

P-①

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## CS-575 - Design & Analysis of Algorithms

### Home Work - II

Sol'n 1)  $\rightarrow$  let Consider

The log's

$\lg^2 n, n \log n, \ln n, \ln \ln n$

Order of log's -

$\ln \ln n < \ln n < n \log n < \lg^2 n$

Now Considering

$(n+1)!$  and  $n!$

$(n+1)! > n!$

following function which are equal to each other

$$\begin{aligned} i) \quad \sqrt[n]{n!} &= 2^{\frac{1}{n} \log n} \\ &= 2^{\log n^{\frac{1}{n}}} \\ &= n^{\frac{1}{n}} \end{aligned}$$

$$ii) \quad 2^{\log n} = n^{\log 2}$$

$$= n$$

$$\begin{aligned} iii) \quad n^{\frac{1}{\lg n}} \text{ by taking log} \\ &= \log n^{\frac{1}{\lg n}} \\ &= \frac{1}{\lg n} \lg n \end{aligned}$$

iv) Now  $e^n$  and  $2^n$

value of  $e \approx 2.718$

Thus  $2^n < e^n$

o - ②

v) ~~relaxation~~ ~~to show~~ ~~obtained~~ [v]

Consider  $\lg n!$  &  $(\lg n)^{\lg n}$

~~act~~ let  $\lg n = x$  ~~act~~ add

Thus  $(\lg n)! = x! \& (\lg n)^{\lg n} = x^x$

Thus  $x! < x^x$  ~~act~~ add

Therefore  $(\lg n)! < \lg n^{\lg n}$

The order is :-

$$n^{2^n} > 2^{2^n} > (n+1)! > n! > e^n >$$

$$n^{2^n} > 2^n > \left(\frac{3}{2}\right)^n > \frac{n^{\lg n}}{(\lg n)^{\lg n}} > (\lg n)! > n^3$$

$$> \{ \lg n, n^2 \} > n \lg n, \lg n! > n, 2^{\lg n} > (\sqrt{2})^{\lg n} >$$

$$2^{\sqrt{2} \lg n} > \lg^2 n > \ln n > \sqrt{\lg n} > \ln \ln n >$$

$$2^{\lg n} > \lg(\lg n), \lg n > \lg(\lg n) > \frac{1}{n} \frac{1}{\lg n}$$

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(Q2) ~~sqn~~

$$T(n) = \alpha T(n/b) + f(n) \stackrel{n=b^k}{=} \dots \xrightarrow{\text{Master Theorem}} \Theta(n^{\log_b a})$$

- a)  $T(n) = 2T\left(\frac{n}{2}\right) + n^3$  — given eqn  
 $a = 2$     $b = 2$     $f(n) = n^3$

$$n \log n = n \log_2^2 n = n' = n \leq n^3$$

$$f(n) = \Omega(n \log_b a + c) \quad \text{--- 3rd Case of Master Theorem}$$

$$a f(n)_b = 2 f(n)_2$$

$$= 2(n)_2^3 \stackrel{WTS}{=} \text{const} \quad (b)$$

$$\Rightarrow a = m^3 / s \quad \delta = d, \tau = s$$

$$\frac{n^3}{3} \leq cn^3$$

$$\Rightarrow y_g \leq c$$

$$\Rightarrow C = 0.25$$

$$T(n) = \Theta(f(n))$$

$$T(n) = \Theta(n^3)$$

- $$\bullet \text{ b) } T(n) = T\left(\frac{9n}{10}\right) + n = \dots \quad \text{--- given eqn} \\ a = 1 \quad b = \frac{9}{10} \quad f(n) = n \quad \text{--- from (1)}$$

$$n \log_b a = n \log_{\frac{1}{10}} \frac{1}{10} = n^0 = 1 = \Theta(1) - 2^{nd} \text{ case of Master Theorem}$$

$$\Rightarrow T(n) = \Theta(\log n)$$

P-④

④-4

c)  $T(n) = 16T(n/4) + n^2$  — given  
 $a=16 \quad b=4 \quad f(n)=n^2$  — from ①

