P-(1)

Date - 15-09-2021

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CS575 - Design / Analysis of Algorithms.

Home Work - I

(1) Find the mean, median and variance of following numbers. 1, 2, -1, 4, 10. [5 points]

→ Sol^n :→

input numbers given → 1, 2, -1, 4, 10.

We need to find mean, median, and variance

① mean ($\bar{x}$) $= \frac{1}{n} \sum_{i=1}^{n} x$

$= \frac{1+2+(-1)+4+10}{5}$

$= \frac{\cancel{1}+2-\cancel{1}+4+10}{5}$

$= \frac{16}{5}$

$\boxed{\text{mean} = 3.2}$

② median → for median 1st we have to sort the numbers in increasing order as below

⇒ -1, 1, 2, 4, 10

2 is middle number & that will be the median

$\boxed{\text{median} = 2}$

P-2

③ variance $\sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x-\bar{x})^2$

$= \frac{1}{5}\left[(1-3.2)^2+(2-3.2)^2+(-1-3.2)^2+(4-3.2)^2 + (10-3.2)^2\right]$

$= \frac{1}{5}(4.84+1.44+17.64+0.64+46.24)$

$= \frac{1}{5}(70.8)$

$\boxed{\text{variance} = 14.16}$

② let X, Y and Z be three random variable $E(X)=2$, $var(X)=1$ and $E(Y)=3$. X and Y are independent of each other $Z = x^2 Y$. Find $E(Z)$

→ Soln →

Random variable given are X, Y, Z

and $E(X)=2$, $\sigma^2=1$, $E(Y)=3$

& also X and Y are ~~not~~ independent of each other

and given $Z = x^2 y$

To find $E[Z]$

$Z^2 = x^2 y$

$E[Z] = E[x^2]\,E[Y]$ —— ①

also we have $\sigma^2 = E[x^2]-(E[x])^2$

$E[x^2] = \sigma^2 + (E[x])^2$

$= 1+(2)^2$

$= 1+4$

$= 5 \quad \text{--- } ②$

from eqⁿ ① and ② we get

$E[Z] = E[x^2]\,E[Y]$

$= (5)(3)$

$\boxed{E[Z] = 15}$

③ let x and Y be two dependent random variable $P[X,Y] = 0.2$ and $P[X] = 0.4$ find $P[Y|X]$

Solⁿ →

X & Y are independent random variables when $P[X,Y] = 0.2$ and $P[X] = 0.4$

$$P[Y/x] = \frac{P[XY]}{P[x]}$$

$$= \frac{0.2}{0.4}$$

$$= \frac{2}{4}$$

$$= \frac{1}{2}$$

$$\boxed{P[Y|x] = 0.5}$$

P-4

(4) Compute $\log_2(45)$ given that $\log_2(5) = 2.32$ and $\log_2(3) = 1.58$

Sol^n →

given $\log_2(5) = 2.32$
and $\log_2(3) = 1.58$

Now $\log_2 45 = \log_2 (9 \times 5)$
$= \log_2 9 + \log_2 5$
$= \log_2 (3)^2 + \log_2 5$
$= 2\log_2(3) + \log_2 5$
$= 2(1.58) + 2.32$
$= 5.48$

$$\boxed{\log_2 45 = 5.48}$$

(5) A man wants to enter a lottery. To enter a lottery he purchased ticket worth \$10. He has 5% chance of winning the grand price of \$100. He has 100% chance of winning 2nd price of \$20. if he does not win any of these prices. he will lose his \$10. Mathematically. explain what should he do

Sol^n →

price of lottery ticket is = \$10
probablity of winning grand price = 0.05
probablity of winning 2nd price = 0.1
probablity. of Not winning any pric = P(Nwin)

P-5

expected value of what will happen
$$E(x) = \Sigma x p(x)$$
here x is the net value

$\Rightarrow E(x) = $ (prob of winning 1st price)(6100-10) + p(2nd price)(20-100) + p(Nwin)(0-10)

we know sum of probablity of winning = 1

$\Rightarrow$ p(grand price) + p(2nd price) + p(Nwin) = 1
0.5 + 0.1 + p(Nwin) = 1
p(Nwin) = 1 − 0.15
p(Nwin) = 0.85

Here we get
$$E(x) = 0.05 \times 90 + 0.1 \times 10 - 0.85 \times 10$$
$$= 4.5 + 1 - 8.5$$
$$= -3$$
$$\boxed{E(x) = -3}$$

if he enter he will lose so he should not play

(6) A drunk person is walking on road with problity 0.6 he take a step forword and with problity 0.4 he take a step backword. after 10 step, what is the problity that he ~~will~~ is at his starting position.

