The "BIG PICTURE" of nonlinear FEA

Top Level (The Body/System)

$$\min_{\underline{x} \in \mathbb{R}^{3N}} \mathcal{I}[\chi_i = \sum_a x_{ia} N_a]$$

$$\hookrightarrow r_{ia} = \frac{\partial \mathcal{I}}{\partial x_{ia}} = f^{int}_{ia}(\underline{x}) - f^{ext}_{ia} = 0$$

(nonlinear eqns.)

Iterative solution

initial guess $\underline{x}^{(0)}$

Iterate on n: $x^{(n)} \rightarrow$ Solver ($\mathcal{I}$, $r = D\mathcal{I}$, $k = D^2\mathcal{I}$) $\rightarrow x^{(n+1)}$

Element Level

$$\mathcal{I}_e = \int_{\Omega_0^e} W \, dV - \int_{\Omega_0^e} B_i x_i \, dV - \int_{\partial\Omega_0^e} T_i x_i \, dV$$

$$(f^{int}_{ia})^e = \int_{\Omega^e} P_{iJ} N_{a,J} \, dV$$

$$k^e_{iakb} = \int_{\Omega_0^e} C_{iJkL} N_{a,J} N_{b,L} \, dV$$

Quadrature Point Level

Shape Functions

$N_a(\underline{X}_p)$ $N_{a,J}(\underline{X}_p)$

↑ quad pt.

Material Response

$F_{iJ}(\underline{X}_p) \rightarrow W \rightarrow P_{iJ} = \frac{\partial W}{\partial F_{iJ}}$

$C_{iJkL} = \frac{\partial P_{iJ}}{\partial F_{kL}}$