

P-①

CS-575

Homework 3

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Q1) input array = $[10, 7, 3, 8, 1, 9, 0]$
sort array using insertion sort

$[10, 7, 3, 8, 1, 9, 0]$
 $\uparrow \quad \uparrow$
 $i \quad j, j+1 = \text{key}$
 $A[i] > \text{key}$
 $\text{as } 10 > 7$

$[7, 10, 3, 8, 1, 9, 0]$
 $\uparrow \quad \uparrow$
 $i \quad \text{key}$
 $A[i] > \text{key}$
 $\text{as } 10 > 3$

$[3, 7, 10, 8, 1, 9, 0]$
 $\uparrow \quad \uparrow$
 $i \quad \text{key}$
 $A[i] > \text{key}$
 $\text{as } 10 > 8$

$[3, 7, 8, 10, 1, 9, 0]$
 $\uparrow \quad \uparrow$
 $i \quad \text{key}$
 $A[i] > \text{key}$
 $\text{as } 10 > 1$

$[1, 3, 7, 8, 10, 9, 0]$
 $\uparrow \quad \uparrow$
 $i \quad \text{key}$
 $A[i] > \text{key}$
 $\text{as } 10 > 9$

$[1, 3, 7, 8, 9, 10, 0]$
 $\uparrow \quad \uparrow$
 $i \quad \text{key}$
 $A[i] > \text{key}$
 $\text{as } 10 > 0$

$[0, 1, 3, 7, 8, 9, 10]$

Sorted array = $[0, 1, 3, 7, 8, 9, 10]$

1-(2)

Q2] input array = [13, 57, 39, 85, 70, 22, 64, 48]

Sort using merge sort

13, 57, 39, 85, 70, 22, 64, 48

[13, 57, 39, 85] [70, 22, 64, 48]

[13, 57]

[39, 85]

[70, 22]

[64, 48]

[13]

[57]

[39]

[85]

[70]

[22]

[64]

[48]

[13, 39, 57, 85]

[22, 48, 64, 70]

[13, 22, 48, 64]

[13, 22, 39, 48, 57, 64, 70, 85]

Sorted array →

[13, 22, 39, 48, 57, 64, 70, 85]

Q3]

given

A = 2, 3

B = 5, 6

4, 5

1, 3

$$\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix} ; \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} = \begin{pmatrix} 5 & 6 \\ 1 & 3 \end{pmatrix}$$

Creating matrix

$$S_1 = B_{12} - B_{22} = 6 - 3 = 3$$

$$S_2 = A_{11} - A_{12} = 2 - 3 = -1$$

$$S_3 = A_{21} - A_{22} = 4 - 5 = -1$$

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$$S_4 = B_{21} - B_{11} = 1 - 5 = -4$$

