{n-i}) + 1$$

which can probably be simplified to 3^n but it became apparent that it was 3^n when $n = 1 \to 3, n = 2 \to 9, n = 3 \to 27$

4. Q(A)

   Consider a situation where you have $2n$ people and room and you want to know how many ways you can form groups of 2 (pairs). Clearly, there are $\binom{2n}{2}$ ways to create the pairs.

   Now decompose this set into 2 groups of size n where A has n elements and B has n elements. Imagine you want to create pairs from these 2 decomposed sets There are three cases:

   (a) You choose a pair both from A. There are $\binom{n}{2}$ ways to do this

   (b) You choose a pair both from B. There are $\binom{n}{2}$ ways to do this.

   (c) You choose a pair, one from A and one from B. There are n elements in A and n elements in B so there are $n \cdot n = n^2$ ways to pick one from A and one from B to form a pair.

   Clearly, there are $2 \cdot \binom{n}{2} + n^2$ ways to count the number of pairs.

   Because $\binom{2n}{2}$ and $2 \cdot \binom{n}{2} + n^2$ are both ways to count the same set of forming pairs:

$$\binom{2n}{2} = 2 \cdot \binom{n}{2} + n^2$$

5. Q(B)

   For the clarity of this question, consider the same equation:

$$\sum_{k=0}^{n} \binom{n}{k}\binom{k}{m} = \binom{n}{m} \cdot 2^{n-m}$$

   .

   Consider a set of n people. Imagine you are attempting to count how many ways there are to select a group of people from n and then a sub group of m people from that selected group.

   For a group size of 0, there are 0 ways to select this section. For a group of size 1, there are $\binom{n}{1}$ ways to select the first group and then $\binom{1}{m}$ ways to select the sub-group so $\binom{n}{1}\binom{1}{m}$ ways total. This pattern can continue as follows which represents the LHS.

   Consider this same set of grouping. There are clearly $\binom{n}{m}$ ways of creating subgroups of size m. For the rest of the members (which there are $n - m$ of, they are either in the original selected group or not, so there are 2^{n-m} possibilities as they are in the group or not which yields $\binom{n}{m}2^{n-m}$ possibilities total.

   Because the LHS and RHS count the same exact set, they must be equal.

   Citation: Consulted Eric Liu on how he approached this problem for the RHS as I was a bit stuck. Was able to derive the LHS from the lecture.