$n \log_b a = n \log_{16} 4 = n^{1/2} = n^2 = \Theta(n)^2$  — 2<sup>nd</sup> case

$$T(n) = \Theta(n^{\log_3 9} \log n)$$

$$T(n) = \Theta(n^2 \log n)$$

d)  $T(n) = 7T(n/3) + n^2$   
 $a=7 \quad b=3 \quad f(n)=n^2$  — ④-①

$n \log_b a = n \log_3 7 = n^{1.77} < n^2$  — 3<sup>rd</sup> case

$$af(n/b) = 7f(n/3) = 7(n^2/9) = \frac{7n^2}{9}$$

$$\frac{7n^2}{9} < cn^2 \Rightarrow 7/9 < 1$$

$$T(n) = \Theta(f(n)) = \Theta(n^2)$$

e)  $T(n) = 7T(n/2) + n^2$   
 $a=7 \quad b=2 \quad f=n^2$

$n \log_b a = n \log_2 7 = n^{2.8} > n^2$  — 4<sup>th</sup> case

$$T(n) = \Theta(n^2)$$

P-5

Q+9

f)  $T(n) = 2T(n/4) + n^{1/2}$

$a = 2$ ,  $b = 4$ ,  $f(n) = n^{1/2}$

$n \log_b a = n \log_4 2 = n^{0.5}$

(real)  $= \Theta(n^{0.5})$  — 2<sup>nd</sup> Case

$$T(n) = \Theta(n \log_a b \log n)$$

$$T(n) = \Theta(n^{0.5} \log n)$$

unit

g)  $T(n) = T(n-1) + n$  (by using recursion method)

$$\begin{aligned} T(n-1) &= T(n-2) + (n-1) \\ T(n-2) &= T(n-3) + (n-2) \dots \therefore T(n-(k+1)) + \\ T(n) &= T(n-(k+1)) + (n-k) + \dots (n-2) + (n-1) + n \end{aligned}$$

Assume  $n-(k+1) = 0$

$$[n-(k+1)] + [n-(k-1)] + \dots + [n-2] + [n-1] = \text{cost}$$

$$[n-(k+1)] + [n-(k-1)] + \dots + [n-2] + [n-1] = \frac{n(n+1)}{2}$$

i.e. longest n term is  $n^2$

$$\boxed{\text{Complexity} = \Theta(n^2)}$$

h)  $T(n) = T(n^{1/2}) + 1$

let  $n = 2^x$  therefore  $x = \log_2 n$

$$T(n) = T(n^{1/2}) + 1$$

$$T(2^x) = T(2^{x/2}) + 1$$

P - ⑥

⑦-4

alt reduced to  $+ C_1 n T \leq C_1 T$  (A)

$$T(x) = T(x/2) + 1 \quad s = p$$

$$a = 1 \quad b = 2 \quad f(m) = 1$$

from ①

$$n \log_3 a = n \log_2^2 = n^0 = 1 \quad (2^{\text{nd}} \text{ case})$$

$$T(n) = \Theta(n \log_b^a \log n)$$

$$\boxed{T(n) = \Theta(\log_2 n)}$$

Q 3)  $\xrightarrow{\text{so r}}$

$$\boxed{(n \log^{2-3} n) \Theta = (nT)}$$

a)

Cost

time

$$1) \text{ for } i=1 \text{ to } n \quad C_1 = \text{cost} \quad (n+1)$$

$$2) \quad k[i] = 0 \quad C_2 \quad n$$

$$3) \quad \text{for } i=1 \text{ to } n \quad C_3 \quad (n+1)$$

$$4) \dots \text{for } j=1 \text{ to } n-aT = C_4 \quad (n-aT) \quad [C_4(n-aT)] + n$$

$$5) \quad k[i] = k[i] + j \quad C_5 = (nT) \quad n \cdot (n+1)$$

$$T(n) = C_1(n+1) + C_2(n) + C_3(n+1) + C_4 \left[ \frac{n(n+1)}{2} \right] + n$$

$$(1+2+3)n = C_1(n+1) + C_2(n) + C_3(n+1) + C_4 \left[ \frac{n(n+1)}{2} \right] + n \cdot \left[ \frac{n+1}{2} \right]$$

$$= C_1 n + C_1 + C_2 n + C_3 n + C_3 + C_4 \left( \frac{n^2+n}{2} + n \right) + C_5 \left( \frac{n^2+n}{2} \right)$$

$$= (C_1 + C_2 + C_3) n + C_1 + C_3 + C_4 \left( \frac{n^2+3n}{2} \right) + C_5 \frac{n^2}{2} + C_5 \frac{n}{2}$$

