मोतीलाल नेहरू राष्ट्रीय प्रौद्योगिकी संस्थान इलाहाबाद
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Department of Mathematics
Mid Semester (Odd) Examination 2022-23

Programme Name: B. Tech.
Course Code: MA11101
Branch:.....
Duration: 90 Minutes

Semester: I
Course Name: Mathematics I
Student Reg. No.: 2 0 2 2 3 0 3 4
Max. Marks.: 20

Instructions: (i) Attempt all the questions. Marks are indicated on write side.
(ii) Attempt each part of a question in continuation.

			Marks
Q1	a	State and prove Lagrange's Mean Value theorem. Discuss the geometrical interpretation of it.	2
	b	Using Cauchy Mean Value theorem, show that $\frac{2\log_e x}{2\sin^{-1}x-\pi} < \frac{\sqrt{1-x^2}}{x}$.	2
Q2	a	Define ε- δ definition of continuity of a function of two variables $f : D \subset R^2 \to R$. Using ε- δ definition, discuss the continuity of the function $f(x,y) = \begin{cases} \frac{x^2+y^2}{3+\sin x}, & if\ (x,y) \neq (0,0) \\ 0, & if\ (x,y) = (0,0) \end{cases}$ at $(0,0)$.	2
	b	Define the differentiability of function of two variables $f : D \subset R^2 \to R$ at the point (x,y). Check the differentiability of the function $f(x,y) = \begin{cases} (x^2+y^2)\sin\left(\frac{1}{x^2+y^2}\right), & if\ (x,y) \neq (0,0) \\ 0, & if\ (x,y) = (0,0), \end{cases}$ at $(0,0)$.	2
Q3	a	Show that for the function $f(x,y) = \begin{cases} \frac{y(x^2-y^2)}{x^2+y^2}, & if\ (x,y) \neq (0,0) \\ 0, & if\ (x,y) = (0,0), \end{cases}$ $f_{xy}(0,0) \neq f_{yx}(0,0)$.	2
	b	If $u = \sin^{-1}\sqrt{\frac{x^{1/3}+y^{1/3}}{x^{1/2}+y^{1/2}}}$, then show that $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = \frac{\tan u}{12}\left(\frac{13}{12} + \frac{\tan^2 u}{12}\right)$.	2
Q4	a	If $xyz - u = 0, x^2+y^2+z^2-v = 0$, and $x+y+z-w = 0$, find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$.	2
	b	Obtain a quadratic Taylor's series approximation to the function $f(x,y) = \cos x \cos y$ about the point $(0,0)$. Also, estimate the maximum absolute error in the region $\lvert x\rvert < 0.2, \lvert y\rvert < 0.1$.	3
Q5	a	The temperature T at any point (x,y,z) in space is given by $T(x,y,z) = kxyz^2$, where k is a positive constant. Find the highest temperature on the surface of the sphere $x^2+y^2+z^2 = a^2$.	3

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