$$S_5 = A_{11} + A_{22} = 2 + 5 = 7$$

$$S_6 = B_{11} + B_{22} = 5 + 3 = 8$$

$$S_7 = A_{12} + A_{22} = 3 - 5 = -2$$

$$S_8 = B_{21} + B_{22} = 1 + 3 = 4$$

$$S_9 = A_{11} - A_{21} = 2 - 4 = -2$$

$$S_{10} = B_{11} + B_{12} = 5 + 6 = 11$$

$$P_1 = A_{11} S_1 = 2 \cdot 3 = 6$$

$$P_2 = S_2 B_{22} = 5 \cdot 3 = 15$$

$$P_3 = S_3 A_{11} = 5 \cdot 2 = 10$$

$$P_4 = A_{22} S_4 = 5 \cdot (-4) = -20$$

$$P_5 = S_5 B_{11} = 7 \cdot 5 = 35$$

$$P_6 = A_{22} S_6 = 5 \cdot 8 = 40$$

$$P_7 = S_7 S_8 = (-2) \cdot 4 = -8$$

$$P_8 = S_9 S_{10} = (-2) \cdot 11 = -22$$

$$C_{11} = P_5 + P_4 - P_2 + P_6 = 35 + (-20) - 15 + 40$$

$$= 35 - 43$$

$$= 13$$

$$C_{12} = P_1 + P_2 = 6 + 15 = 21$$

$$C_{21} = P_3 + P_4 = 10 - 20 = -10$$

$$C_{22} = P_5 + P_1 - P_3 - P_7 = 35 + 6 - 10 + 8$$

$$= 39$$

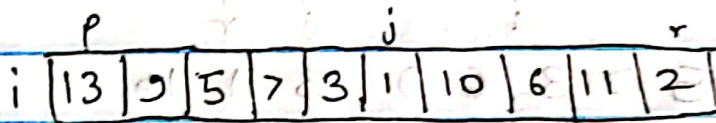
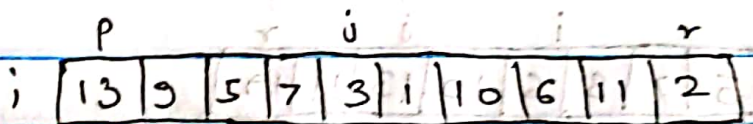
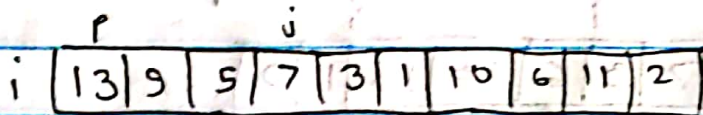
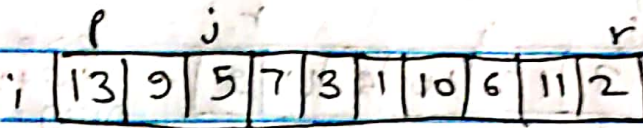
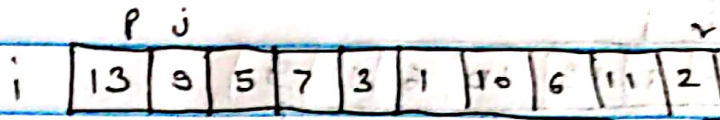
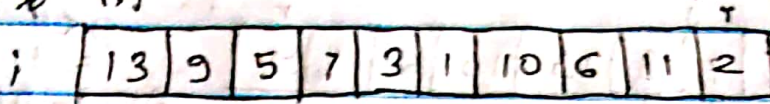
$$\therefore \text{Product of given matrices} = \begin{pmatrix} 13 & 21 \\ -10 & 39 \end{pmatrix}$$

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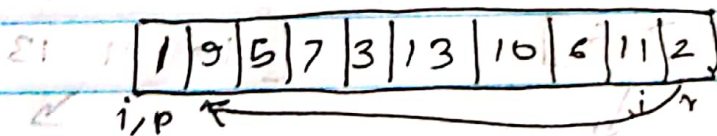
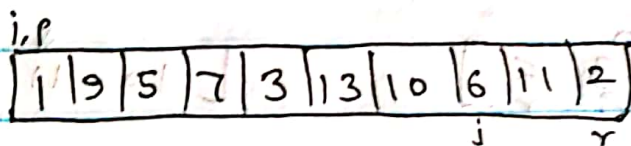
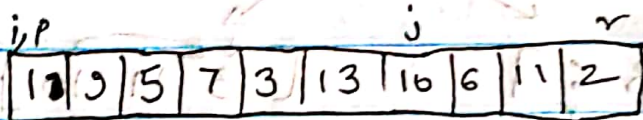
Q-1

Q4) input array = [13, 9, 5, 7, 3, 1, 10, 6, 11, 2]

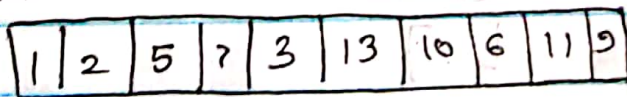
a) P, j



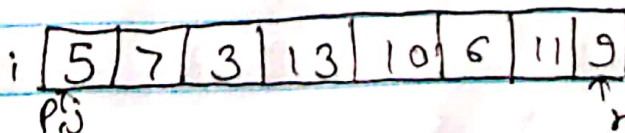
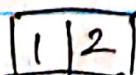
$A[j] < r$
 $\therefore i = i + 1$
 $A[i] = A[j]$



exchange
 $A[i+1]$ with $A[r]$



b) Considering next sub array



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5 7 3 13 10 6 11 9

5 7 3 13 10 6 11 9

5 7 3 13 10 6 11 9

5 7 3 13 10 6 11 9

5 7 3 6 10 13 11 9

5 7 3 6 10 13 11 9

5 7 3 6 9 13 11 10

[5 7 3 6] [13 11 10]