$$= (C_1 + C_2 + C_3) n + (C_4 + C_3) + (C_4 + C_5) \cdot n^2 + n \left( \frac{3C_4}{2} + \frac{C_5}{2} \right)$$

$$TC(n) = n^2$$

$$\boxed{T(n) = \Theta(n^2)}$$

P-7

b)

i=1	$c_1$	1
while i < n	$c_2$	$\log_2 k$
i = 2*i	$c_3$	$\log_2 k - 1$

$$T(n) = c_1 + c_2 \log_2 k + (\log_2 k - 1) c_3$$

$$(a) = (c_2 + c_3) \log_2 k + (c_1 - c_3)$$

$$\boxed{T(n) = (c_2 + c_3) \log_2 k + (c_1 - c_3)}$$

Q5) Sol:

a)  $10000n^2 + 5000 = \Theta(n^2)$

As  $\Theta$  is tight bound & if  $n$  increases  $n^2$  will increase rapidly & in such cases constant doesn't matter so

$10000n^2 + 5000 = \Theta(n^2)$  is true

b)  $n^3 + n^2 + 100n = \Omega(n^3)$

Max value of above statement is  $n^3$

so

$n^3 + n^2 + 100n = \Omega(n^3)$  is true

c)  $\log_n 100 = O(\log n)$

Arper o design

$O \leq f(n) \leq c g(n)$  for all  $n \geq n_0$

$$O \leq \log n \leq c (\log_n 100)$$

P - ⑧

④ - 4

$$O \leq \log n \leq C \cdot \log(n)$$

for any value above statement is true

d)  $2^{n+1} = O(2^n)$

As per O design

$$O \leq f(n) \leq g(n)$$

$$O \leq 2^n \leq C \cdot 2^{n+1}$$

$$O \leq 2^n \leq C \cdot 2^n \cdot 2^1$$

divide  $c_2 \cdot 2^n$  by  $2^n$

$$O \leq 2 \leq c_2$$

for any value  $C > 0$  above statement is true

e)  $n^3 = O(n^2)$

As per O design

$$O \leq f(n) \leq g(n)$$

$$O \leq n^2 \leq C \cdot n^2$$

for any value of  $n$  above statement will be false

Q5  $\xrightarrow{\text{Sol'n}}$

i)

1	3	9	2	8	0	1	5	7	6
↑	↑								

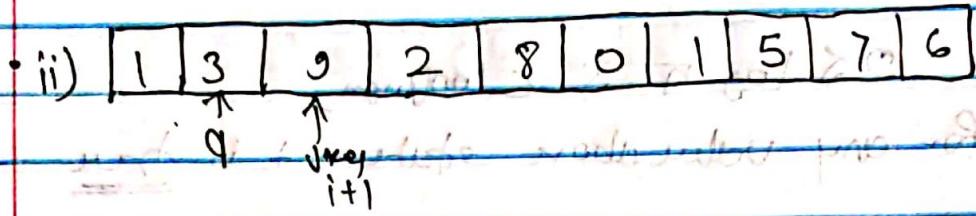
key  
 $j, i+1$

$$A[i] = 1 > 3 \text{ false}$$

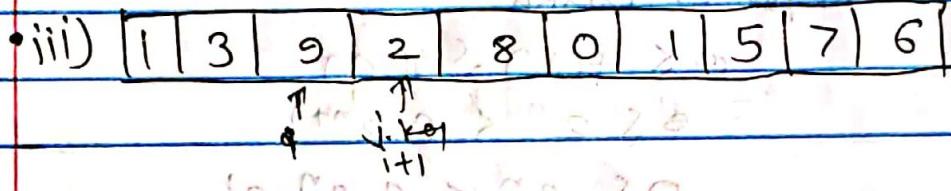
therefore element sorted before key

p - ⑨

⑧ - 9

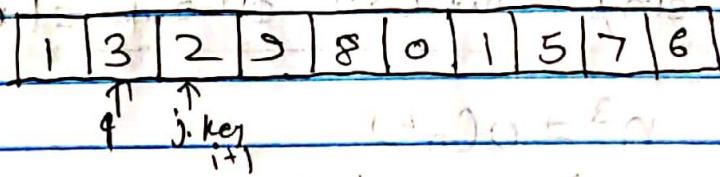
ii) 

$A[i] = 3 > 9$  (false)

iii) 

