

The "BIG PICTURE" of nonlinear FEA

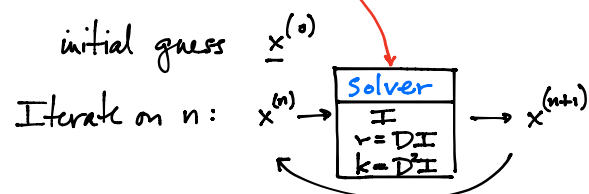
Top Level (The Body/System)

$$\min_{\underline{x} \in \mathbb{R}^{3N}} I[\underline{x}] = \sum_{\alpha} \underline{x}_{\alpha} N_{\alpha}$$

$$\hookrightarrow \underline{r}_{ia} = \frac{\partial I}{\partial x_{ia}} = f_{ia}^{int}(\underline{x}) - f_{ia}^{ext} = 0$$

(nonlinear eqns.)

Iterative solution



Element Level

$$I_e = \int_{\Omega_e^e} w dV - \int_{\Sigma_e} \underline{B}_i \underline{x}_i dV - \int_{\Sigma_e} \underline{T}_i \underline{x}_i dV$$

$$(f_{ia}^{int})^e = \int_{\Sigma_e} P_{i,J} N_{a,J} dV$$

$$K_{iakb}^e = \int_{\Sigma_e} C_{iJkL} N_{a,J} N_{b,L} dV$$

Quadrature Point Level

Shape Functions
$N_a(\underline{x}_p) \quad N_{a,J}(\underline{x}_p)$ \uparrow quad pt.

Material Response
$\underline{F}_{i,J}(\underline{x}_p) \rightarrow$ <div style="display: inline-block; vertical-align: middle;"> w $P_{i,J} = \frac{\partial w}{\partial F_{i,J}}$ $C_{iJkL} = \frac{\partial P_{i,J}}{\partial F_{kL}}$ </div>