[5 7 3 6] [10 11 13]

[5 3 7 6] [10 11 13]

[5, 3] [7] [10, 11, 13]

[3, 5]

$i = i + 1$
 $A[i] = A[j] \Rightarrow A[i] = 7$

$3 < 9$
 $i = i + 1$
 $A[i] = A[j] \Rightarrow A[i] = 3$

$6 > 9$
 $i = i + 1$
 $A[i] = A[j] \Rightarrow A[i] = 6$

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Q-9

Q5] Sorted list of distinct integers in array
 If we need to find index i such that $A[i] = i$
 provide algorithm that runs in $O(\log n)$

For above scenario if the element search the element in right half of array if the element is less than middle element search the element in left half of array so we will basically use binary search for this

pseudo code:-

```
int binsearch(int array[], int min, int max)
if (max >= min)
```

```
    int mid =  $\frac{\text{max} + \text{min}}{2}$ 
```

```
    if (array[mid] == mid)
        return mid
```

```
    else if (array[mid] < mid)
```

```
        return binsearch(array, mid+1, max);
```

```
    else
```

```
        return binsearch(array, midmin, mid-1)
```

Correctness:-

loop invariant:- An index i is in the array $A[\text{min} \dots \text{max}]$ that is sorted in ascending order when length of array is greater than 1

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initialization :-

if the array is of length 1 then element is present in the 1st index only i.e $A[1] = 1$ else it will return -1

maintenance :-

a) if $i \in A[\text{mid}+1, \text{max}]$ then i must be greater than middle element. As the array is not changed it is still sorted and we will recursively call binary search on $(\text{array}, \text{mid}+1, \text{max})$ till $\text{array}[\text{mid}] = \text{mid}$

b) if $i \in A[\text{min}, \text{mid}] \Rightarrow i \leq \text{array}[\text{mid}]$ i.e

array is not changed and sorted & by recursively calling binary search on $(\text{array}, \text{min}, \text{mid}-1)$ at one point $\text{array}[\text{mid}] = \text{mid}$

termination :-

if element is not present in array then it will return -1

if we calculate time complexity :-

time complexity :- Analyzing runtime of search function

The length of array is n , the recursion function is called it will get $\text{array}/2$ i.e. $n/2$

So -- 1st recursion is n

2nd recursion is $n/2$

3rd recursion is $n/4$

k^{th} recursion will be $n/2^k$

$$n/2^k = 1$$

$$\log_2 n = \log_2 2^k = k \log_2 2$$

hence proved $\boxed{\text{Complexity is } O(\log n)}$

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Q6)

probability of getting 1 is $P(1) = \frac{1}{1}(x)$

where x is the Probability

$$P(2) = \frac{1}{2}(x)$$

$$P(3) = \frac{1}{3}(x)$$

$$P(4) = \frac{1}{4}(x)$$

$$P(5) = \frac{1}{5}(x)$$

$$P(6) = \frac{1}{6}(x)$$

$$P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1$$

$$\frac{1}{1} + \frac{1}{2}(x) + \frac{1}{3}(x) + \frac{1}{4}(x) + \frac{1}{5}(x) + \frac{1}{6}(x) = 1$$

$$\frac{60x + 30x + 20x + 15x + 12x + 10x}{60} = 1$$

$$\frac{147x}{60} = 1$$

$$\boxed{x = \frac{60}{147}}$$

↓ probability of number greater than 3 is

$$P(4) + P(5) + P(6) = \frac{1}{4}x + \frac{1}{5}x + \frac{1}{6}x$$

$$= \frac{60}{147} \left(\frac{1}{4} + \frac{1}{5} + \frac{1}{6} \right)$$

$$= \frac{60}{147} \frac{(15+12+10)}{60}$$

$$\boxed{= \frac{37}{147}}$$

$$= x \left(\frac{1}{4} + \frac{1}{5} + \frac{1}{6} \right)$$

$$= x \left(\frac{30+24+20}{120} \right)$$

$$= x$$