Soln → probablity of drunk person take step forword
p(fwd) = 0.6

p-6

probablity of drunk person take step backward

$$p(\text{backword}) = 0.4$$

after 10 step probabity to be at start position of the person takes equal number of forward step and equal number of backword step then he will be at start position

$$p = {}^{10}C_5 (0.6)^5 (0.4)^5$$

$$= \frac{10!}{5!\,5!} (0.6)^5 (0.4)^5$$

$$\boxed{p = 0.2}$$

7

→ Sol^n →

Two players A & B consist of 10 rounds

x = sum of two dice

and if $x > 5$ then A wins

and if $x < 5$ then B wins

probablity that B win the game

$$p(x<5) = 1 - p(x>5)$$

$$p(x<5) = \{(1,5)\ (1,6)\ (2,4)\ (2,5)\ (2,6)$$
$$(3,3)\ (3,4)\ (3,5)\ (3,6) \dots (6,6)\}$$

p - 7

$$p(x<5) = 1 - \frac{\cancel{26}\ 13}{\cancel{36}\ 18}$$

$$= 1 - \frac{13}{18}$$

$$= \frac{5}{18}$$

probablity that B to win game = atleast 6 round or 7, 8, 9 or 10

$$\Rightarrow P(B) = \left(\frac{5}{18}\right)^6\left(\frac{13}{18}\right)^7 + \left(\frac{5}{18}\right)^7\left(\frac{13}{18}\right)^3 + \left(\frac{5}{18}\right)^8\left(\frac{13}{18}\right)^2 + \left(\frac{5}{18}\right)^9\left(\frac{13}{18}\right)^1 + \left(\frac{5}{18}\right)^{10}\left(\frac{13}{18}\right)^1$$

Final expression

$$= (0.28)^6(0.73)^7 + (0.28)^7(0.73)^3 + (0.28)^8(0.73)^2 + (0.28)^9 * (0.73)^1 + (0.28)^7 (0.73)^0$$

~~P(B) =~~

$$\boxed{p(B) = (0.28)^6(0.73)^7 + (0.28)^7(0.73)^3 + (0.28)^8(0.73)^2 + (0.28)^9(0.73)^1 + (0.28)^7(0.73)^0}$$

8 sol^n →

given 5 red balls, 5 green balls, 4 yellow balls, 6 white balls

1] P[W] be probability of drawing white ball

$$p[W] = \frac{6_{C_1}}{20_{C_1}} = \frac{6}{20} = \frac{3}{10}$$

$$\boxed{p[W] = 0.3}$$

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P - 8

2] $p[G]$ be probablity of drawing 3^{rd} ball as green

$$p[G] = \frac{{}^{3}C_1}{{}^{18}C_1} = \frac{3}{18}$$

$$= \frac{1}{6}$$

$$\boxed{p[G] = 0.16}$$

3] $p[4W]$ be probablity of drawing 4^{th} ball as white

$$p[4W] = \frac{{}^{6}C_1}{17} = \frac{6}{17}$$

$$\boxed{p[4W] = 0.35}$$

4] $p[5W]$ be the probabity of drawing 5^{th} ball as white

$$p[5W] = \frac{{}^{5}C_1}{{}^{17}C_1} \times \frac{{}^{6}C_1}{{}^{16}C_1} + \frac{{}^{2}C_1}{{}^{17}C_1} \times \frac{{}^{6}C_1}{{}^{16}C_1} + \frac{{}^{4}C_1}{{}^{17}C_1} \times \frac{{}^{6}C_1}{{}^{16}C_1} + \frac{{}^{6}C_1}{{}^{17}C_1} \times \frac{{}^{5}C_1}{{}^{16}C_1}$$

$$= \frac{15}{136} + \frac{6}{136} + \frac{6}{68} + \frac{15}{136}$$

$$= 0.11 + 0.044 + 0.088 + 0.110$$

$$\boxed{p[5W] = 0.352}$$

p-9

9 sol^n →

if team lost both matches then there is 80% chance of loosing 3rd game

then 20% chance of winning 3rd game

$p[w] = 0.2$ if previous two matches lost

if one of match is won and after one lost

$p[w] = 0.5$

if both the two matches won

$p[w] = 0.9$

sample space = {(w,w), (w,L), (L,w), (L,L)}

probability of winning 3rd match =

$$p(3^{rd}_{match}) = \frac{1}{4}(0.2) + \frac{1}{4}(0.5) + \frac{1}{4}(0.5) + \frac{1}{4}(0.9)$$

$$= (0.25)(0.2) + (0.25)(0.5) + (0.25)(0.5) + (0.25)(0.9)$$

$$= 0.05 + 0.25 + 0.225$$

$$= 0.525$$

probability of winning the next match

$$\boxed{p[w] = 0.525}$$

p-10

10

→ Sol^n →

Now let bet for 5 games.

$n=5$

20% of $n = 1$

80% of $n = 4$

so we will win \$10 in one game and losses

$\$(4\times5) = \$20$

so total gain $= 10-20 = \$10$

Now, let bet 100 games

$n=100$

20% of $n=20$

80% of $n = 80$

Total wining amount $= 20\times10 = \$200$

Total loss $= 80\times5 = \$400$

Total gain $= 200-400 = \$-200$

after 100 games $= \$-200$

ie $-2n$

The expected gain is $\boxed{(-2)(n)\,\$}$ where n is the number of games plays

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