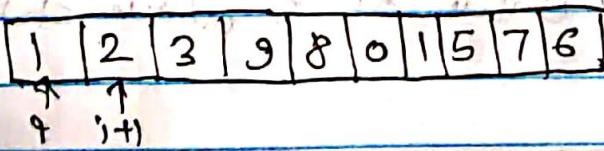
$A[i] = 9 > 2$  (true)

$$A[i+1] = A[i] \text{ if } i = i-1$$



$$A[i] > key \Rightarrow 3 > 2 \text{ (true)}$$

$$\Rightarrow i = i-1 \text{ and } A[i+1] = A[i]$$



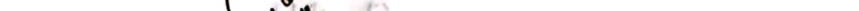
$$A[i] > key \Rightarrow 3 > 2 \text{ (false)}$$

$$A[i+1] = key$$



p-10

6

- iv) 
  
 q → 9

$A[i] > \text{key} \Rightarrow 9 > 8$  (true)

$\rightarrow A[i+1] = A[i]$  if  $i = i-1$

## What constitutes the Social

1	2	3	8	9	0	1	3	?	6
↑	↑							151	

$A[i] > key \Rightarrow 3 > 8$  (false)

After  $A[i+1] = \text{key}$

1238001576 0 (110)

- v)  $(1|2|3|8|9|0|15|7|6)$

$A[i] > key \Rightarrow i+1$  is goal

~~970 true~~ 970 bnd

$$A[i+1] = A[i]$$

j=j-

1 2 3 8 0 9 1 5 7 6

loop ① continues till

~~870, 370, 270~~ 170

0 1 2 3 8 9 1 5 7 6

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P-11

• vi)

0	1	2	3	8	9	1	5	7	6
↑									

+ i+1  
key

$$A[i] \geq key \Rightarrow 9 \geq 1 \Rightarrow \text{True} \quad \rightarrow ②$$

(i=1, A[1]=1, 1 < 9)

loop 2 will continues till

$8 > 1, 3 > 1, 2 > 1, \dots \Rightarrow$  as this true  
 $1 > 1 \Rightarrow \text{False}$

0	1	1	2	3	8	9	5	7	6
↑									

• vii)

0	1	1	2	3	8	9	5	7	6
↑									

+ i  
key  
i+1

$$A[i] \geq key \Rightarrow 9 \geq 5 \Rightarrow \text{true} \quad \rightarrow ③$$

loop 3 will continues till  $8 > 5 \Rightarrow \text{true}$   
and  $3 > 5 \Rightarrow \text{false}$

0	1	1	2	3	5	8	9	7	6
↑									

• viii)

0	1	1	2	3	5	8	9	> k
↑								

+ i  
key  
i+1

$$A[i] \geq key \Rightarrow 9 \geq 7 \text{ true } 8 \geq 7 \text{ true } 5 \geq 7 \text{ false}$$

0	1	1	2	3	5	7	8	9	6
↑									

+ i  
key  
i+1

P (12)

(12+9)

h) 

0	1	1	2	3	5	7	8	9	6
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initialization :-  
while  $A[i] > key$  and  $i < 9$  do  $i = i + 1$

value which has  $8 > 6$ . true so  $i = 8$

a maxed [i] item  $> 6$  true so

$5 > 6$  false so

then  $i = i - 1$  now  $i = 5$

condition not holds so

value 

0	1	1	2	3	5	6	7	8	9
---	---	---	---	---	---	---	---	---	---

Q6  $\rightarrow$  Soln

Condition

Linear Search (Array, Value)

for  $i = 1$  to  $n$  [DATA]

if ( $Array[i] == value$ )

then into  $i$  formula returning  $i$  if  $i < n$

such that return  $Nil$ ; good or break

loop invariant :-

for each iteration of for loop  $A[i] \neq value$

for every value of  $i$

initialization :-

for  $i = 1$  array is single ~~statement~~ element i.e  
at start of 1st iteration  $A[i] \neq value$

Maintenance :-

Let assume at 1<sup>st</sup> iteration  $i=0, i=1$  is

-1 if we check  $\text{array}[i] == \text{value}$

if its true. we return index value which is or we iterate array until  $[i]$  become  $n$

As per assumption

if  $i=2$  then array  $[i-1]$  is not a number which are searching

i.e  $P=2$  the  $\text{array}[i] \neq \text{value}$

Termination :-

loop termination if

a)  $A[i] = \text{value}$  or if

b)  $P=n+1$

&  $P \geq n+1$  iteration the element is still not found so loop invariant holds true